2017 VJC Prelim Paper 1

1. Without using a calculator, solve the inequality $\frac{6x-13}{x^2-4} \geqslant 1$. [4]

2. The Singapore Utility Board charges the residential users based on the usage for electricity, water and gas. Electricity and gas are charged by kilowatt hour (kWh) used while water usage is charged by cubic meters (CuM). Below are the monthly utility statements for Mr Pandy from May to August 2017.

SP Utility Bill (May 2017)

Mr Pandy
Blk 20 Marine ...

Current month charges		
Electricity	514 kWh	***.**
Water	18.8 CuM	**.**
Gas	134 kWh	**.**
Total		**$155.54**

SP Utility Bill (June 2017)

Mr Pandy
Blk 20 Marine ...

Current month charges		
Electricity	309 kWh	***.**
Water	11.3 CuM	**.**
Gas	89 kWh	**.**
Total		**$ 94.99**

SP Utility Bill (July 2017)

Mr Pandy
Blk 20 Marine ...

Current month charges		
Electricity	639 kWh	***.**
Water	21.7 CuM	**.**
Gas	108 kWh	**.**
Total		**$ 208.40**

SP Utility Bill (August 2017)

Mr Pandy
Blk 20 Marine ...

Current month charges		
Electricity	555 kWh	***.**
Water	??? CuM	**.**
Gas	128 kWh	**.**
Total		**$ 184.84**

It is known that the unit costs for electricity, water and gas remain unchanged for May and June. The unit cost for electricity was increased by 20% with effect from July 2017, while the unit cost for gas and water remain unchanged.

(i) Calculate the unit cost for electricity, water and gas for June 2017, giving your answers correct to the nearest 4 decimal places. [3]

(ii) The water usage for August 2017 was not clearly printed on the bill. Using your answers in part (i), calculate the water usage for August 2017 to the nearest CuM. [2]

3. It is given that

$$f(x)=\begin{cases}(x-2)^2-1, & \text{for } 0< x\leqslant 3,\\ x-3, & \text{for } 3<x\leqslant 6,\end{cases}$$

and that $f(x)=f(x+6)$ for all real values of x.

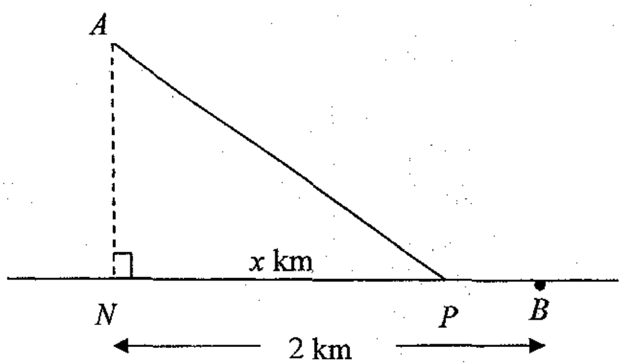
(i) Sketch the graph of $y=f(x)$ for $0<x\leqslant 10$. [3]

(ii) On a separate diagram, sketch the graph of $y = 1 + \mathrm{f}\left(\frac{1}{2}x\right)$ for $0 < x \leqslant 10$. [2]

4. The curve C has equation $(y+4)^2 - (x+3)^2 = 4$. Sketch C, giving the coordinates of any turning points and the equations of any asymptotes. [3]

Hence find the set of values of m such that the straight line with gradient m that passes through the point $(-3,-4)$ intersects C at least once. [2]

5.



Alvin is at the point A on a floating platform in the sea. He wants to reach point B located on a straight stretch of beach. N is the point on the beach nearest to A and NB = 2 km. Alvin swims at a constant speed in a straight line from A to P and then runs at a constant speed from P to B, where P is a point on the straight stretch of beach from N to B. $NP = x$ km and T minutes is the time taken for Alvin to complete the journey.

T and x satisfy the differential equation

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{5\sqrt{5}x}{\sqrt{x^2+4}} - 5.$$

(i) Solve the differential equation. [3]

(ii) Given that the minimum time taken for Alvin to complete this journey is 30 minutes, find T in terms of x. [3]

(iii) Using your answer in part **(ii)**, find the longest time taken by Alvin to complete the journey. [2]

6. The function h is defined by

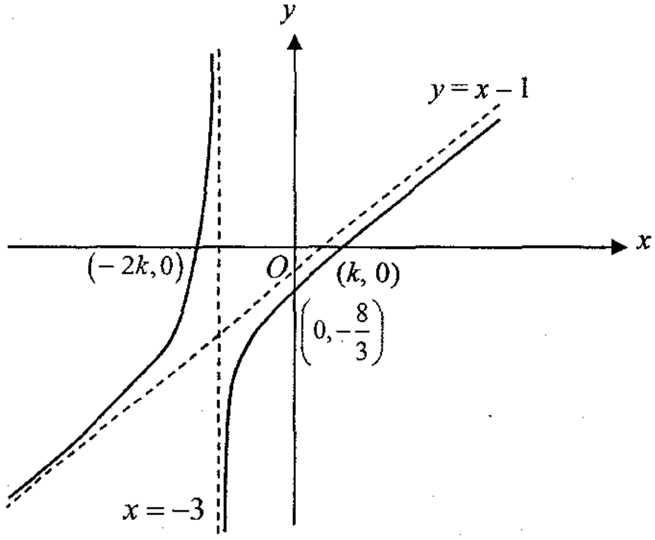
$$\mathrm{h}: x \mapsto \mathrm{e}^{x-2} - 1, \qquad \text{for } x \in \mathbb{R}.$$

(i) Find $\mathrm{h}^{-1}(x)$ and state the domain of h^{-1}. [3]

(ii) Sketch, on the same diagram, the graphs of $y = \mathrm{h}(x)$ and $y = \mathrm{h}^{-1}(x)$, giving the equations of any asymptotes and the exact coordinates of any points where the curves cross the x- and y-axes. [3]

(iii) Find the set of values of x such that $h^{-1}(x) > h(x)$. [2]

7. The diagram below shows the curve with equation $y = f(x)$. The curve crosses the x- and y-axes at the points $(-2k, 0)$, $(k, 0)$ and $\left(0, -\frac{8}{3}\right)$ where $k > 0$. The curve has an oblique asymptote $y = x - 1$ and vertical asymptote $x = -3$.



(i) On separate diagram, sketch the graph of $y = \frac{1}{f(x)}$, including the coordinates of the points where the graph crosses the axes and the equations of any asymptotes. [3]

(ii) It is further known that $f(x) = \frac{x^2 + ax + b}{x + c}$ where a, b and c are constants. Find the values of a, b and c. [4]

8. It is given that $\sum_{r=1}^{n} \frac{r^2}{3^r} = \frac{3}{2} - \frac{n^2 + 3n + 3}{2(3^n)}$.

(i) Find $\sum_{r=1}^{\infty} \frac{r^2 + (-1)^r}{3^r}$. [3]

(ii) Show that $\sum_{r=4}^{n} \frac{(r-2)^2}{3^{r-2}} = \frac{p}{q} - \frac{an^2 - an + a}{2(3^{n-2})}$, where a, p and q are integers to be determined. [5]

9. **(a)** Given that $\int_0^a x \sin x \, dx = 0.5$, where $0 < a < 2$, find an equation that is satisfied by a and use it to find the value of a. [5]

 [Turn Over

(b) Write down a definite integral that represents the area of the region bounded by the curve with equation $y=\frac{\sqrt{x}}{3-\sqrt{x}}$, the two axes and the line $x = 4$.

Use the substitution $u = 3-\sqrt{x}$ to find the exact value of the area. [6]

10. It is given that z_1, z_2 and z_3 are the roots of the equation

$$2z^3+pz^2+qz-4=0$$

such that $\arg z_1 < \arg z_2 < \arg z_3$ and $z_1 = 1-\mathrm{i}\sqrt{3}$. Find the values of the real numbers p and q. [3]

(i) Without using the calculator, find z_2 and z_3. [3]

In an Argand diagram, points P, Q and R represent the complex numbers z_1, $w=\sqrt{2}+\mathrm{i}\sqrt{2}$ and z_1+w respectively and O is the origin.

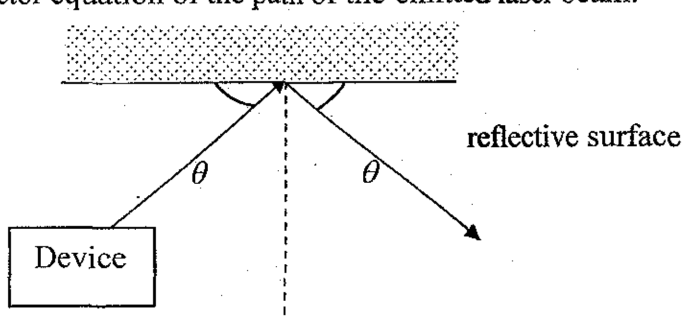
(ii) Express each of z_1 and w in the form $r\,\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leqslant\pi$. Give r and θ in exact form. [2]

(iii) Indicate P, Q and R on the Argand diagram and identify the type of the quadrilateral $OPRQ$. [3]

(iv) Find the exact value of $\arg\left(z_1{}^4 w^*\right)$. [3]

11. Physicists are investigating the reflective property of a particular reflective surface. The diagram below shows the set-up of a particular experiment, where a laser emitting device was placed at the point with coordinates (1, 2, 3). A laser beam was emitted in the direction parallel to $\mathbf{i}+\mathbf{k}$. The path of the emitted laser beam and its reflected path make the same angle θ with the reflective surface. The plane containing these two paths is perpendicular to the reflective surface.

Write down the vector equation of the path of the emitted laser beam. [1]



It is known that the reflective surface has equation $x+y+z=4$.

(i) Find θ. [3]

(ii) Show that the laser beam meets the reflective surface at the point $(0, 2, 2)$. [3]

(iii) Find the vector equation of the path of the reflected laser beam. [5]

12. A curve C has equation $y = \ln(x^2),\ x \neq 0$.

(i) Sketch C. [2]

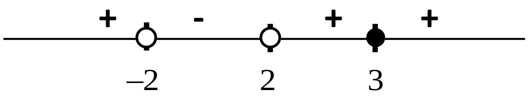
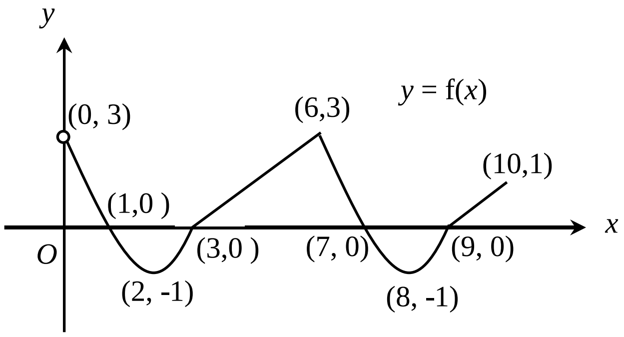
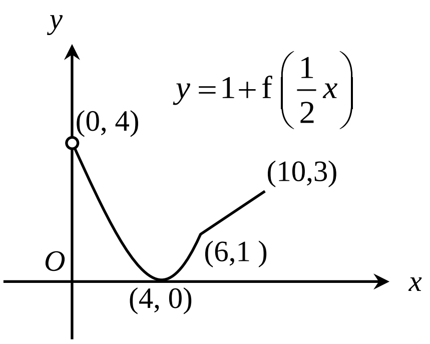
(ii) The part of C from the point $A\,(e^{-1}, -2)$ to the point $B(e^{\frac{k}{2}}, k)$, $k > 4$, and the line $y = -2$ is rotated about the y-axis to form the curved surface and the circular base of an open vase. Find the volume of the vase, giving your answer in terms of π and k, in exact form. [2]

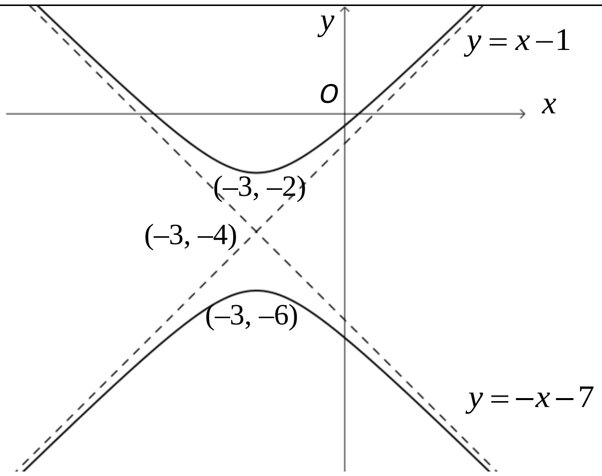
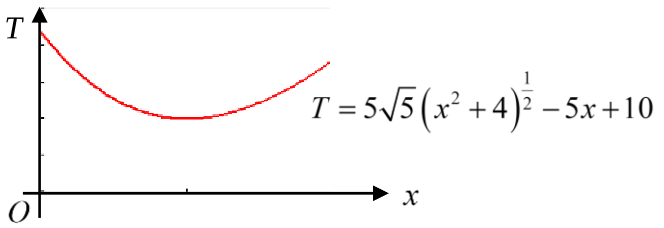
(iii) Water flows into the vase at a constant rate of 2 cm^3 per second. By first showing that the volume of water in the vase is given by $V = \pi\left(x^2 - e^{-2}\right)$ when the radius of the water surface is x cm, find the rate at which x is increasing, giving your answer in terms of x. [4]

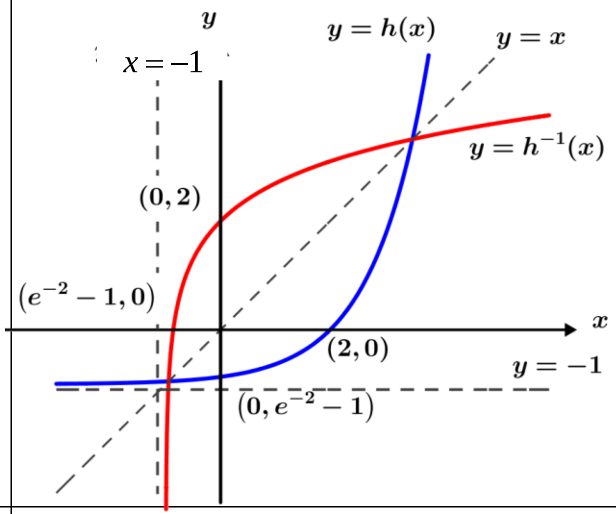
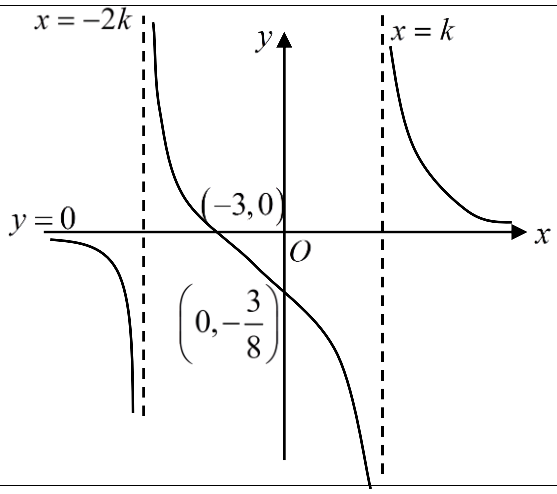
(iv) An insect lands on the inner surface of the vase at the point (e, 2) just as the in-coming water reaches the depth of 2 cm. It immediately starts to crawl along C such that the x-coordinate of its location increases by a constant value of 0.03 cm per second. Find the coordinates of the point on C at which the insect will first come into contact with water. [5]

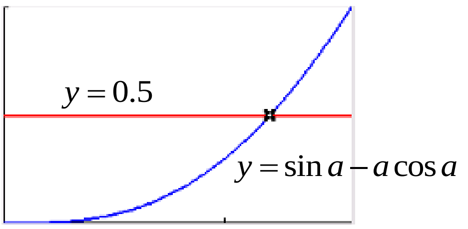
[Turn Over

VJC H2 Maths Preliminary Examination P1 2017 Solutions

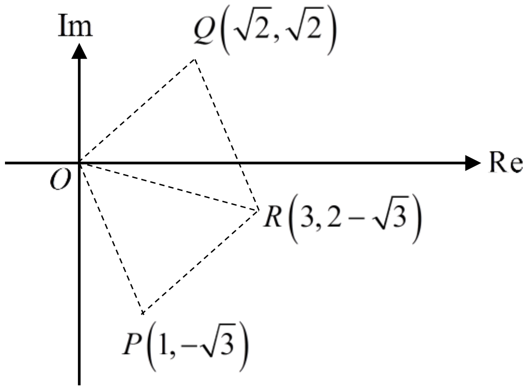
Q	Solution	Comments
1	$\dfrac{6x-13}{x^2-4} \geqslant 1$ $\dfrac{6x-13-x^2+4}{x^2-4} \geqslant 0$ $\dfrac{(x-3)^2}{(x+2)(x-2)} \leqslant 0$  $\therefore -2 < x < 2$ or $x = 3$	
2i	Let $\$E$, $\$W$ and $\$G$ be the unit cost of electricity, water and gas, respectively. $514E + 18.8W + 134G = 155.54$ $309E + 11.3W + 89G = 94.99$ $639(1.2)E + 21.7W + 108G = 208.40$ Using G.C, $E = 0.2137,\quad W = 1.1749,\quad G = 0.1761.$	
2ii	Let w be the water usage for August 2017 $(0.2137)(1.2)(555) + 1.1749w + 0.1761(128) = 184.84$ $w = 17$	
3i		
3ii		

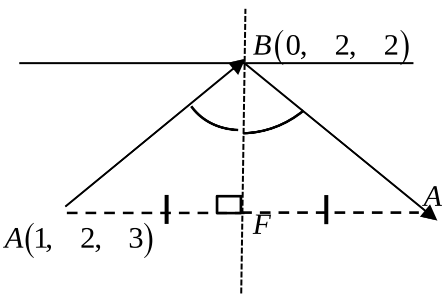
Q	Solution	Comments
4	 Intersect at least once: $\{m\in\mathbb{R}:\ \ m<-1\ \text{ or }\ m>1\}$	
5i	$\frac{\mathrm{d}T}{\mathrm{d}x}=\frac{5\sqrt{5}x}{\sqrt{x^2+4}}-5$ $T=\int\left(\frac{5\sqrt{5}x}{\sqrt{x^2+4}}-5\right)\mathrm{d}x$ $=\frac{5\sqrt{5}}{2}\int 2x\left(x^2+4\right)^{-\frac{1}{2}}\mathrm{d}x-\int 5\,\mathrm{d}x$ $=\frac{5\sqrt{5}}{2}\frac{\left(x^2+4\right)^{\frac{1}{2}}}{\frac{1}{2}}-5x+C$ $=5\sqrt{5}\left(x^2+4\right)^{\frac{1}{2}}-5x+C \qquad ---(1)$	
5ii	When t = 30, $\frac{\mathrm{d}T}{\mathrm{d}x}=0:\ \frac{5\sqrt{5}x}{\sqrt{x^2+4}}=5\Rightarrow\sqrt{5}x=\sqrt{x^2+4}$ $5x^2=x^2+4\Rightarrow x=\pm1$ Since $x>0,\ x=1$ Substitute $x=1$ and $T=30$ into equation (1) $30=5\sqrt{5}(1+4)^{\frac{1}{2}}-5+C\Rightarrow C=10$ $T=5\sqrt{5}\left(x^2+4\right)^{\frac{1}{2}}-5x+10$	
5iii	 When x =0, $T=32.361$. When x =2, $T=31.623$ Longest time taken by Alvin is 32.4 mins.	

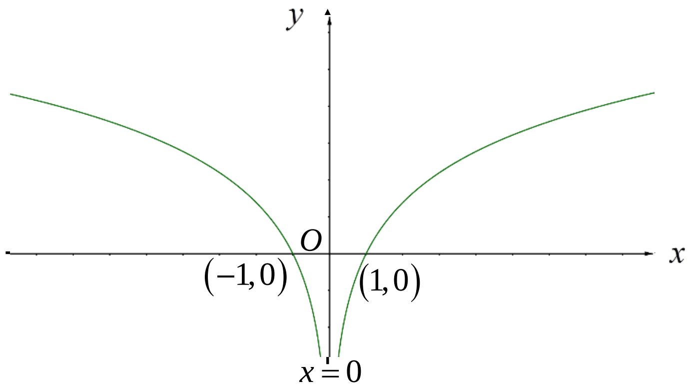
Q	Solution	Comments
6i	$y = e^{x-2} - 1$ $x = \ln(y+1) + 2$ $h^{-1}(x) = \ln(x+1) + 2$ Domain of h^{-1} = range of $h = (-1, \infty)$	
6ii		
6iii	Using G.C, $y = h^{-1}(x)$ and $y = h(x)$ intersects at $x = -0.94753$ and $x = 3.50524$ Set of values of x = $\{ x \in \mathbb{R} : -0.948 < x < 3.51 \}$.	
7i		
ii	Since $x = -3$ is the vertical asymptote, $c = 3$ Given that $y = x - 1$ is an oblique asymptote, $f(x) = x - 1 + \frac{A}{x+3}$ $= \frac{(x-1)(x+3) + A}{x+3} = \frac{x^2 + 2x - 3 + A}{x+3}$ By comparing coefficient of x with $\frac{x^2 + ax + b}{x+3}$: $a = 2$	

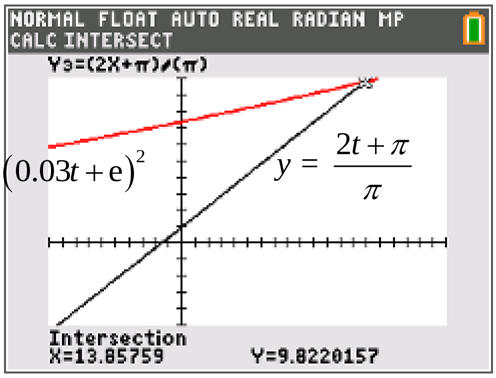
Q	Solution	Comments
	Since $\left(0,-\frac{8}{3}\right)$ is on the curve, $\frac{(0)^2+2(0)+b}{(0)+3}=-\frac{8}{3}$ $b=-8$	
8i	$\sum_{r=1}^{\infty}\frac{r^2+(-1)^r}{3^r}=\sum_{r=1}^{\infty}\frac{r^2}{3^r}+\sum_{r=1}^{\infty}\left(-\frac{1}{3}\right)^r$ $=\frac{3}{2}+\frac{\left(-\frac{1}{3}\right)}{1-\left(-\frac{1}{3}\right)}$ $=\frac{5}{4}$	
8ii	$\sum_{r=4}^{n}\frac{(r-2)^2}{3^{r-2}}=\sum_{r+2=4}^{r+2=n}\frac{r^2}{3^r}$ (replace r with $r+2$) $=\sum_{r=2}^{n-2}\frac{r^2}{3^r}$ $=\sum_{r=1}^{n-2}\frac{r^2}{3^r}-\frac{(1)^2}{3^1}$ $=\frac{3}{2}-\frac{(n-2)^2+3(n-2)+3}{2\left(3^{n-2}\right)}-\frac{1}{3}$ $=\frac{7}{6}-\frac{n^2-4n+4+3n-6+3}{2\left(3^{n-2}\right)}$ $=\frac{7}{6}-\frac{n^2-n+1}{2\left(3^{n-2}\right)}$ $\therefore p=7,\quad q=6,\quad a=1$	
9a	$\int_0^a x\sin x\,dx = 0.5$ $\left[-x\cos x\right]_0^a+\int_0^a\cos x\,dx=0.5$ $\left[-a\cos a+0\right]+\left[\sin x\right]_0^a=0.5$ $-a\cos a+\sin a=0.5$ --- (1)  Using GC, a = 1.20249 =1.20 (3 s.f.)	

Q	Solution	Comments
9b	Area = $\displaystyle\int_0^4 \frac{\sqrt{x}}{3-\sqrt{x}}\ \mathrm{d}x$ Let $u=3-\sqrt{x}$ $\dfrac{\mathrm{d}u}{\mathrm{d}x}=-\dfrac{1}{2\sqrt{x}} \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}u}=-2(3-u)$ When $x=0,\ u=3$ When $x=4,\ u=1$ $\displaystyle\int_0^4 \frac{\sqrt{x}}{3-\sqrt{x}}\ \mathrm{d}x=\int_3^1\left(\frac{3-u}{u}\right)\left[(-2)(3-u)\right]\ \mathrm{d}u$ $\displaystyle=\int_1^3 \frac{2(3-u)^2}{u}\ \mathrm{d}u$ $\displaystyle=2\int_1^3 \frac{9-6u+u^2}{u}\ \mathrm{d}u$ $\displaystyle=2\int_1^3\left(\frac{9}{u}-6+u\right)\ \mathrm{d}u$ $\displaystyle=2\left[9\ln u-6u+\frac{u^2}{2}\right]_1^3$ $\displaystyle=2\left(9\ln 3-18+\frac{9}{2}\right)-2\left(-6+\frac{1}{2}\right)$ $=18\ln 3-16$	
10	Since $1-\sqrt{3}\mathrm{i}$ is a root, $2\left(1-\mathrm{i}\sqrt{3}\right)^3+p\left(1-\mathrm{i}\sqrt{3}\right)^2+q\left(1-\mathrm{i}\sqrt{3}\right)-4=0$ $2(-8)+p\left(-2-2\sqrt{3}\mathrm{i}\right)+q\left(1-\mathrm{i}\sqrt{3}\right)-4=0$ $(-20-2p+q)+\left(-2\sqrt{3}p-\sqrt{3}q\right)\mathrm{i}=0$ Compare real and imaginary parts: $-2p+q=20 \qquad ---(1)$ $-2\sqrt{3}p-\sqrt{3}q=0 \qquad ---(2)$ $\therefore p=-5, \qquad q=10$	
10i	Since $1-\sqrt{3}\mathrm{i}$ is a root, and all coefficients are real $\Rightarrow 1+\sqrt{3}\mathrm{i}$ is also a root. $2z^3-5z^2+10z-4=\left(z-\left(1-\sqrt{3}\mathrm{i}\right)\right)\left(z-\left(1+\sqrt{3}\mathrm{i}\right)\right)(2z+a)$ $=\left(z^2-2z+4\right)(2z+a)$	

Q	Solution	Comments
	By observation: $a=-1$ $\therefore z_2=\frac{1}{2},\qquad z_3=1+\sqrt{3}\mathrm{i}$	
10ii	$\lvert z_1\rvert=\sqrt{1+3}=2 \qquad \lvert w\rvert=\sqrt{2+2}=2$ $\arg z_1=\arg\left(1-\sqrt{3}\mathrm{i}\right)$ $=-\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$ $=-\frac{\pi}{3}$ $\arg w=\arg\left(\sqrt{2}+\mathrm{i}\sqrt{2}\right)$ $=\frac{\pi}{4}$ $\therefore z_1=2\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}, \quad w=2\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$	
10iii	 Quadrilateral $OPRQ$ is a rhombus	
10iv	$4\arg\left(z_1\right)+\arg\left(w^*\right)=4\arg\left(z_1\right)-\arg\left(w\right)$ $=-\frac{4\pi}{3}-\frac{\pi}{4}$ $=-\frac{19\pi}{12}$ $\arg\left(z_1{}^4w^*\right)=-\frac{19\pi}{12}+2\pi$ $=\frac{5\pi}{12}$	
11	$\underset{\sim}{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\1\end{pmatrix}, \qquad \lambda\in\mathbb{R}$	

Q	Solution	Comments
11i	$\cos\alpha = \dfrac{\begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{2}\sqrt{3}}$ $\alpha = 35.3^{o}$ $\theta = 90° - 35.3°$ $= 54.7°$	
11ii	Intersection of light beam with reflective surface: $\left[\begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\1\end{pmatrix}\right].\begin{pmatrix}1\\1\\1\end{pmatrix} = 4$ $6 + 2\lambda = 4$ $\lambda = -1$ Coordinates of point of intersection = $(0,\ \ 2,\ \ 2)$.	
11iii	Let F be the foot of perpendicular from device to normal line and A be the point $(1,\ \ 2,\ \ 3)$:  $\overrightarrow{BF} = \left(\overrightarrow{BA} \cdot \hat{\underset{\sim}{n}}\right)\hat{\underset{\sim}{n}}$ $= \left[\dfrac{\begin{pmatrix}1-0\\2-2\\3-2\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{3}}\right]\dfrac{\begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{3}}$ $= \dfrac{2}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}$ Using Ratio Theorem,	

Q	Solution	Comments
	$\overrightarrow{BF}=\dfrac{\overrightarrow{BA}+\overrightarrow{BA'}}{2}$ $\overrightarrow{BA'}=\dfrac{4}{3}\begin{pmatrix}1\\1\\1\end{pmatrix}-\begin{pmatrix}1\\0\\1\end{pmatrix}=\dfrac{1}{3}\begin{pmatrix}1\\4\\1\end{pmatrix}$ Equation of reflected light path: $r=\begin{pmatrix}0\\2\\2\end{pmatrix}+\alpha\begin{pmatrix}1\\4\\1\end{pmatrix}, \quad \alpha\in\mathbb{R}$	
12i		
ii	Volume of the vase = $\pi\int_{-2}^{k}x^2\,\mathrm{d}y$ $=\pi\int_{-2}^{k}\mathrm{e}^y\,\mathrm{d}y$ $=\pi\left[\mathrm{e}^y\right]_{-2}^{k}$ $=\pi\left[\mathrm{e}^k-\mathrm{e}^{-2}\right]$	
iii	Volume of water , $V=\pi\int_{-2}^{y}\mathrm{e}^y\,\mathrm{d}y$ $=\pi\left[\mathrm{e}^y-\mathrm{e}^{-2}\right]$ $=\pi\left[\mathrm{e}^{\ln x^2}-\mathrm{e}^{-2}\right]$ $=\pi\left[x^2-\mathrm{e}^{-2}\right]$ Given $\dfrac{\mathrm{d}V}{\mathrm{d}t}=2$, $\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}t}\times\dfrac{\mathrm{d}x}{\mathrm{d}V}$ $=2\times\dfrac{1}{2\pi x}$ $=\dfrac{1}{\pi x}$	

Q	Solution	Comments
	Hence the rate at which the radius of the water surface is increasing is $\frac{1}{\pi x}$ cm per second.	
iv	For the insect, $\frac{dx}{dt}=0.03$. t seconds later, the location of the insect is at $x=0.03t+e$ For the movement of the water, $\frac{dx}{dt}=\frac{1}{\pi x}$ $\int \pi x\ dx=\int 1\ dt$ $\frac{\pi x^2}{2}=t+C$ When $t=0$, $x=1$ $C=\frac{\pi}{2}$ $\frac{\pi x^2}{2}=t+\frac{\pi}{2}$ When the insect first comes into contact with water, $\frac{\pi(0.03t+e)^2}{2}-\frac{\pi}{2}=t$ $\pi(0.03t+e)^2-\pi=2t$ $(0.03t+e)^2=\frac{2t+\pi}{\pi}$  Using GC, $t=13.858$ $x=0.03(13.858)+e=3.1340$ $y=\ln(3.1340)^2=2.28$ Hence coordinates of the point = $(3.13, 2.28)$	

2017 VJC Prelim Paper 2

Section A: Pure Mathematic [40 marks]

1. A curve C is defined by the parametric equations

$$x = \frac{t}{1+t}, \qquad y = \frac{t^2}{1+t},$$

where t takes all real values except -1.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, leaving your answer in terms of t. [3]

(i) Show that the equation of the tangent to C at the point $\left(\frac{p}{1+p}, \frac{p^2}{1+p}\right)$ is

$$y = p(p+2)x - p^2.$$ [2]

(ii) Find the acute angle between the two tangents to C which pass through the point $(2,5)$. [3]

2. Referred to the origin O, the points A, B and D are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OD} = \mathbf{d}$. The point C is such that $OACB$ is a parallelogram and angle OAC is $\frac{2\pi}{3}$ radians.

(i) Given that $\mathbf{a}$ is a unit vector and $|\mathbf{b}| = 4$, find the length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OA}$. [3]

(ii) Given that $\lambda\mathbf{a} + \mu\mathbf{b} + \mathbf{d} = \mathbf{0}$ and $\lambda + \mu + 1 = 0$, show that A, B and D are collinear. [3]

If $\mu = 4$, find the area of triangle OBD, leaving your answer in the form $k|\mathbf{a} \times \mathbf{b}|$, where k is a constant to be determined. [3]

3. A geometric series has common ratio r, and an arithmetic series has first term a and common difference d, where a and d are non-zero and $a > 0$. The first three terms of the geometric series are equal to the first, eighth and thirteenth terms respectively of the arithmetic series.

(i) Show that $7r^2 - 12r + 5 = 0$. [2]

(ii) Deduce that the geometric series is convergent. [2]

(iii) The sum of the first n terms of the geometric series is denoted by S_n. Find the smallest value of n for S_n to be within 0.1% of the sum to infinity of the geometric series. [4]

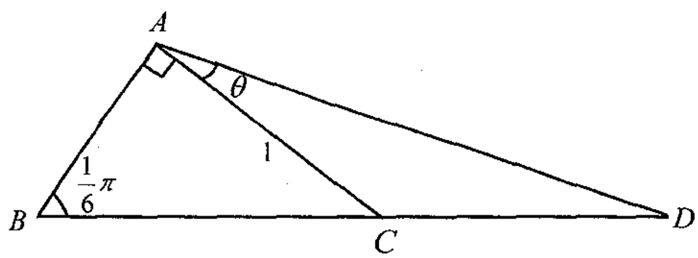
(iv) Find exactly the sum of the first 2017 terms of the arithmetic series, leaving your answer in terms of a. [3]

4. (a) It is given that $y = f(x)$ is such that $my^2 \frac{dy}{dx} - y^3 = -e^x \sin x$ and that the Maclaurin series for $f(x)$ is given by $1 + \frac{1}{3}x + nx^2 + \ldots$, where m and n are some real constants.

(i) State the values of $f(0)$ and $f'(0)$. [2]

(ii) Find the values of m and n. [3]

(b) In the triangle ABC, $AC = 1$, angle $BAC = \frac{\pi}{2}$ radians and angle $ABC = \frac{\pi}{6}$ radians. D is a point on BC produced such that angle $CAD = \theta$ radians (see diagram).



(i) Show that $AD = \frac{\sqrt{3}}{\sqrt{3}\cos\theta - \sin\theta}$. [4]

(ii) Given that θ is a sufficiently small angle, show that

$$AD \approx 1 + a\theta + b\theta^2,$$

for constants a and b to be determined exactly. [3]

Section B: Statistics [60 marks]

5. John and Peter play a game of chess. It is equally likely for either player to make the first move. If John makes the first move, the probability of him winning the game is 0.3 while the probability of Peter winning the game is 0.2. If Peter makes the first move, the probability of him winning the game is 0.5 while the probability of John winning the game is 0.4. If there is no winner, then the game ends in a draw.

(i) Find the probability that Peter made the first move given that he won the game. [3]

(ii) John and Peter played a total of three games. Assuming that the results of the three games are independent, find the probability that John wins exactly one game. [3]

6. An experiment to determine the effect of a fertilizer on crop yield was carried out. A field was divided into eight plots of equal area and eight different amounts of fertilizer, one for each plot, were used. The table below shows the amount of fertilizer, x grams, and the crop yield, y grams, for each plot.

Amount of fertilizer (x)	15	22	37	55	62	69	78	90
Yield (y)	101	123	137	150	150	154	158	160

(i) Draw the scatter diagram for these values, labelling the axes. [1]

It is thought that the yield of a crop, y grams, can be modelled by one of the formulae

$$y = a + bx \quad \text{or} \quad y = c + d\ln x$$

where a, b, c and d are constants.

(ii) Find the value of the product moment correlation coefficient between
(a) x and y,
(b) $\ln x$ and y. [2]

(iii) Use your answers to parts (i) and (ii) to explain which of $y = a + bx$ or $y = c + d\ln x$ is the better model. [2]

(iv) For a plot of land, the yield of the crop was 144 grams. Using a suitable regression line estimate the amount of fertilizer used, giving your answer to the nearest gram. [2]

(v) Comment on the reliability of the model in part (iv) in predicting the value of y when $x = 110$. [1]

7. Four digits are randomly selected from the set $\{1,2,3,4,5,6,7,8,9\}$ to form a four-digit number. Repetitions are not allowed.

(i) Find the probability that none of the digits in the four-digit number are odd. [2]

The random variable X denotes the number of odd digits in the four-digit number formed.

(ii) Show that $P(X = 1) = \frac{10}{63}$, and find the rest of the probability distribution of X, giving each probability as a fraction in its lowest terms. [3]

(iii) Find the expectation and variance of X. [3]

(iv) Two independent observations of X are denoted by X_1 and X_2.
Find $P(|X_1 - X_2| < 3)$. [4]

8. In this question, you should state clearly the values of the parameters of any normal distribution you use.

A supermarket sells two types of durians, D25 and Musang Queen. The durians are sold by weight. The masses, in kilograms, of D25 and Musang Queen are modelled as having normal distributions. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean (kg)	Standard deviation (kg)	Selling price (\$ per kg)
D25	1.5	0.02	9
Musang Queen	1.8	0.035	18

(i) A customer buys 3 D25 durians and 2 Musang Queen durians. Find the probability that the total cost of his purchase is more than \$107. [5]

(ii) State an assumption needed for your calculations in part **(i)**. [1]

(iii) The probability that the average weight of n randomly chosen D25 durians exceeding m kg is at least 0.1. Show that n satisfies the inequality

$$(m-1.5)\sqrt{n} \leqslant 0.025631.$$

Hence find the largest possible value of n when $m = 1.51$. [4]

9. Ryde, a leading private hire car company, announced JustRyde, a new service that promises more affordable fixed fare rides and shorter waiting times. In their advertisement, Ryde claimed that the mean waiting time, in seconds, was 240. A random sample of 50 JustRyde customers is taken and their waiting times, x seconds, is recorded. The data are summarised by

$$\Sigma(x-240) = 120, \quad \Sigma(x-240)^2 = 11200.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 10% significance level, whether the population mean waiting time is more than 240 seconds. [5]

(iii) State, giving a valid reason, whether any assumptions about the population are needed in order for the test to be valid. [1]

(iv) Explain, in the context of the question, the meaning of 'at the 10% significance level'. [1]

(v) In another test, using the same data and also at the 10% significance level, the hypotheses are as follows:

H_0: the population mean waiting time is equal to k seconds.
H_1: the population mean waiting time is not equal to k seconds.

Given that the null hypothesis is rejected in favour of the alternative hypothesis, find the set of possible values of k. [3]

10. It is a common practice for airlines to sell more plane tickets than the number of seats available. This is to maximise their profits as it is expected that some passengers will not turn up for the flight.

The plane used by Victoria Airline for her daily 10 am flight from Singapore to Hong Kong has a maximum capacity of 150 seats. For this particular flight, 154 tickets are sold every day. On average, p out of 100 customers who have purchased a plane ticket for this flight turn up. Customers who turn up after the flight is full will be turned away. The number of customers who turn up for the 10 am flight, on a randomly chosen day, is denoted by X.

(i) State, in the context of this question, two assumptions needed to model X by a binomial distribution. [2]

(ii) Explain why one of the assumptions stated in part **(i)** may not hold in this context. [1]

Assume now that these assumptions do in fact hold.

(iii) It is known that there is a 0.05 probability that at least 153 customers will turn up for the 10 am flight. Write down an equation for the value of p, and find this value numerically. [3]

It is given instead that $p = 94$.

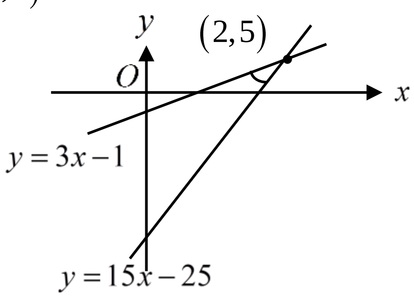
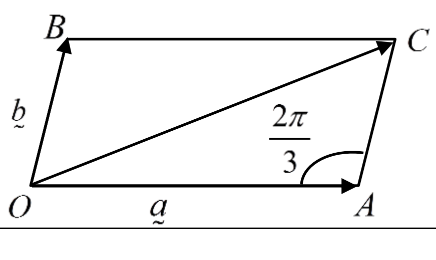
(iv) Find the probability that, on a randomly chosen day,

(a) there are at least 141 but not more than 148 customers who turn up for the 10 am flight, [2]

(b) every customer who turns up gets a seat on the 10 am flight. [1]

(v) Find the probability that every customer who turns up gets a seat on the 10 am flight on more than 5 days in a week. [3]

VJC H2 Maths Prelim P2 2017 Solutions/Mark Scheme

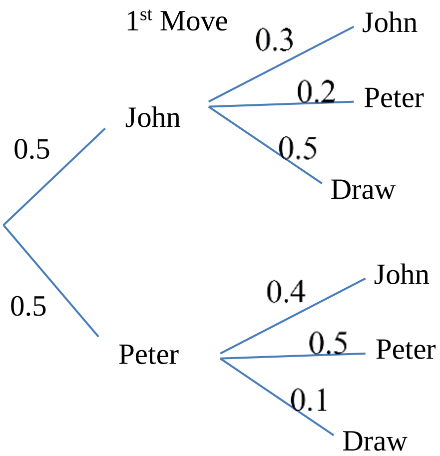
Q	Solution	
Section A: Pure Mathematics [40 marks]		
1	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$ $=\frac{(1+t)2t-t^2}{(1+t)^2}\div\frac{(1+t)(1)-t}{(1+t)^2}$ $=t^2+2t$	
1i	At point $\left(\frac{p}{1+p},\frac{p^2}{1+p}\right)$, $t=p$ Equation of tangent at point $\left(\frac{p}{1+p},\frac{p^2}{1+p}\right)$, $y-\frac{p^2}{1+p}=\left(p^2+2p\right)\left(x-\frac{p}{1+p}\right)$ $y=p(p+2)x+\frac{p^2}{1+p}-\frac{p^3}{1+p}-\frac{2p^2}{1+p}$ $y=p(p+2)x-\frac{p^2(p+1)}{1+p}$ $y=p(p+2)x-p^2$	
1ii	Tangents pass through $(2,5)$ $\Rightarrow 5=p(p+2)(2)-p^2$ $p^2+4p-5=0$ $p=-5$ or $p=1$  Equations of tangents are $y=3x-1$ and $y=15x-25$ Required acute angle between the 2 tangents $=\tan^{-1}(15)-\tan^{-1}(3)$ $=0.255$ rad or $14.6°$	
2	$\overrightarrow{OC}=\underset{\sim}{a}+\underset{\sim}{b}$  	

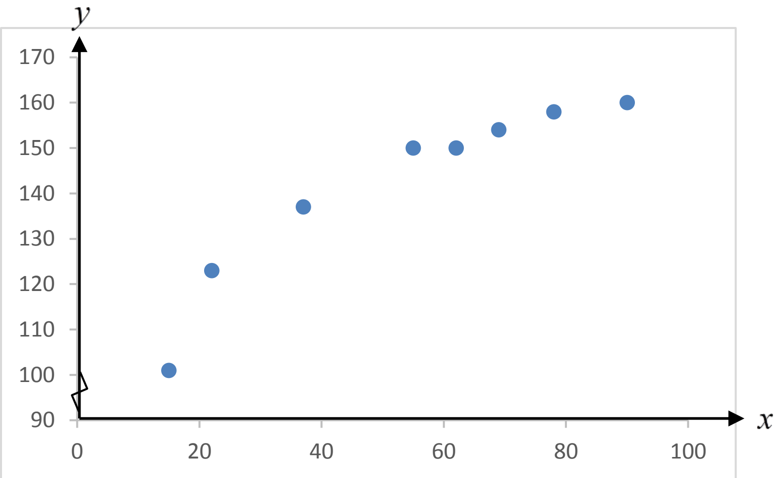
Q	Solution	
2i	Length of projection of $\overrightarrow{OC}$ onto $\overrightarrow{OA}$ $= \left\|(\underset{\sim}{a}+\underset{\sim}{b})\cdot\hat{\underset{\sim}{a}}\right\|$ $= \left\|\underset{\sim}{a}\cdot\hat{\underset{\sim}{a}}+\underset{\sim}{b}\cdot\hat{\underset{\sim}{a}}\right\| = \left\|\underset{\sim}{a}\cdot\underset{\sim}{a}+\underset{\sim}{b}\cdot\underset{\sim}{a}\right\| \quad \because \underset{\sim}{a}=\hat{\underset{\sim}{a}}$ $= \left\|\left\|\underset{\sim}{a}\right\|^2+\left\|\underset{\sim}{b}\right\|\left\|\underset{\sim}{a}\right\|\cos\left(\pi-\frac{2\pi}{3}\right)\right\|$ $= \left\|1+4\left(\frac{1}{2}\right)\right\|$ $=3$	
2ii	$\lambda\underset{\sim}{a}+\mu\underset{\sim}{b}+\underset{\sim}{d}=\underset{\sim}{0} \qquad ---(1)$ $\lambda+\mu+1=0 \qquad ---(2)$ Sub (2) into (1): $(-1-\mu)\underset{\sim}{a}+\mu\underset{\sim}{b}+\underset{\sim}{d}=\underset{\sim}{0}$ $\mu(\underset{\sim}{b}-\underset{\sim}{a})=\underset{\sim}{a}-\underset{\sim}{d}$ $\mu\overrightarrow{AB}=\overrightarrow{DA}$ Since $AB // DA$ and A is a common point, A, B and D are collinear	
	Given $\mu=4$, $\underset{\sim}{d}=5\underset{\sim}{a}-4\underset{\sim}{b}$ Area of triangle OBD $=\frac{1}{2}\left\|\underset{\sim}{b}\times\underset{\sim}{d}\right\|$ $=\frac{1}{2}\left\|\underset{\sim}{b}\times(5\underset{\sim}{a}-4\underset{\sim}{b})\right\|$ $=\frac{1}{2}\left\|5\underset{\sim}{b}\times\underset{\sim}{a}-4\underset{\sim}{b}\times\underset{\sim}{b}\right\|$ $=\frac{5}{2}\left\|\underset{\sim}{b}\times\underset{\sim}{a}\right\| \qquad (\because \underset{\sim}{b}\times\underset{\sim}{b}=\underset{\sim}{0})$ $=\frac{5}{2}\left\|\underset{\sim}{a}\times\underset{\sim}{b}\right\|$ $\therefore k=\frac{5}{2}$	
3i	$ar=a+(8-1)d \Rightarrow d=\frac{ar-a}{7}$ $ar^2=a+(13-1)d \Rightarrow d=\frac{ar^2-a}{12}$ $\frac{ar-a}{7}=\frac{ar^2-a}{12}$ $12r-12=7r^2-7$ $7r^2-12r+5=0$	

Q	Solution	
3ii	From the GC, $r=\frac{5}{7}$ or $r=1$. Since $d\neq 0$, the terms of the geometric series are distinct we conclude that $r\neq 1$. Hence, $r=\frac{5}{7}$. As $\lvert r\rvert=\left\lvert\frac{5}{7}\right\rvert<1$, the geometric series is convergent.	
3iii	$\lvert S_\infty - S_n\rvert < 0.001 S_\infty$ $\left\lvert \frac{a}{1-\frac{5}{7}} - \frac{a\left(1-\left(\frac{5}{7}\right)^n\right)}{1-\frac{5}{7}} \right\rvert < 0.001\left(\frac{a}{1-\frac{5}{7}}\right)$ $\left\lvert \frac{a}{1-\frac{5}{7}} \right\rvert \left\lvert 1-\left(1-\left(\frac{5}{7}\right)^n\right)\right\rvert < 0.001\left(\frac{a}{1-\frac{5}{7}}\right)$ $\left(\frac{5}{7}\right)^n < 0.001 \quad (\because a>0)$ $n\ln\left(\frac{5}{7}\right) < \ln 0.001$ $n > \frac{\ln 0.001}{\ln\frac{5}{7}}$ $n > 20.53$ Smallest value of *n* is 21.	
3iv	$d=\frac{ar-a}{7}=\frac{a\left(\frac{5}{7}\right)-a}{7}=-\frac{2}{49}a$ The sum of the first 2017 terms of the arithmetic series $=\frac{2017}{2}\left[2a+(2017-1)\left(-\frac{2}{49}a\right)\right]$ $=-\frac{566777}{7}a$	
4ai	$\mathrm{f}(x)=1+\frac{1}{3}x+nx^2+\ldots$ Comparing with $\mathrm{f}(x)=\mathrm{f}(0)+\mathrm{f}'(0)x+\frac{\mathrm{f}''(0)}{2!}x^2+\ldots$ $\Rightarrow \mathrm{f}(0)=1$	

Q	Solution	
	$\Rightarrow f'(0)=\frac{1}{3}$	
4aii	Given $my^2\frac{dy}{dx}-y^3=-e^x\sin x$ ---(2) When $x=0$, $m(1)^2\left(\frac{1}{3}\right)-(1)^3=-e^0\sin 0$ $\frac{1}{3}m=1$ $m=3$ Differentiate (2) w.r.t. x: $3y^2\frac{d^2y}{dx^2}+6\left(\frac{dy}{dx}\right)^2-3y^2\frac{dy}{dx}=-e^x\sin x-e^x\cos x$ When $x=0$, $3(1)^2(2n)+6\left(\frac{1}{3}\right)^2-3(1)^2\left(\frac{1}{3}\right)=-1$ $6n=-\frac{2}{9}$ $n=-\frac{1}{9}$	
4bi	**Method 1** $\measuredangle ACD=\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}$ (ext angle of a triangle) Using Sine Rule in $\triangle ACD$ $\frac{AD}{\sin\frac{2\pi}{3}}=\frac{AC}{\sin\left(\pi-\frac{2\pi}{3}-\theta\right)}$ $AD=\frac{\sqrt{3}/2}{\sin\left(\frac{\pi}{3}-\theta\right)}$	

<table>
<tr><th>Q</th><th>Solution</th><th></th></tr>
<tr><td></td><td>$$= \frac{\sqrt{3}/2}{\sin\frac{\pi}{3}\cos\theta - \cos\frac{\pi}{3}\sin\theta}$$
$$= \frac{\sqrt{3}/2}{\left(\sqrt{3}/2\right)\cos\theta - \left(1/2\right)\sin\theta}$$
$$= \frac{\sqrt{3}}{\sqrt{3}\cos\theta - \sin\theta}$$
<u>Method 2</u>
In right-angled $\triangle ABC$, $AB = \frac{1}{\tan\frac{\pi}{6}} = \sqrt{3}$.
$\measuredangle ADB = \pi - \frac{\pi}{6} - \left(\frac{\pi}{2} + \theta\right) = \frac{\pi}{3} - \theta$ (angle sum of a triangle)
Using Sine Rule in $\triangle ABD$
$$\frac{AD}{\sin\frac{\pi}{6}} = \frac{AB}{\sin\left(\frac{\pi}{3} - \theta\right)}$$
$$AD = \frac{\sqrt{3}\sin\frac{\pi}{6}}{\sin\left(\frac{\pi}{3} - \theta\right)}$$
$$= \frac{\sqrt{3}/2}{\sin\frac{\pi}{3}\cos\theta - \cos\frac{\pi}{3}\sin\theta}$$
$$= \frac{\sqrt{3}/2}{\left(\sqrt{3}/2\right)\cos\theta - \left(1/2\right)\sin\theta}$$
$$= \frac{\sqrt{3}}{\sqrt{3}\cos\theta - \sin\theta}$$</td><td></td></tr>
<tr><td>4bii</td><td>When θ is a sufficiently small angle,</td><td></td></tr>
</table>

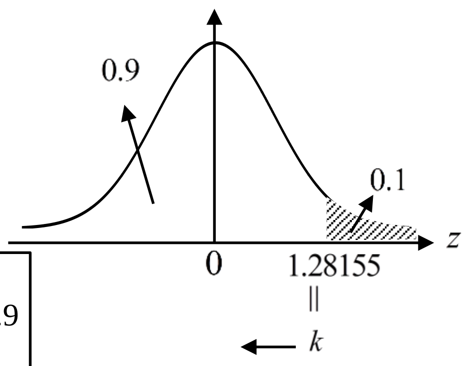
Q	Solution	
	$AD \approx \dfrac{\sqrt{3}}{\sqrt{3}\left(1-\dfrac{\theta^2}{2}\right)-\theta}$ $=\sqrt{3}\left(\sqrt{3}-\theta-\dfrac{\sqrt{3}}{2}\theta^2\right)^{-1}$ $=\left[1+\left(-\dfrac{\theta}{\sqrt{3}}-\dfrac{\theta^2}{2}\right)\right]^{-1}$ $\approx 1-\left(-\dfrac{\theta}{\sqrt{3}}-\dfrac{\theta^2}{2}\right)+\dfrac{(-1)(-2)}{2!}\left(-\dfrac{\theta}{\sqrt{3}}-\dfrac{\theta^2}{2}\right)^2$ $\approx 1+\dfrac{\theta}{\sqrt{3}}+\dfrac{\theta^2}{2}+\dfrac{\theta^2}{3}$ $=1+\dfrac{1}{\sqrt{3}}\theta+\dfrac{5}{6}\theta^2$ $\therefore a=\dfrac{1}{\sqrt{3}}, \qquad b=\dfrac{5}{6}$	
Section B: Statistics [60 marks]		
5		
5i	$\text{P}(\text{Peter made first move} \mid \text{Peter won the game})$ $=\dfrac{\text{P}(\text{Peter made first move and Peter won the game})}{\text{P}(\text{Peter won the game})}$ $=\dfrac{0.5\times0.5}{0.5\times0.2+0.5\times0.5}$ $=\dfrac{5}{7}$	
5ii	$\text{P}(\text{John wins})=0.5\times0.3+0.5\times0.4=0.35$	

Q	Solution	
	$\text{P}(\text{John wins in exactly 1 game})$ $=(0.35)(0.65)(0.65)\times\dfrac{3!}{2!}$ $=0.443625 \text{ or } \dfrac{3549}{8000}$ $=0.444(\text{to 3 s.f.})$ **<u>Alternative</u>** Let X be the number of games won by John out of 3 games. $X \sim B(3, 0.35)$ $\text{P}(\text{John wins in exactly 1 game})$ $=\text{P}(X=1)$ $=0.443625 \text{ or } \dfrac{3549}{8000}$ $=0.444(\text{to 3 s.f.})$	
6i		
6iia	From GC, r = 0.93639 = 0.936 (3 s.f)	
6iib	From GC, r = 0.98775 = 0.988 (3 s.f)	
6iii	Since 1) the points on the scatter diagram seem to lie close to an increasing curve with decreasing gradient (or close to a curve in which y increases by decreasing amounts as x increases), and 2) the product moment correlation coefficient between ln x and y of 0.988 is closer to 1 than the product moment correlation coefficient between x and y of 0.936, hence $y=c+d\ln x$ is the better model.	
6iv	From (iii), we should use the regression line of y on ln x. From GC, the equation of the regression line of y on ln x is	

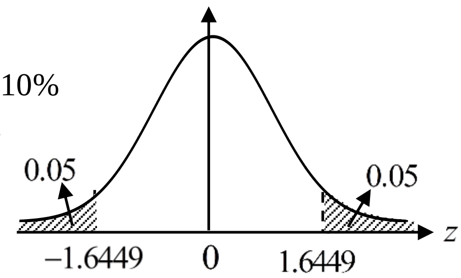
<table>
<tr><th>Q</th><th>Solution</th><th></th></tr>
<tr><td></td><td>$y = 20.8496 + 31.539 \ln x$
$y = 20.8 + 31.5 \ln x$ (3 s.f)
When y = 144, $144 = 20.8496 + 31.539 \ln x$
$\therefore x = 49.635 = 50$ (nearest gram)</td><td></td></tr>
<tr><td>6v</td><td>Since x = 110 is outside the range of data values ($15 \le x \le 90$), hence the estimated value of y may not be reliable.</td><td></td></tr>
<tr><td>7i</td><td>P(no odd digits) = P(all even digits)
$= \frac{{}^4C_4}{{}^9C_4} \left(\text{or } \frac{{}^4P_4}{{}^9P_4} \right)$
$= \frac{1}{126}$</td><td></td></tr>
<tr><td>7ii</td><td>$\mathrm{P}(X = 1) = \frac{{}^5C_1 \, {}^4C_3}{{}^9C_4} = \frac{10}{63}$

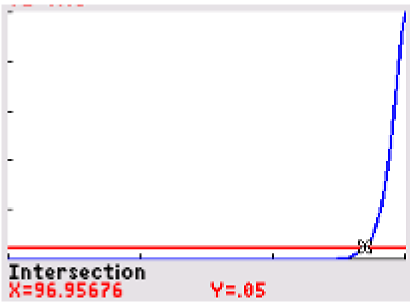
x	0	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{1}{126}$	$\frac{10}{63}$	$\frac{10}{21}$	$\frac{20}{63}$	$\frac{5}{126}$

</td><td></td></tr>
<tr><td>7iii</td><td>$\mathrm{E}(X) = \sum_{x=0}^{4} x\mathrm{P}(X = x)$
$= 1\left(\frac{10}{63}\right) + 2\left(\frac{10}{21}\right) + 3\left(\frac{20}{63}\right) + 4\left(\frac{5}{126}\right)$
$= \frac{20}{9}$
$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
$= \sum_{x=0}^{4} x^2\mathrm{P}(X = x) - \left(\frac{20}{9}\right)^2$
$= 1\left(\frac{10}{63}\right) + 4\left(\frac{10}{21}\right) + 9\left(\frac{20}{63}\right) + 16\left(\frac{5}{126}\right) - \left(\frac{20}{9}\right)^2$
$= \frac{50}{81}$</td><td></td></tr>
</table>

Q	Solution	
7iv	$\text{P}(\lvert X_1 - X_2\rvert < 3) = \text{P}(-3 < X_1 - X_2 < 3)$ $= 1 - 2\text{P}(X_1 = 0 \quad \& \quad X_2 = 4)$ $-2\text{P}(X_1 = 0 \quad \& \quad X_2 = 3)$ $-2\text{P}(X_1 = 1 \quad \& \quad X_2 = 4)$ $= 1 - 2\left(\frac{1}{126}\right)\left(\frac{5}{126}\right) - 2\left(\frac{1}{126}\right)\left(\frac{20}{63}\right)$ $-2\left(\frac{10}{63}\right)\left(\frac{5}{126}\right)$ $= \frac{7793}{7938} \quad (\text{or } 0.982)$	
8	Let X kg and Y kg be the mass of a randomly chosen D25 durian and Musang Queen durian respectively. $X \sim \text{N}(1.5, 0.02^2), \qquad Y \sim \text{N}(1.8, 0.035^2)$	
8ii	Let $T = 9(X_1 + X_2 + X_3) + 18(Y_1 + Y_2)$ $\text{E}(T) = \text{E}[9(X_1 + X_2 + X_3) + 18(Y_1 + Y_2)]$ $= 9(3)(1.5) + 18(2)(1.8)$ $= 105.3$ $\text{Var}(T) = \text{Var}[9(X_1 + X_2 + X_3) + 18(Y_1 + Y_2)]$ $= (9)^2(3)(0.02)^2 + (18)^2(2)(0.035)^2$ $= 0.891$ $T \sim \text{N}(105.3, 0.891)$ $\text{P}(T > 107) = 0.035852$ $= 0.0359$ (3 s.f)	
8ii	The masses of all the durians are independent of each other.	
8iii	Let $\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$ $\overline{X} \sim \text{N}\left(1.5, \frac{0.02^2}{n}\right)$ Given $\text{P}(\overline{X} > m) \geqslant 0.1$ $\text{P}\left(Z > \frac{m - 1.5}{0.02/\sqrt{n}}\right) \geqslant 0.1$ $\Rightarrow \text{P}\left(Z < \frac{m - 1.5}{0.02/\sqrt{n}}\right) \leqslant 0.9$ 	

Q	Solution	
	From the GC, $\mathrm{P}\left(Z<1.28155\right)=0.9$ $\therefore \dfrac{m-1.5}{0.02/\sqrt{n}} \leqslant 1.28155$ $\Rightarrow \left(m-1.5\right)\sqrt{n} \leqslant 0.025631$ when $m=1.51$ $\Rightarrow \left(1.51-1.5\right)\sqrt{n} \leqslant 0.025631$ $\Rightarrow n \leqslant 6.5695$ Largest value of n is 6	
9i	Let $y=x-240$ unbiased estimate of population mean $=\bar{x}$ $=\bar{y}+240$ $=\dfrac{\sum y}{n}+240$ $=\dfrac{120}{50}+240$ $=242.4$ Unbiased estimate of population variance $=s^2$ $=\dfrac{1}{n-1}\left(\sum y^2-\dfrac{\left(\sum y\right)^2}{n}\right)$ $=\dfrac{1}{49}\left(11200-\dfrac{120^2}{50}\right)$ $=222.69=223$ (3 s.f)	
9ii	Let μ be the population mean of X. $\mathrm{H}_0 : \mu=240$ $\mathrm{H}_1 : \mu>240$ Level of significance: 10% Test Statistic : since n = 50 is sufficiently large, By Central Limit Theorem, $\bar{X}$ is approximately normal. When H_0 is true, $Z=\dfrac{\bar{X}-240}{\frac{S}{\sqrt{50}}} \sim \mathrm{N}(0,1)$ approximately	

Q	Solution	
	Computation : $\bar{x} = 242.4$ $s = \sqrt{222.69} = 14.923$ $p-$value $= 0.128$ (3 s.f) Conclusion : Since p-value = 0.128 > 0.10, H_0 is not rejected at the 10% significance level. So there is insufficient evidence that the population mean waiting time is more than 240 seconds.	
9iii	No assumption is needed. Since the sample size is large, by Central Limit Theorem, the distribution of the sample mean ($\bar{X}$) is approximately normal.	
9iv	There is a probability of 0.10 that the test will conclude the population mean waiting time is more than 240 seconds when it is actually 240 seconds.	
9v	$H_0 : \mu = k$ $H_1 : \mu \neq k$ Level of significance: 10% For H_0 to be rejected,  $z \le -1.6449$ or $z \ge 1.6449$ $\dfrac{\bar{x}-k}{s/\sqrt{50}} \le -1.6449$ or $\dfrac{\bar{x}-k}{s/\sqrt{50}} \ge 1.6449$ $\dfrac{242.4-k}{14.923/\sqrt{50}} \le -1.6449$ or $\dfrac{242.4-k}{14.923/\sqrt{50}} \ge 1.6449$ $242.4-k \le -3.4714$ or $242.4-k \ge 3.4714$ $k \ge 245.87$ or $k \le 238.93$ $\{k \in \mathbb{R} : k \le 239 \text{ (3 s.f)} \text{ or } k \ge 246 \text{ (3 s.f)}\}$	
10i	The assumptions are (1) The probability that a customer turn up for the flight is $\dfrac{p}{100}$ <u>for all the 154 customers</u>. (2) Customers turn up independently of each other.	
10ii	Customers may be travelling in a group or as a family. Therefore, customers may not turn up independently of the others in their group.	

Q	Solution	
10iii	$X \sim \mathrm{B}\left(154, \frac{p}{100}\right)$ Given $\mathrm{P}(X \geqslant 153) = 0.05$ $\mathrm{P}(X = 153) + \mathrm{P}(X = 154) = 0.05$ $\binom{154}{153}\left(\frac{p}{100}\right)^{153}\left(1-\frac{p}{100}\right) + \binom{154}{154}\left(\frac{p}{100}\right)^{154} = 0.05$ $154\left(\frac{p}{100}\right)^{153}\left(1-\frac{p}{100}\right) + \left(\frac{p}{100}\right)^{154} = 0.05$  From the GC, $p = 96.9568 = 97.0$ (to 3 s.f.)	
10iv a	$X \sim \mathrm{B}(154, 0.94)$ $\mathrm{P}(141 \leqslant X \leqslant 148) = \mathrm{P}(X \leqslant 148) - \mathrm{P}(X \leqslant 140)$ $= 0.825$	
10iv b	$\mathrm{P}(X \leqslant 150) = 0.98443 = 0.984$ (to 3 s.f.)	
10v	Let Y be the number of days (out of 7) in which every customer who turns up gets a seat on the flight $Y \sim \mathrm{B}(7, 0.98443)$ $\mathrm{P}(Y > 5) = 1 - \mathrm{P}(Y \leqslant 5)$ $= 0.995$	