O

JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS — 4049/01

PAPER 1
Candidates answer on the Question Paper.
Additional Materials : Writing Paper (1 sheet)

29 August 2022
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **17** printed pages including this page.

[Turn Over]

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 It is given that $\sin A = 0.3$, where A is obtuse.
Find the following trigonometric ratios.

(a) $\sec A$, [2]

(b) $\cos 2A$, [2]

(c) $\tan(A + 45°)$. [4]

2 **(a)** Find the range of values of k such that $3x^2 - 5x + k$ is always positive. [2]

(b) Hence, solve the inequality $\dfrac{x^2 - x - 2}{3x^2 - 5x + 4} < 0$. [3]

3 It is given that $f(x) = Ax(e^{kx})$, where A and k are constants.

Find the exact values of A and k such that $f'(x) + 2ke^{kx} + 6f(x) = 0$. [6]

4 A curve has equation $y = \frac{4}{\sqrt{x+3}}$. A point (x, y) is moving along the curve.

Find the coordinates of the point at the instant where the y-coordinate is decreasing at a rate twice of the rate of increase of the x-coordinate. [5]

5 A metal cube is heated to a temperature of 212°C before being dropped into a liquid. As the cube cools, its temperature T°C, t minutes after it enters the liquid is given by $T = P + 180e^{-kt}$, where P and k are constants.
It is recorded that when $t = 5$, $T = 185$.

(a) Find the value of P and of k. [4]

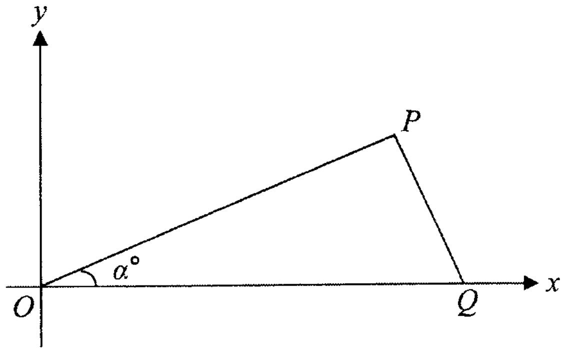
(b) By sketching the graph of T against t, explain why T cannot be 30. [3]

6 It is given that the first three terms, in ascending powers of x, of the binomial expansion of $(2+ax)^6$ are $64-960x+bx^2$ respectively.

(a) Find the value of a and of b. [3]

(b) Using the values found in part **(a)**, find the coefficient of x^3 in the expansion of $(1+3x^2)^5(2+ax)^6$. [4]

7 In the diagram below, the line OP makes an angle of $\alpha°$ with the positive x-axis such that $\tan\alpha = 0.2$ and Q lies on the x-axis.



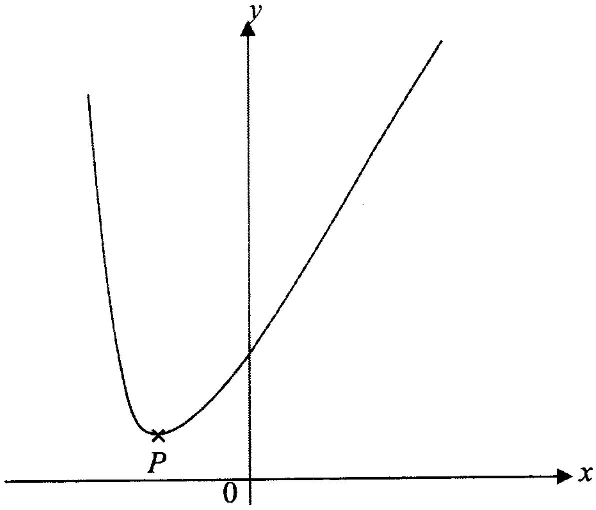
(a) Given that $OP = \dfrac{\sqrt{26}}{2}$ units, show that $P = (2.5, 0.5)$. [3]

(b) Given that the area of ΔOPQ is 0.65 units^2, find the coordinates of Q. [2]

(c) Explain, with calculations, why ΔOPQ is a right-angled triangle. [3]

(d) Find the coordinates of R such that $OPQR$ is a rectangle. [2]

8 The diagram below shows part of the graph of $y = \dfrac{x^2 + 2x + 5}{x + 3}$.



Find the coordinates of the minimum point P. [7]

9 **(a)** It is given that $\frac{2^x \times 32(2^x)}{8^{x+1}} = \frac{9(5^{2x}) - 5^{2x+1}}{5^x - 5^{x-1}}$.

Evaluate 10^x without using a calculator. [5]

(b) Solve $\sqrt{x+7} - x - 1 = 0$. [3]

10 A particle moves in a straight line so that its velocity, v m/s is given by $v = t^2 - 4t + 3$, where t is the time in seconds after passing a fixed point O.

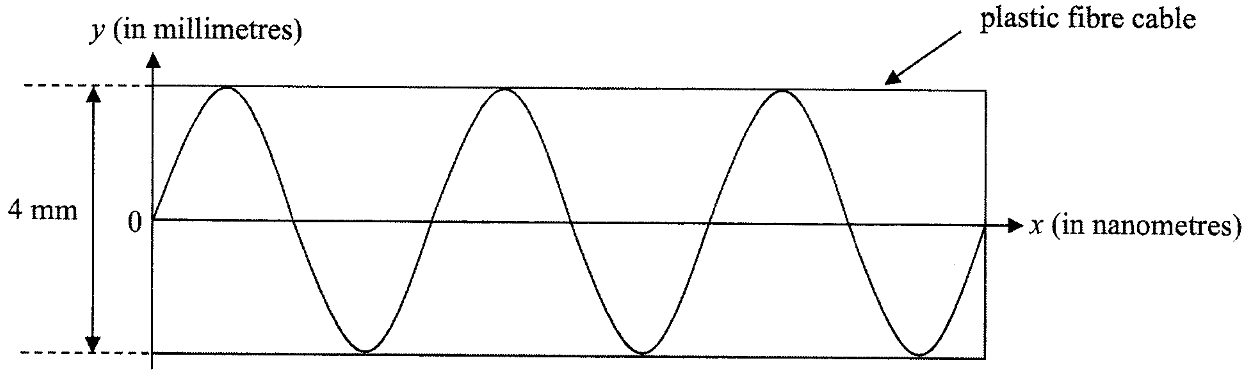
(a) Find the acceleration of the particle when $t = 1$. [2]

(b) Find the value(s) of t when the particle comes to instantaneous rest. [2]

(c) Find the displacement(s) of the particle at the instant when it comes to rest. [3]

(d) Find the average speed of the particle for the first 4 seconds. [3]

11 The diagram below shows a portion a plastic fibre cable, which allows light waves to pass through. The path of the light wave can be modelled by a trigonometric function.



(a) It is given that the diameter of the cable is 4 millimetres.
Find the amplitude of the light wave. [1]

(b) It is given further that the period of the light wave is 500 nanometres.
Find the length of the portion of cable shown in the diagram. [1]

(c) Which of the following can be a suitable model for the light wave?

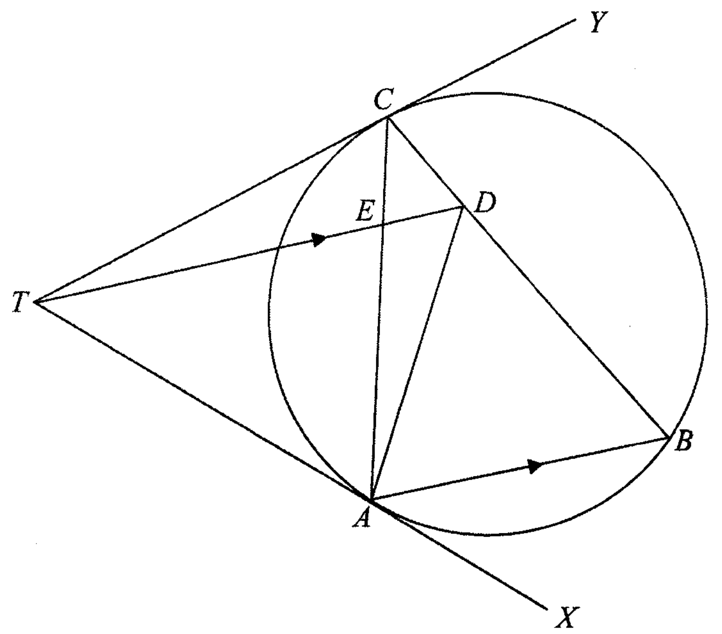
$y = 2\sin(\pi x)$ $\qquad$ $y = 2\cos(\pi x)$ $\qquad$ $y = 2\sin(\frac{\pi}{250}x)$

Explain your answer. [3]

12 **(a)** Sketch the graph of $y=-\sin x+1$ and $y=3\cos 2x$ for $0°\leq x\leq 360°$ on the same axes. [4]

(b) Hence, state the number of solutions to the equation $-\sin x+1=3\cos 2x$ for $0°\leq x\leq 360°$. [1]

13 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB.



(a) Prove that angle ACB is equal to angle ATD. [2]

(b) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

(c) Hence, prove that $CE \times EA = DE \times TE$. [4]

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JURONG SECONDARY SCHOOL
2022 GRADUATION EXAMINATION 2
SECONDARY 4 EXPRESS

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/02

Paper 2 **25 August 2022**

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1 **(a)** Differentiate $2x\cos 3x$ with respect to x. [2]

(b) Hence, find $\int x\sin 3x \, dx$. [3]

2 **(a)** Factorise a^3+b^3. [1]

(b) Show that $\dfrac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=1-\dfrac{\sin 2x}{2}$. [2]

(c) Hence, solve the equation $\dfrac{\sin^3 x+\cos^3 x}{\sin x+\cos x}=1-\sin^2 2x$ for $0\le x\le \pi$. [5]

3 **(a)** Express $\dfrac{3x^2-4}{x^2(3x-2)}$ in partial fractions. [4]

(b) Hence, evaluate $\displaystyle\int \frac{3x^2-4}{x^2(3x-2)}\,\mathrm{d}x$. [3]

4 It is given that $\log_2(4-x^2)-\log_{\sqrt{2}}(x-1)=1$.

(a) Explain clearly why $1<x<2$. [4]

(b) Hence, solve the equation and show that it has only one solution. [5]

5 f(x) is such that $f''(x) = 18x - 4$. Given that f(0) = 1 and f(2) = 9,

(a) find f(x). [4]

(b) Show that $x + 1$ is a factor. [1]

(c) Solve $f(x) = 0$. [4]

6 It is given that $x = -2$ and $y = -1$ are tangents to a circle.
The x-coordinate and y-coordinate of the centre of the circle are positive.
The line $3y = 2x + 5$ is a normal to the circle.

(a) Show that the centre of the circle is (2, 3). [4]

(b) Find the equation of the circle. [2]

7 The height of a coin from the ground, h meters, after it has been flipped in the air for t seconds, can be represented by $h = -6t^2 + 24t + 12$.

(a) By completing the square, find the greatest height which the coin reaches. [3]

(b) Find the exact duration of the coin from the time it is flipped in the air till it lands on the ground. [3]

8 A pot of melted chocolate is cooled from its initial temperature to a temperature of T °C in x minutes and follows the equation of the form $T = A(B^x)$, where A and B are constants. The freezing point of the chocolate is 17°C. The table below shows the corresponding values of T and x recorded.

x	5	10	15	20	25
T	35.429	20.921	12.353	7.295	4.307

(a) Draw the graph of lg T plotted against x, using a scale of 2 cm for 5 unit on the x-axis and a scale of 1 cm for 0.1 unit on the lg T-axis. [3]

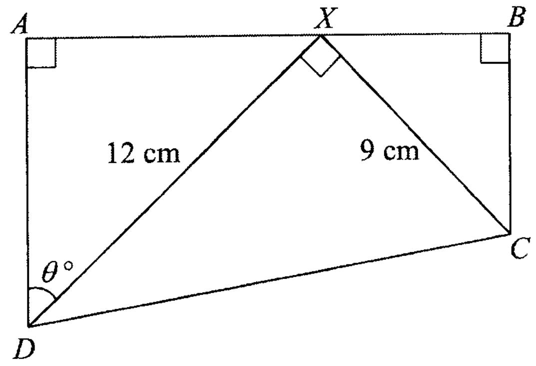
Using your graph,

(b) state whether the chocolate is frozen at 13 minutes and justify your answer, [2]

(c) estimate the value of each of the constants A and B, [5]

(d) find the amount of time taken for the chocolate to cool to half its original temperature. [2]

9 The diagram shows a trapezium $ABCD$. The point X lies on line AB such that $DX = 12$ cm and $CX = 9$ cm. $\angle ADX = \theta°$ and $\angle DAX = \angle DXC = \angle XBC = 90°$.

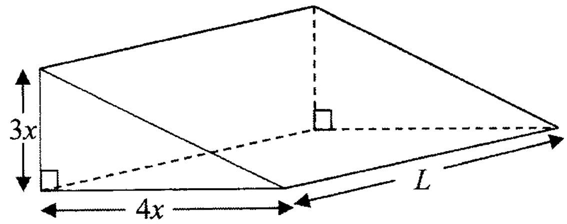


(a) Show that $AB = 9\cos\theta + 12\sin\theta$. [2]

(b) Express AB in the form $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum length of AB and its corresponding value of θ. [5]

(c) Find the value of θ for which $AB = 11$ cm. [3]

10 The figure below shows a right-angled triangular prism.
The height and base of the triangular faces of the prism are $3x$ meters and $4x$ meters respectively. The length of the prism is L meters.

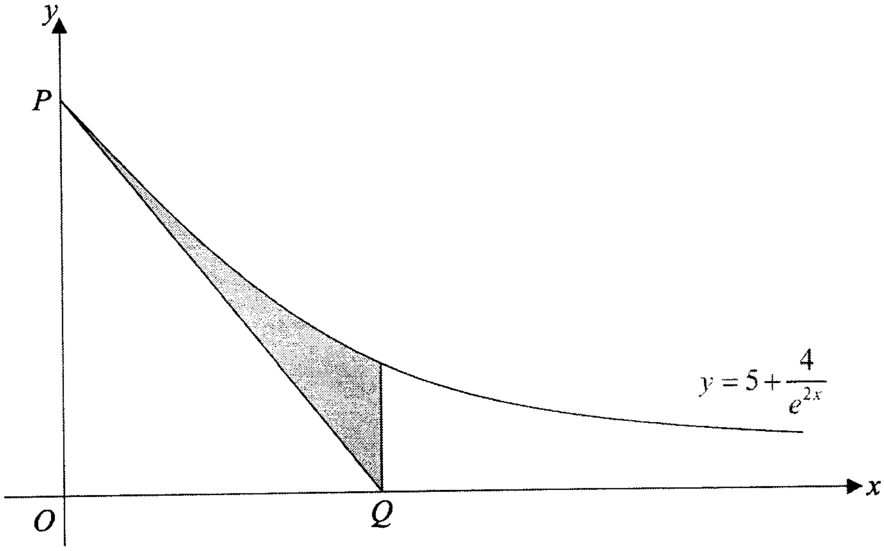


(a) Given that the volume of the prism is 240 m^3, show that the surface area of the prism, A m^2, is given by

$$A = 12x^2 + \frac{480}{x}.$$ [4]

(b) Given that x and L can vary, find the value of x for which A has a stationary value and determine whether this value of A is maximum or minimum. [5]

11



The diagram shows part of the curve $y = 5 + \frac{4}{e^{2x}}$ intersecting the y-axis at point P.

The tangent to the curve at point P intersects the x-axis at Q.

Find the area of the shaded region. [9]

End of Paper