HUA YI SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024

4-G3 / 5-G2

NAME	

CLASS			INDEX NUMBER	

MATHEMATICS 4052/01

PAPER 1

13 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use	
	90

Setter: Ms Jasmine Tan

Mathematical Formulae

Compound interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Calculate $\dfrac{-(-11.8)-\sqrt{(-11)^2-7\times16\times(-40)}}{2\times16}$.

Answer .... [1]

(b) There are 800 people in an auditorium, correct to the nearest hundred.

State the minimum number of people that could be in the auditorium at this time.

Answer .... people [1]

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2 **(a)** Express 1400 as the product of its prime factors.

Answer .... [1]

(b) Write down the smallest positive integer k such that $1400k$ is a perfect cube.

Answer $k =$ .... [1]

(c) n is a number between 300 and 400.

The highest common factor of n and 1400 is 35.

Find the largest possible value for n.

Answer $n =$ .... [2]

3 **(a)** Simplify $y^0 \div 9x^{-2} \times x^7$.

Answer .... [2]

(b) Simplify $\left(81a^{12}\right)^{\frac{5}{4}}$.

Answer .... [1]

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4 **(a)** Express as a single fraction in its simplest form $\dfrac{18b^7}{5c^2} \div \dfrac{3b^4}{81}$.

Answer .... [2]

(b) Use the laws of indices to show that $6^4 \times 100 + 116 \times 36^2$ can be expressed as **a single power of six**.

Answer [2]

5   In a greenhouse, the estimated number of flowering plants increased from 4100 in January 2024 to 4980 in June 2024. The number increased by c% every month.

Find the value of c.

Answer $c =$ .... [3]

6   Kyle runs a tennis club. 54 of the members are adults and 31 are children.

His aim is that **at least** 60% the members should be children.

Form an inequality to find the smallest number of children that Kyle would still need to recruit achieve his aim.

Answer   .... children [3]

7   A car travels at an average speed of 74.5 km/h for 2.25 hours.

   (a)   Convert 75 km/h to m/s.

Answer .... m/s [1]

   (b)   By rounding the numbers correct to 1 significant figure, find an estimate of the distance travelled by the car. Show your working clearly.

Answer .... km [2]

   (c)   Without doing any calculation, explain why the actual distance travelled by the car is greater than the answer to **(b)**.

Answer [1]

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8   Isha has written down five numbers.

   The mean of these numbers is 13.2, the median is 12 and the mode is 7.

   The largest number is three times the smallest number.

   Find the five numbers in **ascending** order.

Answer .... , ... , ... , ... , ... [2]

9 Factorise completely

(a) $2p^4 - 32s^4$,

Answer .... [3]

(b) $12cd - 9cx + 6xy - 8dy$.

Answer .... [2]

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10 (a) Express $9 - 8x + x^2$ in the form $a + (x + b)^2$. Find the value of a and of b.

Answer $a =$ .... [1]

$b =$ .... [1]

(b) Explain why when $x = 4$, the expression $9 - 8x + x^2$ has its minimum value.

Answer [1]

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11 Solve the equation $5 + 2x = \dfrac{20}{1+x}$.

Answer $x =$ .... or .... [3]

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12 The points $(4, 20)$ and $(10, -4)$ satisfy the curve given by the equation $y = ax^2 + bx - 4$.

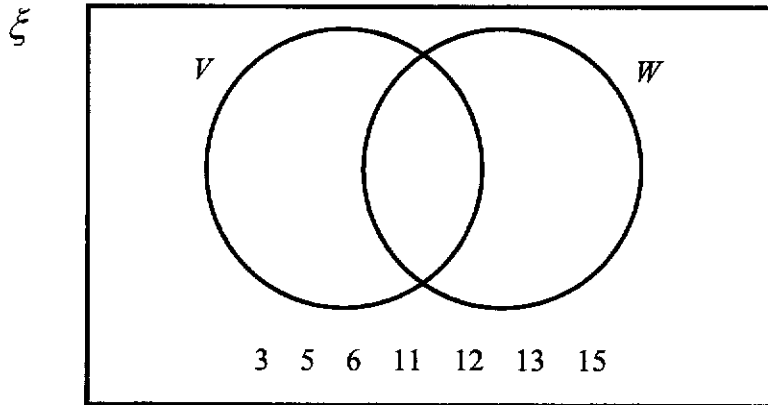Use an algebraic method to determine the values of a and b.

Answer $a =$ ...., $b =$ .... [4]

13 $\xi = \{\text{integers } x : 1 \le x \le 16\}$

$V = \{1, 2, 4, 8, 16\}$

$W = \{1, 4, 7, 9, 10, 14\}$

Some of the information is shown on the Venn diagram.

ξ

V W

3 5 6 11 12 13 15

(a) Complete the Venn diagram by representing all the elements in the given sets.

Answer in above Venn diagram [1]

(b) Describe the elements of Set V.

Answer [1]

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(c) List the elements contained in the set $V' \cap W$.

Answer .... [1]

(d) Find the number of elements in $(V \cap W') \cup (V \cup W)'$.

Answer .... [1]

(e) Use one of the symbols below to complete each statement.

$$= \quad \varnothing \quad \subset \quad \not\subset \quad \notin \quad \in \quad \xi$$

(i) $\{2, 16\}$ V

(ii) 13 .... W

Answer (e) (i) .... [1]

(ii) .... [1]

14 In grocery mart G, water costs $1.80 per litre, fresh milk costs $2.90 per litre and juice costs $2.30 per litre.

In grocery mart H, water costs $0.20 more per litre, fresh milk costs $0.40 less per litre and juice costs $0.10 less per litre.

This information can be represented by the matrix $\mathbf{Q} = \begin{pmatrix} 1.8 & 0.2 \\ 2.9 & -0.4 \\ 2.3 & -0.1 \end{pmatrix} \begin{matrix} W \\ M \\ J \end{matrix}$ with column headings $G \quad H$.

(a) Rayyen and Zinnie went shopping together.

Rayyen bought 4 litres of water, 2 litres of milk and 3 litres of juice.

Zinnie bought 3 litres of water and 4 litres of juice.

Represent their purchases in a 2×3 matrix $\mathbf{P}$.

Answer $\mathbf{P} =$ .... [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{PQ}$.

Answer $\mathbf{R} =$ .... [2]

(c) State what the elements in the second column of matrix $\mathbf{R}$ represent.

Answer [1]

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(d) Rayyen shopped in grocery mart H. He got a discount coupon that entitled him to a discount of 15%. How much did he pay altogether for his items?
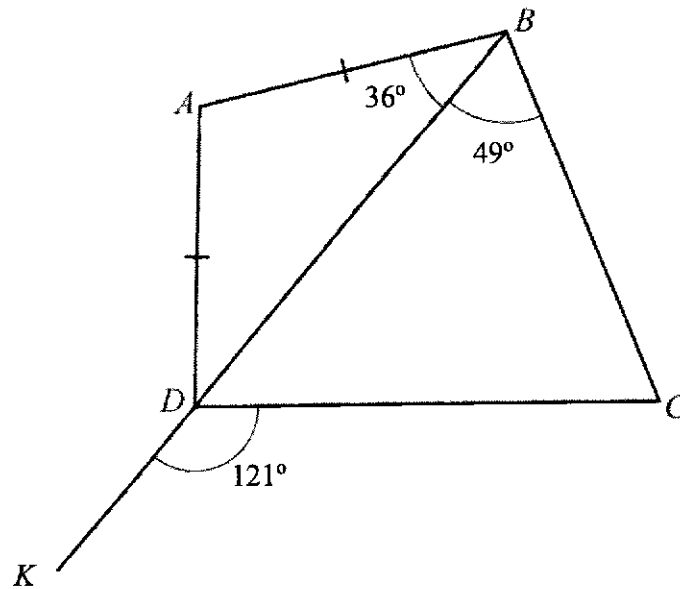
Answer $ .... [2]

15   In a regular polygon, the ratio of an interior angle : exterior angle = 14 : 1.

   (a)   Find the number of sides of the polygon.

   Answer   .... sides [3]

   (b)   Find the sum of the interior angles of the polygon.

   Answer   ....° [1]

16   In the diagram, BDK is a straight line and $AB = AD$.

Angle $ABD = 36°$, angle $CBD = 49°$ and angle $CDK = 121°$.

Explain why it is possible to draw a circle that passes through the points A, B, C and D.

Give reasons for each step of your working.

Answer   [3]

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17   The diagram represents a plot of land, *EFGH*, which is to be used for an observatory.
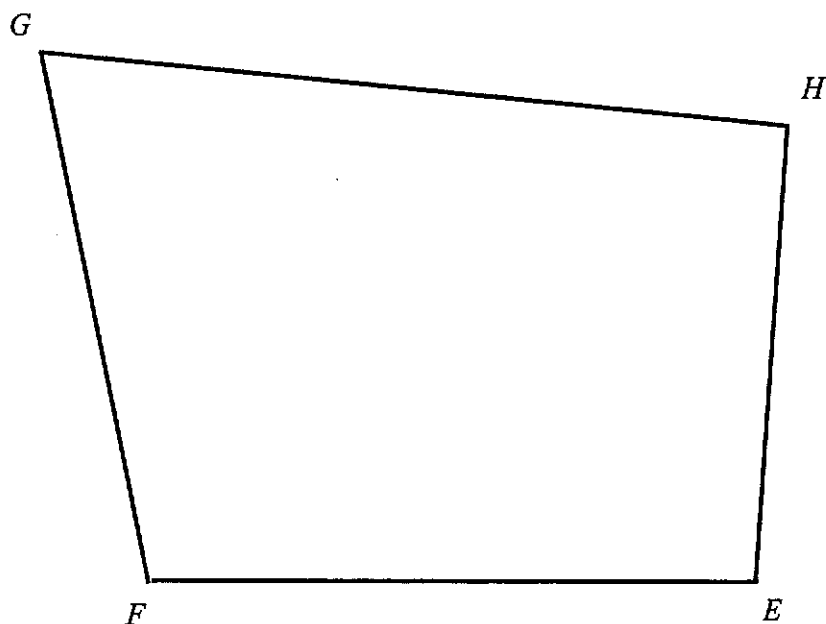
(a)   Construct the bisector of the angle *EHG*.   [1]

(b)   Construct the perpendicular bisector of *EF*.   [1]

(c)   A café is to be built in the observatory, nearer to *E* than to *F* and nearer to *GH* than to *EH*.

Shade the region where the café is to be built.   [1]

18 **(a)** A cargo ship has an average fuel consumption of 0.000 892 kilometres per litre.

Write this consumption in litres per kilometre.

Answer .... l / km [1]

(b) A model of another cargo ship is made to a scale of 1 : 60.

The length of this model cargo ship is 550 cm.

(i) Find the actual length of this cargo ship in metres.

Answer .... m [1]

The capacity of the fuel tanks in the model cargo ship is 7.75 litres.

(ii) Find the actual capacity of the fuel tanks of the cargo ship.

Express your answer in standard form.

Answer .... l [2]

19 A composite solid is made from a cylinder and two hemispheres.

The cylinder has radius p and length $2p$, while the hemispheres have radius p.
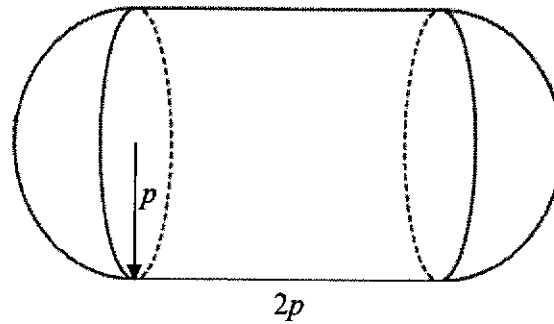
The total surface area of the solid is six times the total surface area of a solid cone with radius p and slant height l.

Find l in terms of p.

Answer $l = $ .... [3]

20 Gino can paint 6 fence panels in 4 hours, while Danish can paint 7 fence panels in 5 hours. Gino and Danish work together to paint a total of 21 panels.

If they continue to paint at the same rate, how long will it take them to paint 21 panels?
Give your answer in hours and minutes, correct to the nearest minute.

Answer .... h .... min [3]

21 A fitness centre has 16 employees.

One of the 16 employees is selected at random.

The probability that it is a woman working part time is $\dfrac{1}{8}$.

Two of the 16 employees are selected at random.

The probability that they are both men working full time is $\dfrac{1}{8}$.

Complete the table of information below about the 16 employees of the fitness centre.

Show all supporting calculations clearly.

Answer [4]

	Part-time employees	*Full-time employees*
Women		5
Men		

22   A librarian wants to find out how much time patrons spend at the library in a week.

He uses the questionnaire.

How many hours do you spend at the library in a week (including weekends)?

Please tick one box.

1 – 2   *3 – 4*   *5 – 7*   *over 7*

List **two** ways to improve the questionnaire.

Answer   [2]

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23   K is the point $(7,-8)$ and L is the point (x,y).

The gradient of the line KL is $\dfrac{2}{3}$.

Maverick claims that it is possible to express x in terms of y, such that $x = a + by$, where a and b are constants.

Explain why Maverick is correct.

Answer   [2]

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24   *A*, *B* and *C* are points on the circle with centre *O*.

Q and R are points on the circle with centre *P*.

CSR and BSQ are tangents on to both circles.

AOB is a straight line.
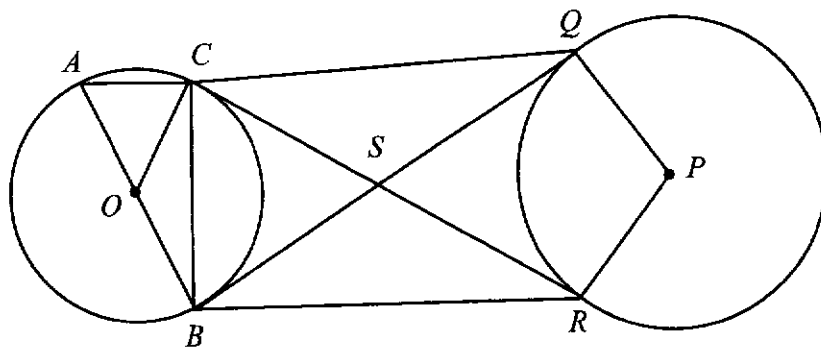
(a)   Show that triangle *CQS* is congruent to triangle *BRS*.

Give a reason for each statement you make.

Answer   [3]

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24 **(b)** Angle $ABC = y°$

Find, in terms of y,

(i) angle BAC,

Answer ....° [1]

(ii) angle QPR.

Answer ....° [2]

25 *ORS* is a triangle.

J is a point on *RS* such that *RJ* : *JS* = 3 : 2.

$\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{OS} = \mathbf{s}$.
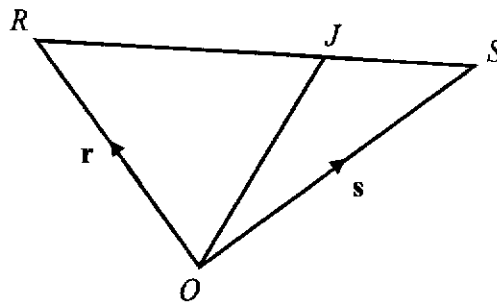
(a) Show that $\overrightarrow{OJ} = \dfrac{1}{5}(2\mathbf{r} + 3\mathbf{s})$.

Answer   [2]

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(b) *X* is a point such that $\overrightarrow{RX} = \dfrac{1}{5}(\mathbf{r} + 9\mathbf{s})$.

Explain why *O*, *J* and *X* lie on a straight line.

Answer   [2]

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END OF PAPER

HUA YI SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024

4-G3 / 5-G2

NAME

CLASS

INDEX NUMBER

MATHEMATICS

PAPER 2

4052/02

19 August 2024

2 hour 15 minutes

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For Examiner's Use	
	90

Setter: Ms Jasmine Tan

[Turn Over

Mathematical Formulae

Compound interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = $\pi r l$

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Express as a single fraction in its simplest form $\dfrac{2}{7-3x} - \dfrac{1}{6-x}$.

Answer .... [2]

(b) It is given that $v = \dfrac{3-5w}{w+2} + 9$.

    (i) Find v when $w = -6$.

Answer .... [1]

    (ii) Rearrange the formula to make w the subject.

Answer $w = $ .... [3]

(c) Solve the equation $\dfrac{5}{x-2} - \dfrac{3}{x^2-4} = \dfrac{1}{7}$.

Give your solutions correct to two decimal places.

Answer $x =$ .... , $x =$ .... [5]

2 C is the point $(-9,1)$ and D is the point $(7,4)$.

$$\overrightarrow{CE} = \begin{pmatrix} -2 \\ 8 \end{pmatrix} .$$

(a)   Calculate the length of the line CD.

Answer   .... units [2]

(b)   Determine the coordinates of point E.

Answer E (.........., ...) [1]

(c)   Find the equation of the line DE.

Leave your answer in the form $ax + by = c$, where a, b and c are constants.

Answer   .... [3]

3 **(a)** The first four terms of a sequence are 5, $\dfrac{9}{4}$, $\dfrac{13}{9}$, $\dfrac{17}{16}$.

   (i) State the fifth term of the sequence.

   Answer .... [1]

   (ii) Find an expression, in terms of n, for the nth term, T_n, of this sequence.

   Answer $T_n =$ .... [2]

(b) Elijah finds a number grid from his board game.
The diagram shows part of a number grid.

A rectangle outlining four numbers, as shown, can be placed anywhere on the grid.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37					

(i) If p represents the number in the top right corner of the rectangle, write down an expression, in terms of p, for the number in the bottom left corner of the rectangle.

Answer .... [1]

(ii) Show that the difference between the products of the numbers in the opposite corners of the rectangle is always -7 .

Answer [2]

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(iii) Elijah says it is impossible for the sum of the four numbers in the rectangle to be 199.

Justify with relevant working, why he is correct.

Answer [3]

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4 (a) Sophie earns a monthly salary of $6875.

She gives 15% of this amount to her parents.

She puts 35% of the remainder into a savings account.

Calculate the amount she has left after giving to her parents and putting into her savings account. Leave your answer correct to the nearest dollar.

Answer $ .... [3]

(b) The cash price of a sofa is $830.

Sophie buys this sofa on credit.

She pays a deposit of one quarter of the cash price.

She then pays 3 monthly payments of $260.

Calculate the total amount Sophie pays for the sofa.

Answer $ .... [2]

(c) Sophie pays a monthly rent of $3174.20.

This is 18% more than her monthly rent last year.

Calculate her monthly rent last year.

Answer $ .... [2]

(d) During her vacation, Sophie visits her friend in Wellington.

Sophie spends NZD 940 in New Zealand using her credit card.

She is charged a 2.6% fee for the currency conversion.

The exchange rate between Singapore dollars (SGD) and New Zealand dollars (NZD) is SGD 100 = NZD 120.7206.

Calculate the total amount on Sophie's credit card bill, including the fee.

Give your answer in Singapore dollars, correct to the nearest cent.

Answer SGD .... [3]
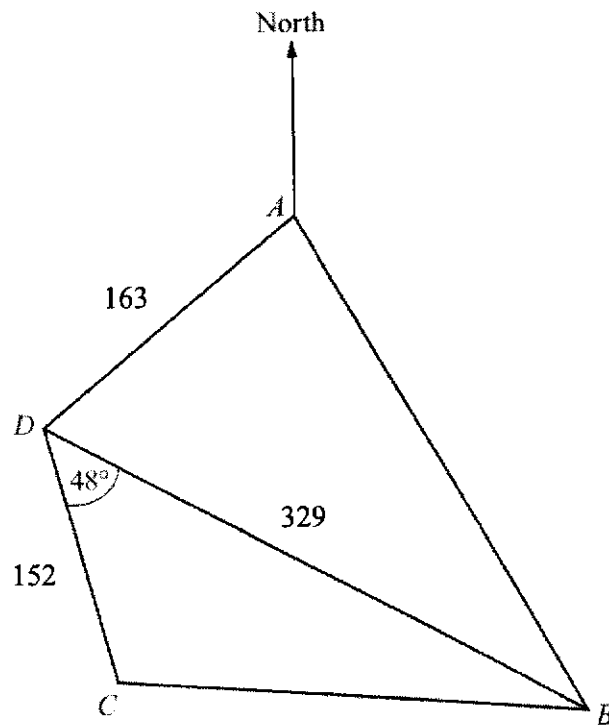
5

North

ABCD is a field on horizontal ground.

AD = 163 m, *BD* = 329 m, *CD* = 152 m and angle *BDC* = 48°.

The bearing of *B* from *A* is 151° and the bearing of *D* from *A* is 237°.

(a) Calculate the bearing of *D* from *B*.

Answer ....° [3]

(b) Calculate the distance from B to C.

Answer .... m [2]

(c) An aircraft is flying above D.

Find the angle of elevation of the aircraft from C when it is 245 m vertically above D.

Answer .... ° [2]

6 The diagram shows a semicircle, centre *O*, radius 16 cm.
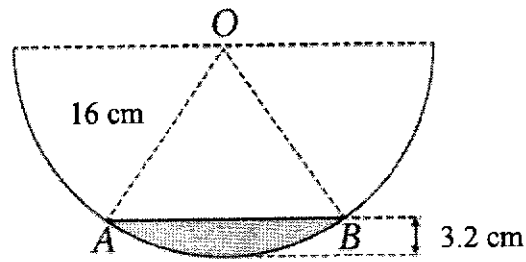
(a) Show that angle *AOB* = 1.287 radians, correct to 3 decimal places.

Answer [2]

(b) Calculate the area of the shaded region.

Answer .... cm² [3]

(c) The semicircle is the cross section of a water trough of length 2.8 m, standing on level ground.

The shaded area represents the water in the trough.

(i) Calculate the volume of water, in cm^3, in the trough.

Leave your answer in standard form.

Answer .... cm^3 [3]

(ii) Calculate the number of litres of water that must be added to fill the trough.

Answer .... litres [3]

7   A researcher wants to conduct a study to find out if there is a correlation between the analytical skills of adults with age.

The researcher invited 80 adults to solve a number puzzle.

The cumulative frequency curve shows the distribution of the time taken.
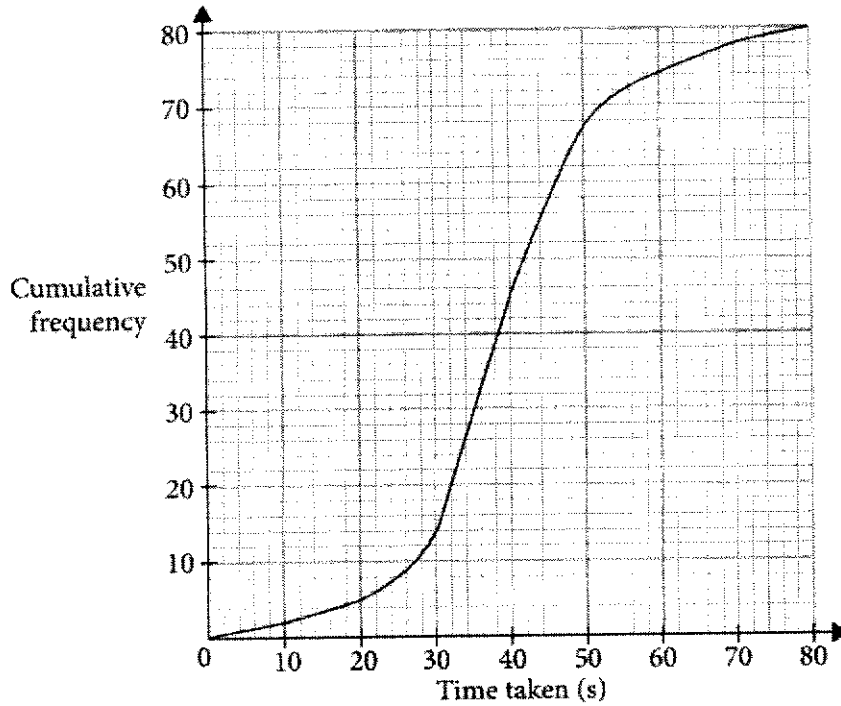
(a)   Use the curve to estimate

(i)   the median time taken,

Answer   .... s [1]

(ii)   the interquartile range of the time taken.

Answer   .... s [2]

(b)   20% of the adults took more than *n* seconds to solve the puzzle.

Find *n*.

Answer   *n* = ... [2]

(c) The frequency distribution of the data for the 80 adults invited to solve the number puzzle is shown in the table.

Time taken, x (seconds)	Frequency
$0 < x \le 20$	5
$20 < x \le 40$	40
$40 < x \le 60$	29
$60 < x \le 80$	6

Find an estimate of the standard deviation of the time taken by the 80 adults to solve the number puzzle.

Answer .... s [2]

(d) The same group of adults were each given a word puzzle to solve.
The box-and-whisker plot shows the distribution of the time taken.

Time taken (s)
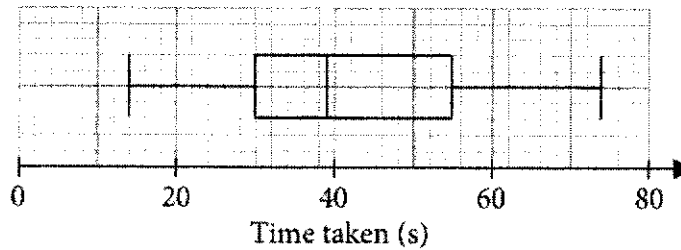
Make two comparisons between the performances of the adults in solving the two puzzles.

Answer [2]

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8   In the diagram, *XYZ* is a straight line.

$AY = 20$ m, $AZ = 12$ m and $YZ = 16$ m.

The ratio of *XY* : *YZ* is 1 : 4.
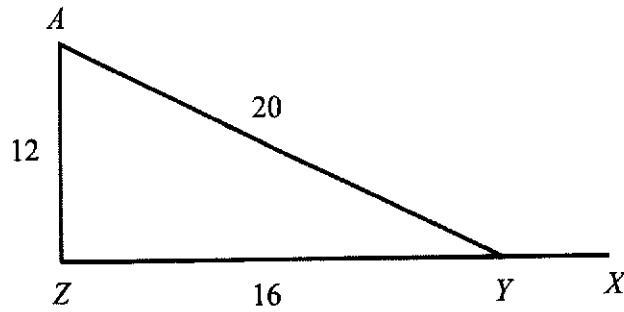
(a)   Prove that triangle *AYZ* is right-angled.

Answer   [2]

(b)   Find the value of $\cos \angle AYX$, giving your answer as a fraction in its lowest terms.

Answer .... [1]

(c)   The area of triangle *AXY* is 24m².

Lenard says: In another triangle *AYW*, whereby the length *YW* = *YX* and its area is the same as that of triangle *AXY*, it is possible for angle *Y* to be acute.

Explain why Lenard is correct. Use calculations to support your answer.

Answer   [2]

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9   The variables x and y are connected by the equation $y = \dfrac{x^3}{2} - 5x - 2$.

Some corresponding values of x and y are given in the table.

x	−3	−2	−1	0	1	2	3	4
y	−0.5	4	2.5	−2	−6.5	k	−3.5	10

(a) Find the value of k.

Answer $k =$ .... [1]

(b) On the grid on **page 19**, draw the graph of $y = \dfrac{x^3}{2} - 5x - 2$ for $-3 \le x \le 4$.   [2]

(c) The equation $\dfrac{x^3}{2} - 5x = 7$ has only one solution.

Explain how this can be deduced from your graph.

Answer   [2]

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(d) By drawing a tangent, estimate the gradient of the curve at $(1, -6.5)$.

Answer   Gradient = ... [2]

(e) **(i)** On the same grid, draw the line $y = 4 - x$ for $-1 \le x \le 4$. [2]

(ii) Write down the x-coordinate of the point where this line intersects the curve.

Answer $x = $ .... [1]

(iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$.

Find the value of A and the value of B.

Answer $A = $ ...., $B = $ .... [2]

BP~429

10   Javier and Stella are planning their exercise routines.

Javier and Stella have a body mass of 70 kg and 60 kg respectively.

The speed for walking and jogging of the two individuals are represented in the chart.

Speed comparison for Javier and Stella

They research the following information about the benefits of exercise.

HEALTH ADVICE

For optimal health benefits, adults should do at least 150 minutes of moderate-intensity aerobic activity, or at least 75 minutes of vigorous-intensity aerobic activity per week.

For additional health benefits, adults should increase their moderate-intensity aerobic activity to 300 minutes per week, or an equivalent combination of moderate-intensity and vigorous-intensity aerobic activity.

1 minute of vigorous-intensity aerobic activity is equivalent to 2 minutes of moderate-intensity aerobic activity.

E.g. 10 minutes of jogging = 20 minutes of brisk walking.

Muscle-strengthening activities should be done on 2 or more days per week.

Approximate calories burned during 30 minutes of aerobic exercise

	Body mass			
	60 kg	70 kg	80 kg	90 kg
Walking 5 km/h	105	120	135	150
Walking 6.5 km/h	130	150	170	190
Jogging 8 km/h	240	280	315	350
Jogging 9.5 km/h	300	345	390	435

Non-aerobic muscle-strengthening activities

❖ 1 hour of yoga burns approximately 3 calories per kilogram of body mass

❖ 1 hour of weight training burns approximately 4 calories per kilogram of body mass

(a) In their first week of exercise, they each plan to go for four brisk walks.

They will walk the same route each time.

The four walks together meet the minimum target for the time for optimal health benefits in one week.

(i) Find the distance of one of these walks.

Answer .... km [2]

(ii)   Find out how many more calories Javier burns in these four walks than Stella.

Answer   .... calories [1]

(b)   After one month, they change their routines.

Javier wants to gain additional health benefits.

He decides to do a 4 km walk 3 times per week and do a 6 km jog 2 times per week.

He will also attend a 45-minute yoga class 2 times per week.

Stella wants to maintain optimal health benefits.

She decides to do an 8 km jog 2 times per week.

She will also attend a 30-minute weight training session 2 times per week.

Javier says:

We will both meet our targets for exercise.

However, I will burn about 50% more calories than Stella during our exercise per week.

Is Javier correct?

Justify your decision with clear calculations.

Answer   [7]

Answer space for **10 (b)**

Answer space for **10 (b)**

Javier is ....

....

....

....

....

END OF PAPER

2

Mathematical Formulae

Compound interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

HUA YI SECONDARY SCHOOL

4-G3 / 5-G2

PRELIMINARY EXAMINATION 2024

NAME

CLASS

INDEX NUMBER

MATHEMATICS
PAPER 1

4052/01

13 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper.

MARKING SCHEME

| For Examiner's Use | 90 |

Setter: Ms Jasmine Tan

[Turn Over

3

1 (a) Calculate

$$\frac{-(-11.8)-\sqrt{(-11)^2-7\times16\times(-40)}}{2\times16}.$$

$-1.750958 = -1.75$ (3 s.f.) ----B1

Answer .... [1]

(b) There are 800 people in an auditorium, correct to the nearest hundred.
State the minimum number of people that could be in the auditorium at this time.

750 people ----B1

Answer .... people [1]

2 (a) Express 1400 as the product of its prime factors.

$1400 = 2^3 \times 5^2 \times 7$ ----B1

Answer .... [1]

(b) Write down the smallest positive integer k such that $1400k$ is a perfect cube.

$1400k = 2^3 \times 5^2 \times 7 \times k = 2^3 \times 5^2 \times 7 \times (5\times7^2)$

$k = 245$ ----B1

Answer $k =$ .... [1]

(c) n is a number between 300 and 400.
The highest common factor of n and 1400 is 35.
Find the largest possible value for n.

$$n = \quad 2^3 \times 5 \times 7 \times ?$$
$$1400 = \quad 2^3 \times 5^2 \times 7$$
$$\overline{\text{HCF} = \quad 5\times 7 = 35}$$

By guess-and-check, $n = 5\times7\times11 = 385$ ----B2
[Condition: largest possible value between 300 – 400]

Answer $n =$ .... [2]

4

3 (a) Simplify $y^0 \div 9x^{-2} \times x^7$.

$$y^0 \div 9x^{-2} \times x^7 = 1\times\frac{x^2}{9}\times x^7 \quad ---M1$$

$$= \frac{x^9}{9} \quad ---A1$$

Answer .... [2]

(b) Simplify $(81a^{12})^{\frac{5}{4}}$.

$$(81a^{12})^{\frac{5}{4}} = (\sqrt[4]{81})^5 \times a^{12\times\frac{5}{4}}$$

$$= 243a^{15} \quad ---B1$$

Answer .... [1]

4 (a) Express as a single fraction in its simplest form $\dfrac{18b^7}{5c^2} \div \dfrac{3b^4}{81}$.

$$\frac{18b^7}{5c^2} \div \frac{3b^4}{81} = \frac{18b^7}{5c^2} \times \frac{81}{3b^4}$$

$$= \frac{486b^3}{5c^2} \quad ---\left[\text{M1 for } \frac{486}{5}, \text{ A1 for full answer}\right]$$

Answer .... [2]

(b) Use the laws of indices to show that $6^4 \times 100 + 116 \times 36^2$ can be expressed as a single power of six.

Answer

$$6^4 \times 100 + 116 \times 36^2 = 6^4 \times 100 + 116 \times 6^4$$

$$= 6^4[100+116] \quad ---M1$$

$$= 6^4 \times 216$$

$$= 6^7 \quad ---A1 \ (shown)$$

5 In a greenhouse, the estimated number of flowering plants increased from 4100 in January 2024 to 4980 in June 2024. The number increased by $c\%$ every month.

Find the value of c.

$$4980 = P\left(1+\frac{r}{100}\right)^n$$

$$4980 = 4100(1+0.01c)^5 \quad ---M1$$

$$\sqrt[5]{\frac{249}{205}} \approx 1+0.01c$$

$$c = \frac{\sqrt[5]{\dfrac{249}{205}}-1}{0.01} \quad ---M1$$

$$= 3.965 \approx 3.97 \quad (3\text{s.f.}) \quad ---A1$$

Answer c = ... [3]

6 Kyle runs a tennis club. 54 of the members are adults and 31 are children.

His aim is that **at least 60%** the members should be children.

Form an inequality to find the smallest number of children that Kyle would still need to recruit to achieve his aim.

Let c be the no. of children needed to join the club.

$$\frac{31+c}{54+31+c} \geq 60\% \quad ---M1 \text{ (forms inequality)}$$

$$31+c \geq 0.6(85+c)$$

$$31+c \geq 51+0.6c$$

$$c-0.6c \geq 20 \quad ---M1 \text{ (isolate unknown)}$$

$$0.4c \geq 20$$

$$c \geq 50$$

$$\therefore \text{Smallest number} = 50 \quad ---A1$$

Answer .... children [3]

6

7 A car travels at an average speed of 74.5 km/h for 2.25 hours.

(a) Convert 75 km/h to m/s.

$$\frac{75\,000}{3600} = 20\frac{5}{6} = 20.8 \text{ m/s} \quad ---B1 \quad \text{(Accept either 1 form)}$$

Answer .... m/s [1]

(b) By rounding the numbers correct to 1 significant figure, find an estimate of the distance travelled by the car. Show your working clearly.

$$\text{Distance} = 74.5 \times 2.25 = 70 \times 2 \quad ---M1 \text{ (must show rounding to 1 s.f. each)}$$

$$= 140 \text{ km} \quad ---A1$$

Answer .... km [2]

(c) Without doing any calculation, explain why the actual distance travelled by the car is greater than the answer to **(b)**.

Answer [1]

Both the *average speed and time is greater than the 1 significant figure rounded values of speed and time* respectively. Thus, the actual distance travelled is greater than the answer in part **(b)**.

—— B1

8 Isha has written down five numbers.

The mean of these numbers is 13.2, the median is 12 and the mode is 7.

The largest number is three times the smallest number.

Find the five numbers in **ascending** order.

Smallest		Median		largest
7	7	12	y	21

Mean = 13.2

$$\frac{Sum}{5} = 13.2$$

$Sum = 66$, so $y = 19$.

> Any **3** consecutive values correct → B1
> ALL **5** values correct → Full mark, B2

Answer .... [2]

7

9 Factorise completely

(a) $2p^4 - 32s^4$.

$2p^4 - 32s^4 = 2[(p^2)^2 - (4s^2)^2]$ ---M1 [factorise by HCF]

$= 2[(p^2 + 4s^2)(p^2 - 4s^2)]$ ---M1 [factorise by $a^2 - b^2$]

$= 2[(p^2 + 4s^2)(p + 2s)(p - 2s)]$ ---A1 [factorise by $a^2 - b^2$ again]

Answer .... [3]

(b) $12cd - 9cx + 6xy - 8dy$.

$3c(4d - 3x) + 2y(3x - 4d)$

$= 3c(4d - 3x) - 2y(4d - 3x)$ ---M1 [Factorise out $-ve$ sign]

$= (3c - 2y)(4d - 3x)$ ---A1

Answer .... [2]

10 (a) Express $9 - 8x + x^2$ in the form $a + (x + b)^2$. Find the value of a and of b.

$9 - 8x + x^2 = (x - 4)^2 + 9 - 16 = -7 + (x - 4)^2$

$a = -7$ ---B1, $b = -4$ ---B1

Answer $a = $ .... [1]

$b = $ .... [1]

(b) Explain why when $x = 4$, the expression $9 - 8x + x^2$ has its minimum value.

Answer

For <u>any perfect square</u>, the <u>smallest value</u> is always <u>equal to zero or greater than zero</u>.

When $x = 4$, $(x - 4)^2 = 0$ thus, $(x - 4)^2 - 7 = -7 < 0$ and the coefficient of x^2 is positive,

implying that the quadratic expression will have a minimum value at $x = 4$. ——B1   [1]

8

11 Solve the equation $5 + 2x = \dfrac{20}{1+x}$.

$5 + 2x = \dfrac{20}{1+x}$

$5 + 5x + 2x + 2x^2 = 20$

$2x^2 + 7x - 15 = 0$ ---M1

$(2x - 3)(x + 5) = 0$

$x = 1.5$ or $x = -5$ ---A1 [BOTH ans correct]

Answer $x = $ .... or .... [3]

12 The points $(4, 20)$ and $(10, -4)$ satisfy the curve given by the equation $y = ax^2 + bx - 4$.

Use an algebraic method to determine the values of a and b.

$20 = a(4)^2 + b(4) - 4$ $-4 = a(10)^2 + b(10) - 4$

$24 = 16a + 4b$ $-4 = 100a + 10b - 4$

$6 = 4a + b$ ---Eqn(1) $100a = -10b$

                                  $10a + b = 0$ ---Eqn(2)   ---M1 [any 1 eqn correct]

*Solve by either elimination or substitution — Award M1

$a = -1$, $b = 10$ ---A2

Answer $a = $ ...., $b = $ .... [4]

Page 9

13 $\xi = \{$integers $x : 1 \leq x \leq 16\}$

$V = \{1, 2, 4, 8, 16\}$

$W = \{1, 4, 7, 9, 10, 14\}$

Some of the information is shown on the Venn diagram.

3 5 6 11 12 13 15
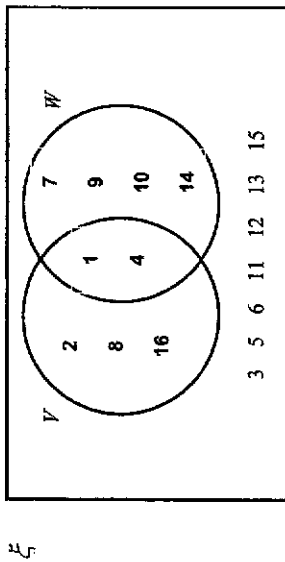
┌─────────────────────────────┐
│ (a) All parts of elements to│
│   be correctly represented│
│   for B1   │
└─────────────────────────────┘

(a) Complete the Venn diagram by representing all the elements in the given sets.

Answer in above Venn diagram [1]

(b) Describe the elements of Set V.

Answer

Elements of Set V are the factors of 16. —— **B1** [1]

(c) List the elements contained in the set $V' \cap W$.

Answer {7, 9, 10, 14} —— **B1** [1]

(d) Find the number of elements in $(V \cap W') \cup (V \cup W)'$.

Answer 10 —— **B1** [1]

(e) Use one of the symbols below to complete each statement.

$= \quad \varnothing \quad \subset \quad \not\subset \quad \notin \quad \in \quad \xi$

(i) {2, 16} V

(ii) 13 .... W

Answer (e) (i) ... $\subset$ ... [1]

(ii) ... $\notin$ ... [1]

[Turn Over]

Page 10

14 In grocery mart G, water costs $1.80 per litre, fresh milk costs $2.90 per litre and juice costs $2.30 per litre.

In grocery mart H, water costs $0.20 more per litre, fresh milk costs $0.40 less per litre and juice costs $0.10 less per litre.

This information can be represented by the matrix $\mathbf{Q} = \begin{pmatrix} 1.8 & 0.2 \\ 2.9 & -0.4 \\ 2.3 & -0.1 \end{pmatrix} \begin{matrix} W \\ M \\ J \end{matrix}$ with columns G H.

(a) Rayyen and Zinnie went shopping together.

Rayyen bought 4 litres of water, 2 litres of milk and 3 litres of juice.

Zinnie bought 3 litres of water and 4 litres of juice.

Represent their purchases in a 2×3 matrix $\mathbf{P}$.

$\mathbf{P} = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 0 & 4 \end{pmatrix} \begin{matrix} Rayyen \\ Zinnie \end{matrix}$ —— **B1**

with columns W M J

Answer $\mathbf{P} =$ [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{PQ}$.

$\mathbf{R} = \begin{matrix} Rayyen \\ Zinnie \end{matrix} \begin{pmatrix} 4 & 2 & 3 \\ 3 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1.8 & 0.2 \\ 2.9 & -0.4 \\ 2.3 & -0.1 \end{pmatrix}$

$= \begin{pmatrix} 19.9 & -0.3 \\ 14.6 & 0.2 \end{pmatrix}$ —— **B1** [Any 2 correct values, B1, all correct B2]

Answer $\mathbf{R} =$ [2]

(c) State what the elements in the second column of matrix $\mathbf{R}$ represent.

Answer

They represent the difference in the total spendings of Rayyen and of Zinnie at grocery mart G and H respectively for water, fresh milk and juice. —— **B1** [1]

(d) Rayyen shopped in grocery mart H. He got a discount coupon that entitled him to a discount of 15%. How much did he pay altogether for his items?

Amount Rayyen spent $= 19.90 - 0.30 = \$19.60$ ——M1

Amount to pay after 15% discount $= 19.60 \times 85\% = \$16.66$ ——A1

Answer $ [2]

[Turn Over]

15

(a) In a regular polygon, the ratio of an interior angle : exterior angle = 14 : 1.

Find the number of sides of the polygon.

15 units ——— 180°

1 unit ——— 12° ——M1

Sum of exterior angles = 360°

No. of sides = $\dfrac{360°}{12}$ ——M1

= 30 sides ——A1

Answer .... sides [3]

(b) Find the sum of the interior angles of the polygon.

Sum of interior angles = $(n-2)180°$

No. of sides = $(30-2)180°$

= 5040° ——A1 (*ECF*)

Answer ....° [1]

16 In the diagram, *BDK* is a straight line and *AB* = *AD*.

Angle *ABD* = 36°, angle *CBD* = 49° and angle *CDK* = 121°.
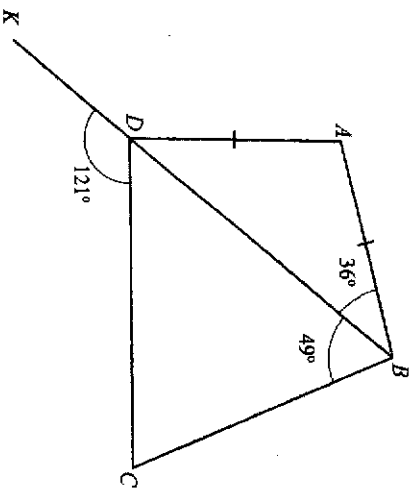
Explain why it is possible to draw a circle that passes through the points *A*, *B*, *C* and *D*.

Give reasons for each step of your working.

Answer [3]

If a circle can pass through all points *A*, *B*, *C* and *D*, then

Angle *DAB* + Angle *DCB* = 180° (angles in opp. segment)

CHECK:

∠*ADB* = 36° (isos. Δ)

∠*DAB* = 180° − 36° − 36° = 108° (sum of angles in Δ) ——M1

∠*BDC* = 180° − 121° = 59° (adj. angles on str. line)

∠*DCB* = 180° − 59° − 49° = 72° (sum of angles in Δ) ——M1

∴ ∠*DAB* + ∠*DCB* = 108° + 72° = 180°

(angles in opp. segment) ——A1

Since the above two angles add up to 180°, it means that it will be possible to draw a circle that passes through all points *A*, *B*, *C* and *D*.

**Any step without/ supported with incorrect reason more than once, DEDUCT 1 mark from overall point score for Q16.

14

18 **(a)** A cargo ship has an average fuel consumption of 0.000 892 kilometres per litre.
Write this consumption in litres per kilometre.

$$0.000892 \text{ km} \rightarrow 1l$$
$$1 \text{ km} \rightarrow \frac{1}{0.000892}$$
$$= 1121.076$$
$$= 1120l \quad (3 \text{ s.f.}) \quad ---B1$$

Answer .... *l* / km [1]

(b) A model of another cargo ship is made to a scale of 1 : 60.
The length of this model cargo ship is 550 cm.
(i) Find the actual length of this cargo ship in metres.

$$1 \text{ cm} : 0.6 \text{ m}$$
$$550 \text{ cm} : 330 \text{ m} \quad ---B1$$

Answer .... m [1]

The capacity of the fuel tanks in the model cargo ship is 7.75 litres.
(ii) Find the actual capacity of the fuel tanks of the cargo ship.
Express your answer in standard form.

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$
$$\frac{7.75}{V_{ship}} = \left(\frac{1}{60}\right)^3 \quad ---M1$$
$$V_{ship} = 1.674 \times 10^6 \, l \quad ---A1$$

Answer .... *l* [2]

13

17 The diagram represents a plot of land, *EFGH*, which is to be used for an observatory.

(a) Construct the bisector of the angle *EHG*. [1]

(b) Construct the perpendicular bisector *EF*. [1]

(c) A café is to be built in the observatory, nearer to *E* than to *F* and nearer to *GH* than to *EH*.

Shade the region where the café is to be built. [1]

GUIDANCE:

** Must show construction arcs clearly for both parts (i) and (ii) to award B1 mark EACH.

** If parts (i) and/ or (ii) wrong, but able to shade based on descriptions, award A1 (ECF). [2]

19 A composite solid is made from a cylinder and two hemispheres.

The cylinder has radius p and length $2p$, while the hemispheres have radius p.
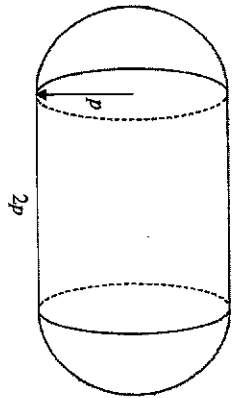
The total surface area of the solid is six times the total surface area of a cone with radius p and slant height l.

Find l in terms of p.

Since cone is a **SOLID (i.e. closed) cone,**

Total S.A. of solid = 6 × Total S.A. of cone

$$4\pi r^2 + 2\pi r h = 6\left(\pi r l + \pi r^2\right)$$

$$4\pi p^2 + 2\pi p\left(2p\right) = 6\left(\pi p l + \pi p^2\right) \quad \text{-----M1 [forms equation to connect S.A.]}$$

$$8\pi p^2 - 6\pi p^2 = 6\pi p l \quad \text{-----M1 [isolate } l \text{ term]}$$

$$l = \frac{2\pi p^2}{6\pi p}$$

$$= \frac{p}{3} \quad \text{-----A1}$$

Answer $\quad l = \text{...........................}$ [3]

---

20 Gino can paint 6 fence panels in 4 hours, while Danish can paint 7 fence panels in 5 hours.
Gino and Danish work together to paint a total of 21 panels.

If they continue to paint at the same rate, how long will it take them to paint 21 panels?
Give your answer in hours and minutes, correct to the nearest minute.

For Gino,
$$4\text{ h} \rightarrow 6\text{ panels}$$
$$1\text{ h} \rightarrow 1.5\text{ panels}$$

For Danish,
$$5\text{ h} \rightarrow 7\text{ panels}$$
$$1\text{ h} \rightarrow 1.4\text{ panels}$$

So in 1 hour, both will finish 2.9 panels ----- M1

With 2 people,
$$2.9\text{ panels} \rightarrow 1\text{ h}$$
$$21\text{ panels} \rightarrow \frac{21}{2.9} \quad \text{----M1 [ECF]}$$
$$= 7.241379\text{ h} = 7\text{h }14\text{min} \quad \text{----A1}$$

*ALTERNATIVE acceptable answer

- Accept also rounded value of <u>7h 15 min</u>, since some students considered that the work can only be completed after 7h 14 min 29 sec.

Answer $\quad \text{.......... h .... min}$ [3]

17

21 A fitness centre has 16 employees.

One of the 16 employees is selected at random.

The probability that it is a woman working part time is $\frac{1}{8}$.

Two of the 16 employees are selected at random.

The probability that they are both men working full time is $\frac{1}{8}$.

Complete the table of information below about the 16 employees of the fitness centre.

Show all supporting calculations clearly.

Answer

	Part-time employees	Full-time employees
Women	a = 2 ----B1	5
Men	3 ----B1	b = 6 ---- A1

$\frac{a}{16} = \frac{1}{8} \implies a = 2$

$P(\text{both men full time}) = \frac{1}{8}$

$\frac{b}{16} \times \frac{b-1}{15} = \frac{1}{8}$

$b^2 - b = 30$

$b^2 - b - 30 = 0$

$(b-6)(b+5) = 0$ ---- M1

$b = 6 \quad or \quad b = -5 \text{ (N.A.)}$

[4]

18

22 A librarian wants to find out how much time patrons spend at the library in a week.

He uses the questionnaire.

How many hours do you spend at our library in a week (including weekends)?

Please tick one box.

1 - 2	3 - 4	5 - 7	over 7

List two ways to improve the questionnaire.

Answer

[2]

*Award full marks for any 2 logical improvements for questionnaire.

List of possible suggestions

- The 3rd option has a duration of 2 hours resulting in the options, which is different from the other options. Each option to <u>contain a duration that is consistent</u>.
- The <u>options are not continuous</u>, as there are no options for patrons who may spend 2-3 hours or 4-5 hours. Options can be rephrased in the idea of inequality. E.g. Less than 2 hours, 2 or more hours but less than 3 hours.
- Add in the option of 0 – 1 hour as there may be patrons who spend lesser than 1 hour.
- Rephrase the question to:
On average, how many hours do you spend at our library in a week (including weekends)?
This would make the question more accurately phrased as patrons do not always spend the same amount of time in the library during every visit.

23 K is the point $(7, -8)$ and L is the point (x, y).

The gradient of the line KL is $\frac{2}{3}$.

Maverick claims that it is possible to express x in terms of y, such that $x = a + by$, where a and b are constants.

Explain why Maverick is correct. [2]

Answer

Gradient $KL = \dfrac{y - (-8)}{x - 7}$

$\dfrac{2}{3} = \dfrac{y+8}{x-7}$

$2x - 14 = 3y + 24$ ----M1

$2x = 3y + 38$

$x = \dfrac{3y + 38}{2}$

$x = 19 + \dfrac{3}{2}y$ ----A1

We see that it is possible to express the equation in the form $x = a + by$, where a and b are constants. Thus, he is correct.

24 A, B and C are points on the circle with centre O.
Q and R are points on the circle with centre P.
CSR and BSQ are tangents on to both circles.
AOB is a straight line.
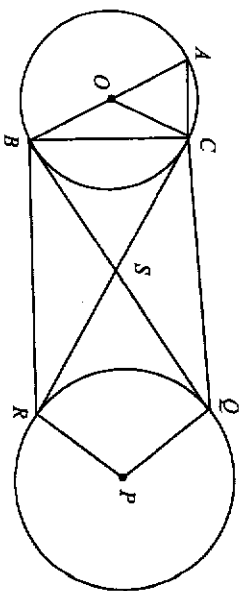
(a) Show that triangle CQS is congruent to triangle BRS.
Give a reason for each statement you make. [3]

Answer

Since CSR and BSQ are tangents on both circles, applying the property of <u>tangents from external points</u>, $CS = BS$ and $QS = RS$. ------ M1

$\angle CSQ = \angle BSR$ (vert. opp. $\angle$s) ------M1

By <u>SAS property</u>, $\triangle CQS \equiv \triangle BRS$ ------ A1

**Any missing reason, <u>deduct 1 mark from overall mark scored for Q19</u>.

21

24 (b) Angle $ABC = y°$

Find, in terms of y,

(i) angle BAC,

$\angle ACB = 90°$ (angle in semicircle)
$\angle BAC = 180° - 90° - y°$ (angle sum of triangle)
$= (90 - y)°$ ---B1

Answer .... ° [1]

(ii) angle QPR.

> Any 2 correct statements to conclude 2 quads are similar
> → Award M1
> *Reasons optional

$\angle OCS = \angle OBS = 90°$ (tan $\perp$ rad.)
$\angle PSR = \angle PQS = 90°$ (tan $\perp$ rad.)
$\angle CSB = \angle RSQ$ (vert. opp. $\angle s$)
Quad. $PQRS$ similar to Quad. $OBSC$

$\angle OCB = \angle OBC = y°$ (isos.Δ)
$\angle QPR = \angle BOC = (180 - 2y)°$ ---A1

*ALTERNATIVE acceptable answer

- $\angle BOC = (180 - 2y)°$ ---B1
- Award next **B1** mark for any correct supporting explanation to arrive at above answer.

Answer .... ° [2]

22

25 ORS is a triangle.

J is a point on RS such that $RJ : JS = 3 : 2$.

$\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{OS} = \mathbf{s}$.

(a) Show that $\overrightarrow{OJ} = \dfrac{1}{5}(2\mathbf{r} + 3\mathbf{s})$.

[2]
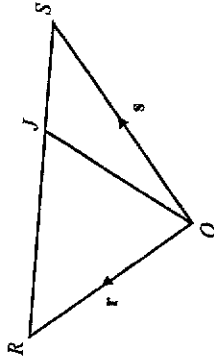
Answer

$\overrightarrow{RS} = -\mathbf{r} + \mathbf{s}$ ---M1
$\overrightarrow{OJ} = \overrightarrow{OR} + \overrightarrow{RJ}$
$= \mathbf{r} + \dfrac{3}{5}\overrightarrow{RS}$
$= \mathbf{r} + \dfrac{3}{5}(\mathbf{s} - \mathbf{r})$ ---M1
$= \dfrac{2}{5}\mathbf{r} + \dfrac{3}{5}\mathbf{s} = \dfrac{1}{5}(2\mathbf{r} + 3\mathbf{r})$ (shown)

(b) X is a point such that $\overrightarrow{RX} = \dfrac{1}{5}(\mathbf{r} + 9\mathbf{s})$.

Explain why O, J and X lie on a straight line.

[2]

Answer

$\overrightarrow{RX} = \dfrac{1}{5}(\mathbf{r} + 9\mathbf{s})$

$\overrightarrow{OX} - \overrightarrow{OR} = \dfrac{1}{5}\mathbf{r} + \dfrac{9}{5}\mathbf{s}$

$\overrightarrow{OX} = \dfrac{1}{5}\mathbf{r} + \dfrac{9}{5}\mathbf{s} + \mathbf{r}$

$\quad\quad = \dfrac{6}{5}\mathbf{r} + \dfrac{9}{5}\mathbf{s}$

$\overrightarrow{OX} = 3\left[\dfrac{2}{5}\mathbf{r} + \dfrac{3}{5}\mathbf{s}\right]$ ---M1

$\overrightarrow{OX} = 3\,\overrightarrow{OJ}$

Since the vectors $\overrightarrow{OX}$ and $\overrightarrow{OJ}$ **differ by a constant of 3** and have a **common point** O, it means that the **vectors are parallel.**

This further implies that OX and OJ are parallel lines. ---- **A1**

Thus, the points O, J and X lie on the same straight line.

~ END OF PAPER ~

2

Mathematical Formulae

Compound interest

$$\text{Total Amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

HUA YI SECONDARY SCHOOL   **4-G3 / 5-G2**

PRELIMINARY EXAMINATION 2024

NAME

CLASS

INDEX NUMBER

MATHEMATICS   **4052/02**

PAPER 2

19 August 2024

2 hour 15 minutes

Candidates answer on the Question Paper.

MARKING SCHEME

For Examiner's Use	90

This document consists of **25 printed pages** and **1 blank page**.

Setter: Ms Jasmine Tan

[Turn Over

3

1 (a) Express as a single fraction in its simplest form $\dfrac{2}{7-3x}-\dfrac{1}{6-x}$.

$$\dfrac{2}{7-3x}-\dfrac{1}{6-x}=\dfrac{2(6-x)-(7-3x)}{(6-x)(7-3x)}$$

$$=\dfrac{12-2x-7+3x}{(6-x)(7-3x)} \quad\text{---}M1\ [\text{expand numerator correctly}]$$

$$=\dfrac{5+x}{(6-x)(7-3x)} \quad\text{---}A1$$

Answer .... [3]

(b) It is given that $v=\dfrac{3-5w}{w+2}+9$.

(i) Find v when $w=-6$.

$$v=\dfrac{3-5(-6)}{-6+2}+9=\dfrac{3}{4}\quad or\quad 0.75 \quad\text{---}B1$$

Answer .... [1]

(ii) Rearrange the formula to make w the subject.

$$v=\dfrac{3-5w}{w+2}+9$$

$$(w+2)(v-9)=3-5w \quad\text{---}M1\ [\text{remove bases}]$$

$$wv-9w+2v-18=3-5w$$

$$wv-4w=21-2v \quad\text{---}A1\ [\text{group }w\text{-terms on 1 side}]$$

$$w=\dfrac{21-2v}{v-4}$$

$$=-\dfrac{21-2v}{4-v} \quad\text{---}A1$$

Answer $w=$ .... [3]

4

(c) Solve the equation $\dfrac{5}{x-2}-\dfrac{3}{x^2-4}=\dfrac{1}{7}$.

Give your solutions correct to two decimal places.

$$\dfrac{5}{x-2}-\dfrac{3}{x^2-4}=\dfrac{1}{7}$$

$$\dfrac{5(x+2)-3}{(x+2)(x-2)}=\dfrac{1}{7} \quad\text{---}M1\ [\text{puts to common base}]$$

$$7(5x+10-3)=x^2-4$$

$$x^2-35x-53=0 \quad\text{---}M1\ [\text{simplifies accurately quad. eqn}]$$

$$x=\dfrac{-(-35)\pm\sqrt{(-35)^2-4(1)(-53)}}{2(1)} \quad\text{---}M1$$

$$x=-1.45389 \quad or\quad x=36.45389$$

$$x=-1.45 \quad\text{---}A1\quad or\quad x=36.45 \quad\text{---}A1\ [\text{accept 2 d.p. only}]$$

Answer $x=$ .... , $x=$ .... [5]

5

2   C is the point $(-9,1)$ and D is the point $(7,4)$.

$\overrightarrow{CE} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}$.

(a)   Calculate the length of the line CD.

Length $CD = \sqrt{(-9-7)^2 + (1-4)^2}$ ----M1
$= \sqrt{265} = 16.3$ ----A1

Answer   .... units [2]

(b)   Determine the coordinates of point E.

$\overrightarrow{CE} = \overrightarrow{OE} - \overrightarrow{OC}$

$\begin{pmatrix} -2 \\ 8 \end{pmatrix} + \begin{pmatrix} -9 \\ 1 \end{pmatrix} = \overrightarrow{OE}$

$\therefore E(-11,9)$ ----B1

Answer E (.........., ...) [1]

(c)   Find the equation of the line DE.
Leave your answer in the form $ax+by=c$, where a, b and c are constants.

Gradient $DE = \dfrac{4-9}{7-(-11)} = -\dfrac{5}{18}$ ----M1(ECF)

$y = -\dfrac{5}{18}x + c$
Subst. $D(7,4)$
$c = \dfrac{107}{18}$ ----M1

So, equation of line DE: $\dfrac{5}{18}x + y = \dfrac{107}{18}$
$5x + 18y = 107$ ----A1

6

Answer   .... [3]

$\overrightarrow{CE} = (\ )$

3   (a)   The first four terms of a sequence are 5, $\dfrac{9}{4}$, $\dfrac{13}{9}$, $\dfrac{17}{16}$.

(i)   State the fifth term of the sequence.

5th term $= \dfrac{21}{25}$ ----B1

Answer   .... [1]

(ii)   Find an expression, in terms of n, for the nth term, T_n, of this sequence.

Rule for numerator = $4n+1$
Rule for base = n^2

$T_n = \dfrac{4n+1}{n^2}$ ----B1

← Award B1 for ANY 1 correct rule

Answer $T_n = $ .... [2]

(b)   Elijah finds a number grid from his board game.
The diagram shows part of a number grid.

A rectangle outlining four numbers, as shown, can be placed anywhere on the grid.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37					

7

(i) If p represents the number in the top right corner of the rectangle, write down an expression, in terms of p, for the number in the bottom left corner of the rectangle.

$$p+6 \quad ---B1$$

Answer .... [1]

(ii) Show that the difference between the products of the numbers in the opposite corners of the rectangle is always -7 .

Answer

$(p-1)(p+7)-p(p+6) \quad ---M1$ [form BOTH set of products, take difference]

$= p^2+7p-p-7-p^2-6p \quad ---A1$ [expand all terms correctly]

$= -7 \quad (shown)$

....

[2]

(iii) Elijah says it is impossible for the sum of the four numbers in the rectangle to be 199.

Justify with relevant working, why he is correct.

Answer

[3]

$$\text{Sum} = p-1+p+p+6+p+7$$
$$= 4p+12 \quad ---M1 \text{ [simplied sum correctly]}$$

If sum = 199, then we must see that p is an integer,

$$4p+12=199$$
$$4p=187 \quad ---M1 \text{ [isolated } p\text{-term]}$$
$$p=46.75$$

From above, we see that p is **not an integer**, so 199 cannot be the sum of four numbers on the grid. ----- A1 (**attempt to explain and make relation to the workings**)

8

4 (a) Sophie earns a monthly salary of $6875.
She gives 15% of this amount to her parents.
She puts 35% of the remainder into a savings account.

Calculate the amount she has left after giving to her parents and putting into her savings account. Leave your answer correct to the nearest dollar.

....

Amount given to parents = $15\% \times 6875 = \$1031.25$
Amount left after giving parents = $\$5843.75$

Amount put into her savings = $35\% \times 5843.75 = \$2045.3125 \quad ---M1$

Amount left after giving parents and paid to savings
$5843.75 - 2045.3125 = 3798.4375$
$$= \$3798 \quad \text{(nearest dollar)} \quad ---A1$$

[dashed box: ← Award M1 for any 1 correct computation]

Answer $ .... [3]

(b) The cash price of a sofa is $830.
Sophie buys this sofa on credit.
She pays a deposit of one quarter of the cash price.
She then pays 3 monthly payments of $260.

Calculate the total amount Sophie pays for the sofa.

Total amount for sofa
$$= \left(\frac{1}{4} \times 830\right) + (3 \times 260) \quad ---M1 \text{ [both sets of computation to be shown]}$$
$$= \$987.50 \quad ---A1$$

9

(c) Sophie pays a monthly rent of $3174.20.

This is 18% more than her monthly rent last year.

Calculate her monthly rent last year.

$$118\% \;\to\; \$3174.20$$
$$100\% \;\to\; \frac{100}{118}\times3174.20 \quad ----M1$$
$$= \$2690 \quad ----A1$$

Answer \$ .... [2]

(d) During her vacation, Sophie visits her friend in Wellington.
Sophie spends NZD 940 in New Zealand using her credit card.
She is charged a 2.6% fee for the currency conversion.
The exchange rate between Singapore dollars (SGD) and New Zealand dollars (NZD) is
SGD 100 = NZD 120.7206.

Calculate the total amount on Sophie's credit card bill, including the fee.
Give your answer in Singapore dollars, correct to the nearest cent.

NZD	SGD
120.7206	100
940	$\frac{100}{120.7206}\times940 = 778.657\,495$ ----M1

Total amount on credit card bill
$$= 102.6\%\times778.657\,495 \quad ----M1$$
$$= SGD\ 798.90 \quad (\text{nearest cent}) \quad ----A1$$

Answer \$ .... [2]
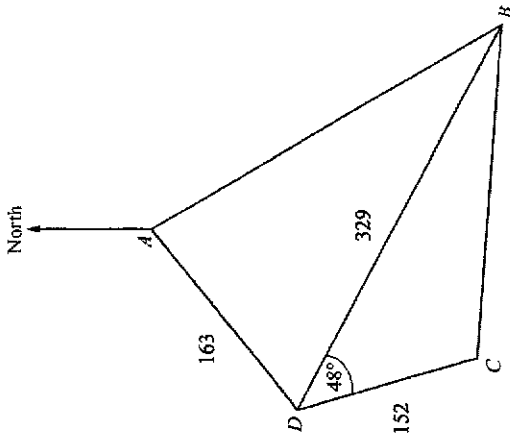
10

5

Answer SGD .... [3]

ABCD is a field on horizontal ground.

$AD = 163$ m, $BD = 329$ m, $CD = 152$ m and angle $BDC = 48°$.

The bearing of B from A is 151° and the bearing of D from A is 237°.

(a) Calculate the bearing of D from B.

$$\angle BAD = 237° - 151° = 86°$$
$$\frac{\sin\angle ABD}{163} = \frac{\sin86°}{329} \quad ----M1$$
$$\angle ABD = 29.61924°$$

Bearing of D from B
$$360° - \angle ABD - (180° - 151°) \quad ----M1$$
$$= 360° - 29.61924° - 29°$$
$$= 301.4° \quad ----A1$$

11

(b) Calculate the distance from *B* to *C*.

By cosine rule,

$$BC^2 = 152^2 + 329^2 - 2(152)(329)\cos 48° \quad ----M1$$

$$= \sqrt{64421.23327}$$

$$= 253.81338 = 254m \quad ----A1$$

Answer .... ° [3]

(c) An aircraft is flying above *D*.

Find the angle of elevation of the aircraft from *C* when it is 245 m vertically above *D*.

Let the angle of elevation be θ.

$$\tan\theta = \frac{245}{152} \quad ---M1$$

$$\theta = 58.2° \quad ---A1$$

Answer .... ° [2]

12

6   The diagram shows a semicircle, centre *O*, radius 16 cm.
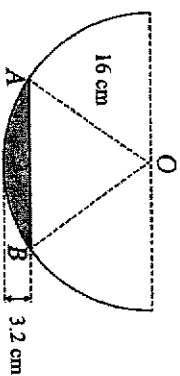
(a) Show that angle *AOB* = 1.287 radians, correct to 3 decimal places.   [2]

Answer

Since triangle *OAB* is isosceles, vertical line from *O* to *AB* will be perpendicular.

Let this point be *P* and *OP* = 16 − 3.2 = 12.8 cm.

$$\cos\angle AOP = \frac{12.8}{16}$$

$$\angle AOP = 36.86989° = 0.643501 \text{ rad.} \quad ---M1$$

$$\angle AOB = 2 \times 0.643501 \quad ---M1$$

$$= 1.287 \text{ rad. } (shown)$$

(b) Calculate the area of the shaded region.

Area of sector $OAB = \frac{1}{2} \times 16^2 \times 1.287 = 164.736 \text{ cm}^2 \quad ---M1$

Area of triangle $OAB = \frac{1}{2} \times 16^2 \times \sin 1.287 = 122.87992 \text{ cm}^2 \quad ---M1$

Area of shaded segment = 164.736 − 122.87992 = 41.9 cm² $\quad ----A1$

13

Answer .... cm² [3]

(c) The semicircle is the cross section of a water trough of length 2.8 m, standing on level ground.

The shaded area represents the water in the trough.

(i) Calculate the volume of water, in cm³, in the trough. Leave your answer in standard form.

Volume of water = Segment area [Part b] × length

= 41.85608×280 ---$M2$ [M1 conversion, M1 (ECF) expression]

= 1.17×10^4 cm³ (standard form) ----$A1$

Answer .... cm³ [3]

Max mark earned from ECF = 1 ONLY

(ii) Calculate the number of litres of water that must be added to fill the trough.

Volume of trough = $\left(\frac{1}{2} \times \pi \times 16^2 \times 280 \right)$ ---$M1$

= 112594.6807 cm³

Additional water to be added = Vol of trough – Vol of water already present [part ci]

= $112\,594.6807 - 11\,719.7024$ ---$M1$ [ECF]

= $100\,874.9788$ cm³

= $101l$ (3 s.f.) ---$A1$

Answer .... litres [3]

Max mark earned from ECF = 1 ONLY

14

7 A researcher wants to conduct a study to find out if there is a correlation between the analytical skills of adults with age.

The researcher invited 80 adults to solve a number puzzle.

The cumulative frequency curve shows the distribution of the time taken.
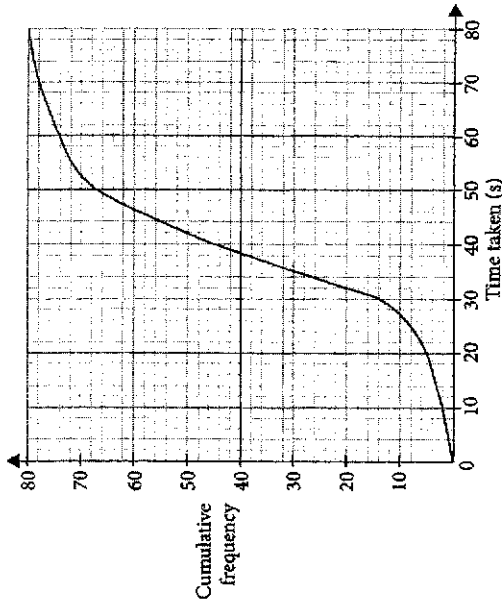
(a) Use the curve to estimate

(i) the median time taken,

Median = 38 or 39 seconds ----B1

Answer .... s [1]

(ii) the interquartile range of the time taken.

Upper quartile = 46 seconds , Lower quartile = 32 seconds ---- M1 (for any 1 correct)

IQR = 46 – 32 = 14s ---- A1

Answer .... s [2]

(b) 20% of the adults took more than n seconds to solve the puzzle. Find n.

$P_{80} = 64^{th}$ term ---- M1

= 48s ---- A1

15

(c) The frequency distribution of the data for the 80 adults invited to solve the number puzzle is shown in the table.

Time taken, x (seconds)	Frequency
$0 < x \leq 20$	5
$20 < x \leq 40$	40
$40 < x \leq 60$	29
$60 < x \leq 80$	6

Answer $n = $ .... [2]

Find an estimate of the standard deviation of the time taken by the 80 adults to solve the number puzzle.
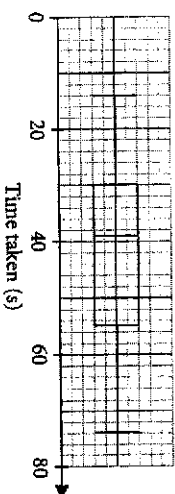
Standard deviation

$$= \sqrt{\frac{5(10)^2 + 40(30)^2 + 29(50)^2 + 6(70)^2}{80} - \left(\frac{5(10)+40(30)+29(50)+6(70)}{80}\right)^2}$$

$$= \sqrt{\frac{138400}{80} - \left(\frac{3120}{80}\right)^2} \quad ---M1$$

$$= 14.4568 = 14.5 \quad ---A1$$

Answer .... s [2]

16

(d) The same group of adults were each given a word puzzle to solve. The box-and-whisker plot shows the distribution of the time taken.

Time taken (s)

Make two comparisons between the performances of the adults in solving the two puzzles.

Answer [2]

	Median	IQR
Word Puzzle (Box Plot)	39 s	25 s
Number Puzzle (From earlier)	38 or 39 s	14 s

- There was **no significant difference in the average performance** of the adults when solving for word and number puzzles. The median values were about the same (or equal). ---A1

- Comparing the interquartile range, the adults had a **greater spread of timings** (i.e. less consistency) when doing word puzzles as the IQR value was higher. ---A1

***Max ECF mark: 1 ONLY

18

9 The variables x and y are connected by the equation $y = \frac{x^3}{2} - 5x - 2$.

Some corresponding values of x and y are given in the table.

x	-3	-2	-1	0	1	2	3	4
y	-0.5	4	2.5	-2	-6.5	k	-3.5	10

(a) Find the value of k.

$k = -8$ ----B1

Answer $k =$ .... [1]

(b) On the grid on page 19, draw the graph of $y = \frac{x^3}{2} - 5x - 2$ for $-3 \le x \le 4$. [2]

- All 8 points correct —— **B1**
- Smooth curve with turning points well-drawn —— **B1**

(c) The equation $\frac{x^3}{2} - 5x = 7$ has only one solution.

Explain how this can be deduced from your graph. [2]

Answer

To solve the equation, we will **look for the intersection point of the curve** $y = \frac{x^3}{2} - 5x - 2$ and **the line** $y = 5$. —— **B1**

Since the graphs will **only intersect once**, it means that there is only one solution. —— **B1**

*Student answers to **contain the keywords** in **BOLD**.

(d) By drawing a tangent, estimate the gradient of the curve at $(1, -6.5)$. [2]

Gradient of tangent $= \dfrac{-1.4 + 10.4}{-0.7 - 2.3}$

$= -3$

**Actual gradient $= -3.5$

- Draw acceptable tangent → **B1**
- Calculated tangent lies in acceptable range (____ to ____) → **B1**

Answer Gradient = ... [2]

17

8 In the diagram, XYZ is a straight line.

$AY = 20$ m, $AZ = 12$ m and $YZ = 16$ m.

The ratio of $XY : YZ$ is $1 : 4$.

(a) Prove that triangle AYZ is right-angled. [2]

Answer

$AY^2 = 20^2 = 400$

$AZ^2 + YZ^2 = 12^2 + 16^2 = 400$ ---- M1 [BOTH correct]

Since $AY^2 = AZ^2 + YZ^2$, by converse of Pythagoras' Theorem triangle AYZ is right-angled. [proven] —— A1

(b) Find the value of $\cos \angle AYX$, giving your answer as a fraction in its lowest terms.

$\cos \angle AYX = -\cos \angle AYZ$

$= \dfrac{16}{20} = -\dfrac{4}{5}$ ---B1

Answer .... [1]

(c) The area of triangle AXY is $24 \, \text{m}^2$.

Lenard says: In another triangle AYW, whereby the length $YW = YX$ and its area is the same as that of triangle AXY, it is possible for angle Y to be acute.

Explain why Lenard is correct. Use calculations to support your answer. [2]

Answer

$0.5 \times AZ \times YX = 24$

$YX = (24 \times 2) \div 12 = 4$

Then, $0.5 \times 4 \times 20 \sin \angle AYW = 24$

$\angle AYW = 36.9°$ (1 d.p)

From above, we see that it is possible to obtain an acute angle for angle Y, thus Lenard is correct.

(e) (i) On the same grid, draw the line $y = 4 - x$ for $-1 \le x \le 4$. [2]

x	-1	0	4
y	5	4	0

- Plot at least 2 correct points → B1

- Draw smooth line → B1

(ii) Write down the x-coordinate of the point where this line intersects the curve.

$x = 3.4$ ---- B1

Answer $x = $ .... [1]

(iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$.
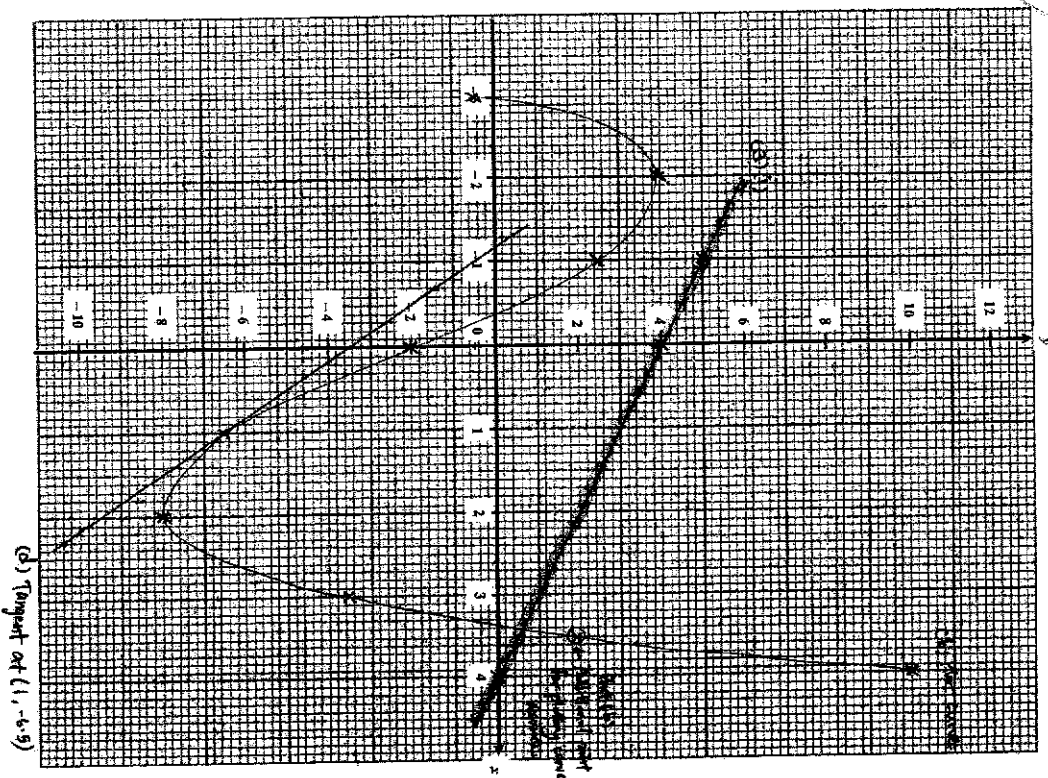Find the value of A and the value of B.

$$4 - x = \frac{x^3}{2} - 5x - 2$$

$$8 - 2x = x^3 - 10x - 4$$

$$x^3 - 8x - 12 = 0$$

$A = -8$ ---- B1 or $B = -12$ ---- B1

Answer $A = $ ...., $B = $ .... [2]

(d) Tangent at (1, -6.5)

10 Javier and Stella are planning their exercise routines.

Javier and Stella have a body mass of 70 kg and 60 kg respectively.

The speed for walking and jogging of the two individuals are represented in the chart.

Speed comparison for Javier and Stella

Speed (km/h) — axis 0 to 10
Javier, Stella
■ Brisk walking   ▨ Jogging

They research the following information about the benefits of exercise.

HEALTH ADVICE

For optimal health benefits, adults should do at least 150 minutes of moderate-intensity aerobic activity, or at least 75 minutes of vigorous-intensity aerobic activity per week.

For additional health benefits, adults should increase their moderate-intensity aerobic activity to 300 minutes per week, or an equivalent combination of moderate-intensity and vigorous-intensity aerobic activity.

1 minute of vigorous-intensity aerobic activity is equivalent to 2 minutes of moderate-intensity aerobic activity.

E.g. 10 minutes of jogging = 20 minutes of brisk walking.

Muscle-strengthening activities should be done on 2 or more days per week.

Approximate calories burned during 30 minutes of aerobic exercise

	Body mass			
	60 kg	70 kg	80 kg	90 kg
Walking 5 km/h	105	120	135	150
Walking 6.5 km/h	130	150	170	190
Jogging 8 km/h	240	280	315	350
Jogging 9.5 km/h	300	345	390	435

Non-aerobic muscle-strengthening activities

❖ 1 hour of yoga burns approximately 3 calories per kilogram of body mass

❖ 1 hour of weight training burns approximately 4 calories per kilogram of body mass

(a) In their first week of exercise, they each plan to go for four brisk walks.

They will walk the same route each time.

The four walks together meet the minimum target for the time for optimal health benefits in one week.

(i) Find the distance of one of these walks.

*NOTE:

- In total, 4 walks will take 150 minutes each week for both Javier and Stella.
- Brisk-walking is moderate intensity activity.
- Both Javier and Stella have the same brisk-walking speed (6.5 km/h).

$60 \text{ min} \rightarrow 6.5 \text{ km}$

$150 \text{ min} \rightarrow \dfrac{6.5}{60} \times 150 = 16.25 \text{ km}$ --- M1

Distance of 1 walk $= \dfrac{16.25}{4} = 4\dfrac{1}{16} = 4.0625 \text{ km}$ --- A1

Answer .... km [2]

23

(ii) Find out how many more calories Javier burns in these four walks than Stella.

In every 30 minutes, difference in calories burnt = 150−130 = 20

$$30 \text{ min} \rightarrow 20 \text{ calories}$$
$$150 \text{ min} \rightarrow 100 \text{ calories} \quad \text{— — B1}$$

Answer .... calories [1]

(b) After one month, they change their routines.

Javier wants to gain additional health benefits.

He decides to do a 4 km walk 3 times per week and do a 6 km jog 2 times per week.

He will also attend a 45-minute yoga class 2 times per week.

Stella wants to maintain optimal health benefits.

She decides to do an 8 km jog 2 times per week.

She will also attend a 30-minute weight training session 2 times per week.

Javier says:

We will both meet our targets for exercise.

However, I will burn about 50% more calories than Stella during our exercise per week.

Is Javier correct?

Justify your decision with clear calculations.

Answer

[7]

24

JAVIER (gain *additional* health benefits)

- Yoga plan: 45 min, 2 times per week

From **Health Advice**	From **Health Advice**	
• Brisk-walking is moderate-intensity	• Jogging is vigorous-intensity	
• Walk plan: 4 km, 3 times per week	• Jog plan: 6 km, 2 times per week	
Total distance per week = 12 km	Total distance per week = 12 km	From non-aerobic (Pg 21)
Walking speed = 6.5 km/h	Jogging speed = 8 km/h	Total time per week = 90 min
6.5 km → 60 min	8 km → 60 min	60 min → 3 cal/kg
12 km → $110\frac{10}{13}$ min	12 km → 90 min	90 min → 4.5 cal/kg
	Ratio (convert to moderate-intensity)	From 90 min yoga per week, total calories burnt for 70 kg Javier,
Award J1 for Javier's walking/Jogging	1 min jog → 2 min walk	$4.5 \times 70 = 315$ cal*
	90 min jog → 180 min walk	*Award JC1* for calories counted correctly once*

Calories burned

See calories table	See calories table	Overall calories burned
30 min → 150 cal	30 min → 280 cal	$553\frac{11}{13} + 840 + 315 = 1709$ cal (nearest int.)
$110\frac{10}{13}$ min → $553\frac{11}{13}$ cal*	90 min → 840 cal*	*Award J1 for Javier's total time*

❖ Total moderate-intensity time (e.g brisk-walk) = $180 + 110\frac{10}{13} = 290\frac{10}{13}$ min (<300 min)

STELLA (gain *optimal* health benefits)

From Health Advice	• Weight-training: 30 min, 2 times per week
• Jogging is vigorous-intensity	
• Jog plan: 8 km, 2 times per week	**From non-aerobic (Pg 21)**
Total distance per week = 16 km	Total time per week = 60 min
Jogging speed = 9.5 km/h	60 min → 4 cal/ kg
9.5 km → 60 min	From 60 min yoga per week, total calories burnt for 60 kg
16 km → $101\frac{1}{19}$ min (> 75 min)	Stella, $4 \times 60 = 240$ cal
Award S1 for Stella's jogging time	
See calories table	**Calories burned**
30 min → 300 cal	**Overall calories burned**
$101\frac{1}{19}$ min → $1010\frac{10}{19}$ cal	$1010\frac{10}{19} + 240 = 1251$ cal (nearest int.)
	Award SC1 for Stella's total calories

Comparing % of calories burned for Javier to Stella,

$$= \frac{Javier}{Stella} \times 100\% = \frac{1709 - 1251}{1251} \times 100\%$$

$= 37\%$ (nearest int.) (< 50%)

Award M1 (ECF) for comparing calories in percentage

❖ From above, we see that <u>Javier will only burn 37% more</u>, not 50% more than Stella.

❖ Comparing the timings, we see <u>that only Stella will achieve her target</u> to maintain optimal health benefits. Thus, <u>Javier is incorrect</u>.

Award A1 (ECF) accurate conclusion, based on calculations

- END OF PAPER -

[Turn Over