SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 1

MATHEMATICS

8864/01

Paper 1

14 September 2016

3 hours

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

Section A: Pure Mathematics [35 marks]

1 **(i)** Sketch the curve with equation $y=\frac{3x}{x^2-4}+1$, stating the equations of any asymptotes and the coordinates of the points where the curve crosses the axes. [3]

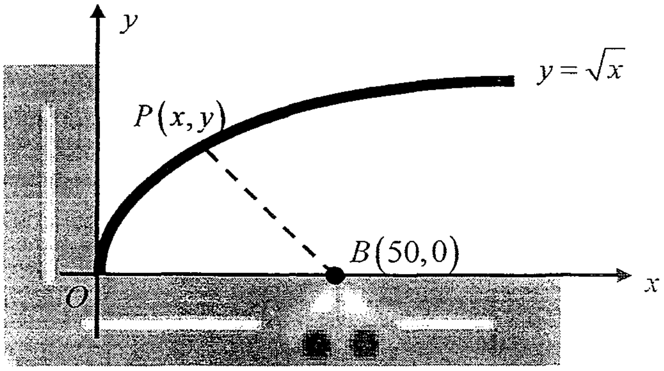
(ii) Hence solve the inequality $\frac{x}{x^2-4}>x+1$. [3]

2 **(i)** The curve C has equation $y=\ln\left(\frac{x}{3-x}\right)$, for $0<x<3$. Show that the values of x for which the gradient of C is equal to the constant k satisfy the equation $kx^2-3kx+3=0$. [4]

(ii) Find the exact range of values of k for which the equation $kx^2-3kx+3=0$ has no real roots. [2]

(iii) Hence explain whether there exists a point on curve C with gradient 2. [1]

3 The diagram below shows a particular section of a park connector which can be modelled by the curve with equation $y=\sqrt{x}$. The point B on the x-axis represents a bus stop situated 50 metres away from the origin, O. The National Parks Board wishes to increase accessibility by constructing a shortest straight-line path joining the bus stop to the park connector.



(i) Let D be the straight-line distance (in metres) between the bus stop and any point P on the park connector. Show that $D=\sqrt{x^2-99x+2500}$. [2]

(ii) Find, using differentiation, the minimum value of D. [4]

4 **(a)** **(i)** Differentiate $\sqrt{3}x-\frac{\pi}{\sqrt{x}}$. [1]

(ii) Given that $\mathrm{f}(x)=\mathrm{e}^{\left[\left(k^2\right)\sqrt{x}+k\right]}$, where k is a constant. Find $\mathrm{f}'(x)$ in terms of k. [2]

(b) Given that $y=2^x\left(x^3-1\right)$, find the numerical value of the derivative at $x=-1$. Leave your answer correct to 2 decimal places. [1]

(c) Use a non-calculator method to find the exact value of $\int_0^1\left(\mathrm{e}^{\frac{x}{2}}+3\right)^2\,\mathrm{d}x$. [4]

5 The curve C has equation $y=1+2\ln(4-x)$.

(i) Sketch C, labelling the equations of any asymptotes and the points where the graph crosses the axes. [2]

(ii) Find the exact equation of the normal to C at the point where $x=\frac{1}{2}$. [3]

(iii) Find the area bounded by the curve C, the normal found in part **(ii)** and the y-axis, giving your answer to 4 decimal places. [3]

Section B: Statistics [60 marks]

6 A school comprises a large number of students. A sample comprising 2% of the student population is to be selected to take part in a survey on their opinions about the school facilities.

(a) Describe briefly how this sample can be obtained via systematic sampling. [2]

(b) Give one advantage and one disadvantage of quota sampling in this context. [2]

7 Events A and B are such that $\mathrm{P}(A)=3p-1$ and $\mathrm{P}(B)=\mathrm{P}(A\mid B)=p$.

(i) Given that $\mathrm{P}(A\cup B)=0.8$, find a quadratic equation satisfied by p and find the value(s) of p, correct to 2 decimal places. [3]

(ii) Find $\mathrm{P}(A'\cup B')$. [2]

Determine whether A and B are independent events. [1]

8 The table below shows the ages of teak trees, t years, with trunk diameters, d inches. It can be assumed that the diameters of teak trees depend on their ages.

Age t (years)	11	15	28	45	52	64	75	81	88	97
Diameter d (inches)	7.5	11.5	14.5	19	20.5	21	21.5	21.9	22.2	22 .22

(i) Draw a scatter diagram for these values, labelling the axes. [2]

(ii) Calculate the product moment correlation coefficient. [1]

(iii) Find the equation of the regression line of d on t and sketch the regression line on your scatter diagram in part **(i)**. Interpret, in context, the value of the gradient of the regression line. [3]

(iv) Use the regression line found in part **(iii)** to estimate the diameter of a 40 year-old teak tree. Comment on the reliability of your answer. [2]

(v) It is desired to predict the diameters of very old trees (of over hundred years old). Explain why, in context, a linear model is not likely to be appropriate. [1]

(vi) It was found that due to an error in the calibration of the measurement instrument, each trunk diameter recorded in the table above should be k inches longer, where k is a positive constant. Write down, in terms of k, the corrected equation of the new regression line of d on t. [1]

9 It has been estimated that only $100p\%$ of the world's population has blue eyes. A group of 70 people are randomly selected from all over the world. The number of people in this group who have blue eyes is the random variable Y.

(i) State, in the context of this question, one assumption needed to model Y by a binomial distribution. [1]

Assume now that Y indeed follows a binomial distribution, and that $p > 0.05$.

(ii) Given that the probability that exactly 3 people in the group have blue eyes is 0.105. Write down an equation in p. Hence show that $p = 0.0800$, correct to 4 decimal places. [3]

(iii) Find the probability that at least 5 but less than 21 people in the group will have blue eyes. [2]

(iv) Use a suitable approximation to find the probability that more than 9 people in the group have blue eyes. You should state the parameters of the distribution you have used. [4]

10 **(i)** Two fair dice are tossed. Find the probability that

(a) the sum of the scores of the two tosses is at least 8, and [1]

(b) the difference between the scores of the two tosses is at least 4. [1]

In a game, a player draws a ball from a box that contains 3 red balls and 4 white balls. If a red ball is drawn, the player will add the scores obtained from tossing two fair dice. If a white ball is drawn, the player will take the difference of the scores obtained from tossing two fair dice.

The player wins the game if the sum of the scores is at least 8 or the difference of the scores is at least 4.

(ii) Find the probability that the player wins the game. [2]

(iii) Given that the player wins the game, find the probability that a red ball is drawn. [2]

11 An accountant wishes to investigate the figures provided by credit card companies for the amount of loans borrowed by each client, \$$x$. He carries out an online survey from clients in a certain company. The responses from a random sample of 40 clients are summarised by

$$\sum x = 38100, \ \sum (x - \overline{x})^2 = 731800.$$

(i) Calculate unbiased estimates of the population mean and variance of the amount of loans borrowed by each client, correct to 1 decimal place. [2]

The company claims that its clients will borrow \$1000 on average.

(ii) Test, at the 5% level of significance, whether the mean amount of loans borrowed by the clients differs from \$1000. [4]

(iii) Explain, in the context of the question, the meaning of 'at the 5% level of significance'. [1]

The company revised their loan policy and the new population standard deviation is known to be 250. A new random sample of 40 clients is taken and the mean amount of loans borrowed for this sample is \$$k$. A test at the 5% significance level indicates that the null hypothesis would not be rejected for this revised loan policy.

(iv) Find the range of values of k. [3]

12 **(a)** The continuous random variable X has the distribution $N(\mu,\sigma^2)$. It is known that $P(X<18.1)=P(X>21.9)=0.2$. Find the value of μ and σ. [3]

(b) The battery life (in hours) of DuraSell batteries and Energise batteries are modelled as having independent normal distributions with means and standard deviations as shown in the table below.

	Mean	Standard Deviation
DuraSell	m	1.5
Energise	19.8	2.7

(i) Find the probability that the battery life of a randomly selected DuraSell battery is within 1 hour of its population mean. [3]

(ii) Three DuraSell batteries are chosen at random. Find the probability that exactly two DuraSell batteries have battery life within 1 hour of its population mean. [3]

(iii) Assume now that $m=22.4$. Find the probability that a randomly selected DuraSell battery will outlast a randomly selected Energise battery. [2]

To ensure that Energise batteries produced have satisfactory battery lives, a plant manager collects random samples of 50 Energise batteries for quality testing. A sample passes the quality check if its mean lifetime exceeds 19 hours.

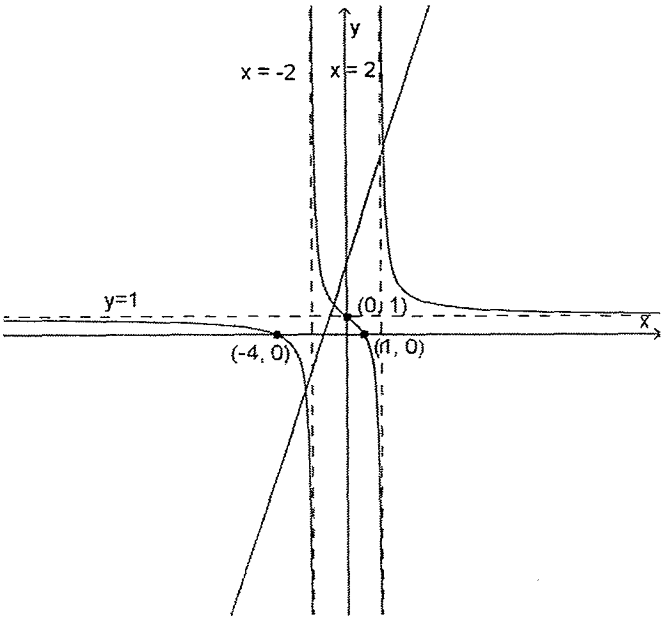
(iv) Suppose 100 such samples are collected, find the expected number of samples that pass the quality check. [3]

– END OF PAPER –

Suggested Solution: 2016 SH2 H1 Mathematics Preliminary Examination

1 (i)

$$y=\frac{3x}{x^2-4}+1$$



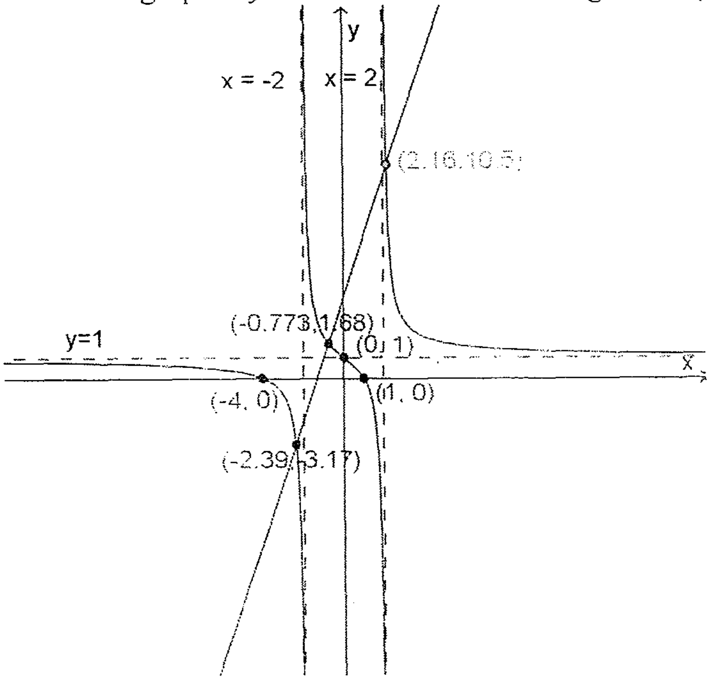
1 (ii)

$$\frac{x}{x^2-4}>x+1$$

$$\frac{3x}{x^2-4}>3x+3$$

$$\frac{3x}{x^2-4}+1>3x+4$$

Sketch the graph if $y = 3x + 4$ on the same diagram in (i):



From graph.

$x<-2.39$ or $-2<x<-0.773$ or $2<x<2.16$

2(i)	$y=\ln\left(\frac{x}{3-x}\right)$ $=\ln x-\ln(3-x)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}-\frac{-1}{3-x}=\frac{1}{x}+\frac{1}{3-x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=k$ $\frac{1}{x}+\frac{1}{3-x}=k$ $\frac{3-x+x}{x(3-x)}=k$ $3=kx(3-x)$ $3=3kx-kx^2$ $kx^2-3kx+3=0$ (shown)
2(ii)	No real roots implies Discriminant < 0 $(-3k)^2-4k(3)<0$ $9k^2-12k<0$ $3k(3k-4)<0$ Therefore, $0<k<\frac{4}{3}$.
2(iii)	Since 2 is not in the range $0<k<\frac{4}{3}$, there is a point on the curve C with gradient 2.

3 (i)

$$D=\sqrt{(x-50)^2+y^2}$$
$$=\sqrt{(x-50)^2+\left(\sqrt{x}\right)^2}$$
$$=\sqrt{x^2-100x+2500+x}$$
$$=\sqrt{x^2-99x+2500}$$

3 (ii)

$$D=\sqrt{x^2-99x+2500}$$

Applying Chain Rule:

$$\frac{\mathrm{d}D}{\mathrm{d}x}=\frac{1}{2}\left(x^2-99x+2500\right)^{-\frac{1}{2}}(2x-99)$$
$$=\frac{2x-99}{2\sqrt{x^2-99x+2500}}$$

For stationary point(s), $\frac{\mathrm{d}D}{\mathrm{d}x}=0$, so we have

$$2x-99=0$$
$$x=49.5$$

Method 1 (1st Derivative Test)

x	49.4	49.5	49.6
$\frac{\mathrm{d}D}{\mathrm{d}x}$	-0.0141762	0	0.0141762
	<0	$=0$	>0

Thus, D is a minimum at $x = 49.5$.

Method 2 (2nd Derivative Test)

$$\frac{\mathrm{d}^2D}{\mathrm{d}x^2}=-\frac{(2x-99)^2}{4\left(x^2-99x+2500\right)^{\frac{3}{2}}}+\frac{2}{2\sqrt{x^2-99x+2500}}$$

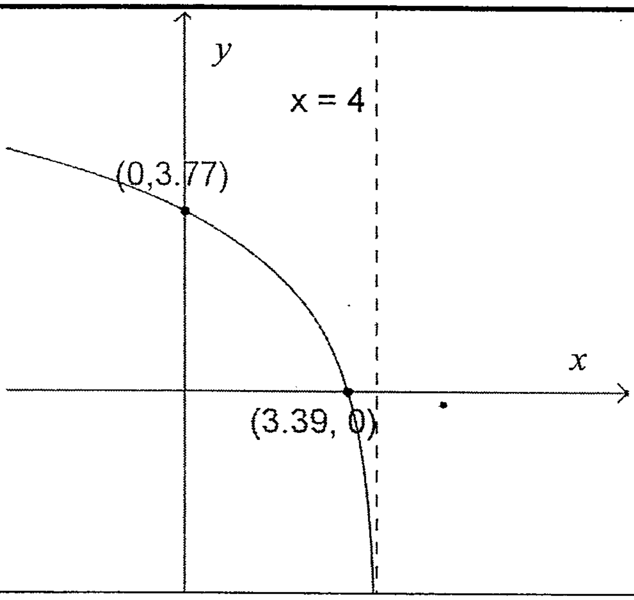
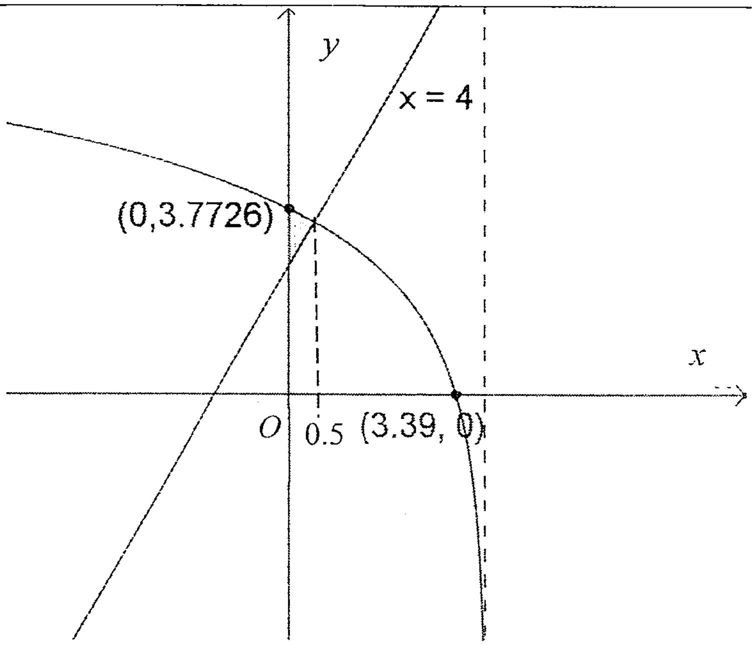
When $x=49.5$, $\frac{\mathrm{d}^2D}{\mathrm{d}x^2}=0.14177>0$.

Thus, D is a minimum at $x = 49.5$.

The minimum D is

$$\sqrt{(49.5)^2-99(49.5)+2500}=7.05 \text{ metres.}$$

4 (a)(i)	Let $y=\sqrt{3}x-\dfrac{\pi}{\sqrt{x}}$ $=\sqrt{3}x-\pi x^{0.5}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{3}-\dfrac{\pi}{2\sqrt{x}}$
4 (a)(ii)	$\mathrm{f}\left(x\right)=\mathrm{e}^{\left(k^2\sqrt{x}+k\right)}$ $\mathrm{f}\,'\left(x\right)=\dfrac{k^2}{2\sqrt{x}}\mathrm{e}^{\left(k^2\sqrt{x}+k\right)}$
4 (b)	By GC, $\dfrac{\mathrm{d}y}{\mathrm{d}x}\approx 0.80685=0.81$ (2 d.p)
4 (c)	$\int_0^1\left(\mathrm{e}^{\frac{x}{2}}+3\right)^2\mathrm{d}x=\int_0^1\left(\mathrm{e}^x+6\mathrm{e}^{\frac{x}{2}}+9\right)\mathrm{d}x$ $=\left[\mathrm{e}^x+12\mathrm{e}^{\frac{x}{2}}+9x\right]_0^1$ $=\left(\mathrm{e}+12\mathrm{e}^{\frac{1}{2}}+9\right)-\left(1+12+0\right)$ $=\mathrm{e}+12\mathrm{e}^{\frac{1}{2}}-4$

5 (i)	
5 (ii)	$\frac{dy}{dx}=\frac{-2}{4-x}$ When $x=0.5$, $\frac{dy}{dx}=\frac{-2}{{}^{7}\!/_{2}}=-\frac{4}{7}$ Gradient of normal $=-\frac{1}{-4/7}=\frac{7}{4}$ When $x=0.5$, $y=1+2\ln(3.5)$: Equation of normal is $y-(1+2\ln(3.5))=\frac{7}{4}(x-0.5)$ $y=\frac{7}{4}x+\frac{1}{8}+2\ln\left(\frac{7}{2}\right)$
5 (iii)	

Integrating with respect to x-axis:

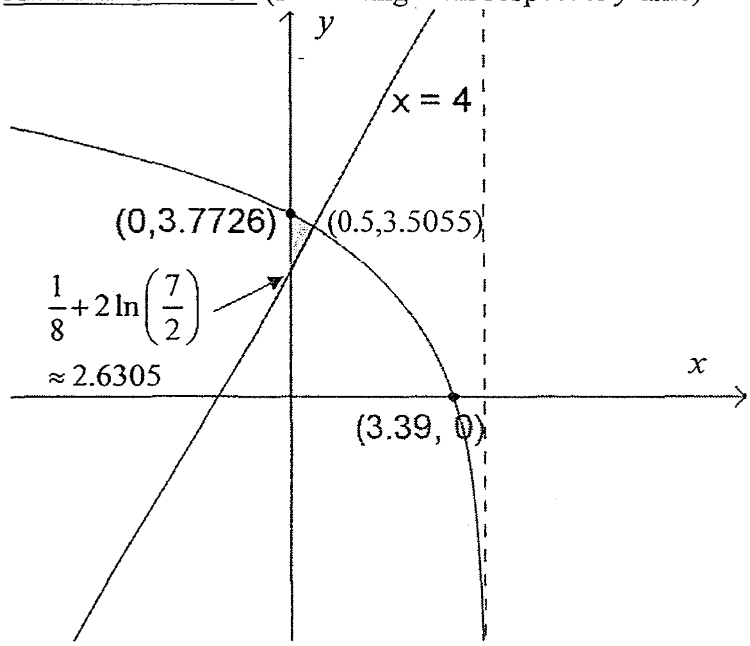
Area of required region (shaded)

= "Area bounded between curve C and the normal between $x = 0$ and $x = 0.5$"

$$= \int_0^{0.5} \left(1 + 2\ln(4-x) - \left(\frac{7}{4}x + \frac{1}{8} + 2\ln\left(\frac{7}{2}\right)\right)\right) \mathrm{d}x$$

$= 0.2870$ (4d.p) by GC

Alternative Method (Ir ting with respect to y-axis)



$$y = 1 + 2\ln(4-x)$$

$$\Rightarrow y - 1 = 2\ln(4-x)$$

$$\Rightarrow \frac{y-1}{2} = \ln(4-x)$$

$$\Rightarrow \mathrm{e}^{\frac{y-1}{2}} = 4 - x$$

$$\Rightarrow x = 4 - \mathrm{e}^{\frac{1}{2}y - \frac{1}{2}}$$

Area of required region (shaded)

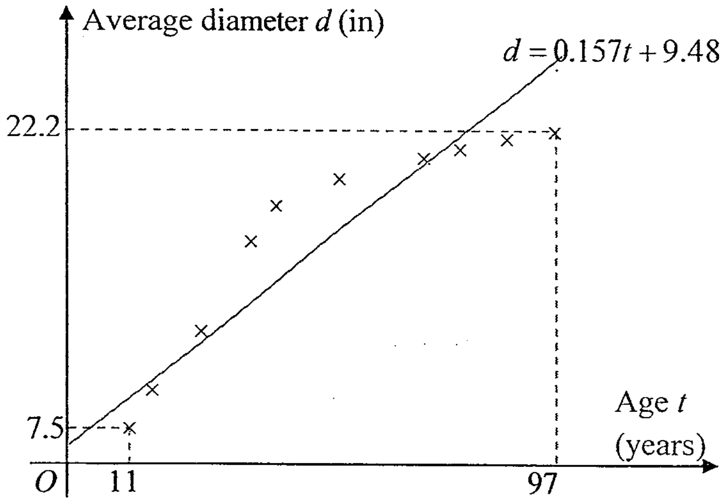
$$= \int_{3.5055}^{3.7726} \left(4 - \mathrm{e}^{\frac{1}{2}y - \frac{1}{2}}\right) \mathrm{d}y + \frac{1}{2}\left(\frac{1}{2}\right)(3.5055 - 2.6305)$$

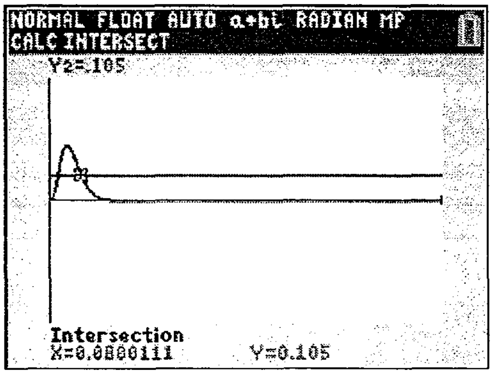
$\approx 0.0682611 + 0.21875$

$= 0.2870$ (4d.p) by GC

6 (a)	Assign a number from 1 to N to each of the students, where N represents the student population size OR obtain a list of the students from the administration office in order of their identification numbers or registration numbers. Next, determine the sampling interval size $k = \frac{1}{0.02} = 50$. Randomly select a student from the 1st 50 students. Select every 50th student thereafter until the required sample is obtained.
6 (b)	**Advantages:** • **<u>Representativeness of Sample</u>** Quota sampling allows the survey to capture the responses that represent various groups of students (e.g. different PM classes, or CCAs); this may be preferred as certain homeroom or sports facilities may not be in as good a condition as others, and the representation of each group will ensure that the results will not be biased towards those who are often using these less functional facilities or towards those who are often using the more functional facilities. • **<u>Efficiency of Collecting the Sample</u>** Quota sampling may be more efficient as systematic sampling in this case requires the surveyor to identify the selected respondents and to contact them, which can be time consuming (e.g. student selected may be on MC on day of survey, selected students do not respond to online survey etc) **Disadvantages:** • **<u>Non-randomness/Selection Bias</u>** Quota sampling is non-random and may contain selection bias, where the surveyor chooses people who may appear more friendly or choose students in the canteen only at a selected time period. This results in certain students having no chance of being selected at all, which may affect the validity of the survey results. • **<u>Non-representativeness of Sample</u>** Quota sampling may result in a group (e.g. one entire cohort, or people coming later to the canteen etc) being excluded entirely from the selection, which may result in the data collected being an inaccurate representation of the entire school population.

7 (i)	$\mathrm{P}(A\mid B)=p$ $\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=p$ $\mathrm{P}(A\cap B)=p^2$ $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $0.8=3p-1+p-p^2$ $-p^2+4p-1=0.8$ Using GC, p = 0.5167603 or p = 3.4832397 (reject since $p>1$) Therefore, p = 0.517 (2 d.p)
7 (ii)	$\mathrm{P}(A'\cup B')=1-\mathrm{P}(A\cap B)$ $=1-p^2$ $=1-(0.51676)^2$ $=0.733$ (3s.f)
7 last part	$\mathrm{P}(A)=(3(0.52)-1)=0.56$ $\mathrm{P}(A\mid B)=0.52$ A and B are not independent since $\mathrm{P}(A)\neq\mathrm{P}(A\mid B)$. **Alternative Method** $\mathrm{P}(A)\mathrm{P}(B)=(3p-1)p$ $=(3(0.52)-1)0.52$ $=0.2912$ $\mathrm{P}(A\cap B)=p^2=(0.52)^2=0.2704$ A and B are not independent since $\mathrm{P}(A)\mathrm{P}(B)\neq\mathrm{P}(A\cap B)$

8 (i)	
(ii)	$r = 0.918$ (to 3 s.f)
(iii)	The regression line of d on t is $d = 0.15657t + 9.4765$ $d = 0.157t + 9.48$ (3s.f) For every increase in 1 year of the age of a teak tree, the trunk diameter increases **approximately** by 0.157 inch.
(iv)	When $t = 40$, $d = 0.15657(40) + 9.4765 = 15.7393$ $d = 15.7$ (3 s.f) Hence the diameter is 15.7 inches (to 3 s.f.) Since $t = 40$ is within the range of values of t, $[11, 97]$ and $r = 0.918$ is close to 1, suggesting a strong positive linear correlation between d and t, therefore the estimate is reliable.
(v)	A linear model predicts the diameter will keep increasing indefinitely. Therefore a linear model is not appropriate. **OR** The diameter of any teak tree should approach a constant value over time, hence a linear model is not suitable.
(vi)	The regression line of d on t is $d = 0.15657t + 9.4765 + k$ $d = 0.157t + 9.48 + k$ (3s.f)

9 (i)	The eye colour of a person is independent of that of another person. **OR** The probability that the eye colour of a person is blue is constant.
9 (ii)	$Y \sim \mathrm{B}(70, p)$ $\mathrm{P}(Y=3)=0.105$ $\binom{70}{3}p^3(1-p)^{67}=0.105$ from MF15 $54740p^3(1-p)^{67}=0.105$  NORMAL FLOAT AUTO a+bi RADIAN MP CALC INTERSECT Y2=.105 Intersection X=0.0800111 Y=0.105 (*Tip: Set the window for* x_{min} = 0 and x_{max} = 1 since x (which is p in the above equation) represents probability) From graph, since $p > 0.05$, $p = 0.080011$ $= 0.0800$ (4 d.p)
9 (iii)	$Y \sim \mathrm{B}(70, 0.08)$ Then $\mathrm{P}(5 \le Y < 21)$ $= \mathrm{P}(Y \le 20) - \mathrm{P}(Y \le 4)$ $= 0.668$ (3 s.f)
9 (iv)	Since $n = 70$ is sufficiently large, $np = 5.6 > 5$ and $nq = 64.4 > 5$, hence $Y \sim \mathrm{N}(5.6, 5.152)$ approximately. $\mathrm{P}(Y > 9) = \mathrm{P}(Y > 9.5)$ $= 0.0429$ (to 3 s.f.)

10 (i)(a)

Let E and F denote the event that the sum of scores is at least 8 and the event that the difference between the scores is at least 4 respectively.

Sum of Scores

1st die \ 2nd die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\mathrm{P}(E)=\frac{1}{6}\times\frac{1}{6}\times 15=\frac{5}{12}$$

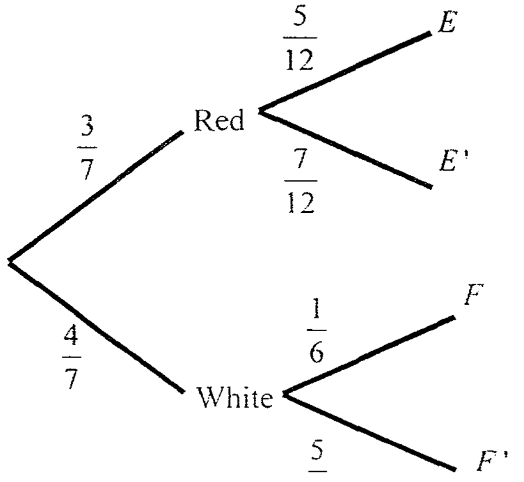
10 (i)(b)

Difference between Scores

1st die \ 2nd die	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

$$\mathrm{P}(F)=\frac{1}{6}\times\frac{1}{6}\times 6=\frac{1}{6}$$

10 (ii)



$\mathrm{P}(\text{player wins})$

$=\mathrm{P}(\text{red and sum at least } 8)+\mathrm{P}(\text{white and diff at least } 4)$

$=\left(\frac{3}{7}\right)\left(\frac{5}{12}\right)+\left(\frac{4}{7}\right)\left(\frac{1}{6}\right)$

$=\frac{23}{84}$

10 **(iii)**	P(red ball selected | player wins) $=\dfrac{\text{P(red ball selected }\cap\text{player wins)}}{\text{P(player wins)}}$ $=\dfrac{\left(\frac{3}{7}\right)\left(\frac{5}{12}\right)}{\frac{23}{84}}=\dfrac{15}{23}$

11 (i)

Unbiased estimate of μ, $\bar{x}=\frac{\sum x}{n}=\frac{38100}{40}=952.50$

Unbiased estimate of σ^2, $s^2=\frac{1}{39}(731800)$
$=18764.10$ (2 d.p.)

11 (ii)

H_0: $\mu=1000$
H_1: $\mu\neq1000$

Level of Significance: 5%

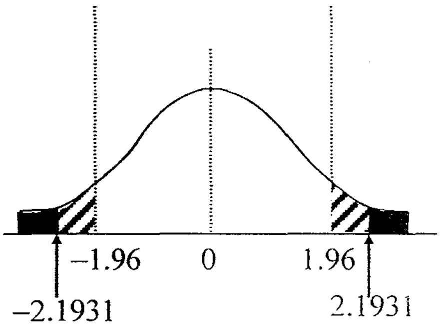
Under H_0, $Z=\dfrac{\overline{X}-1000}{S/\sqrt{40}}\sim N(0,1)$ approximately by Central Limit Theorem

Method 1: *Compare critical region and observed test statistic*

Critical region: $|z|>1.960$

$$z=\frac{952.5-1000}{s/\sqrt{40}}\approx-2.1931 \qquad \left(\text{where } s=\sqrt{\frac{731800}{39}}\right)$$

Since $|z|=2.1931>1.960$, we reject H_0.



Method 2: *Using p-value*

p-value $=0.028299$

Since p-value $=0.0283<0.05$, we reject H_0.

We conclude that there is sufficient evidence at 5% level of significance that the mean amount of loans borrowed by its clients differs from \$1000.

11 (iii)

The meaning of 'at the 5% significance level' is that there is a probability of 0.05 of rejecting the claim that the **mean** amount of loans borrowed by its clients is \$1000 given that it is true.

OR

	The meaning of ‘at the 5% significance level’ is that there is a probability of 0.05 that it was **wrongly concluded** that the **mean** amount of loans borrowed by its clients differs from \$1000.
11(iv)	Test Statistic: $z=\dfrac{k-1000}{250/\sqrt{40}}$ Do not reject $\mathrm{H}_0 \Rightarrow -1.96<z<1.96$ $-1.96<\dfrac{k-1000}{250/\sqrt{40}}<1.96$ $-77.4758<k-1000<77.4758$ $922.524<k<1077.4758$ $922.53 \le k \le 1077.47$

12 (a)	$X \square \mathrm{N}(\mu,\sigma^2)$ By symmetry, $\mu=\frac{18.1+21.9}{2}=20$ $\mathrm{P}(X<18.1)=0.2$ $\mathrm{P}\left(Z<\frac{18.1-20}{\sigma}\right)=0.2$ $\frac{18.1-20}{\sigma}=-0.841621$ --- (1) ***OR*** $\mathrm{P}(X>21.9)=0.2$ $\mathrm{P}(X<21.9)=0.8$ $\mathrm{P}\left(Z<\frac{21.9-20}{\sigma}\right)=0.8$ $\frac{21.9-20}{\sigma}=0.841621$ --- (2) $\sigma=2.26$ (3s.f)
12 (bi)	Let D and E be the battery life of a DuraSell and Energise battery respectively. $D\sim\mathrm{N}(m,1.5^2)$ and $E\sim\mathrm{N}(19.8,2.7^2)$ $D-m\sim\mathrm{N}(0,1.5^2)$ $\mathrm{P}(m-1<D<m+1)$ $=\mathrm{P}(-1<D-m<1)$ $=0.495015066$ $=0.495$ (3s.f) <u>Alternative Method (Standardisation)</u> $\mathrm{P}(m-1<D<m+1)$ $=\mathrm{P}\left(\frac{-1}{1.5}<\frac{D-m}{1.5}<\frac{1}{1.5}\right)$ $=\mathrm{P}\left(\frac{-2}{3}<Z<\frac{2}{3}\right)$ $=0.495015066$ $=0.495$ (3s.f)
12 (bii)	Let X be the number of DuraSell batteries, out of 3, with battery life within 1 hour of its mean. $X\sim\mathrm{B}(3,\ 0.49502)$ $\mathrm{P}(X=2)=0.371$ (3s.f) <u>Alternative Method</u>

	Let A be the event that a DuraSell battery has battery life within 1 hour of its mean. P(exactly 2 DuraSell batteries with battery life within 1 hour of its mean) $=\mathrm{P}(A'AA)+\mathrm{P}(AA'A)+\mathrm{P}(AAA')$ $=\mathrm{P}(A')\mathrm{P}(A)\mathrm{P}(A)+\mathrm{P}(A)\mathrm{P}(A')\mathrm{P}(A)+\mathrm{P}(A)\mathrm{P}(A)\mathrm{P}(A')$ $=3(1-0.49502)(0.49502)^2$ $=0.371$ (3s.f)
12 (biii)	$D-E\sim\mathrm{N}(2.6,9.54)$ $\mathrm{P}(D>E)$ $=\mathrm{P}(D-E>0)$ $=0.800$ (3s.f)
12 (biv)	$\overline{E}\sim\mathrm{N}\left(19.8,\frac{2.7^2}{50}\right)$ $\mathrm{P}(\overline{E}>19)\approx 0.98192$ Expected number of samples that passes quality check = 100(0.98192) = 98.2 (3 s.f)