ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 1

MATHEMATICS 8865/01

Paper 1

16 August 2017

3 hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2017

MATHEMATICS 8865
Higher 1
Paper 1

/ 100

Index No: ☐☐☐☐ Form Class: ____________

Name: ____________________________

Calculator model: ____________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/4
2	/4
3	/4
4	/7
5	/8
6	/13
7	/5
8	/6
9	/6
10	/9
11	/10
12	/12
13	/12

Summary of Areas for Improvement			
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Presentation (P)

Section A: Pure Mathematics [40 marks]

1 A company with businesses in Canada, the United States and Mexico makes calls to these countries on a regular basis. Calls made to Canada, the United States and Mexico through the telco Singcall are charged at 28, 30 and 84 cents per minute respectively.
With Singcall, the bill for all the calls made to these three countries during a particular week was \$90. It was also recorded that the calls made to Mexico was twice as long as those to the United States, and that the total duration of calls made to Canada and Mexico was 120 minutes.
For calls made to each of the three countries, another telco, Sunhub, offers the first 10 minutes of talktime free. Subsequently, calls made to Canada, the United States and Mexico are charged at 38, 40 and 94 cents per minute respectively. For the same call duration made to each country in that particular week, determine if Sunhub's rates are better for the company. [4]

2 Use a non-calculator method to find $\int_{-1}^{1}\left(\mathrm{e}-\mathrm{e}^{-x}\right)^2\,\mathrm{d}x$ in the form $a+b(\mathrm{e}^2-\mathrm{e}^{-2})$ where a and b are constants. [4]

3 A bank pays interest on a fixed deposit at a rate of r% per annum. With a principal amount of \$$P$ placed in the bank, if interest is compounded m times per year, after t years, the amount in the bank will be \$$A$ where

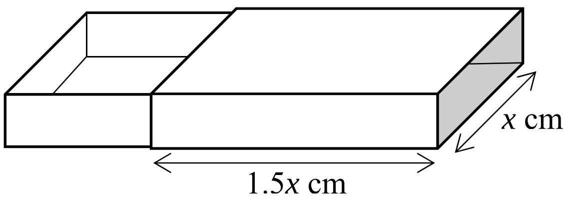
$$A = P(1+\frac{r}{100m})^{tm}.$$

Mr Tan deposits \$7 800 in a fixed deposit that pays 3.2% interest per annum.

(i) Mr Tan intends to use \$10 000 to make the down-payment of a new car after 80 months. If interest is compounded monthly, how much more money does he need in order to do so? Leave your answer to the nearest cent. [2]

(ii) If interest is compounded yearly, find the minimum number of years that it will take him to save \$10 000. [2]

4



A matchbox, shown in the diagram above, consists of an outer cover which is open at opposite ends and an inner box with an open top which slides into the outer cover. The length of the outer cover is $1.5x$ cm, where x cm is the breadth of the cover. Assume that the entire matchbox is made of cardboard with negligible thickness, and that the inner box has the same dimensions as the outer cover. If the volume of the matchbox is 30 cm^3, show that the area A cm^2 of the cardboard used to make both the inner box and outer cover is given by

$$A = 4.5x^2 + \frac{160}{x}.$$

If the amount of cardboard used is minimum, find the exact dimensions of the matchbox, justifying that they give the minimum amount of cardboard used. [7]

5 The curve C has equation $y = x - \ln(x^2 + 1)$.

(i) Use a non-calculator method to find the coordinates of the stationary point on C and determine its nature. [4]

(ii) Find the area bounded by C, the positive x-axis, and the tangent to C at the point where $x = 5$. [4]

6 **(a)** The petrol consumption of a car, in millilitres per kilometre, is advertised to be $P(x) = \dfrac{2500}{x} + \dfrac{2x}{3}$ where x is the speed of the car in km/h.

(i) Find the exact speed of the car when petrol consumption is minimum. [3]

(ii) Sketch the graph of $y = P(x)$ for $x > 0$. [1]

Hence find the range of values of x for which the petrol consumption is at most 90 millilitres per kilometre. [1]

(b) At any time t seconds, a tank is being filled with fuel at a rate given by

$$\frac{dV}{dt} = 0.15\pi\sqrt{\pi t + 1},$$

where V is the volume of fuel in cm^3. Given that the tank is empty initially, find the amount of fuel in the tank after 1 minute, correct to 3 decimal places. [3]

The tank is shaped such that when the petrol in it is at a height of h cm, the volume of petrol, V, is given by

$$V = \pi h^3 .$$

Find the rate of change of h after 1 minute. [5]

Section B: Statistics [60 marks]

7 Durians are sold by weight at a fruit stall. The popular variety, Black Pearl, sells at \$21 per kg. It is found that the probability that a randomly chosen Black Pearl durian is heavier than 2 kg is 0.322, and that the probability that a randomly chosen Black Pearl durian costs less than \$20 is 0.215. Assuming that the weights of Black Pearl durians follow a normal distribution, find the mean and standard deviation of the weight of a randomly chosen Black Pearl durian. [5]

8 In a game of penalty kicks, a player is given three attempts at scoring. Once the player scores, he wins and the game ends. Henry, who has a 0.7 chance of scoring on any penalty kick, plays the game.

Draw a probability tree diagram to illustrate one such game. [1]

Find the probability that in one game, Henry

(a) scores on the second attempt, [1]

(b) made two attempts, given that he wins. [2]

In 3 such games, find the probability that Henry scores on the first attempt in exactly one game, and on the second attempt in exactly one game. [2]

9 A school is asked to send a delegation of 7 students to attend the opening ceremony of the Asian Youth Games. They are chosen from 8 swimmers, 5 basketball players and 5 tennis players where no student plays more than one game.
How many different delegations can be formed? [1]
One of the swimmers is the brother of a basketball player. How many different delegations can be formed which include exactly one of the two brothers? [2]
Find the probability that the delegation consists of at least 2 students from each sport. [3]

10 In a wet market, tuna steak and leather jacket steak are sold by mass. The mass, in kg, of tuna steak and leather jacket steak follow independent normal distributions with means and standard deviations as shown in the table below.

	Mean mass	Standard deviation
Tuna steak	1.2	0.3
Leather jacket steak	1.6	0.2

(i) Find the probability that the average mass of one randomly chosen leather jacket steak and two randomly chosen tuna steaks is less than 1.5 kg. [3]

Tuna steaks cost \$17 per kg and leather jacket steaks \$10 per kg.

(ii) Find the probability that a randomly chosen tuna steak costs more than \$22.50 and a randomly chosen leather jacket steak costs more than \$16.50. [2]

(iii) Find the probability that the total cost of a randomly chosen tuna steak and a randomly chosen leather jacket steak is more than \$39. [3]

(iv) Explain why your answer to **(ii)** is smaller than your answer to **(iii)**. [1]

11 A biased cube with exactly one face painted red is thrown n times. Denoting the number of times the red face appears by X, it is found that $\mathrm{E}(X)=\dfrac{40}{7}$ and $\mathrm{Var}(X)=\dfrac{200}{49}$.
Find the value of n and hence find the probability that more than one quarter of the throws showed the red face. [5]
Each person in a large group of N people is asked to throw the same cube n times. Using a suitable approximation, determine the least value of N so that there is a probability of more than 0.9 that the mean number of red faces obtained per person is less than 6. [5]

12 Seven primary school boys took the standing board jump test. The weight, w kg, of each boy and the distance he jumped on the test, x cm, are given in the table below.

w (kg)	42	30	34	37	40	55	45
x (cm)	138	157	158	152	148	126	136

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the product moment correlation coefficient and comment on its value in the context of the question. [2]

(iii) Find the equation of the least squares regression line of x on w in the form $x = a + bw$, leaving the values of a and b correct to 5 significant figures.
Give an interpretation of the value of b in the context of the question. [2]

(iv) Use the equation of your least squares regression line to calculate an estimate for the standing board jump distance of a boy who weighs

(a) 35 kg,

(b) 15 kg.

Comment on the reliability of your answers. [3]

(v) Aaron also took the test, but it was found that his standing broad jump result was not recorded. After including his weight and the distance he jumped on the test, a new least squares regression line of x on w is calculated to be $x = 202.98 - 1.4249w$.
Given that Aaron weighs 39 kg, find the distance he jumped on the test. [3]

13 **(a)** A manager claims that a cup of coffee brewed by a particular barista contains at least 35 ml of coffee on average. A random sample of 80 cups of coffee brewed by the barista is examined and the quantity x ml of espresso coffee in each cup is measured. The results are summarized by $\sum(x-35) = -40$ and $\sum(x-35)^2 = 950$.

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Suggest a reason why, in this context, the given data is summarised in terms of $(x-35)$ rather than x. [1]

(iii) Test at the 10% significance level whether the manager's claim is valid. [5]

(b) A product designer claims that a new coffee machine brews coffee that contains 35 ml of coffee on average. The variance of the quantity of coffee in each cup is known to be 10.1 ml^2. A random sample of 80 cups of coffee made by the machine is measured. A test at the 10% significance level revealed that the product designer's claim that each cup of coffee is 35 ml on average is valid.
Find the range of values of the mean quantity of coffee in this sample, giving your answer correct to 3 decimal places. [4]

2017 H1 Prelim Solutions

1	Let the time he spend making calls within Canada, the Unites States and Mexico be *x*, *y* and *z* (in min). $0.28x+0.30y+0.84z=\$90$ $2y=z$ $x+z=120$ $x=40.56,\ \ y=39.71,\ \ z=79.44$ Total bill by Sunhub $=0.38(x-10)+0.4(y-10)+0.94(z-10)$ $=88.77$ Sunhub is cheaper.
2	$\int_{-1}^{1}\left(e-e^{-x}\right)^2\mathrm{d}x=\int_{-1}^{1}e^2-2e^{1-x}+e^{-2x}\mathrm{d}x$ $=\left[e^2x+2e^{1-x}-\frac{1}{2}e^{-2x}\right]_{-1}^{1}$ $=\left(e^2+2-\frac{1}{2}e^{-2}\right)-\left(-e^2+2e^2-\frac{1}{2}e^2\right)$ $=2+\frac{1}{2}\left(e^2-e^{-2}\right)$
3(i)	$A=7800\left(1+\frac{3.2}{100(12)}\right)^{80}=9652.077\approx 9652.08$ 10 000 – 9652.08 = 347.92
(ii)	$10000=7800(1+\frac{3.2}{100})^t$ $\frac{50}{39}=(\frac{129}{125})^t$ $t=\frac{\ln 1.282}{\ln 1.032}=7.888\approx 8$ It will take 8 years.
4	Let *h* be height of matchbox. $V=x(1.5x)h=30\Rightarrow h=\frac{30}{1.5x^2}=\frac{20}{x^2}$ $A=3xy+2xh+4yh$ $\quad=3x(1.5x)+2xh+4(1.5x)h$ $\quad=4.5x^2+8xh$ $\quad=4.5x^2+8x(\frac{20}{x^2})$ $\quad=4.5x^2+\frac{160}{x}$ $\frac{\mathrm{d}A}{\mathrm{d}x}=9x-\frac{160}{x^2}=0$ $x^3=\frac{160}{9}\Rightarrow x=\sqrt[3]{\frac{160}{9}}\approx 2.6099$ $y=1.5\times\sqrt[3]{\frac{160}{9}}=\sqrt[3]{\frac{27}{8}}\times\sqrt[3]{\frac{160}{9}}=\sqrt[3]{60}$ 

$$h=\frac{20}{\left(\sqrt[3]{\frac{160}{9}}\right)^2}=20\left(\frac{9}{160}\right)^{\frac{2}{3}}$$

$$\frac{d^2A}{dx^2}=9+\frac{320}{x^3}>0 \text{ for all } x \therefore \text{Min } A$$

x	2.4	2.61	2.8
$\frac{dA}{dx}$	-6.1778	0	4.7918

5(i)

$$\frac{d}{dx}\left[x-\ln(x^2+1)\right]=1-\frac{2x}{x^2+1}$$

$$y=x-\ln(x^2+1)$$

$$\frac{dy}{dx}=0\Rightarrow 1-\frac{2x}{x^2+1}=0$$

$$\frac{2x}{x^2+1}=1$$

$$x^2-2x+1=0$$

$$(x-1)^2=0$$

$$\therefore x=1,\ y=1-\ln 2$$

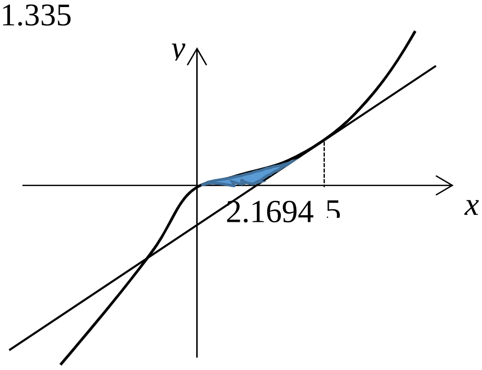
x	1^- 0.5(0.9)	1	1^+ 1.5(1.1)
$\frac{dy}{dx}$	0.2(0.0055)	0	0.0769(0.0045)

$\therefore$(1, 1 – ln2) is a stationary point of inflexion.

(ii)

Equation of tangent at x = 5 is

$y=0.61538x-1.335$



when y = 0, x = 2.1694

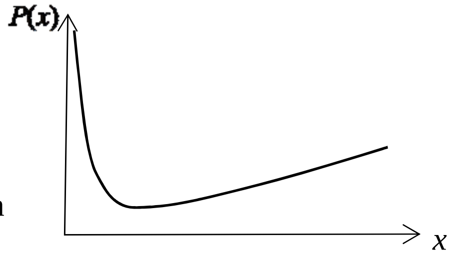
when x = 5, y = 1.7419

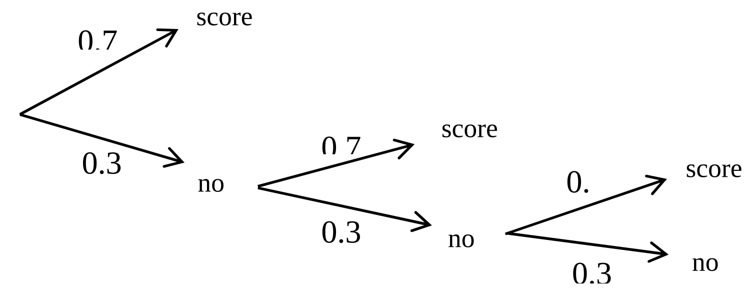
$\int_0^5 y\ dx-$ Area of triangle

$=\int_0^5[x-\ln(x^2+1)]\ dx-0.5(5-2.1694)(1.7419)$

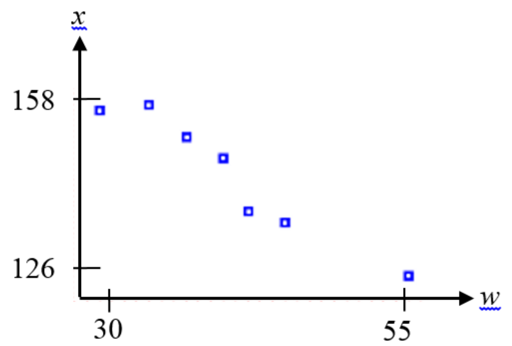
$=3.4627-2.4653$

$=0.99739\approx 0.997$ (3 s.f.)

6(a)	$P'(x) = -\frac{2500}{x^2} + \frac{2}{3} = 0$ $x^2 = 3750$ $\therefore x = 25\sqrt{6}$  Min point (61.237, 81.65) Optimal speed = 81.7 km/h When $P(x) \le 90$, $39.1 \le x \le 95.9$.
6(b)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.15\pi\sqrt{\pi t + 1}$ $V = \int 0.15\pi\sqrt{\pi t+1}\ \mathrm{d}t = \frac{0.15\pi}{1.5\pi}\left(\pi t+1\right)^{1.5} + C = 0.1\left(\pi t+1\right)^{1.5} + C$ When $t = 0$, $V = 0$ $\therefore C = -0.1$ $\therefore V = 0.1\left(60\pi+1\right)^{1.5} - 0.1 = 260.755$ OR $V = \int_0^{60} 0.15\pi\sqrt{\pi t+1}\ \mathrm{d}t = 260.755$ fr GC $\therefore$ After 1 min, amount of fuel in the tank is 260.755 litres. Vol of fuel = 260.755 $\Rightarrow \pi h^3 = 260.755$ $\therefore h = 4.36208$ $V = \pi(h)^2 h = \pi h^3$ $\frac{\mathrm{d}V}{\mathrm{d}h} = 3\pi h^2$ $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ $= \frac{1}{3\pi h^2} \times 0.15\pi\sqrt{\pi(60)+1}$ when $t = 60$ $= 0.036173 \approx 0.0362$ cm/s
7	Let M be the mass of a randomly chosen Black Pearl durian, and S be the cost of a randomly chosen Black Pearl durian. $S = 21M$ $M \sim N(\mu,\ \sigma^2)$ $S \sim N\left(21\mu,\ (21\sigma)^2\right)$ $P(M > 2) = 0.322$ $P(M < 2) = 0.678$ $P(Z < \frac{2-\mu}{\sigma}) = 0.678$ $\frac{2-\mu}{\sigma} = 0.46211$

	$P(S<20)=0.215$ $P(Z<\frac{20-21\mu}{21\sigma})=0.215$ $\frac{20-21\mu}{21\sigma}=-0.78919$ Therefore we have two simultaneous equations: $\mu+0.46211\sigma=2$ ----(1) $21\mu-16.573\sigma=20$ -----(2) Solving (1) and (2) $\mu=1.6131=1.61$ (3sf) $\sigma=0.83722=0.837$ (3sf)
8	
(a)	0.3 × 0.7
(b)	$\frac{0.3\times0.7}{1-\text{P(not scoring)}}=\frac{0.3\times0.7}{1-0.3^3}$ or $\frac{0.3\times0.7}{0.3+0.3\times0.7+0.3^2\times0.7}=\frac{0.21}{0.973}=\frac{30}{139}$
(c)	$3!\times0.7\times0.21\times(1-0.7-0.21)=0.07938\approx0.0794$
9	${}^{18}C_7=31\ 824$ ${}^{2}C_1\,{}^{16}C_6=2\times8\ 008=16\ 016$ Probability = P(3 S + 2 B + 2 T) + P(2 S + 3 B + 2 T) + P(2 S + 2 B + 3 T) $=\frac{{}^8C_3\,{}^5C_2\,{}^5C_2+{}^8C_2\,{}^5C_3\,{}^5C_2+{}^8C_2\,{}^5C_2\,{}^5C_3}{{}^{18}C_7}=\frac{11200}{31\ 824}=\frac{700}{1989}\approx0.352$ (3 s.f.)
10(i)	Let *L* be the mass of a randomly chosen leather jacket steak. Let *T* be the mass of a randomly chosen tuna steak. Let *A* be the average mass of 1 randomly chosen leather jacket and 2 randomly chosen tuna steak. $A=\frac{L+T_1+T_2}{3}$ $\text{E}(A)=E\left(\frac{L+T_1+T_2}{3}\right)$ $=\frac{1}{3}[\text{E}(L)+2\text{E}(T)]=\frac{1}{3}(1.6+2\times1.2)=\frac{4}{3}$

	$\text{Var}(A) = \text{Var}\left(\dfrac{L_1 + T_1 + T_2}{3}\right)$ $= \dfrac{1}{3^2}\left[\text{Var}(L) + 2\text{Var}(T)\right]$ $= \dfrac{1}{9}\left(0.2^2 + 2\times 0.3^2\right) = \dfrac{11}{450}$ $A \sim N\left(\dfrac{4}{3},\ \dfrac{11}{450}\right)$ $P(A < 1.5) = 0.857$ (3sf)
(ii)	Let X be the cost of a randomly chosen tuna steak, and Y be the cost of a randomly chosen leather jacket steak. $X = 17T$ $Y = 10L$ $X \sim N\left(17\times 1.2,\ (17\times 0.3)^2\right) = \left(20.4,\ 26.01\right)$ $Y \sim N\left(10\times 1.6,\ (10\times 0.2)^2\right) = \left(16, 4\right)$ $P(X > 22.50)$□$P(Y > 16.50)$ = (0.34026)(0.40130) = 0.13654 = 0.137 (3sf)
(iii)	$X + Y \sim N\left(20.4 + 16,\ 26.01 + 4\right) = \left(36.4,\ 30.01\right)$ $P(X + Y > 39) = 0.31753 = 0.318$ (3sf)
(iv)	because the event in (ii) is a subset of the event in (iii), <u>i.e., a tuna steak cost $24 and a leather jacket steak cost $16 is not included in (ii) but is included in (iii).</u>
11	Let p be the probability of obtaining a red in a throw. Then $X \sim \text{B}(n, p)$ $\text{E}(X) = np = \dfrac{40}{7}$ and $\text{Var}(X) = np(1-p) = \dfrac{200}{49}$ $\Rightarrow \dfrac{40}{7}(1-p) = \dfrac{200}{49}$ $1 - p = \dfrac{5}{7} \Rightarrow p = \dfrac{2}{7}$ $\therefore np = \dfrac{40}{7}$ $n = \dfrac{40}{7}\times\dfrac{7}{2} = 20$ $P(X > 5) = 1 - P(X \le 5) \approx 0.528$ (3 s.f.)
	Let $\overline{X}$ denotes the mean number of reds obtained per person. $\overline{X} = \dfrac{X_1 + X_2 + ... + X_N}{N}$

	Since N is large, $\overline{X} \sim N\left(\frac{40}{7}, \frac{200}{49N}\right)$ approximately by Central limit Theorem. Let $Z = \frac{\overline{X} - \frac{40}{7}}{\sqrt{\frac{200}{49N}}} \sim N(0, 1)$ $\mathrm{P}(\overline{X} < 6) > 0.9$ $\mathrm{P}\left(Z \le \frac{6 - \frac{40}{7}}{\sqrt{\frac{200}{49N}}}\right) > 0.9$ Method 1: $\frac{6 - \frac{40}{7}}{\sqrt{\frac{200}{49N}}} > 1.28155$ $N > \frac{200}{49}\left(\frac{1.28155}{6 - \frac{40}{7}}\right)^2$ $N > 82.11$ The least value of N is 83. **OR**
	Method 2: From GC: $N = 82, \quad \mathrm{P}(\overline{X} < 6) = 0.8998$ $N = 83, \quad \mathrm{P}(\overline{X} < 6) = 0.9012$ The least value of N is 83
12(i)	
(ii)	$r = -0.962$ (3s.f.) There is a strong negative linear relationship between the weight of a boy and the distance of the standing board jump that he can make, i.e., as the weight of a boy increases, the distance of his standing board jump decreases.
(iii)	From G.C., $x = 202.23 - 1.4156w$ (5 s.f.) Meaning of b:

	$b=-1.4156$ means that 1 unit (i.e., kg) increase in the weight of a boy (w) will mean a decrease of 1.4156 units (i.e., cm) in the distance of his standing board jump (x).
(iv)	When $w=35$ kg, $x=153$cm (3sf) Since the value of r is close to 1 and w is within the range of data, i.e., $30 \le w \le 55$, the estimate is reliable. When $w=15$ kg, $x=181$cm (3sf) Since w is outside the given range of data, i.e., $30 \le w \le 55$, the linear relation may no longer hold, therefore the estimate is unreliable.
(v)	Let the distance of Aaron's standing board jump be d. $\bar{w}=\frac{322}{8}=40.25$ Since $(\bar{w},\bar{x})$ lies on the regression line, sub $\bar{w}$ to get $\bar{x}$ $\bar{x}=202.98-1.4249(40.25)$ $\bar{x}=145.6$ $\bar{x}=\frac{1015+d}{8}$ $145.6=\frac{1015+d}{8}$ $d=150$ cm (3s.f.)
13 (a) (i)	Unbiased estimate of the population mean, $\bar{x}=35+\frac{\sum(x-35)}{80}=35+\frac{-40}{80}=34.5$ Unbiased estimate of the population variance, $s^2=\frac{1}{80-1}\left[\sum(x-35)^2-\frac{\left(\sum(x-35)\right)^2}{80}\right]$ $=\frac{1}{79}\left[950-\frac{(-40)^2}{80}\right]=11.772=11.8$ (3sf)
(ii)	"Keeping the recorded values small since they are around 35 ml" or "Giving an indication of the variations around the hypothesized mean of 35 ml".
(iii)	To test H_0: $\mu=35$ against H_1: $\mu<35$ 1-tail Z-test at the 10% significance level. Since $n=80$ is large, by Central Limit Theorem, Under Ho, $\bar{X}\sim N\left(35,\frac{11.772}{80}\right)$ approximately. From GC, p-value = 0.096213

	Since p-value < 0.10, reject H_0 We conclude that there is <u>sufficient evidence at the 10% level of significance</u> that average cup of single-shot expresso coffee is less than 35 ml, the manager's claim is not valid.
(b)	To test H_0: $\mu = 35$ against H_1: $\mu \neq 35$ 2-tail Z-test at the 10% significance level. Since n = 80 is large, by Central Limit Theorem, Under Ho, $\bar{X} \sim N\left(35, \frac{10.1}{80}\right)$ approximately. Since the product designer's claim is valid, H_0 is not rejected, $-1.6449 < \dfrac{\bar{x}-35}{\sqrt{\frac{10.1}{80}}} < 1.6449$ Range of sample mean $34.416 \leq \bar{x} \leq 35.584$ (3 dp)