1 It is given that $\mathrm{f}(x)=5x^3+7x^2-kx+2$, where k is a constant.

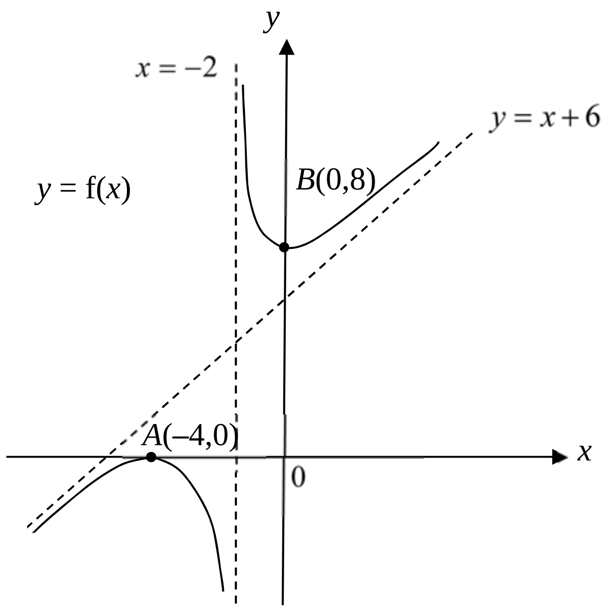
(i) If the graph of $y=\mathrm{f}(x)$ is strictly increasing, find the range of values of k. [3]

(ii) If k = 0, state the range of values of x where the graph of $y=\mathrm{f}(x)$ is concave downward. [1]

2 **(a)** Find $\int \frac{1}{1-4x^2}\,\mathrm{d}x$. [2]

(b) By writing $\sin^3 x$ as $\sin x(1-\cos^2 x)$, find $\int \sin^3 x\,\mathrm{d}x$. [2]

3. The diagram below shows the graph of $y=\mathrm{f}(x)$. The curve has a maximum point at A and a minimum point at B. The lines $x=-2$ and $y=x+6$ are asymptotes of the graph.



Sketch, on separate diagrams, the graphs of

(i) $y=\frac{1}{\mathrm{f}(x)}$, [3]

(ii) $y=\mathrm{f}'(x)$, [3]

showing clearly in each case, where appropriate, the asymptotes and co-ordinates of the points corresponding to A and B.

4 **(i)** Obtain the expansion of $\sqrt{1 - x + x^2}$ up to and including the term in x^2. [3]

(ii) Given that f(x) = ln (a + bx), where a is a positive constant and b is a non-zero constant, find the first three terms in the Maclaurin series for f(x). [3]

(iii) The first two terms of the series in (i) are equal to the first two terms in the series expansion of f(x), find a and b, leaving your answers in the exact form. [2]

5 **(i)** Sketch the graphs of $ay = x - b$ and $y = \frac{x-b}{x}$ on the same diagram, where a and b are constants such that $0 < a < 1 < b$. State clearly the exact coordinates of any points of intersection with the axes and the equations of any asymptotes. [4]

(ii) Hence, solve the inequality $\frac{x-b}{x} > \frac{x-b}{a}$. [3]

(iii) Deduce the solution to the inequality $\frac{x}{x+b} > \frac{x}{a}$. [2]

6 A curve C has equation $3y^2 - 2xy + 3x^2 - 96 = 0$.

(i) Find the exact x- coordinates of the stationary points of C. [4]

(ii) For the stationary point with x < 0, determine whether it is a maximum or minimum. [3]

(iii) Find the equation(s) of the tangent(s) to C which are parallel to the y-axis. [3]

7 A particle moves on the Cartesian plane in such a way that at time t, its position is given by the parametric equations $x = t\cos t$, $y = t\sin t$. The particle starts moving from the origin.

(i) Find the equation of the normal at which the particle first crosses the y-axis, leaving your answer in the exact form. [5]

(ii) It is given that the instantaneous speed of the particle at time t is defined as $\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}$. Find the exact instantaneous speed of the particle when it crosses the x-axis for the second time. [4]

(iii) Explain if the particle will ever come to rest. [1]

8 **(a)** The shape of the mould used to make Ah Gong lava cake is formed by rotating the region bounded by the line $y = 2$, the x-axis and the curves $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$, where $a > 2$ and $1 < b < 2$, through π radians about the y-axis. Find the volume of the mould, giving your answer in the form $\pi\left(pa^2 + q + rb^3\right)$, where p, q, r are constants to be determined. [3]

(b) A curve has parametric equations

$$x = 2\sin\theta, \quad y = 3\sin\left(2\theta + \frac{\pi}{2}\right), \quad \text{for } 0 \le \theta \le \frac{\pi}{2}.$$

(i) Sketch the curve. [2]

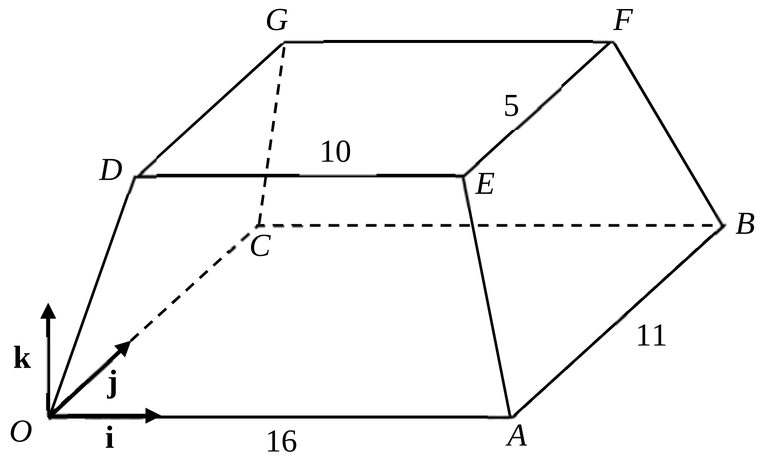
(ii) Find the exact area of the region bounded by the curve and the axes. [6]

9 **(a)** The product of the first three terms of a convergent geometric progression is 1000. If 6 is added to the second term and 7 is added to the third term, the three terms are now consecutive terms of an arithmetic progression. Find the first term and common ratio of the geometric progression. [4]

(b) **(i)** A man takes a loan of \$$P$ for a house from a bank at the beginning of a month. The interest rate is 0.5 % per month so that at the start of every month, the amount of outstanding loan is increased by 0.5%. Equal instalment is paid to the bank at the end of every month. Find his monthly instalment if he would like to repay the loan in 20 years, leaving your answer in the form $0.005P\dfrac{k^n}{k^n - 1}$, where k and n are constants to be determined. [5]

(ii) If the man takes a loan of \$1 000 000 for the house from another bank which charges interest at the start of every month, he will have to pay a monthly instalment of \$10 000 at the end of every month over 20 years to repay the loan. Find the monthly interest rate charged by this bank. [3]

10



The diagram shows a container with a horizontal rectangular base $OABC$, where $OA = 16$ cm and $AB = 11$ cm. The top of the container $DEFG$ is also a horizontal rectangle, where $DE = 10$ cm and $EF = 5$ cm. The 4 sloping faces (e.g. $OAED$) of the container, each a trapezium, are inclined at the same angle to the horizontal such that the distance between the base and the top is 6 cm. The point O is taken as origin and perpendicular unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ are such that $\mathbf{i}$ and $\mathbf{j}$ are parallel to OA and OC respectively.

(i) Show that a cartesian equation of plane BCD is $3y + 4z = 33$. [3]

(ii) Find the cartesian equations of the planes such that the perpendicular distance from each plane to plane BCD is 10 cm. [4]

(iii) Find the coordinates of the point on plane BCD which is closest to point G. [3]

(iv) Hence or otherwise, determine the acute angle between CG and plane BCD. [3]

12 Environmental conditions such as acidity, temperature, oxygen levels and toxins influence the rate of growth of microorganisms. A biologist investigates the change of population of a particular type of microorganism of size n thousand at time t days under different conditions. In both models I and II, the initial population of the microorganism is 3000 and the population reaches 2000 after 1 day.

(i) Under model I, the biologist observes that the rate of growth of microorganism is a constant whereas the death rate is proportional to its population. He also observes that when the population of the microorganism is 1000, it remains at this constant value. By setting up and solving a differential equation, show that $n = 1 + 2^{1-t}$. [8]

(ii) Under model II, the biologist observes that n and t are related by the differential equation $\dfrac{\mathrm{d}^2 n}{\mathrm{d}t^2} = 4 - 6t$. Find the particular solution of this differential equation.

[3]

(iii) By sketching the graphs of n against t for both model I and II, state and explain which of the two models is more harmful for the growth of this type of microorganism. [2]

Section A: Pure Mathematics [40 marks]

1 Referred to the origin O, points A and B have position vectors **a** and **b** respectively. The point C lies on OA produced and is such that $OC = \lambda OA$, where $\lambda > 1$. The point D lies on OB, between O and B, such that $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{OB}$. It is given that $|\mathbf{a}| = 4$, $|\mathbf{b}| = 8$ and $\angle AOB = 60^\circ$.

(i) Show that $\overrightarrow{OD} = \frac{1}{4}\mathbf{b}$. [2]

(ii) Show that the vector equation of the line BC can be written as $\mathbf{r} = \lambda\mu\mathbf{a} + (1-\mu)\mathbf{b}$, where μ is a parameter. [2]

The point E lies on the line BC.

(iii) Find the values of μ, in terms of λ, such that the area of triangle ODE is $\sqrt{300}$. [4]

2 **Do not use a calculator in answering this question.**

(a) Solve the equation $z^2 = 5 - 12\mathrm{i}$, giving your answers in the form $x + \mathrm{i}y$. [4]

Hence, find the roots of the equation $w^4 - 10w^2 + 169 = 0$. [2]

(b) The complex number p has modulus r and argument θ, where $0 < \theta < \frac{1}{2}\pi$. State the argument of q, where $q = \dfrac{(-1+\mathrm{i})\,p^*}{p^2}$. Given that q^2 is real and negative, find the possible values of θ. [4]

3 The function f is defined by

$$\mathrm{f}(x) = \begin{cases} 2x^2 + 5, & x < 1, \\ |x - 8|, & x \geq 1. \end{cases}$$

(i) Sketch the graph of f and state the range of f. [3]

(ii) State whether the inverse function f^{-1} exists, justifying your answer. [2]

The function g is defined by

$$\mathrm{g}(x) = \sqrt{x} + 1, \qquad 0 < x \leq 16.$$

(iii) Find $\mathrm{g}^{-1}(4)$. [2]

(iv) State the range of g and find fg(x), stating the domain of fg. [3]

4 The rth term of a sequence is given by $u_r = \dfrac{1}{r!}$.

(i) Show that $u_r - u_{r+1} = \dfrac{1}{r! + (r-1)!}$. [2]

(ii) Hence find $\sum_{r=1}^{N} \dfrac{1}{r! + (r-1)!}$. [2]

(iii) Give a reason why the series in **(ii)** is convergent and state the sum to infinity. [2]

(iv) Use your answer to **(ii)** to find $\sum_{r=1}^{N+1} \dfrac{1}{r! + (r+1)!}$. [3]

(v) Deduce that $\sum_{r=1}^{N} \dfrac{1}{r!} < 2$. [3]

Section B: Statistics [60 marks]

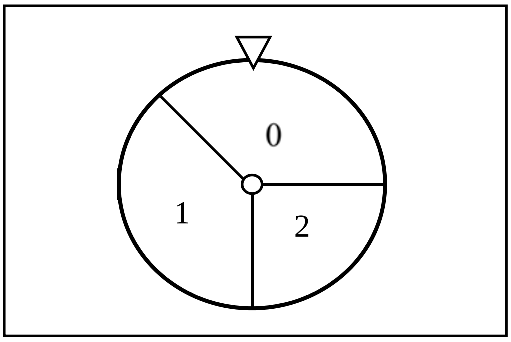
5 **(a)** Three boys and three girls are to be seated in two rows of three chairs each. How many ways can it be done if

(i) boys and girls must alternate, [2]

(ii) two particular girls must sit next to each other? [2]

(b) How many ways can six people be seated around two identical round tables such that there is at least one person at each table? [3]

6 A circular card is divided into 3 sectors scoring 0, 1, 2 and having angles 135°, 135°, 90° respectively. The card is mounted onto a wall at its centre such that it can rotate freely. A pointer is fixed on the wall just above the card, as shown in the diagram.



In a game, a participant will spin the card twice, and the random variable X is the product of the scores of the 2 spins.

(i) Tabulate the probability distribution of X. [3]

(ii) Show that $\mathrm{E}(X) = \dfrac{49}{64}$, and find $\mathrm{Var}(X)$. [2]

Dave intends to use the above setup in a fundraising carnival. For every game, the participant will be charged \$$y$ to play, and will be awarded a prize money of \$$5X$ based on the outcome of their spins. Find the least integer value of y such that on average, Dave can expect to earn at least \$1 from each participant. [2]

7 National Aeronautics and Space Administration (NASA) gives some information about the planets in the solar system. The distance from the Sun, x million km, and mean temperature, y Kelvin, of seven of them are as follows

Planet	Mercury	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
x	57.9	149.6	227.9	778.6	1433.5	2872.5	4495.1
y	440	288	208	163	133	78	73

(Source: https://nssdc.gsfc.nasa.gov/planetary/factsheet/)

(i) Draw the scatter diagram for these values, labelling the axes clearly. Explain whether your answer suggests that a linear model is appropriate. [2]

It is thought that the mean temperature y can be modelled by one of the formulae

$$y = a + \frac{b}{x} \quad \text{or} \quad y = c + \frac{d}{\sqrt{x}}$$

where a, b, c and d are constants.

(ii) Find, correct to 4 decimal places, the product moment correlation coefficient between

(a) $\frac{1}{x}$ and y,

(b) $\frac{1}{\sqrt{x}}$ and y. [2]

(iii) Use your answers to part **(ii)** to explain which of $y = a + \frac{b}{x}$ and $y = c + \frac{d}{\sqrt{x}}$ is the better model. [1]

(iv) It is required to estimate the mean temperature of Venus, which is 108.2 million km from the Sun. Find the equation of a suitable regression line, and use it to find the required estimate. Comment on the reliability of your estimate. [4]

(v) Given that the conversion of Kelvin (T_K) to Celsius (T_C) follows the formula $T_K = T_C + 273$, re-write your equation from part **(iv)** so that it can be used to estimate the temperature, in Celsius, when the distance from the Sun is given. [2]

8 **In this question you should state clearly the values of the parameters of any normal distribution you use.**

The masses, in grams, of peaches produced by orchard A and orchard B have the distributions $N(145, 15^2)$ and $N(190, 20^2)$ respectively.

(i) Find the probability that the mass of a randomly chosen peach from A is less than 140 grams. [1]

(ii) Find the probability that of 3 randomly chosen peaches from A, only one will have a mass of less than 140 grams. [3]

(iii) The probability that the mass of a randomly chosen peach from A is at least k grams is at most 0.15. Find the range of values of k. [2]

(iv) Find the probability that the total mass of 4 randomly chosen peaches produced by A will differ from thrice the mass of a randomly chosen peach produced by B by at least 15 grams. [4]

State an assumption needed for your calculations in **(ii)** and **(iv)**. [1]

9 A swimming school administers a swimming assessment for 60 candidates each day, and the number of those candidates who pass the assessment is denoted by S.

(i) State, in context, two assumptions needed for S to be well modelled by a binomial distribution. [2]

Assume now that S has the distribution $\mathrm{B}(60, 0.7)$.

(ii) Find the probability that more than 40 candidates pass the assessment on a randomly chosen day. [1]

(iii) The school claims that the probability of at least m candidates passing the assessment each day is at least 90%. Find the greatest value of m. [3]

A “good” day is a day in which at least 45 candidates pass the assessment.

(iv) Find the probability that a day is good. [1]

(v) Find the probability that less than 50 candidates passed the assessment on a good day. [3]

(vi) Find the probability that there are at most 10 good days over a period of 50 days. [2]

10 A fruit juice seller claims that each bottle of fruit juice he sells contains, on average, 300 ml of fruit juice. A random sample of 35 bottles of fruit juice is selected and the amount of fruit juice, x ml, in each bottle is measured. The results are summarised by:

$$\sum(x-250)=1705\,,\ \sum(x-250)^2=83650\,.$$

(i) Find the unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 5% level of significance, whether the fruit juice seller is overstating the average amount of fruit juice in each bottle. You should state your hypotheses and define any symbols you use. [5]

(iii) Explain why there is no need for the fruit juice seller to know anything about the distribution of the amount of fruit juice in each bottle. [1]

Improvements are made to the bottling process and the variance of the amount of fruit juice in each bottle is now known to be 15.7 ml^2. The fruit juice seller now claims that the amount of fruit juice in each bottle is more than 300ml. A new random sample of 15 bottles of fruit juice is chosen and the mean of this sample is y ml. A test at the 5% significance level indicates that the fruit juice seller's claim is valid for this improved process.

(iv) Find the least possible value of y, giving your answer correct to 2 decimal places. [3]

State any assumption you made in your calculation in **(iv)**. [1]

2018 J2 H2 Maths Prelims Paper 1 Solutions

1(a) $\mathrm{f}(x) = 5x^3 + 7x^2 - kx + 2$

$\mathrm{f}'(x) = 15x^2 + 14x - k$

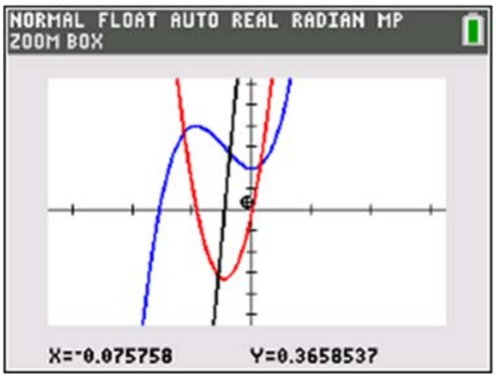
$\mathrm{f}'(x) > 0$, $15x^2 + 14x - k > 0$ for all x

Discriminant < 0 ,

$(14)^2 - 4(15)(-k) < 0$

$k < -\dfrac{49}{15}$ or (-3.27)

1(b) From GC ,



Graph is concave downward when $x < -0.467$

2(a) $$\int \frac{1}{1-4x^2}\,\mathrm{d}x = \frac{1}{4}\int \frac{1}{\left(\frac{1}{2}\right)^2 - x^2}\,\mathrm{d}x$$

$$= \frac{1}{4}\left(\frac{1}{2\left(\frac{1}{2}\right)}\right)\ln\left|\frac{\frac{1}{2}+x}{\frac{1}{2}-x}\right| + C$$

$$= \frac{1}{4}\ln\left|\frac{1+2x}{1-2x}\right| + C$$

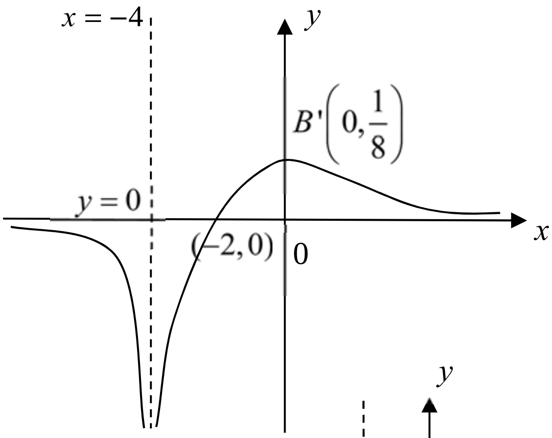
2(b)

$$\int \sin^3 x\,\mathrm{d}x = \int \sin x(1-\cos^2 x)\,\mathrm{d}x$$

$$= \int \sin x - \sin x\cos^2 x\,\mathrm{d}x$$

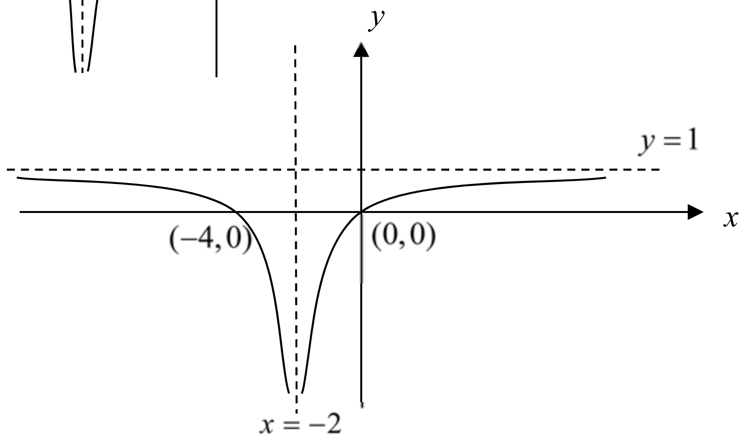
$$= -\cos x + \frac{1}{3}\cos^3 x + C$$

3(i)



$y=\frac{1}{\mathrm{f}(x)}$

3(ii)



$y=\mathrm{f}'(x)$

4(i) $\sqrt{1-x+x^2}=\left[1-(x-x^2)\right]^{\frac{1}{2}}$

$$=\left(1-\frac{1}{2}(x-x^2)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(x-x^2)^2+....\right)$$

$$=\ 1-\frac{1}{2}x+\frac{1}{2}x^2-\frac{1}{8}x^2+....$$

$$=\ 1-\frac{1}{2}x+\frac{3}{8}x^2+....$$

(ii)

$\ln\left(a+bx\right)$ $=\ln a\left(1+\frac{b}{a}x\right)$

$$=\ \ln a+\ln\left(1+\frac{b}{a}x\right)$$

$$=\ln a+\left(\frac{b}{a}x-\frac{\left(\frac{b}{a}x\right)^2}{2}\right)+........$$

$$=\ \ln a+\frac{b}{a}x-\frac{b^2}{2a^2}x^2+........$$

(iii)Comparing first two terms ,

$\ln a = 1 \Rightarrow \quad a = \mathrm{e}$

$\dfrac{b}{a} = -\dfrac{1}{2} \Rightarrow \quad b = -\dfrac{1}{2}\mathrm{e}$

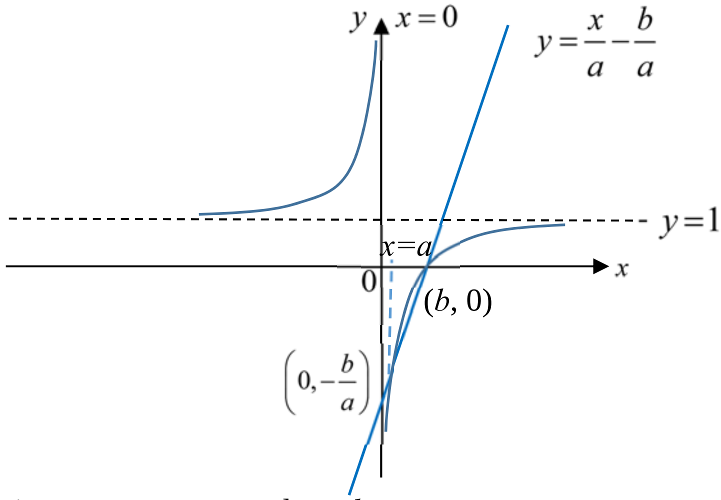
5 (i) $y = \dfrac{x-b}{x} = 1 - \dfrac{b}{x}$

When $y = 0, \ x = b$

$ay = x - b \Rightarrow y = \dfrac{x}{a} - \dfrac{b}{a}$

When $y = 0, \ x = b$

When $x = 0, \ y = -\dfrac{b}{a}$



(ii) By observation, the 2 graphs intersect at $x = a$ and $x = b$.

Hence, for $\dfrac{x-b}{x} > \dfrac{x-b}{a}$, $a < x < b$ or $x < 0$.

(iii) $\dfrac{x}{x+b} > \dfrac{x}{a} \Rightarrow \dfrac{(x+b)-b}{x+b} > \dfrac{(x+b)-b}{a}$

Replace x by $x+b$:

$a < x+b < b$ or $x+b < 0$

$a-b < x < 0$ or $x < -b$

6.(i) $3y^2 - 2xy + 3x^2 - 48 = 0$

Differentiate wrt x , $6y\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2\left(x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\right) + 6x = 0$

$$6y\frac{\mathrm{d}y}{\mathrm{d}x} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 2y + 6x = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y - 6x}{6y - 2x} = \frac{y - 3x}{3y - x}$$

At stationary point , $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$,

$y = 3x$

Substitute into equation of C , $3(3x)^2 - 2x(3x) + 3x^2 - 96 = 0$

$$24x^2 = 96$$
$$x^2 = 4$$
$$x = \pm 2$$

(ii) $6y\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2x\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y + 6x = 0$

Differentiate wrt x , $(6y - 2x)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \dfrac{\mathrm{d}y}{\mathrm{d}x}\left(6\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2\right) - 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 6 = 0$

When $x = -2$, $y = 3x = -6$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$

Substitute : $\left(6(-6) + 2(2)\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6 = 0$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-6}{-32} = \frac{3}{16} > 0$$

Hence the point is a minimum point.

(iii)Tangent // y-axis $\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}$ is undefined: $3y - x = 0$,

$$y = \frac{x}{3}$$

Substitute into equation of C , $3\left(\dfrac{x}{3}\right)^2 - 2x\left(\dfrac{x}{3}\right) + 3x^2 - 96 = 0$

$$\frac{8}{3}x^2 = 96$$
$$x^2 = 36$$
$$x = \pm 6$$

7(i) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -t\sin t + \cos t$, $\dfrac{\mathrm{d}y}{\mathrm{d}t} = t\cos t + \sin t$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{t\cos t + \sin t}{-t\sin t + \cos t}$$

When the particle first crosses the y-axis , $x = 0$

$$t\cos t = 0$$
$$t \neq 0 \text{ (starting point) } \cos t = 0$$
$$t = \frac{\pi}{2}$$

$$y = \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2}$$

$$\text{Gradient of normal} = -\frac{-t\sin t+\cos t}{t\cos t+\sin t}=\frac{\pi}{2}$$

$$\text{equation of normal}: \quad y-\frac{\pi}{2}=\frac{\pi}{2}(x-0)$$

$$y=\frac{\pi}{2}x+\frac{\pi}{2}$$

(ii) When $y = t\sin t = 0$,

$t \neq 0$ (starting point), $\sin t = 0$,

$t = 2\pi$ (crosses the x-axis for the second time)

$t \neq \pi$ (crosses the x-axis for the first time)

$$\text{Speed} = \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}$$

$$= \sqrt{(-t\sin t+\cos t)^2+(t\cos t+\sin t)^2}$$

$$= \sqrt{t^2\sin^2 t-2t\sin t\cos t+\cos^2 t+t^2\cos^2 t+2t\sin t\cos t+\sin^2 t}$$

$$= \sqrt{t^2\left(\sin^2 t+\cos^2 t\right)+\cos^2 t+\sin^2 t}$$

$$= \sqrt{t^2+1}$$

$$= \sqrt{4\pi^2+1}$$

(iii) Since speed at time t is $\sqrt{t^2+1}$, $t^2 \geq 0 \Rightarrow \sqrt{t^2+1} \geq 1$, so speed is never 0 and will never come to rest.

8(a) Required volume

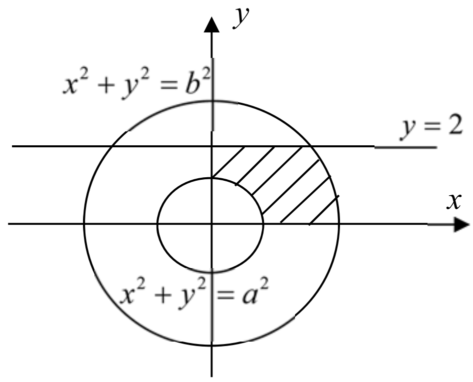
$$=\pi\int_0^2 a^2-y^2\ \mathrm{d}y-\pi\int_0^b b^2-y^2\ \mathrm{d}y$$

$$=\pi\left(\left[a^2y-\frac{1}{3}y^3\right]_0^2-\left[b^2y-\frac{1}{3}y^3\right]_0^b\right)$$

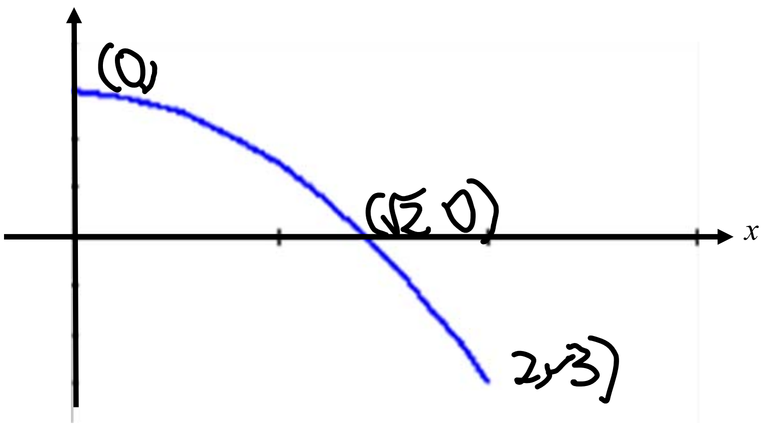
$$=\pi\left(\left(2a^2-\frac{8}{3}\right)-\left(b^3-\frac{1}{3}b^3\right)\right)$$

$$=\pi\left(2a^2-\frac{8}{3}-\frac{2}{3}b^3\right)$$

Hence, $p=2, q=-\frac{8}{3}, r=-\frac{2}{3}$



8(b)(i)



(b)(ii) Area bounded by curve and axes

$$= \int_0^3 x \, \mathrm{d}y$$

$$= \int_{\frac{\pi}{4}}^{0} 2\sin\theta \left(6\cos\left(2\theta + \frac{\pi}{2} \right) \mathrm{d}\theta \right)$$

$$= 6\int_{\frac{\pi}{4}}^{0} 2\cos\left(2\theta + \frac{\pi}{2} \right) \sin\theta \mathrm{d}\theta$$

$$= 6\int_{\frac{\pi}{4}}^{0} \sin\left(3\theta + \frac{\pi}{2} \right) - \sin\left(\theta + \frac{\pi}{2} \right) \mathrm{d}\theta$$

$$= 6\left[-\frac{1}{3}\cos\left(3\theta + \frac{\pi}{2} \right) + \cos\left(\theta + \frac{\pi}{2} \right) \right]_{\frac{\pi}{4}}^{0}$$

$$= 6\left[\left(-\frac{1}{3}\cos\left(\frac{\pi}{2} \right) + \cos\left(\frac{\pi}{2} \right) \right) - \left(-\frac{1}{3}\cos\left(\frac{5\pi}{4} \right) + \cos\left(\frac{3\pi}{4} \right) \right) \right]$$

$$= 6\left[-\left(\frac{1}{3\sqrt{2}} - \frac{1}{\sqrt{2}} \right) \right]$$

$$= 2\sqrt{2}$$

Alternative Solution: Area bounded by curve and axes

$$= \int_0^{\sqrt{2}} y \, \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{4}} 3\sin\left(2\theta + \frac{\pi}{2} \right) 2\cos\theta \mathrm{d}\theta$$

$$= 3\int_0^{\frac{\pi}{4}} 2\sin\left(2\theta + \frac{\pi}{2} \right) \cos\theta \mathrm{d}\theta$$

$$= 3\int_0^{\frac{\pi}{4}} \sin\left(3\theta + \frac{\pi}{2} \right) + \sin\left(\theta + \frac{\pi}{2} \right) \mathrm{d}\theta$$

$$= 3\left[-\frac{1}{3}\cos\left(3\theta + \frac{\pi}{2} \right) - \cos\left(\theta + \frac{\pi}{2} \right) \right]_0^{\frac{\pi}{4}}$$

$$= 3\left[\left(-\frac{1}{3}\cos\left(\frac{5\pi}{4}\right)-\cos\left(\frac{3\pi}{4}\right)\right)-\left(-\frac{1}{3}\cos\left(\frac{\pi}{2}\right)-\cos\left(\frac{\pi}{2}\right)\right)\right]$$
$$= 3\left[\frac{1}{3\sqrt{2}}+\frac{1}{\sqrt{2}}\right]$$
$$= 2\sqrt{2}$$

9(a) GP: a, ar, ar^2

AP: ….. , $a, ar+6, ar^2+7$, …..

$$a(ar)(ar^2) = 1000$$
$(ar)^3 = 1000$ Þ $ar = 10$
$$(ar+6) - a = ar^2 + 7 - (ar+6)$$
$$ar^2 - 2ar + a - 5 = 0$$
$$10r - 20 + a - 5 = 0$$
$$10r + a = 25$$
$$10r + \frac{10}{r} = 25$$
$$10r^2 - 25r + 10 = 0$$
$$r = 2 \text{ or } \frac{1}{2}$$

Reject $r = 2$ since GP is convergent , $-1 < r < 1$

Hence $a = 20$

(b)(i)

	Amount (start of)	**Amount (end of)**
1st month	$1.005P$	$1.005P - x$
2nd month	$1.005(1.005P-x)$	$1.005(1.005P-x)-x$ $=1.005^2P-1.005x-x$
3rd month	$1.005(1.005^2P-1.005x-x)$	$1.005(1.005^2P-1.005x-x)-x$ $=1.005^3P-1.005^2x-1.005x-x$

Hence, amount of money at the end of 20 years

$$1.005^{240}P-1.005^{239}x-1.005^{238}x-......-x=0$$

$$1.005^{240}P=1.005^{239}x+1.005^{238}x+......+x$$
$$1.005^{240}P = \frac{x\left[(1.005)^{240}-1\right]}{1.005-1}$$
$$x = 0.005P\frac{1.005^{240}}{\left[(1.005)^{240}-1\right]}$$

$k = 1.005$, $n = 240$

(ii) $r(1000000)\dfrac{(1+r)^{240}}{\left[(1+r)^{240}-1\right]}=10000$

Using GC , r = 0.00877

Hence the interest rate is 0.877 % per month

10(i)

$$\overrightarrow{OB}=\begin{pmatrix}16\\11\\0\end{pmatrix},\quad \overrightarrow{OC}=\begin{pmatrix}0\\11\\0\end{pmatrix},\quad \overrightarrow{OD}=\begin{pmatrix}3\\3\\6\end{pmatrix}\quad \overrightarrow{CD}=\begin{pmatrix}3\\-8\\6\end{pmatrix}$$

Vector parallel to $\overrightarrow{BC}$ is $\begin{pmatrix}1\\0\\0\end{pmatrix}$

$$\begin{pmatrix}3\\-8\\6\end{pmatrix}\times\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}0\\6\\8\end{pmatrix}=2\begin{pmatrix}0\\3\\4\end{pmatrix}$$

$$\mathbf{r}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=\begin{pmatrix}0\\11\\0\end{pmatrix}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=33$$

Hence, cartesian equation is $3y+4z=33$ (shown).

(i) **<u>Method 1:</u>** (Consider distance of plane from origin)

$$\mathbf{r}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=33$$

$$\mathbf{r}\cdot\frac{\begin{pmatrix}0\\3\\4\end{pmatrix}}{\sqrt{3^2+4^2}}=\frac{33}{\sqrt{3^2+4^2}}$$

The 2 planes are

$$\mathbf{r}\cdot\frac{\begin{pmatrix}0\\3\\4\end{pmatrix}}{\sqrt{3^2+4^2}}=\frac{33}{\sqrt{3^2+4^2}}+10 \quad\text{and}\quad \mathbf{r}\cdot\frac{\begin{pmatrix}0\\3\\4\end{pmatrix}}{\sqrt{3^2+4^2}}=\frac{33}{\sqrt{3^2+4^2}}-10$$

i.e. $\mathbf{r}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=83$ and $\mathbf{r}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=-17$

i.e. $3y+4z=83$ and $3y+4z=-17$

Method 2:
(Obtain distance between 2 planes by considering length of projection of vector between a point on each plane)

Let equation of the plane be $\mathbf{r}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=b$.

Let point H be a point on this plane with coordinates $\left(0,\frac{b}{3},0\right)$.

$$\overrightarrow{CH}=\begin{pmatrix}0\\\frac{b}{3}-11\\0\end{pmatrix}$$

Distance between 2 planes = Length of projection of $\overrightarrow{CH}$ onto normal $=10$

$$\frac{\left|\begin{pmatrix}0\\\frac{b}{3}-11\\0\end{pmatrix}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}\right|}{\sqrt{3^2+4^2}}=10$$

$|b-33|=50$

$b-33=50$ or $b-33=-50$

$b=83$ or $b=-17$

The 2 planes are $3y+4z=83$ and $3y+4z=-17$.

(ii) $\overrightarrow{OG}=\begin{pmatrix}3\\8\\6\end{pmatrix}$

$$\mathbf{r}=\begin{pmatrix}3\\8\\6\end{pmatrix}+\lambda\begin{pmatrix}0\\3\\4\end{pmatrix},\ \lambda\in\mathbb{R}$$

$$\begin{pmatrix}3\\8+3\lambda\\6+4\lambda\end{pmatrix}\cdot\begin{pmatrix}0\\3\\4\end{pmatrix}=33$$

$48+25\lambda=33$

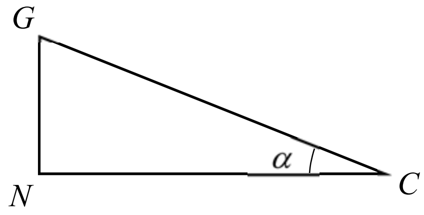
$\lambda=-\frac{3}{5}$

Coordinates of point is $\left(3,\frac{31}{5},\frac{18}{5}\right)$.

(iii) **Method 1:** (Use trigo ratio of right angle triangle CGN)

Let the point obtained in (iii) be N.

$\overrightarrow{GN} = \begin{pmatrix}3\\8\\6\end{pmatrix} - \frac{1}{5}\begin{pmatrix}15\\31\\18\end{pmatrix} = \frac{3}{5}\begin{pmatrix}0\\3\\4\end{pmatrix}$, $\overrightarrow{CG} = \begin{pmatrix}3\\-3\\6\end{pmatrix}$



$\sin\alpha = \frac{GN}{CG} = \frac{3}{\sqrt{3^2+3^2+6^2}}$

$\alpha = 24.1^\circ$

<u>Method 2:</u>

$\overrightarrow{CG} = \begin{pmatrix}3\\-3\\6\end{pmatrix}$

$\begin{pmatrix}3\\-3\\6\end{pmatrix} \cdot \begin{pmatrix}0\\3\\4\end{pmatrix} = \sqrt{3^2+3^2+6^2}\sqrt{3^2+4^2}\cos\beta$

$\cos\beta = \frac{15}{5\sqrt{54}}$

$\beta = 65.905^\circ$

$\alpha = 90^\circ - 65.905^\circ = 24.1^\circ$

11(i) Rate of change of population: $\frac{\mathrm{d}n}{\mathrm{d}t} = a - bn$

Since $\frac{\mathrm{d}n}{\mathrm{d}t} = 0$ when $n = 1$, $0 = a - b$. Hence, $a = b$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = a - an = a(1-n)$$

$$\int \frac{1}{1-n}\,\mathrm{d}n = \int a\ \mathrm{d}t$$

$$-\ln|1-n| = at + C$$

$$\ln|1-n| = -at - C$$

$$|1-n| = \mathrm{e}^{-at-C}$$

$$1 - n = \pm\mathrm{e}^{-C}\mathrm{e}^{-at}$$

$$n = 1 + A\mathrm{e}^{-at},\ A = \mp\mathrm{e}^{-C}$$

Sub $t = 0, n = 3$: $3 = 1 + A\mathrm{e}^{0} \Rightarrow A = 2$

Sub $t = 1, n = 2$: $2 = 1 + 2\mathrm{e}^{-a} \Rightarrow \frac{1}{2} = \mathrm{e}^{-a} \Rightarrow -a = \ln\frac{1}{2}$

Hence, $n = 1 + 2\mathrm{e}^{\ln\frac{1}{2}t} = 1 + 2\left(\mathrm{e}^{\ln\frac{1}{2}}\right)^t = 1 + 2\left(\frac{1}{2}\right)^t = 1 + 2^{1-t}$

(ii) $\dfrac{\mathrm{d}^2 n}{\mathrm{d}t^2} = 4 - 6t$

$\dfrac{\mathrm{d}n}{\mathrm{d}t} = 4t - 3t^2 + C$

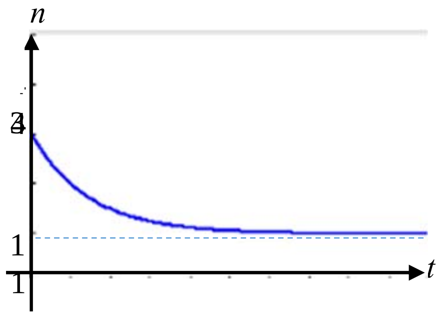
$n = 2t^2 - t^3 + Ct + D$

Sub $t = 0, n = 3$: $D = 3$

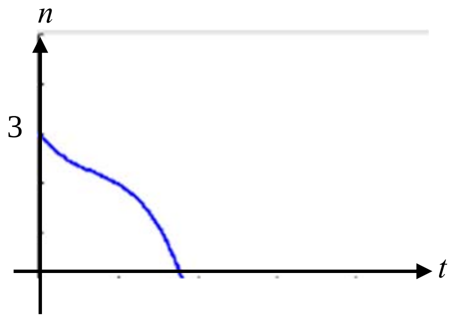
Sub $t = 1, n = 2$: $C = -2$

Hence, $n = 2t^2 - t^3 - 2t + 3$

(iii) Condition I



Condition II

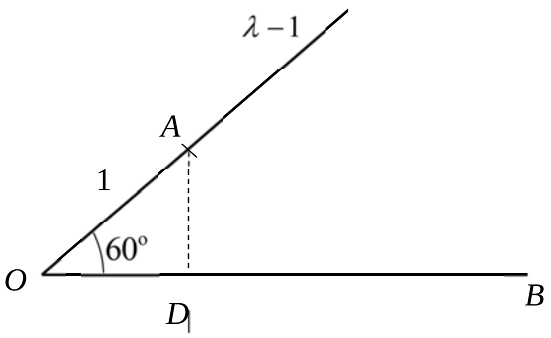


Based on the graphs, the population decreases but approaches a constant of 1000 for Condition I. Whereas for Condition II, the microorganism would die off quickly. Hence Conditions II is more harmful.

PJC J2 H2 Maths Prelim Paper 2 Solution:

1

(i) **Method 1:**



$$\overrightarrow{OD} = \left(4\cos 60^{\circ}\right)\hat{\mathbf{b}}$$

$$= \left(4\cos 60^{\circ}\right)\frac{\mathbf{b}}{|\mathbf{b}|}$$

$$= (2)\frac{\mathbf{b}}{8}$$

$$= \frac{1}{4}\mathbf{b} \text{ (shown)}$$

Method 2:

$$\overrightarrow{OD} = \left|\mathbf{a}\cdot\hat{\mathbf{b}}\right|\hat{\mathbf{b}}$$

$$= \left|\mathbf{a}\cdot\frac{\mathbf{b}}{|\mathbf{b}|}\right|\frac{\mathbf{b}}{|\mathbf{b}|}$$

$$= \frac{1}{|\mathbf{b}|^2}\left|\mathbf{a}\cdot\mathbf{b}\right|\mathbf{b}$$

$$= \frac{1}{|\mathbf{b}|^2}|\mathbf{a}||\mathbf{b}|\left|\cos 60^{\circ}\right|\mathbf{b}$$

$$= \frac{1}{8}(4)\left(\frac{1}{2}\right)\mathbf{b}$$

$$= \frac{1}{4}\mathbf{b} \text{ (shown)}$$

Method 3:

$$\overrightarrow{AD}\cdot\overrightarrow{OB} = 0$$

$$\left(\overrightarrow{OD} - \overrightarrow{OA}\right)\cdot\overrightarrow{OB} = 0$$

$$\overrightarrow{OD}\cdot\overrightarrow{OB} - \overrightarrow{OA}\cdot\overrightarrow{OB} = 0$$

$$\left|\overrightarrow{OD}\right|\left|\overrightarrow{OB}\right| = \left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|\cos 60^{\circ}$$

$$\left|\overrightarrow{OD}\right| = 4\cos 60^{\circ} = 2$$

Hence, $\overrightarrow{OD} = \frac{2}{8}\overrightarrow{OB} = \frac{1}{4}\mathbf{b}$ (Shown)

(ii) $\overrightarrow{BC} = \lambda\mathbf{a} - \mathbf{b}$

Equation of line BC: $\mathbf{r} = \mathbf{b} + \mu\left(\lambda\mathbf{a} - \mathbf{b}\right)$

$$\mathbf{r} = \lambda\mu\mathbf{a} + (1-\mu)\mathbf{b} \text{ (shown)}$$

(iii) Area of $\Delta ODE = \frac{1}{2}\left|\overrightarrow{OD} \times \overrightarrow{OE}\right| = \sqrt{300}$

$$\frac{1}{2}\left|\frac{1}{4}\mathbf{b} \times \left[\lambda\mu\mathbf{a} + (1-\mu)\mathbf{b}\right]\right| = \sqrt{300}$$

$$\frac{1}{8}\left|\mathbf{b} \times \lambda\mu\mathbf{a}\right| = \sqrt{300}$$

$\lambda\left|\mathbf{b} \times \mu\mathbf{a}\right| = 8\sqrt{300}$ (since $\lambda > 0$)

$$\lambda|\mu||\mathbf{b}||\mathbf{a}|\left|\sin 60^\circ\right| = 8\sqrt{300}$$

$$\lambda|\mu|(8)(4)\left(\frac{\sqrt{3}}{2}\right) = 8\sqrt{300}$$

$$|\mu| = \frac{5}{\lambda}$$

$\mu = \frac{5}{\lambda}$ or $\mu = -\frac{5}{\lambda}$

2 Let $z = x + \mathrm{i}y$

$$(x + \mathrm{i}y)^2 = 5 - 12\mathrm{i}$$

$$x^2 - y^2 + \mathrm{i}2xy = 5 - 12\mathrm{i}$$

Comparing real and imaginary components,

Real: $x^2 - y^2 = 5$

Imaginary: $2xy = -12 \Rightarrow y = \frac{-6}{x}$

$$x^2 - \left(\frac{-6}{x}\right)^2 = 5$$

$$x^4 - 5x^2 - 36 = 0$$

$$x^2 = \frac{5 \pm \sqrt{5^2 - 4(1)(-36)}}{2(1)} = \frac{5 \pm 13}{2}$$

$x^2 = -4$ (rej since $x^2 \geq 0$) or $x^2 = 9$

$x = 3$ or $x = -3$

$y = -2$ or $y = 2$

Hence, $z = 3 - 2\mathrm{i}$, $z = -3 + 2\mathrm{i}$

$$w^4 - 10w^2 + 169 = 0$$

$$w^2 = \frac{10 \pm \sqrt{10^2 - 4(1)(169)}}{2(1)} = \frac{10 \pm 24\mathrm{i}}{2} = 5 \pm 12\mathrm{i}$$

Since $w^4 - 10w^2 + 169 = 0$ is a polynomial with real coefficients, the roots occurs in conjugate pairs.

Hence, the roots are $w = 3 - 2\mathrm{i}$, $w = 3 + 2\mathrm{i}$, $w = -3 - 2\mathrm{i}$, $w = -3 + 2\mathrm{i}$

(a) $\arg(q) = \arg\left[\dfrac{(-1+\mathrm{i})\,p^*}{p^2}\right] = \arg(-1+\mathrm{i}) + (-\theta) - (2\theta) = \dfrac{3}{4}\pi - 3\theta$

Since q^2 is real,

$\sin\left(\dfrac{3}{2}\pi - 6\theta\right) = 0$

$\dfrac{3}{2}\pi - 6\theta = -\pi, 0, \pi$

$6\theta = \dfrac{1}{2}\pi, \dfrac{3}{2}\pi, \dfrac{5}{2}\pi$

$\theta = \dfrac{1}{12}\pi, \dfrac{1}{4}\pi, \dfrac{5}{12}\pi$

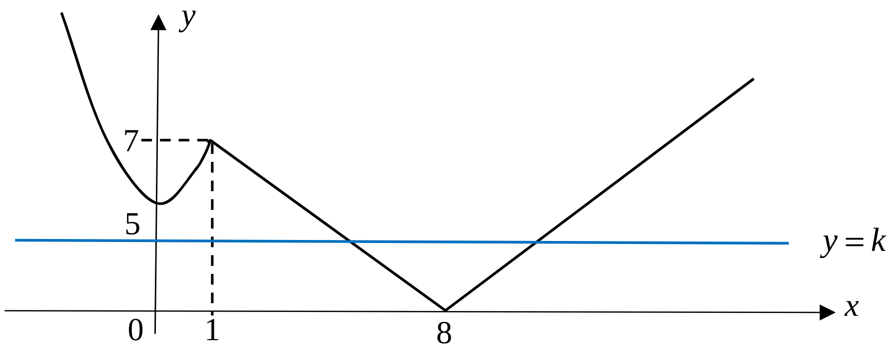
Since q^2 is negative,

$\cos\left(\dfrac{3}{2}\pi - 6\theta\right)$ is negative $\Rightarrow\ \theta \neq \dfrac{1}{4}\pi$.

Hence $\theta = \dfrac{1}{12}\pi, \dfrac{5}{12}\pi$

3

(i)



$\mathrm{R_f} = [0, \infty)$

(ii) Since there exists a horizontal line $y = k$ that cuts the graph of f at more than one point, f is not one-one. Hence, f^{-1} does not exist.

(ii) Let $\mathrm{g}^{-1}(4) = x \Rightarrow g(x) = 4$

$\sqrt{x} + 1 = 4 \Rightarrow \sqrt{x} = 3 \Rightarrow x = 9$

Hence, $\mathrm{g}^{-1}(4) = 9$

(iii) $\mathrm{R_g} = (1, 5]$

Hence, $\mathrm{fg}(x) = -\left[\left(\sqrt{x}+1\right) - 8\right] = -\sqrt{x} + 7$, $0 < x \le 16$.

4

(i) $$u_r - u_{r+1} = \frac{1}{r!} - \frac{1}{(r+1)!}$$

$$= \frac{1}{r!} - \frac{1}{(r+1)r!}$$

$$= \frac{(r+1)-1}{(r+1)r!}$$

$$= \frac{r}{(r+1)r!}$$

$$= \frac{1}{(r+1)(r-1)!}$$

$$= \frac{1}{r(r-1)!+(r-1)!}$$

$$= \frac{1}{r!+(r-1)!} \text{ (Shown)}$$

(ii) $$\sum_{r=1}^{N} \frac{1}{r!+(r-1)!} = \sum_{r=1}^{N} u_r - u_{r+1}$$

$$= u_1 - \cancel{u_2}$$

$$+ \quad \cancel{u_2} - \cancel{u_3}$$

$$+ \quad \cancel{u_3} - u_4$$

$$\ldots\ldots$$

$$+ \quad \cancel{u_{N-2}} - \cancel{u_{N-1}}$$

$$+ \quad \cancel{u_{N-1}} - \cancel{u_N}$$

$$+ \quad \cancel{u_N} - u_{N+1}$$

$$= u_1 - u_{N+1}$$

$$= 1 - \frac{1}{(N+1)!}$$

(iii) As $N \to \infty$, $\frac{1}{(N+1)!} \to 0$ $\sum_{r=1}^{N} \frac{1}{r!+(r-1)!} \to 1$ which is finite, hence $\sum_{r=1}^{N} \frac{1}{r!+(r-1)!}$ converges and the sum to infinity is 1

(iv) $$\sum_{r=1}^{N+1}\frac{1}{r!+(r+1)!} = \sum_{r=2}^{N+2}\frac{1}{r!+(r-1)!}$$

$$= \sum_{r=1}^{N+2}\frac{1}{r!+(r-1)!}-\frac{1}{2}$$

$$= 1-\frac{1}{(N+3)!}-\frac{1}{2}$$

$$= \frac{1}{2}-\frac{1}{(N+3)!}$$

(v) Since $r! > (r-1)!$

$$\frac{1}{r!+r!}<\frac{1}{r!+(r-1)!}$$

$$\sum_{r=1}^{N}\frac{1}{r!+r!}<\sum_{r=1}^{N}\frac{1}{r!+(r-1)!}$$

$$\sum_{r=1}^{N}\frac{1}{2r!}<\sum_{r=1}^{N}\frac{1}{r!+(r-1)!}$$

$$\sum_{r=1}^{N}\frac{1}{r!}<2\sum_{r=1}^{N}\frac{1}{r!+(r-1)!}=2\left(1-\frac{1}{(N+1)!}\right) \quad < 2 \text{ since } \frac{1}{(N+1)!}>0.$$

5 (a)(i) There are 2 possible cases: Case 1 Case 2

Case 1	Case 2
GBG	BGB
BGB	GBG

For each case, there are $3!\times3!$ ways

Hence, total number of ways = $2\times3!\times3!=72$

(a)(ii) Group the 2 girls together: 2!

Number of ways to place the group of 2 girls: 4

No of ways to arrange the remaining 4 person: 4!

Hence, number of ways = $2!\times4\times4!=192$

(b) Case 1 ($5+1$): Number of ways = ${}^6C_5\times(5-1)!\times(1-1)!=144$

Case 2 ($4+2$): Number of ways = ${}^6C_4\times(4-1)!\times(2-1)!=90$

Case 3 ($3+3$): Number of ways = $\dfrac{{}^6C_3\times(3-1)!\times(3-1)!}{2}=40$

(divide by 2 because the 2 tables are of identical size)

Hence, total number of ways = $144+90+40=274$

6

(i) $\frac{90}{360}=\frac{1}{4}$, $\frac{135}{360}=\frac{3}{8}$

$\mathrm{P}(X=0)=\mathrm{P}(0,0)+\mathrm{P}(0,1)\times 2!+\mathrm{P}(0,2)\times 2!$

$$=\left(\frac{3}{8}\right)^2+\left(\frac{3}{8}\right)\left(\frac{3}{8}+\frac{1}{4}\right)2!=\frac{39}{64}$$

$\mathrm{P}(X=1)=\mathrm{P}(1,1)=\left(\frac{3}{8}\right)^2=\frac{9}{64}$

$\mathrm{P}(X=2)=\mathrm{P}(1,2)\times 2!=\left(\frac{3}{8}\right)\left(\frac{1}{4}\right)\times 2!=\frac{3}{16}$

$\mathrm{P}(X=4)=\mathrm{P}(2,2)=\left(\frac{1}{4}\right)^2=\frac{1}{16}$

X	0	1	2	4
$\mathrm{P}(X=x)$	$\frac{39}{64}$	$\frac{9}{64}$	$\frac{3}{16}$	$\frac{1}{16}$

(ii) $\mathrm{E}(X)=(0)\left(\frac{39}{64}\right)+(1)\left(\frac{9}{64}\right)+(2)\left(\frac{3}{16}\right)+(4)\left(\frac{1}{16}\right)=\frac{49}{64}$

$\mathrm{E}(X^2)=(0)^2\left(\frac{39}{64}\right)+(1)^2\left(\frac{9}{64}\right)+(2)^2\left(\frac{3}{16}\right)+(4)^2\left(\frac{1}{16}\right)=\frac{121}{64}$

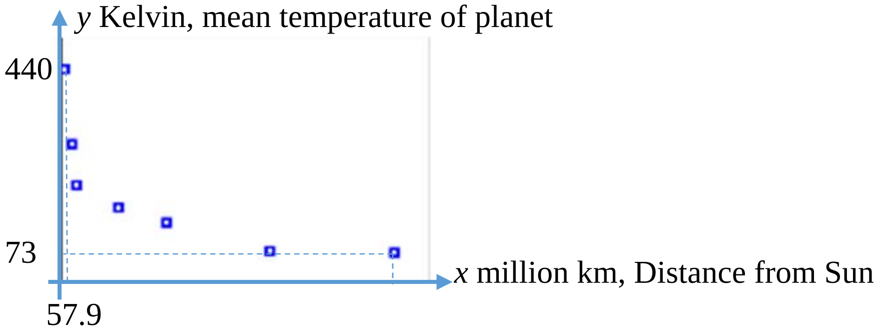
$\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2=\frac{5343}{4096}$

(iii) $\mathrm{E}(5X)=3.8281$

Hence, least integer value of y is 5.

7

(i)



From the diagram, it is observed that as x increases, y decreases by decreasing amounts. Hence, a linear model is not appropriate.

(ii) (a) For $y = a + \frac{b}{x}$, $r = 0.968656 \approx 0.9687$

(b) For $y = c + \frac{d}{\sqrt{x}}$, $r = 0.991301 \approx 0.9913$

(iii) Since the r value of $y = a + \frac{b}{x}$ is nearer to 1 than the r value of $y = c + \frac{d}{\sqrt{x}}$, $y = a + \frac{b}{x}$ is a better model.

(iv) Using GC, $y = \frac{3052.7}{\sqrt{x}} + 33.963 \approx \frac{3050}{\sqrt{x}} + 34.0 \text{ (1dp)}$

$$y = \frac{3052.7}{\sqrt{108.2}} + 33.963 = 327.44 \approx 327$$

Since r-value is near to 1 and this is an interpolation, the estimate is a reliable one.

(v) $y + 273 = \frac{20504.39}{x} + 107.07$

$$y = \frac{20504.39}{x} - 165.93$$

8(i) A: mass of a peach from Orchard A. A: $\mathrm{N}(145, 15^2)$

$\mathrm{P}(X < 140) = 0.36944 \approx 0.369$

(ii) $\mathrm{P}(A < 140)\left[\mathrm{P}(A \geq 140)\right]^2 \times \frac{3!}{2!} = 0.44067 \approx 0.440$

Alternatively

Let Y: number of peaches from Orchard A with mass less than 140g out of 3

Y: $B(3,\ \mathrm{P}(A < 140))$, i.e. Y: $B(3,\ 0.36944)$

$\mathrm{P}(Y = 1) = 0.44067 \approx 0.440$

(iii) $\mathrm{P}(A \geq k) \leq 0.15$

Consider $\mathrm{P}(A \geq k) = 0.15$

From GC, k = 160.55

Hence, for $\mathrm{P}(A \geq k) \leq 0.15$, $k \geq 160.55$

i.e. $k \geq 161$

(iv) Let $X = A_1 + ... + A_4 \sim \mathrm{N}(145 \times 4, 15^2 \times 4) \Rightarrow X \sim \mathrm{N}(580, 900)$

$3B$: $\mathrm{N}(190 \times 3, 20^2 \times 3^2) \Rightarrow 3B$: $\mathrm{N}(570, 3600)$

$X - 3B$: $\mathrm{N}(580 - 570, 900 + 3600) \Rightarrow X - 3B$: $\mathrm{N}(10, 4500)$

$\mathrm{P}(|X - 3B| \geq 15) = 1 - \mathrm{P}(|X - 3B| \leq 15) = 1 - \mathrm{P}(-15 < X - 3B < 15) = 0.824986 \approx 0.825$

Assume that the masses of all peaches are independent of one another.

9(i) 1) The assessment results of the candidates are independent of each other.

2) The probability of each candidate passing the assessment is constant.

(ii) $P(S > 40) = 1 - P(S \le 40) = 0.66916 \approx 0.669$

(iii)
$$P(S \ge m) \ge 0.9$$
$$1 - P(S \le m-1) \ge 0.9$$
$$P(S \le m-1) \le 0.1$$

When $m = 36$, $P(S \le m-1) = 0.0362$ (< 0.1)

When $m = 37$, $P(S \le m-1) = 0.0632$ (< 0.1)

When $m = 38$, $P(S \le m-1) = 0.1041$ (> 0.1)

When $m = 39$, $P(S \le m-1) = 0.1618$ (> 0.1)

Hence, greatest value of m is 37.

(iv) $P(S \ge 45) = 1 - P(S \le 44) = 0.24378 \approx 0.244$

(v)
$$P(S < 50 \mid S \ge 45) = \frac{P(45 \le S < 50)}{P(S \ge 45)}$$
$$= \frac{P(S \le 49) - P(S \le 44)}{1 - P(S \le 44)} = 0.94307 \approx 0.943$$

(vi) $X \sim$ Number of good days, out of 50.

$X \sim B(50, 0.24378)$

$P(X \le 10) = 0.29608 \approx 0.296$

10(i) $\bar{x} = \frac{1705}{35} + 250 = 298.714 \approx 299$ (3s.f.)

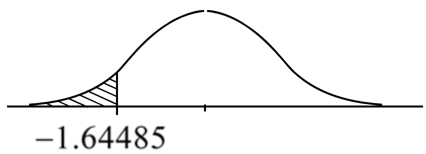
$$s^2 = \frac{1}{34}\left(83650 - \frac{1705^2}{35}\right) = 17.415966 \approx 17.4 \text{ (3 s.f.)}$$

(ii) Let X : amount of fruit juice in a bottle

$H_0 : \mu = 300$ vs $H_1 : \mu < 300$, where μ is the mean amount of fruit juice in a bottle.

Since n = 35 is large, by Central Limit Theorem,

$\bar{X} \sim N\left(300\ ,\ \frac{17.415966}{35}\right)$ approximately.



Level of significance: 5%

Critical region: $z < -1.64485$

Test statistic: $z = \frac{\bar{x} - \text{"claimed value"}}{s/\sqrt{n}} = \frac{298.714 - 300}{\sqrt{\frac{17.415966}{35}}} = -1.82306 < -1.64485$

From GC, p-value = 0.034147 <0.05

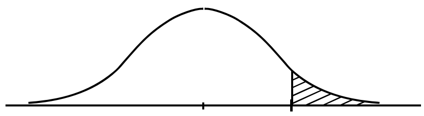
Since the p-value is less than the level of significance, we reject H_0. There is sufficient evidence, at the 5% level, to indicate that the mean amount of fruit juice in each bottle is less than 300ml (OR: to indicate that the fruit juice seller overstated the mean amount of fruit juice in each bottle).

(iii) Central Limit Theorem states that sample means will have a normal distribution when sample size is big enough.

(iv) $H_0: \mu = 300$ vs $H_1: \mu > 300$

Assuming X is normal,

$\overline{X} \sim N\left(300\ ,\ \dfrac{15.7}{15}\right)$ exactly.



1.6449

Level of significance: 5 %

Critical value: $z = 1.6449$

Test statistic: $z = \dfrac{\overline{x} - \text{"claimed value"}}{s/\sqrt{n}} = \dfrac{y-300}{\sqrt{\dfrac{15.7}{15}}}$

Since the seller's claim is valid, we reject H_0, i.e.

$$\frac{y-300}{\sqrt{\dfrac{15.7}{15}}} > 1.6449$$

$y > 301.6828$

Hence, least value of y is 301.68.

(v) Assume that the amount of fruit juice in each bottle follows a normal distribution.