Name:____________________________ Class:__________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2024

MATHEMATICS
Higher 2

9758/01

Paper 1

9 September 2024

3 hours

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given by [ ] at the end of each question or part question.

For Candidate's Use	For Examiner's Use
Question Number	**Marks Obtained**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	/ 100

This document consists of **6** printed pages.

[Turn over

1 The first four terms of a sequence are given by $v_1 = -9$, $v_2 = 7$, $v_3 = 47$ and $v_4 = 141$. It is given that v_n is a cubic polynomial in n. Find v_n in terms of n. [3]

2 Without using a calculator, solve the inequality $\dfrac{11x-19}{x^2+2x-8} \le 2$. [4]

Hence solve $\dfrac{11e^{-x}-19}{e^{-2x}+2e^{-x}-8} \le 2$, leaving your answer in the exact form. [2]

3 The region R is bounded by the lines $y = 0$, $y = \dfrac{5}{2}\sqrt{3}$ and the curve $x^2 = \sqrt{25-y^2}$. R is rotated about the y-axis through π radians. Using the substitution $y = 5\sin\theta$, find the exact volume generated. [6]

4 **(i)** A curve C has equation $y = \dfrac{x-3}{(x-2)(x+5)}$. Sketch C, giving the equations of the asymptotes, the coordinates of the stationary point(s) and the point(s) where C crosses either axis. [4]

(ii) Describe a sequence of transformations that maps C onto the graph of $y = \dfrac{2x-3}{(x-1)(2x+5)}$. [3]

5 The function f is defined by $\mathrm{f}: x \mapsto e^{(x+c)^2}$, $x \in \mathbb{R}$, $x \le k$, $c > 0$.

(i) Find the largest value of k in terms of c for which the function f^{-1} exists. [1]

For the rest of the question, use the value of k found in part **(i)**.

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

(iii) Sketch, on the same diagram, the graphs of f and f^{-1} and state the relationship between the graphs. [3]

(iv) The function g is defined by $\mathrm{g}: x \mapsto \ln x$ for $x \in \mathbb{R}$, $x > 0$. Give a reason why the composite function gf exists. Find $\mathrm{gf}(x)$. [2]

6 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively such that angle $AOB = 30°$ and $|\mathbf{a}| = 4|\mathbf{b}|$.

(i) Point C has position vector $m\mathbf{a} + n\mathbf{b}$, where m and n are positive integers. Find the area of triangle ABC in terms of m, n and $|\mathbf{b}|$. [4]

Point D is the mid-point of OA and point E lies on OB such that $OE : EB = 3 : 2$.

(ii) Find the position vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$, giving your answers in terms of **a** and **b**. [2]

(iii) Show that the vector equation of the line BD can be written as $\mathbf{r} = \lambda\mathbf{a} + (1 - 2\lambda)\mathbf{b}$, where λ is a parameter. By finding the vector equation of the line AE in a similar form in terms of a parameter μ, find, in terms of **a** and **b**, the position vector of the point F where the lines BD and AE meet. [4]

7 The plane p has equation $\mathbf{r} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ a \\ 1 \end{pmatrix}$, and the line l has equation $\frac{x-4}{3} = \frac{y-1}{5}, z = 0$, where a is a constant and λ and μ are parameters.

(a) Find the value of a such that l and p do not meet in a unique point. [4]

(b) In the case when $a = 7$,

(i) find the coordinates of the point at which l and p intersect. [3]

(ii) find the Cartesian equation of line l', the reflection of l in p. [5]

8 Selena decides to borrow \$200 000 from the bank to fund her home purchase on 1 January 2025. On the first day of each month, 0.5% interest is added to the amount owed with the first interest amount added on 1 January 2025. On the last day of each month, she makes a repayment of \$$x$ to the bank, starting from 31 January 2025.

(i) Show that the amount of money Selena owes the bank at the end of n months is $200\,000(1.005)^n - 200x\left(1.005^n - 1\right)$. [3]

(ii) If she repays the bank \$1500 a month on the last day of each month, on which date will she fully repay the loan? What is the amount of the last repayment? [4]

(iii) If she wants to fully repay the loan in 10 years, how much will she need to repay the bank every month? [2]

9 **(a)** One of the roots of the equation $z^4 - z^3 - 9z^2 + sz + t = 0$, where s and t are real, is $1+2\mathrm{i}$. Find the other roots of the equation and the values of s and t. [5]

(b) The complex number w is such that $w = a + \mathrm{i}b$, where a and b are positive real numbers. The complex conjugate of w is denoted by w^*. Given that $\dfrac{w^3}{\mathrm{i}w^*}$ is purely imaginary, find w in terms of a. [5]

10 A Red Lion skydiver jumps off from a helicopter. He leaves the helicopter with zero speed, and his speed v ms^{-1} satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 2\mathrm{e}^{-0.1t}.$$

(i) Find v in terms of t. Hence find the exact time the skydiver takes to reach a speed of 10 ms^{-1}. [5]

(ii) According to this model, explain what happens to the speed of the skydiver eventually. [1]

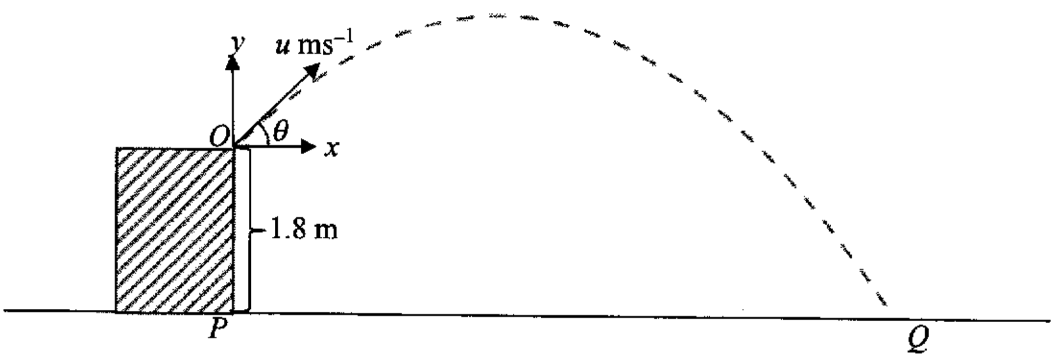
Another Red Lion skydiver jumps off the helicopter and opens his parachute. His speed is 18 ms^{-1} immediately after opening the parachute. His speed t seconds after he opens his parachute is w ms^{-1} and satisfies the differential equation

$$-2\frac{\mathrm{d}w}{\mathrm{d}t} = (w-3)(w+2).$$

(iii) Find w in terms of t. [6]

(iv) Sketch the graph of w against t. State the speed that the skydiver will not fall below. [2]

11



The diagram shows an object being projected from the top of a platform at O at an angle of projection θ made with the horizontal, where $0 < \theta < \frac{\pi}{2}$, with an initial speed of u ms^{-1}. The x-y plane contains the trajectory of the object, with the equation of the trajectory, referred to the horizontal and vertical axes through O, given by

$$y = x\tan\theta - \frac{10x^2}{2u^2\cos^2\theta},$$

where x m and y m are the distances travelled to the right and upwards from O respectively as shown in the diagram above.

The point P is 1.8 m vertically below O on the horizontal ground and Q is the point where the object lands on the ground.

(i) If the angle of projection of the object is $\frac{\pi}{4}$ and it lands at a distance of 15 m on the ground from P, find its initial speed of projection. [3]

It is given that $u = 10$ for the rest of the question.

(ii) Show that at Q, x satisfies the equation

$$x^2 - 10x\sin 2\theta - 18\cos 2\theta - 18 = 0.$$ [3]

(iii) By using differentiation, express x in terms of $\tan 2\theta$ at the stationary value of x. [3]

(iv) Hence calculate θ to 2 decimal places and find the stationary value of x. [4]

Name:______________________________ **Class:**__________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2024

MATHEMATICS **9758/02**
Higher 2 **13 September 2024**

Paper 2 **3 hours**

Candidates answer on the Question Paper.

Additional materials: List of Formulae (MF 26)

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Section A: Pure Mathematics [40 marks]

1 **(a)** Sketch, on the same axes, the graphs of $y = 3x$ and $y = |x^2 - a^2|$ where $a > 0$. [2]

(b) Find the exact solution of $|x^2 - a^2| = 3x$ for $0 < x < a$. [2]

2 **(a)** Find $\int \sin px \cos qx \, dx$, where p and q are constants such that $p \neq q$ and $p \neq -q$. [2]

(b) Given that $m \neq 0$, find $\int x \sin mx \, dx$. [3]

(c) Using the result in part **(b)**, for all positive integers m, evaluate $\int_0^{\pi} x \sin mx \, dx$, giving your answers in the form $\frac{k}{m}\pi$ where the possible value(s) of k are to be determined. [2]

3 It is given that $f(x) = e^{\tan^{-1} x}$, where $\tan^{-1} x$ denotes the principal values.

(i) Show that $(1 + x^2)f'(x) = f(x)$. [1]

(ii) By further differentiation of the above result, find the Maclaurin series for $f(x)$, up to and including the term in x^3. [5]

(iii) Using the series in part **(ii)** find an approximate value of $\int_0^{0.5} f(x)\, dx$, giving your answer to 4 significant figures. [1]

(iv) Comment on the suitability of substituting $x = 1$ into the series in part **(ii)** to estimate the value of $e^{\frac{\pi}{4}}$. [1]

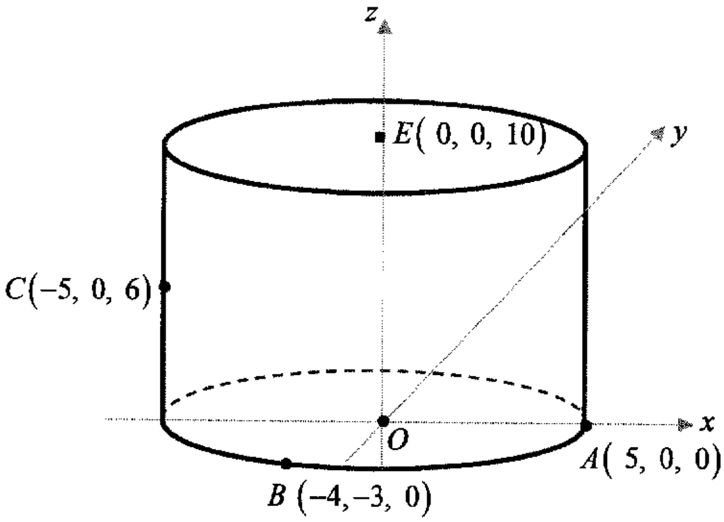
4 **(i)** Show that $\frac{7}{n-2} - \frac{5}{n-1} - \frac{2}{n} = \frac{9n-4}{(n-2)(n-1)n}$. [2]

(ii) Hence find $\sum_{n=3}^{N} \frac{9n-4}{(n-2)(n-1)n}$, giving your answer in the form $k - f(N)$, where k is a constant. [3]

(iii) Show that $\sum_{n=3}^{N} \frac{9n-4}{(n-2)(n-1)n}$ is convergent and state the sum to infinity of this series. [2]

(iv) Use your answer in **(ii)** to find $\sum_{n=2}^{N} \frac{9n+14}{n(n+1)(n+2)}$. [3]

5



A closed cylinder has a base in the shape of a circle with centre O. The coordinates of points A, B and C are $(5, 0, 0)$, $(-4, -3, 0)$ and $(-5, 0, 6)$ respectively. Point E is directly above O with coordinates $(0, 0, 10)$.

(i) Point D lies in the cylinder such that $ABCD$ is a parallelogram. Find the position vector of D and determine the shape of $ABCD$, justifying your answer. [3]

(ii) Find the cartesian equation of the plane ABC. [2]

(iii) Find the acute angle between the plane ABC and the base of the cylinder. [2]

(iv) Point F lies on AE such that $AF : AE = 1:5$. Find the length of projection of AF onto the plane ABC. [4]

Section B: Probability and Statistics [60 marks]

6 The individual letters of the word APPROPRIATE are printed on identical cards and arranged in a straight line.

(a) Find the number of arrangements of all 11 letters of the word such that

(i) the letters are **not** in alphabetical order, [2]

(ii) all the vowels are together and only two of the P′s are together. [3]

(b) The cards are now placed in a bag and 3 cards are drawn without replacement. Find the probability that there are at least two vowels drawn. [2]

7 The number of years (x) an employee has worked for the company and the corresponding salary increment, in dollars (y), received by the employee are given in the table.

Years, x	6	7	9	11	13	15	18
Amount, y	155	170	211	230	248	260	265

(i) Draw the scatter diagram for these values, labelling the axes clearly. [1]

It is thought that the salary increment, \$$y$, can be modelled by one of the formulae

$$y = ax + b \qquad \text{or} \qquad y = c\ln x + d$$

where a, b, c and d are constants.

(ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(a) x and y,

(b) $\ln x$ and y. [2]

(iii) Use your answers to parts **(i)** and **(ii)** to explain which of $y = ax + b$ or $y = c\ln x + d$ is the better model. [2]

It is required to estimate the value of x for which $y = 200$.

(iv) Explain why neither the regression line of x on y nor the regression line of $\ln x$ on y should be used. [1]

(v) Find the equation of a suitable regression line and use it to find the required estimate, commenting on its reliability. [3]

8 A store owner receives a shipment of stationery items, including notebooks, pens, and correction tapes. Historical data indicates that 1% of the notebooks, 2% of the pens, and 4% of the correction tapes are defective. The quality of notebooks, pens and correction tapes is independent of one another. The store owner decides to sell the stationery in 180 packets, each containing one notebook, two pens and one correction tape. A packet is deemed unsatisfactory if any of the four items is defective.

(i) Show that the probability that a randomly selected packet is unsatisfactory is 0.0872, correct to 3 significant figures. [1]

The number of packets that are unsatisfactory is denoted by X. You may assume that X can be modelled by a binomial distribution.

(ii) Find the probability that there are at least 5 but less than 10 packets that are unsatisfactory. [2]

(iii) Find the least value of r such that the probability that there are more than r packets that are unsatisfactory is at most 0.12. [2]

Before selling the packets of stationery, he decided to select a sample of 9 packets to check for unsatisfactory packets.

(iv) How should the packets be selected? Give a reason for this method of selection. [2]

(v) Find the probability that the ninth packet is the third unsatisfactory packet selected. [2]

9 Box A contains five cards numbered 1, 2, 2, 3 and 3.

Box B contains three cards numbered 4, 5 and 5.

Cards that are numbered 2, 3 and 5 are red, while cards that are numbered 1 and 4 are blue. A card is drawn from each of the two boxes. If both cards are of the same colour, then the score will be the sum of the numbers on the two cards. If both cards are of different colours, then the score will be the product of the numbers on the two cards. Let X be the score obtained.

(i) Show that $P(X = 8) = \frac{2}{5}$. [2]

(ii) Find the probability distribution of X. [3]

(iii) Find $E(X)$ and $Var(X)$. [3]

(iv) Find the probability that the mean score of 50 independent observations of X lies between 7.5 and 8.5. [2]

10 In a drinks factory, a machine is programmed to dispense 500 ml of green tea into empty bottles.

(a) After a routine check of the machine, the production manager suspects that the machine is dispensing more green tea than expected. A random sample of 50 bottles is taken for a hypothesis test at $\alpha\%$ level of significance and the data is as follows:

Volume of green tea (correct to nearest ml)	498	499	500	501	502	503	504
Number of bottles	5	11	10	11	10	1	2

Taking $\alpha = 2$, test the production manager's suspicion and state the meaning of the p-value in the context of the question. [5]

(i) Determine the set of values of α for which the production manager's suspicion is valid. [1]

(ii) Explain why the production manager is able to carry out the hypothesis test without knowing anything about the distribution of the volume of green tea dispensed by the machine. [1]

(b) The machine is being recalibrated to dispense 500 ml of green tea. Another random sample of 50 bottles of green tea is taken and the mean and standard deviation of this sample are 502 ml and k ml respectively. A hypothesis test, at the 5% significance level, concluded that the recalibration is done accurately. Find the set of values that k can take. [5]

11 [For this question, you should state clearly the values of the parameters of any normal distribution you use.]

After dinner every day, Benedict gives himself some time for relaxation before starting his revision at 8 pm. He finishes his dinner at D minutes past 7 pm, where D follows the distribution $N(10, 2^2)$. After dinner, he usually spends M minutes on social media, which follows the distribution $N(k, 12^2)$.

(i) Given that Benedict starts his revision late 12.5% of the time if he spends time on social media after his dinner, show that $k = 36.0$. [3]

For the rest of this question, assume that $k = 36$.

(ii) Sketch the distribution of the time that Benedict spends on dinner after 7 pm and on social media till 8 pm. [2]

On some occasions after dinner, when he does not spend time on social media, he plays online games. The time he takes to finish the games, G minutes, follows the distribution $N(45, 10^2)$.

(iii) On a particular day after dinner, Benedict spent time on social media. On another day, he played online games. Find the probability that the time he starts revision on the two days differ by at most 5 minutes. [3]

On average, Benedict plays online games on 20% of the evenings after dinner.

(iv) On a particular evening, Benedict started his revision late. Find the probability that he played online games that evening. [4]

(v) State a necessary assumption for your calculations in **(ii)**, **(iii)** and **(iv)** to be valid. [1]

2024 JPJC J2 H2 Prelims Paper 1 Solutions

1

Let $v_n = an^3 + bn^2 + cn + d$

$v_1 = -9 : a + b + c + d = -9 --- (1)$

$v_2 = 7 : 8a + 4b + 2c + d = 7 --- (2)$

$v_3 = 47 : 27a + 9b + 3c + d = 47 --- (3)$

$v_4 = 141 : 64a + 16b + 4c + d = 141 --- (4)$

Use GC: $a = 5, b = -18, c = 35, d = -31$

$v_n = 5n^3 - 18n^2 + 35n - 31$

2

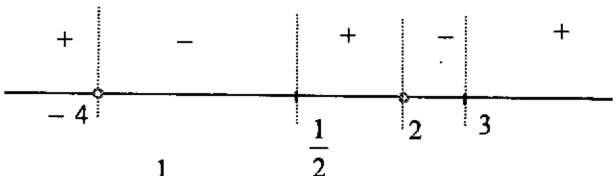
$$\frac{11x-19}{x^2+2x-8} \le 2$$

$$\frac{11x-19}{x^2+2x-8} - 2 \le 0$$

$$\frac{11x-19-2x^2-4x+16}{x^2+2x-8} \le 0$$

$$\frac{-2x^2+7x-3}{x^2+2x-8} \le 0$$

$$\frac{(2x-1)(x-3)}{(x+4)(x-2)} \ge 0$$



$x < -4$ or $\frac{1}{2} \le x < 2$ or $x \ge 3$

Replace x by e^{-x}

$e^{-x} < -4$ or $\frac{1}{2} \le e^{-x} < 2$ or $e^{-x} \ge 3$

(no solution), $\ln\frac{1}{2} \le -x < \ln 2$ or $-x \ge \ln 3$

$-\ln 2 < x \le \ln 2$ or $x \le -\ln 3$

3

$y = 5\sin\theta \Rightarrow \frac{dy}{d\theta} = 5\cos\theta$

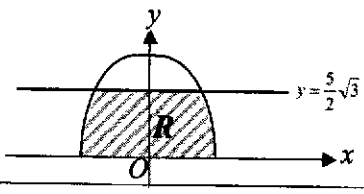
$y = 0, \quad 0 = 5\sin\theta \Rightarrow \theta = 0$

$y = \frac{5}{2}\sqrt{3}, \quad \frac{5}{2}\sqrt{3} = 5\sin\theta \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

Volume

$= \pi\int_0^{\frac{5\sqrt{3}}{2}} x^2 \, dy$

$= \pi\int_0^{\frac{5\sqrt{3}}{2}} \sqrt{25-y^2} \, dy$



$$= \pi \int_0^{\frac{\pi}{3}} \sqrt{25 - 25\sin^2\theta}\,(5\cos\theta)\,\mathrm{d}\theta$$

$$= 5\pi \int_0^{\frac{\pi}{3}} \sqrt{25(1-\sin^2\theta)}\,\cos\theta\,\mathrm{d}\theta$$

$$= 25\pi \int_0^{\frac{\pi}{3}} \sqrt{\cos^2\theta}\,\cos\theta\,\mathrm{d}\theta$$

$$= 25\pi \int_0^{\frac{\pi}{3}} \cos^2\theta\,\mathrm{d}\theta$$

$$= \frac{25}{2}\pi \int_0^{\frac{\pi}{3}} 1 + \cos 2\theta\,\mathrm{d}\theta$$

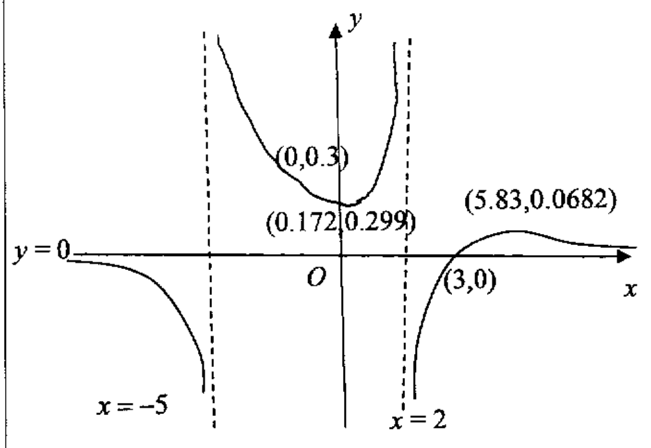
$$= \frac{25}{2}\pi \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{3}}$$

$$= \frac{25}{2}\pi \left[\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3}\right]$$

$$= \frac{25}{2}\pi \left[\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right]$$

$$= \frac{25}{6}\pi^2 + \frac{25}{8}\sqrt{3}\pi \quad \text{(exact)}$$

4(i)



(ii)

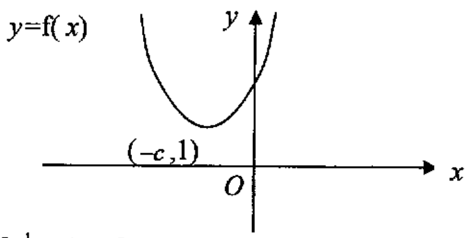
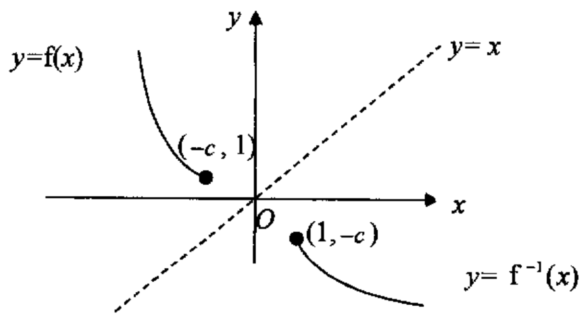
$$y = \mathrm{f}(x) = \frac{x-3}{(x-2)(x+5)}$$

$$\to\ y = \mathrm{f}(2x) = \frac{2x-3}{(2x-2)(2x+5)}$$

$$\to y = (2)\mathrm{f}(2x) = (2)\frac{2x-3}{(2x-2)(2x+5)}\quad y = \frac{2x-3}{(x-1)(2x+5)}$$

Scaling parallel to the x-axis by scale factor $\frac{1}{2}$.

Scaling parallel to the y-axis by scale factor 2.

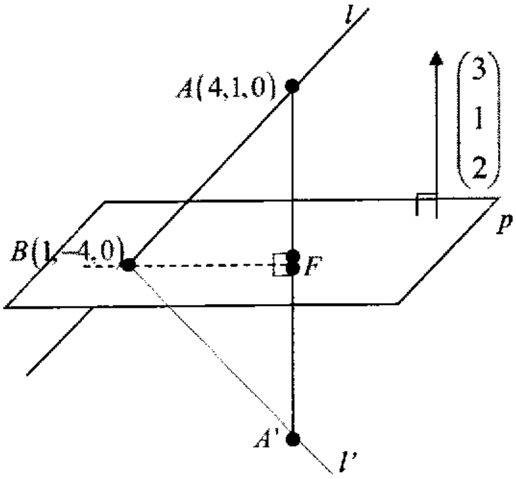
5(i)	 f^{-1} exists for $x \le -c$ $k=-c$
(ii)	$y=e^{(x+c)^2}$ $(x+c)^2=\ln(y)$ $x+c=\pm\sqrt{\ln(y)}$ $x=-c\pm\sqrt{\ln(y)}$ Since $x \le -c$, $\quad x=-c-\sqrt{\ln(y)}$ $f^{-1}(x)=-c-\sqrt{\ln x},\ x\ge 1$
(iii)	 Graphs of f and f^{-1} are reflections of each other about the line $y=x$
(iv)	Range of f $=[1,\ \infty)$ and Domain of g $=(0,\ \infty)$ Range of f $\subseteq$ Domain of g Hence gf exists gf(x)= $\ln\ e^{(x+c)^2}$ $=\ (x+c)^2\ ,\ x\le -c$

6(i)	$\overrightarrow{AB}=\underset{\sim}{b}-\underset{\sim}{a}$ $\overrightarrow{AC}=(m\underset{\sim}{a}+n\underset{\sim}{b})-\underset{\sim}{a}=(m-1)\underset{\sim}{a}+n\underset{\sim}{b}$ Area of triangle ABC $=\frac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AC}\right\rvert$ $=\frac{1}{2}\left\lvert(\underset{\sim}{b}-\underset{\sim}{a})\times((m-1)\underset{\sim}{a}+n\underset{\sim}{b})\right\rvert$

	$=\frac{1}{2}\left\|(m-1)(\underset{\sim}{b}\times\underset{\sim}{a})+n(\underset{\sim}{b}\times\underset{\sim}{b})-(m-1)(\underset{\sim}{a}\times\underset{\sim}{a})-n(\underset{\sim}{a}\times\underset{\sim}{b})\right\|$ $=\frac{1}{2}\left\|(m-1)(\underset{\sim}{b}\times\underset{\sim}{a})+\underset{\sim}{0}+\underset{\sim}{0}+n(\underset{\sim}{b}\times\underset{\sim}{a})\right\|$ $=\frac{1}{2}(m+n-1)\left\|\underset{\sim}{b}\times\underset{\sim}{a}\right\|$ $=\frac{1}{2}(m+n-1)\left\|\underset{\sim}{b}\right\|\left\|\underset{\sim}{a}\right\|\sin 30^\circ$ $=\frac{1}{2}(m+n-1)\left\|\underset{\sim}{b}\right\|(4\left\|\underset{\sim}{b}\right\|)\left(\frac{1}{2}\right)$ $=(m+n-1)\left\|\underset{\sim}{b}\right\|^2$
(ii)	$\overrightarrow{OD}=\frac{1}{2}\underset{\sim}{a}\quad \overrightarrow{OE}=\frac{3}{5}\underset{\sim}{b}$
(iii)	$l_{BD}:\underset{\sim}{r}=\underset{\sim}{b}+\lambda_1\overrightarrow{BD}$ $=\underset{\sim}{b}+\lambda_1(\underset{\sim}{d}-\underset{\sim}{b})$ $=\underset{\sim}{b}+\lambda_1\left(\frac{1}{2}\underset{\sim}{a}-\underset{\sim}{b}\right)$ $=\underset{\sim}{b}+\lambda(\underset{\sim}{a}-2\underset{\sim}{b})$, where $\lambda=\frac{\lambda_1}{2}$ $=\lambda\underset{\sim}{a}+(1-2\lambda)\underset{\sim}{b}$ (shown) $l_{AE}:\underset{\sim}{r}=\underset{\sim}{a}+\mu_1\overrightarrow{AE}$ $=\underset{\sim}{a}+\mu_1(\underset{\sim}{e}-\underset{\sim}{a})$ $=\underset{\sim}{a}+\mu_1\left(\frac{3}{5}\underset{\sim}{b}-\underset{\sim}{a}\right)$ $=\underset{\sim}{a}+\mu(3\underset{\sim}{b}-5\underset{\sim}{a})$, where $\mu=\frac{\mu_1}{5}$ $=(1-5\mu)\underset{\sim}{a}+3\mu\underset{\sim}{b}$ Since lines BD and AE meet, $\lambda\underset{\sim}{a}+(1-2\lambda)\underset{\sim}{b}=(1-5\mu)\underset{\sim}{a}+3\mu\underset{\sim}{b}$ Comparing coefficients of $\underset{\sim}{a}$, $\lambda=1-5\mu\Rightarrow\lambda+5\mu=1$ --- (1) Comparing coefficients of $\underset{\sim}{b}$, $1-2\lambda=3\mu\Rightarrow 2\lambda+3\mu=1$ --- (2) Using GC, $\lambda=\frac{2}{7},\mu=\frac{1}{7}$ $\overrightarrow{OF}=\frac{2}{7}\underset{\sim}{a}+\left[1-2\left(\frac{2}{7}\right)\right]\underset{\sim}{b}$ $=\frac{2}{7}\underset{\sim}{a}+\frac{3}{7}\underset{\sim}{b}$

7(a)	$l: \underset{\sim}{r} = \begin{pmatrix}4\\1\\0\end{pmatrix} + t\begin{pmatrix}3\\5\\0\end{pmatrix}, t \in \mathbb{R}$ ---(1) For plane p, $\underset{\sim}{n} = \begin{pmatrix}1\\-5\\1\end{pmatrix} \times \begin{pmatrix}-3\\a\\1\end{pmatrix} = \begin{pmatrix}-5-a\\-4\\a-15\end{pmatrix}$ Since l and p do not meet in a unique point, $\begin{pmatrix}-5-a\\-4\\a-15\end{pmatrix} \bullet \begin{pmatrix}3\\5\\0\end{pmatrix} = 0$ $3(-5-a) - 20 = 0$ $-3a = 35$ $a = -\frac{35}{3}$
(b)(i)	Given $a = 7$, $\underset{\sim}{n} = \begin{pmatrix}-5-7\\-4\\7-15\end{pmatrix} = -4\begin{pmatrix}3\\1\\2\end{pmatrix}$ $p: \underset{\sim}{r} \bullet \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}-3\\0\\4\end{pmatrix} \bullet \begin{pmatrix}3\\1\\2\end{pmatrix} = -1$ --- (2) Subst. (1) into (2): $\left[\begin{pmatrix}4\\1\\0\end{pmatrix} + t\begin{pmatrix}3\\5\\0\end{pmatrix}\right] \bullet \begin{pmatrix}3\\1\\2\end{pmatrix} = -1$ $13 + 14t = -1$ $t = -1$ Position vector of the point of intersection, $B = \begin{pmatrix}4\\1\\0\end{pmatrix} - \begin{pmatrix}3\\5\\0\end{pmatrix} = \begin{pmatrix}1\\-4\\0\end{pmatrix}$ $\therefore B(1,-4,0)$

(ii)



To find F, the foot of perpendicular from A to p:

$$l_{AF}:\ \underset{\sim}{r} = \begin{pmatrix}4\\1\\0\end{pmatrix} + s\begin{pmatrix}3\\1\\2\end{pmatrix}, s \in \mathbb{R}$$

$$\left[\begin{pmatrix}4\\1\\0\end{pmatrix} + s\begin{pmatrix}3\\1\\2\end{pmatrix}\right] \bullet \begin{pmatrix}3\\1\\2\end{pmatrix} = -1$$

$$13 + 14s = -1$$

$$s = -1$$

$$\overrightarrow{OF} = \begin{pmatrix}4\\1\\0\end{pmatrix} - \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}1\\0\\-2\end{pmatrix}$$

Since F is the mid-point of AA',

$$\overrightarrow{OF} = \frac{\overrightarrow{OA} + \overrightarrow{OA'}}{2}$$

$$\overrightarrow{OA'} = 2\overrightarrow{OF} - \overrightarrow{OA}$$

$$= 2\begin{pmatrix}1\\0\\-2\end{pmatrix} - \begin{pmatrix}4\\1\\0\end{pmatrix} = \begin{pmatrix}-2\\-1\\-4\end{pmatrix}$$

$$\overrightarrow{BA'} = \begin{pmatrix}-2\\-1\\-4\end{pmatrix} - \begin{pmatrix}1\\-4\\0\end{pmatrix} = \begin{pmatrix}-3\\3\\-4\end{pmatrix}$$

$$l':\ \underset{\sim}{r} = \begin{pmatrix}1\\-4\\0\end{pmatrix} + \alpha\begin{pmatrix}-3\\3\\-4\end{pmatrix}, \alpha \in \mathbb{R}$$

$$\frac{1-x}{3} = \frac{y+4}{3} = -\frac{z}{4}$$

8 (i)

Month	Amount owed at beginning of the month	Amount owed at the end of the month
1	$200000(1.005)$	$200000(1.005)-x$
2	$200000(1.005)^2-(1.005)x$	$200000(1.005)^2-(1.005)x-x$
3	$200000(1.005)^3-(1.005)^2x-(1.005)x$	$200000(1.005)^3-(1.005)^2x-(1.005)x-x$

Amount owed at the end of n months

$=200000(1.005)^n-(1.005)^{n-1}x-(1.005)^{n-2}x-...-1.005x-x$

$=200000(1.005)^n-x\left[1+1.005+...+(1.005)^{n-2}+(1.005)^{n-1}\right]$

$=200000(1.005)^n-\dfrac{x\left[1-1.005^n\right]}{1-1.005}$

$=200000(1.005)^n-200x\left[1.005^n-1\right]$

(ii)

$200000(1.005)^n-200x\left[1.005^n-1\right]\le 0$

$200000(1.005)^n-200(1500)\left[1.005^n-1\right]\le 0$

$300000-100000(1.005)^n\le 0$

$(1.005)^n\ge 3$

$n\ge\dfrac{\ln 3}{\ln 1.005}$

$n\ge 220.27$

Alternatively,
Use GC table

n	$200000(1.005)^n-200(1500)\left[1.005^n-1\right]$
219	1896.19
220	$405.67>0$
221	$-1092.30<0$

$n=221$

At the end of 220 months, Selena owed

$200000(1.005)^{220}-200(1500)\left[1.005^{220}-1\right]=\$405.67$

Last repayment amount to be repaid on the 221st month

$=\$405.67\times1.005=$ <u>\$407.70</u> <u>(2d.p.)</u>

221 months = 18 years 5 months

Full repayment on: <u>31 May 2043</u>

(iii)

$200000(1.005)^{120}-200x\left[1.005^{120}-1\right]\le 0$

$363879.3468-163.8793468x\le 0$

$x\ge 2220.410039$

\$2220.42 (2d.p.) [\$2220.41 not accepted]

9(a)	Sub. $z=1+2\mathrm{i}$ into $z^4-z^3-9z^2+sz+t=0$ $(1+2\mathrm{i})^4-(1+2\mathrm{i})^3-9(1+2\mathrm{i})^2+s(1+2\mathrm{i})+t=0$ $(-7-24\mathrm{i})-(-11-2\mathrm{i})-9(-3+4\mathrm{i})+s(1+2\mathrm{i})+t=0$ $(31+s+t)+(2s-58)\mathrm{i}=0$ Comparing imaginary parts, $2s-58=0$ $\underline{\underline{s=29}}$ Comparing real parts, $31+s+t=0$ $t=-31-s$ $=\underline{\underline{-60}}$ Now $z^4-z^3-9z^2+29z-60=0$ Using GC the other roots are $\underline{\underline{1-2\mathrm{i},3,-4}}$. **Alternative solution** Since $z^4-z^3-9z^2+sz+t=0$ is a polynomial equation with real coefficients and $1+2\mathrm{i}$ is a root, $1-2\mathrm{i}$ is another root. Quadratic factor $=[z-(1+2\mathrm{i})][z-(1-2\mathrm{i})]$ $=[(z-1)-2\mathrm{i}][(z-1)+2\mathrm{i}]$ $=(z-1)^2-(2\mathrm{i})^2$ $=z^2-2z+5$ Let $z^4-z^3-9z^2+sz+t=(z^2-2z+5)(z^2+az+b)$. By comparing coefficients, z^3: $-1=a-2 \quad \Rightarrow a=1$ z^2: $-9=b-2a+5 \quad \Rightarrow b=-12$ z: $s=-2b+5a=29$ constant term: $t=5b=-60$ Now $z^4-z^3-9z^2+29z-60=0$ Using GC (polyroot finder), the other roots are $\underline{\underline{1-2\mathrm{i},3,-4}}$.
(b)	$\arg\left(\frac{w^3}{\mathrm{i}w^*}\right)=\arg w^3-\arg(\mathrm{i}w^*)$ $=3\arg w-(\arg i+\arg w^*)$ $=3\arg w-\left(\frac{\pi}{2}-\arg w\right)$ $=4\arg w-\frac{\pi}{2}$

	For $\dfrac{w^3}{\mathrm{i}w^*}$ to be purely imaginary, $\arg\left(\dfrac{w^3}{\mathrm{i}w^*}\right)=\pm\dfrac{\pi}{2},\pm\dfrac{3\pi}{2},\pm\dfrac{5\pi}{2},\ldots$ $4\arg(w)-\dfrac{\pi}{2}=\pm\dfrac{\pi}{2},\pm\dfrac{3\pi}{2},\pm\dfrac{5\pi}{2},\ldots$ $4\arg(w)=0,\ \pi,\ 2\pi,\ -\pi,\ 3\pi,-2\pi$ $\arg(w)=\dfrac{\pi}{4},0,\ \dfrac{\pi}{2},\dfrac{-\pi}{4},\dfrac{3\pi}{4},-\dfrac{\pi}{2}$ Since $w=a+\mathrm{i}b$ and a and b are positive real numbers, $0<\arg(w)<\dfrac{\pi}{2}$. $\therefore \arg(w)=\dfrac{\pi}{4}$ $\tan^{-1}\left(\dfrac{b}{a}\right)=\dfrac{\pi}{4}$ $\dfrac{b}{a}=1$ $b=a$ $w=a+\mathrm{i}a$

10(i)	$\dfrac{\mathrm{d}v}{\mathrm{d}t}=2\mathrm{e}^{-0.1t}$ $v=\int 2\mathrm{e}^{-0.1t}\,\mathrm{d}t$ $v=2\left(\dfrac{\mathrm{e}^{-0.1t}}{-0.1}\right)+c$ $v=-20\mathrm{e}^{-0.1t}+c$ When $t=0$, $v=0$ $0=-20\mathrm{e}^{0}+c$ $c=20$ $v=20-20\mathrm{e}^{-0.1t}$ Subt $v=10$ $10=20-20\mathrm{e}^{-0.1t}$ $20\mathrm{e}^{-0.1t}=10$ $\mathrm{e}^{-0.1t}=\dfrac{1}{2}$ $-0.1t=\ln\dfrac{1}{2}$ $t=-10\ln\dfrac{1}{2}=10\ln 2$ (exact)
(ii)	As $t\to\infty,\mathrm{e}^{-0.1t}\to 0, v\to 20$ Eventually, the speed increases and tend to 20 ms^{-1}

<table>
<tr><td rowspan="3">(iii)</td><td>

$$-2\frac{\mathrm{d}w}{\mathrm{d}t}=(w-3)(w+2)$$

$$\frac{1}{(w-3)(w+2)}\frac{\mathrm{d}w}{\mathrm{d}t}=-\frac{1}{2}$$

$$\int\frac{1}{(w-3)(w+2)}\,\mathrm{d}w=\int-\frac{1}{2}\,\mathrm{d}t$$

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Method 1: Partial fractions

$$\frac{1}{(w-3)(w+2)}=\frac{A}{(w-3)}+\frac{B}{(w+2)}$$

$$1=A(w+2)+B(w-3)$$

$$1=-5B\Rightarrow B=-\frac{1}{5}$$

$$1=5A\Rightarrow A=\frac{1}{5}$$

$$\frac{1}{5}\int\frac{1}{(w-3)}-\frac{1}{(w+2)}\,\mathrm{d}w=\int-\frac{1}{2}\,\mathrm{d}t$$

Method 2: Completing the square

$$\int\frac{1}{w^2-w-6}\,\mathrm{d}w=\int-\frac{1}{2}\,\mathrm{d}t$$

$$\int\frac{1}{\left(w-\frac{1}{2}\right)^2-\left(\frac{5}{2}\right)^2}\,\mathrm{d}w=\int-\frac{1}{2}\,\mathrm{d}t$$

$$\frac{1}{2\left(\frac{5}{2}\right)}\ln\left|\frac{w-\frac{1}{2}-\frac{5}{2}}{w-\frac{1}{2}+\frac{5}{2}}\right|=\int-\frac{1}{2}\,\mathrm{d}t$$

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$$\frac{1}{5}\left[\ln|w-3|-\ln|w+2|\right]=-\frac{1}{2}t+c$$

$$\frac{1}{5}\ln\left|\frac{w-3}{w+2}\right|=-\frac{1}{2}t+c$$

$$\ln\left|\frac{w-3}{w+2}\right|=-\frac{5}{2}t+5c$$

$$\left|\frac{w-3}{w+2}\right|=\mathrm{e}^{-\frac{5}{2}t+5c}$$

$$\frac{w-3}{w+2}=\pm\mathrm{e}^{-\frac{5}{2}t+5c}$$

$$\frac{w-3}{w+2}=A\mathrm{e}^{-\frac{5}{2}t},\quad A=\pm\mathrm{e}^{5c}$$

Sub w = 18,

$$\frac{18-3}{18+2}=A\mathrm{e}^{0}$$

$$A=\frac{15}{20}=\frac{3}{4}$$

$$\frac{w-3}{w+2}=\frac{3}{4}\mathrm{e}^{-\frac{5}{2}t}$$

$$w-3=\frac{3}{4}w\mathrm{e}^{-\frac{5}{2}t}+\frac{3}{2}\mathrm{e}^{-\frac{5}{2}t}$$

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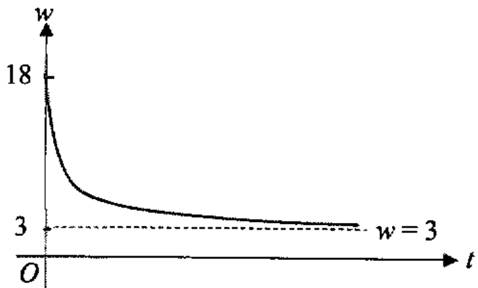
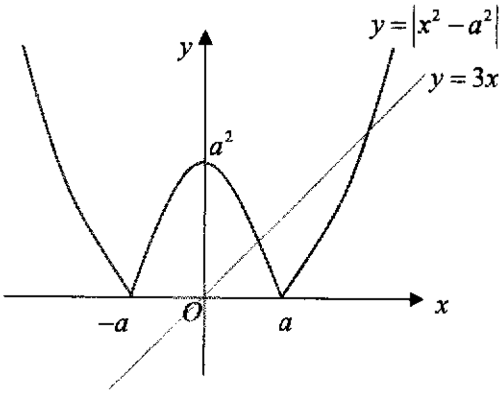
</table>

	$w-\frac{3}{4}we^{-\frac{5}{2}t}=3+\frac{3}{2}e^{-\frac{5}{2}t}$ $w=\dfrac{3+\frac{3}{2}e^{-\frac{5}{2}t}}{1-\frac{3}{4}e^{-\frac{5}{2}t}}$ $w=\dfrac{12+6e^{-\frac{5}{2}t}}{4-3e^{-\frac{5}{2}t}}$
(iv)	 The speed will not fall below 3 ms^{-1}.

11(i)	$y=x\tan\theta-\dfrac{10x^2}{2u^2\cos^2\theta}$ $-1.8=15\tan\left(\frac{\pi}{4}\right)-\dfrac{10(15)^2}{2u^2\cos^2\left(\frac{\pi}{4}\right)}$ $-1.8=15(1)-\dfrac{10(15)^2}{2u^2\left(\frac{1}{2}\right)}$ $\dfrac{10(15)^2}{u^2}=15+1.8$ $u^2=\dfrac{10(15)^2}{16.8}$ $u=11.6$
(ii)	$y=x\tan\theta-\dfrac{10x^2}{2(10)^2\cos^2\theta}$ When $y=-1.8$ $\therefore \quad -1.8=x\tan\theta-\dfrac{x^2}{20\cos^2\theta}$ $-36\cos^2\theta=20x\tan\theta\cos^2\theta-x^2$ $x^2-20x\sin\theta\cos\theta-36\cos^2\theta=0$ $x^2-10x\sin 2\theta-18(1+\cos 2\theta)=0$ $x^2-10x\sin 2\theta-18\cos 2\theta-18=0$ (Shown)

(iii)	Differentiate w.r.t. θ, we have $2x\frac{dx}{d\theta}-10\frac{dx}{d\theta}\sin 2\theta-20x\cos 2\theta+36\sin 2\theta=0$ At stationary value of x, $\frac{dx}{d\theta}=0$ $-20x\cos 2\theta+36\sin 2\theta=0$ $36\sin 2\theta=20x\cos 2\theta$ $x=\frac{36\sin 2\theta}{20\cos 2\theta}$ $x=\frac{9}{5}\tan 2\theta$
(iv)	Sub into equation, we have $\left(\frac{9}{5}\tan 2\theta\right)^2-10\left(\frac{9}{5}\tan 2\theta\right)\sin 2\theta-18\cos 2\theta-18=0$ $\frac{81}{25}\tan^2 2\theta-18\tan 2\theta\sin 2\theta-18\cos 2\theta-18=0$ $81\tan^2 2\theta-450\sin 2\theta\tan 2\theta-450\cos 2\theta-450=0$ Using GC, $\theta=0.70883\approx 0.71$ (2 decimal places), $\frac{\pi}{2}$ (reject) Therefore, stationary value of $x=\frac{9}{5}\tan 2(0.70883)=11.7$

2024 JC2 H2 Maths Prelim Paper 2 Solutions

1((a)	
(b)	At point of intersection where $0<x<a$, $-(x^2-a^2)=3x$ $x^2-a^2=-3x$ $x^2+3x-a^2=0$ $x=\dfrac{-3\pm\sqrt{9+4a^2}}{2}$ $x=\dfrac{-3+\sqrt{9+4a^2}}{2}$ (since $x>0$)

2(a)	$\int \sin px\cos qx\,\mathrm{d}x$ $=\frac{1}{2}\int 2\sin px\cos qx\,\mathrm{d}x$ $=\frac{1}{2}\int \sin(p+q)x+\sin(p-q)x\,\mathrm{d}x$ $=\frac{1}{2}\left[-\frac{1}{p+q}\cos(p+q)x-\frac{1}{p-q}\cos(p-q)x\right]+C$ Alternatively, $\int \sin px\cos qx\,\mathrm{d}x$ $=\frac{1}{2}\int 2\cos qx\sin px\,\mathrm{d}x$ $=\frac{1}{2}\int \sin(q+p)x-\sin(q-p)x\,\mathrm{d}x$ $=\frac{1}{2}\left[-\frac{1}{q+p}\cos(q+p)x+\frac{1}{q-p}\cos(q-p)x\right]+C$	
(b)	$\int x\sin mx\,\mathrm{d}x=-\frac{1}{m}x\cos mx+\int\frac{1}{m}\cos mx\,\mathrm{d}x$ $=-\frac{1}{m}x\cos mx+\frac{1}{m^2}\sin mx+C$	$u=x\quad \frac{\mathrm{d}v}{\mathrm{d}x}=\sin mx$ $\frac{\mathrm{d}u}{\mathrm{d}x}=1\quad v=-\frac{\cos mx}{m}$

(c)	$\int_0^{\pi} x\sin mx\,\mathrm{d}x$ $=\left[-\frac{1}{m}x\cos mx+\frac{1}{m^2}\sin mx\right]_0^{\pi}$ $=\left[-\frac{1}{m}\pi\cos m\pi+\frac{1}{m^2}\sin m\pi\right]-\left[-\frac{1}{m}(0)\cos 0+\frac{1}{m^2}\sin(0)\right]$ $=-\frac{1}{m}\pi\cos m\pi$ When m is odd, $\int_0^{\pi} x\sin mx\,\mathrm{d}x=-\frac{1}{m}\pi(-1)=\frac{1}{m}\pi$ When m is even, $\int_0^{\pi} x\sin mx\,\mathrm{d}x=-\frac{1}{m}\pi(1)=-\frac{1}{m}\pi$ $k=1$ or -1

3(i)	$\mathrm{f}'(x)=\mathrm{e}^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$ $(1+x^2)\mathrm{f}'(x)=\mathrm{f}(x)$ <u>Alternatively,</u> $\mathrm{f}(x)=\mathrm{e}^{\tan^{-1}x}$ $\ln\mathrm{f}(x)=\tan^{-1}x$ Differentiate with respect to x $\frac{1}{\mathrm{f}(x)}\mathrm{f}'(x)=\frac{1}{1+x^2}$ $(1+x^2)\mathrm{f}'(x)=\mathrm{f}(x)$
(ii)	$(1+x^2)\mathrm{f}'(x)=\mathrm{f}(x)$ Differentiate with respect to x $(1+x^2)\mathrm{f}''(x)+2x\mathrm{f}'(x)=\mathrm{f}'(x)$ Differentiate with respect to x $(1+x^2)\mathrm{f}'''(x)+2x\mathrm{f}''(x)+2x\mathrm{f}''(x)+2\mathrm{f}'(x)=\mathrm{f}''(x)$ $(1+x^2)\mathrm{f}'''(x)+4x\mathrm{f}''(x)+2\mathrm{f}'(x)=\mathrm{f}''(x)$ When $x=0,\ y=\mathrm{f}(0)=\mathrm{e}^{\tan^{-1}0}=1$ $(1+0)\mathrm{f}'(x)=1\Rightarrow\mathrm{f}'(x)=1$ $(1+0)\mathrm{f}''(x)+0=1\Rightarrow\mathrm{f}''(x)=1$ $(1+0)\mathrm{f}'''(x)+0+2(1)=1\Rightarrow\mathrm{f}'''(x)=-1$ $y=\mathrm{f}(x)=\mathrm{e}^{\tan^{-1}x}=1+x+\frac{1}{2!}x^2-\frac{1}{3!}x^3+\ldots\qquad\approx 1+x+\frac{1}{2}x^2-\frac{1}{6}x^3$
(iii)	$\int_0^{0.5}\mathrm{f}(x)\,\mathrm{d}x\approx\int_0^{0.5}1+x+\frac{1}{2}x^2-\frac{1}{6}x^3\,\mathrm{d}x\approx 0.6432$ (4s.f.)
(iv)	The series is a good approximation for f(x) if x is close to 0. Since <u>x =1 is not close to 0,</u> it is <u>not suitable</u> to be used to estimate $\mathrm{e}^{\frac{\pi}{4}}$.

4(i)

$$\frac{7}{n-2}-\frac{5}{n-1}-\frac{2}{n}=\frac{7(n-1)n-5(n-2)n-2(n-2)(n-1)}{(n-2)(n-1)n}$$

$$=\frac{7n^2-7n-5n^2+10n-2n^2+6n-4}{(n-2)(n-1)n}$$

Obtain $\dfrac{9n-4}{(n-2)(n-1)n}$ (Shown)

<u>Alternative : By partial fractions</u>

Let $\dfrac{9n-4}{(n-2)(n-1)n}=\dfrac{A}{n-2}+\dfrac{B}{n-1}+\dfrac{C}{n}$

$9n-4=A(n-1)n+B(n-2)n+C(n-2)(n-1)$

Subst $n=2$, obtain $A=7$

Subst $n=1$, obtain $B=-5$

Subst $n=0$, obtain $C=-2$

Obtain $\dfrac{9n-4}{(n-2)(n-1)n}$ (Shown)

(ii)

$$\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}=\sum_{n=3}^{N}\left[\frac{7}{(n-2)}-\frac{5}{(n-1)}-\frac{2}{n}\right]$$

$$=\left[\begin{array}{l}\frac{7}{1}-\frac{5}{2}-\frac{2}{3}\\+\frac{7}{2}-\frac{5}{3}-\frac{2}{4}\\+\frac{7}{3}-\frac{5}{4}-\frac{2}{5}\\+\frac{7}{4}-\frac{5}{5}-\frac{2}{6}\\+\ldots\ldots\\+\frac{7}{N-4}-\frac{5}{N-3}-\frac{2}{N-2}\\+\frac{7}{N-3}-\frac{5}{N-2}-\frac{2}{N-1}\\+\frac{7}{N-2}-\frac{5}{N-1}-\frac{2}{N}\end{array}\right]$$

$$=8-\left[\frac{7}{N-1}+\frac{2}{N}\right]$$

(iii)

$$\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}=8-\left[\frac{7}{N-1}+\frac{2}{N}\right]$$

As $N\to\infty$, $\dfrac{7}{N-1}\to 0$ and $\dfrac{2}{N}\to 0$, $\displaystyle\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}\to 8$ which is finite.

Hence $\displaystyle\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}$ is convergent and the sum to infinity is 8.

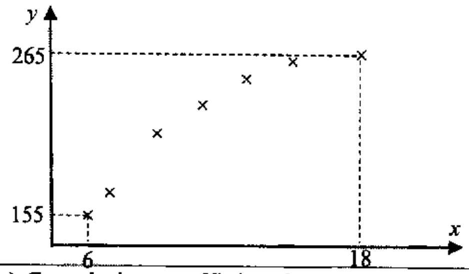
(iv)	$\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}=\frac{23}{(1)(2)(3)}+\left(\frac{32}{(2)(3)(4)}+\frac{41}{(3)(4)(5)}+\ldots\ldots\frac{9N-4}{(N-2)(N-1)N}\right)$ $\sum_{n=2}^{N}\frac{9n+14}{n(n+1)(n+2)}$ $=\left(\frac{32}{(2)(3)(4)}+\frac{41}{(3)(4)(5)}+\ldots+\frac{9N-4}{(N-2)(N-1)(N)}\right)+\frac{9N+5}{(N-1)(N)(N+1)}+\frac{9N+14}{N(N+1)(N+2)}$ $=\sum_{n=4}^{N+2}\frac{9n-4}{(n-2)(n-1)n}$ Note : replace n by $n-2$ $=\sum_{n=3}^{N+2}\frac{9n-4}{(n-2)(n-1)n}-\frac{23}{(1)(2)(3)}$ $=8-\left[\frac{7}{N+2-1}+\frac{2}{N+2}\right]-\frac{23}{6}$ $=\frac{25}{6}-\left[\frac{7}{N+1}+\frac{2}{N+2}\right]$

5(i)	$\overrightarrow{OA}=\begin{pmatrix}5\\0\\0\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}-4\\-3\\0\end{pmatrix}, \overrightarrow{OC}=\begin{pmatrix}-5\\0\\6\end{pmatrix}$ Since $ABCD$ is a parallelogram, $\overrightarrow{AD}=\overrightarrow{BC}$ $\overrightarrow{OD}-\overrightarrow{OA}=\overrightarrow{OC}-\overrightarrow{OB}$ $=\begin{pmatrix}-5\\0\\6\end{pmatrix}-\begin{pmatrix}-4\\-3\\0\end{pmatrix}=\begin{pmatrix}-1\\3\\6\end{pmatrix}$ $\overrightarrow{OD}=\begin{pmatrix}-1\\3\\6\end{pmatrix}+\begin{pmatrix}5\\0\\0\end{pmatrix}$ $=\begin{pmatrix}4\\3\\6\end{pmatrix}$

	$\overrightarrow{BA}=\begin{pmatrix}5\\0\\0\end{pmatrix}-\begin{pmatrix}-4\\-3\\0\end{pmatrix}=\begin{pmatrix}9\\3\\0\end{pmatrix}$ $\overrightarrow{BA}\bullet\overrightarrow{BC}=\begin{pmatrix}9\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-1\\3\\6\end{pmatrix}=0$ Since $ABCD$ is a parallelogram and $AB\perp BC$, $ABCD$ is a rectangle.
(ii)	$\overrightarrow{BA}\times\overrightarrow{BC}=\begin{pmatrix}9\\3\\0\end{pmatrix}\times\begin{pmatrix}-1\\3\\6\end{pmatrix}=3\begin{pmatrix}3\\1\\0\end{pmatrix}\times\begin{pmatrix}-1\\3\\6\end{pmatrix}=6\begin{pmatrix}3\\-9\\5\end{pmatrix}$ Normal to the plane $ABC=\underset{\sim}{n}=\begin{pmatrix}3\\-9\\5\end{pmatrix}$. $\overrightarrow{OA}\bullet\begin{pmatrix}3\\-9\\5\end{pmatrix}=\begin{pmatrix}5\\0\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-9\\5\end{pmatrix}=15$ Vector equation of plane ABC is $\underset{\sim}{r}\bullet\begin{pmatrix}3\\-9\\5\end{pmatrix}=15$ Cartesian equation of plane ABC is $\underline{\underline{3x-9y+5z=15}}$.
(iii)	Normal vector of the base is parallel to $\overrightarrow{OE}$. Hence, normal vector of the base = $\begin{pmatrix}0\\0\\1\end{pmatrix}$ Acute angle between plane ABC and the base $=\cos^{-1}\dfrac{\left\|\begin{pmatrix}3\\-9\\5\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|}{\left\|\begin{pmatrix}3\\-9\\5\end{pmatrix}\right\|\left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\right\|}$ $=\cos^{-1}\dfrac{\|5\|}{\sqrt{3^2+9^2+5^2}\sqrt{1}}$ $=\cos^{-1}\dfrac{5}{\sqrt{115}}$ $=\underline{\underline{62.2°}}$ (1 dp)

(iv)	Since $AF : AE = 1:5$, $\overrightarrow{AF} = \frac{1}{5}\overrightarrow{AE} = \frac{1}{5}\left[\begin{pmatrix}0\\0\\10\end{pmatrix} - \begin{pmatrix}5\\0\\0\end{pmatrix}\right] = \begin{pmatrix}-1\\0\\2\end{pmatrix}$ Length of projection of AF onto plane ABC $= \frac{\lvert\overrightarrow{AF} \times \underset{\sim}{n}\rvert}{\lvert\underset{\sim}{n}\rvert}$ $= \frac{\left\lvert\begin{pmatrix}-1\\0\\2\end{pmatrix} \times \begin{pmatrix}3\\-9\\5\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}3\\-9\\5\end{pmatrix}\right\rvert}$ $= \frac{\left\lvert\begin{pmatrix}18\\11\\9\end{pmatrix}\right\rvert}{\sqrt{3^2 + (-9)^2 + 5^2}}$ $= \frac{\sqrt{526}}{\sqrt{115}}$ $= 2.1387$ ≈ 2.14

6(a)(i)	Since there is only one way the letters are in alphabetical order, Total number of ways $= \frac{11!}{3!2!2!} - 1$ $= 1\,663\,199$
(a)(ii)	↑ [A A O I E] ↑ R ↑ R ↑ T ↑ [PP] P Total number of ways $= \frac{4!}{2!} \times \frac{5!}{2!} \times {}^5P_2 = 14\,400$
(b)	<u>Method 1:</u> Probability $= \frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} \times \frac{3!}{2!} + \frac{5}{11} \times \frac{4}{10} \times \frac{3}{9} = \frac{14}{33}$ (or 0.424) <u>Method 2:</u> Probability $= \frac{{}^6C_1\,{}^5C_2 + {}^5C_3}{{}^{11}C_3}$ <u>Method 3</u>: Probability $= 1 - \frac{{}^6C_3}{{}^{11}C_3} - \frac{{}^5C_1\,{}^6C_2}{{}^{11}C_3} = \frac{14}{33}$

7(i)	
(ii)	**(a)** Correlation coefficient between x and y is $r = 0.9483$ (4 d.p.) **(b)** Correlation coefficient between $\ln x$ and y is $r = 0.9849$ (4 d.p.)
(iii)	From the scatter diagram, as x increases, y increases at a decreasing rate. In addition, the product moment correlation coefficient between $\ln x$ and y, 0.9849, is closer to +1 as compared to that between x and y, 0.9483. Hence $y = c\ln x + d$ is the better model.
(iv)	Since x is the independent variable, neither the regression line of x on y nor the regression line of $\ln x$ on y should be used to estimate the value of x when $y = 200$.
(v)	Equation of regression line of y on $\ln x$ is $y = 106.5611 \ln x - 31.2643$ $y = 107 \ln x - 31.3$ when $y = 200$, $200 = 106.5611 \ln x - 31.2643$ $x = 8.76$ (3 s.f.) Since $y = 200$ is within the given range of data, which is an interpolation, and r is close to +1, indicating a strong positive linear correlation, the estimate is reliable.

8(i)	P(packet is unsatisfactory) $= 1 - (0.99)(0.98)^2(0.96)$ $= 0.087236 \approx 0.0872$ (3 s.f.)
(ii)	$X \sim B(180, 0.087236)$ $P(5 \le X < 10) = P(X \le 9) - P(X \le 4)$ $= 0.042669 \approx 0.0427$
(iii)	$P(X > r) \le 0.12$ $1 - P(X \le r) \le 0.12$ $P(X \le r) \ge 0.88$ From GC, $P(X \le 19) = 0.8429$ (< 0.88) $P(X \le 20) = 0.8947$ (> 0.88) $P(X \le 21) = 0.9323$ (> 0.88) $\therefore$ least $r = 20$
(iv)	Each packet has an equal chance of being selected and the selection of the packets is independent of one another. This method of selection is done so that a random sample will be obtained which is free from bias and will be representative of the population.

(v)

Required probability

$= (0.087236)^2 \times (0.912764)^6 \times \dfrac{8!}{2!6!} \times (0.087236)$

$= 0.010749 \approx 0.0107$

<u>Alternative Method</u>

Let Y be the the number of packets (out of 8) that are unsatisfactory.

$Y \sim B(8, 0.087236)$

Required probability $= P(Y = 2) \times (0.087236)$

$= 0.010749 \approx 0.0107$

9(i)

Box A \ Box B	(B) 4 $(\frac{1}{3})$	(R) 5 $(\frac{2}{3})$
(B) 1 $(\frac{1}{5})$	5	5
(R) 2 $(\frac{2}{5})$	8	7
(R) 3 $(\frac{2}{5})$	12	8

$P(X = 8) = P(\text{2 from Box A and 4 from Box B}) + P(\text{3 from Box A and 5 from Box B})$

$= \left(\dfrac{2}{5}\right)\left(\dfrac{1}{3}\right) + \left(\dfrac{2}{5}\right)\left(\dfrac{2}{3}\right)$

$= \dfrac{2}{5}$

(ii)

$P(X = 5) = \left(\dfrac{1}{5}\right)\left(\dfrac{1}{3}\right) + \left(\dfrac{1}{5}\right)\left(\dfrac{2}{3}\right) = \dfrac{1}{5}$

$P(X = 7) = \left(\dfrac{2}{5}\right)\left(\dfrac{2}{3}\right) = \dfrac{4}{15}$

$P(X = 12) = \left(\dfrac{2}{5}\right)\left(\dfrac{1}{3}\right) = \dfrac{2}{15}$

The probability distribution of X is

x	5	7	8	12
$P(X = x)$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{2}{5}$	$\frac{2}{15}$

(iii)

$E(X) = 5\left(\dfrac{1}{5}\right) + 7\left(\dfrac{4}{15}\right) + 8\left(\dfrac{2}{5}\right) + 12\left(\dfrac{2}{15}\right) = \dfrac{23}{3}$

$E(X^2) = 5^2\left(\dfrac{1}{5}\right) + 7^2\left(\dfrac{4}{15}\right) + 8^2\left(\dfrac{2}{5}\right) + 12^2\left(\dfrac{2}{15}\right) = \dfrac{943}{15}$

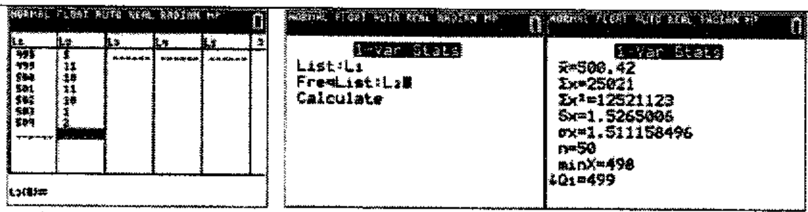
$\mathrm{Var}(X) = \dfrac{943}{15} - \left(\dfrac{23}{3}\right)^2 = \dfrac{184}{45}$

(iv) Since sample size $n = 50$ is large, by Central Limit Theorem,

$\overline{X} \sim \mathrm{N}\left(\frac{23}{3}, \frac{184}{45(50)}\right) = \mathrm{N}\left(\frac{23}{3}, \frac{92}{1125}\right)$ approximately.

$\mathrm{P}\left(7.5 < \overline{X} < 8.5\right) = 0.71821 \approx 0.718$ (3 s.f.)

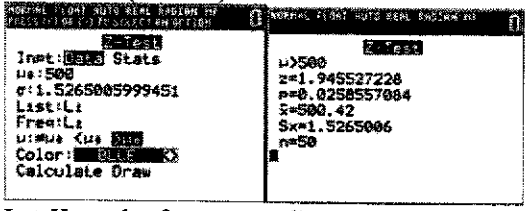
10(i)



Using GC,

$\bar{x} = 500.42$ (exact)

$s = 1.5265$ (to 5 s.f.)



Let X = vol. of green tea dispensed in a bottle and

μ = population mean green tea dispensed in a bottle

$\mathrm{H}_0 : \mu = 500$

$\mathrm{H}_1 : \mu > 500$

Under H_0, since $n = 50$ is large, by Central Limit Theorem,

$$\overline{X} \sim \mathrm{N}\left(500\ ,\ \frac{1.5265^2}{50}\right) \text{ approximately}$$

Test statistic: $z = \dfrac{\bar{x} - 500}{\left(\dfrac{1.5265}{\sqrt{50}}\right)}$

$$= \frac{500.42 - 500}{\left(\dfrac{1.5265}{\sqrt{50}}\right)} = 1.9455 \approx 1.95 \text{ (to 3 s.f)}$$

From GC, p-value $= 0.025856 \approx 0.0259$ (to 3 s.f)

Since p-value $= 0.0259 > 0.02$, we do not reject Ho at 2% level of significance and conclude that there is insufficient evidence that the production manager's suspicion is valid.

The p-value is 0.0259 and it means there is a probability of 0.0259 of observing a test statistic, $z \geq 1.95$, given that the population mean green tea dispensed in a bottle is 500ml.

(ii)

For the production manager's suspicion ($\mu > 500$) to be valid, H_0 is rejected. Hence,

$$p\text{-value} = 0.025856 < \frac{\alpha}{100}$$

$$\therefore 2.59 < \alpha < 100 \text{ (3 s.f)}$$

(iii)

Since the sample size = 50 is large enough, Central Limit Theorem can be applied for sample means to follow a normal distribution approximately.

(b)

$$s^2 = \frac{n}{n-1}\sigma_x^{\ 2} = \frac{50}{49}k^2$$

$H_0 : \mu = 500$

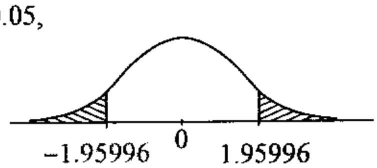
$H_1 : \mu \neq 500$

Under H_0, since $n = 50$ is large, by Central Limit Theorem,

$$\overline{X} \sim N\left(500\ ,\ \frac{\left(\frac{50k^2}{49}\right)}{50}\right) = N\left(500\ ,\ \frac{k^2}{49}\right) \text{ approximately}$$

Test statistic: $z = \dfrac{\bar{x}-500}{\sqrt{\frac{k^2}{49}}} = \dfrac{502-500}{\frac{k}{7}} = \dfrac{14}{k}$

For $\alpha = 0.05$,



If recalibration is done accurately ($\mu = 500$), H_0 is not rejected. Hence,

$-1.95996 < z < 1.95996$

$$-1.95996 < \frac{14}{k} < 1.95996$$

$\frac{k}{14} < -0.51021$ (rejected) or $\frac{k}{14} > 0.51021$

$$k > 7.1430$$

$$\therefore k > 7.14 \text{ (to 3 s.f)}$$

11(i)

Let D be the time in minutes past 7 pm that Benedict finishes his dinner.

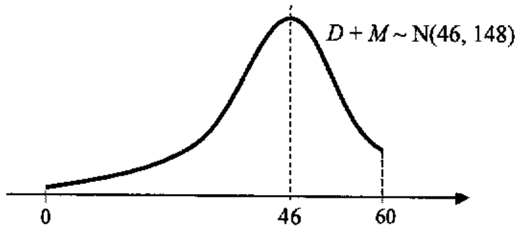
Let M be the time in minutes he spends on social media.

$D \sim N(10, 2^2)$ and $M \sim N(k, 12^2)$,

Then $D + M \sim N(10 + k, 148)$

Given $P(D + M > 60) = 0.125$

$$P\left(Z > \frac{60-(10+k)}{\sqrt{148}}\right) = 0.125$$

	$\frac{50-k}{\sqrt{148}}=1.15035$ $k=50-1.15035\sqrt{148}=36.005\approx 36.0$
(ii)	
(iii)	$X=D+M\sim N(46, 148)$ $Y=D+G\sim N(55, 104)$ Then $Y-X\sim N(9, 252)$ $P(\lvert Y-X\rvert\le 5)=P(-5\le Y-X\le 5)$ $=0.21162\approx 0.212$
(iv)	P(started revision late) $=0.8\ P(D+M>60)+0.2\ P(D+G>60)$ $=0.16232$ P(played online games | started revision late) $=\frac{0.2\ P(D+G>60)}{0.16232}$
	$=0.38438\approx 0.384$
(v)	D, M and G are independent normal variables.