Name	( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2022

Subject : **Mathematics**
Paper : **4048/01**
Level : **Secondary Four Express**
Date : **17 August 2022**
Duration : **2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This paper consists of 23 printed pages.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions.

1 The number of people living in a town is given as 60 000, correct to 2 significant figures. Write down values for the smallest and largest possible number of people who could be in the town.

Answer smallest possible number = ... [1]

largest possible number = ... [1]

2 **(a)** Use prime factors to explain why 135×200 is a perfect cube.

Answer

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.....[2]

(b) The lowest common multiple of x and 135 is $2 \times 3^4 \times 5 \times 7$.
Find the smallest possible value of x.

Answer $x =$ ..... [1]

3 Write the following numbers in order of size, starting with the **smallest**.

$$\left(\frac{3}{10}\right)^2,\ -\frac{21}{90},\ 90\%,\ -0.23\ ,\ \sqrt{0.3}$$

Answer ..... [2]

4 Factorise $\dfrac{6a-15a^2+20ab-8b}{25a^3-4a}$ completely.

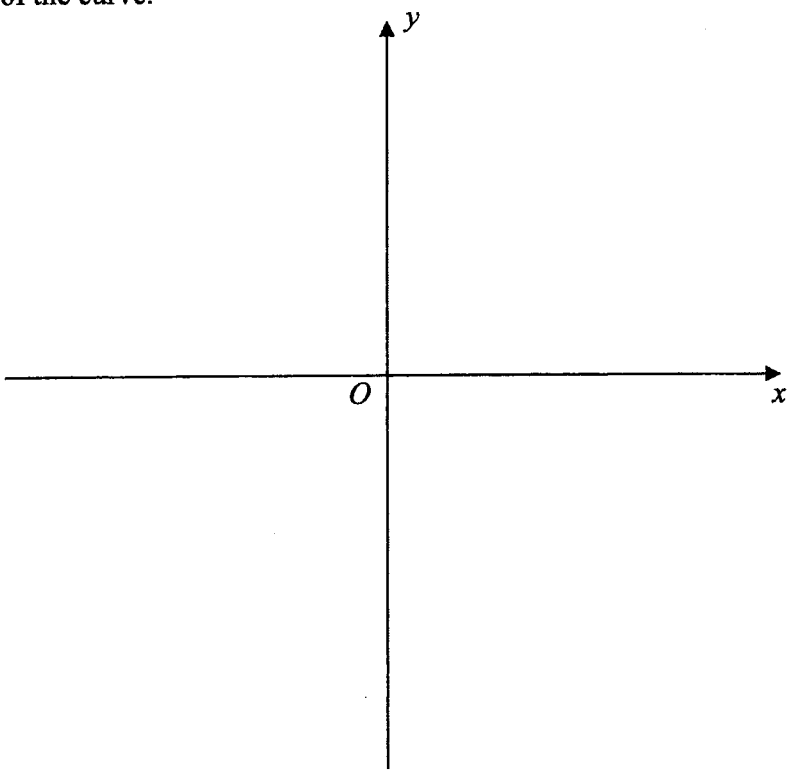
Answer ..... [3]

5 Rearrange the formula $\frac{1}{a}-\frac{1}{2b}=\frac{1}{3c}$ to make b the subject.

Answer $b =$ ..... [2]

6 Sketch the graph of $y=(3-x)(x+2)$ on the axes below.

Indicate clearly the coordinates of the points where the graph crosses the axes and the turning point of the curve.



[2]

7 The scale of a map is 8 cm : 2 km .

(a) Write this scale in the form $1 : n$.

Answer 1 : ..... [1]

(b) The actual area of a lake is 90 000 m^2.
Calculate the area, in square centimetres, of the lake on the map.

Answer .....cm^2 [2]

8 3.8 is the mean of 5 positive numbers a, b, c, d and e.
The sum of their squares is 360. Each of the numbers is now multiplied by 2.
Find the new standard deviation.

Answer ..... [3]

9 One solution of the equation $(k+1)x^2+kx=15$ is $x=-3$.

(a) Find the value of k.

Answer $k=$ ..... [1]

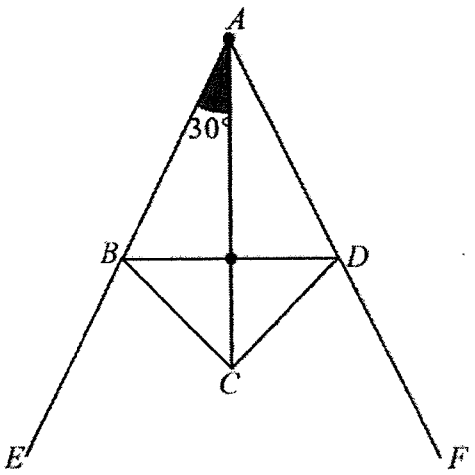
(b) Find the second possible value of x.

Answer $x=$ ..... [1]

10 The cash price of a washing machine is \$840.
If paid by hire purchase scheme, the deposit is 15% of the cash price and the subsequent 24 equal monthly payments is \$33.50.
Calculate the interest rate per annum.

Answer .....% [3]

11



The diagram above shows a kite $ABCD$ where angle $BAC = 30°$.
EB, BD and DF are three sides of a regular polygon.
ABE and ADF are straight lines.
The ratio of angle CBE to angle CBD is 3: 2.

(a) Calculate the number of sides of the polygon.

Answer ..... [2]

(b) Find angle BCD.

Answer ..... [2]

12 **(a)** The first four terms in a sequence of numbers are given below.

$T_1 = 2^2 + 7$

$T_2 = 3^2 + 12$

$T_3 = 4^2 + 17$

$T_4 = 5^2 + 22$

Explain why the value of T_n must be odd for all values of n.

Answer

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.....[2]

(b) The product of the first n terms of a sequence is given by $2n^2 + 3n$.
Find the 12^{th} term of this sequence.

Answer ..... [2]

13 The thickness of a layer of ice in a water body is 0.00205 m.

(a) Write 0.00205 in standard form.

Answer ..... [1]

(b) The ice covers an area of 1.60×10^5 m^2.
Assuming that all the ice melts and ignoring the expansion of volume when water freezes, calculate the volume of water, in litres.

Answer .....*l* [2]

(c) A thunderstorm occurs and rainwater is falling at an average rate of 5×10^{-1} litres per second over the water body.
Calculate the percentage increase in the volume of water in the water body after two hours.

Answer .....% [2]

14 Given that the coordinates of A is $(2,-3)$, $\overrightarrow{AB}=\begin{pmatrix}-5\\7\end{pmatrix}$ and $\overrightarrow{FE}=k\begin{pmatrix}2.5\\-3.5\end{pmatrix}$.

(a) Find the value of k if $ABFE$ is a parallelogram.

Answer ..... [1]

(b) Find the coordinates of B.

Answer ..... [1]

(c) C is the point $(6,-10)$.
Justify if A, B and C are collinear.

Answer [2]

15 A tour agency records the total number of people buying tour packages to Thailand and Vietnam in the months of November and December.
In November, 144 people bought the Thailand tour package and 100 people bought the Vietnam tour package.
In December, 208 people bought the Thailand tour package and 180 people bought the Vietnam tour package.

This information can be represented by the matrix, $\mathbf{M} = \begin{matrix} \\ \text{November} \\ \text{December} \end{matrix} \begin{matrix} \begin{matrix} \text{Thailand} & \text{Vietnam} \end{matrix} \\ \begin{pmatrix} 144 & 100 \\ 208 & 180 \end{pmatrix} \end{matrix}$

(a) The price of the Thailand and Vietnam package is \$890 and \$750 respectively.
Represent the price of the tour package by a 2×1 column matrix **K**.

Answer $\mathbf{K} = \begin{pmatrix} \\ \\ \end{pmatrix}$ [1]

(b) Evaluate the matrix $\mathbf{R} = \mathbf{MK}$.

Answer $\mathbf{R} =$ [1]

(c) State what the elements of **R** represent.

Answer

.....

.....

.....[1]

(d) Evaluate $\frac{1}{2}(1 \quad 1)\,\mathbf{R}$ and explain what the answer represents.

Answer

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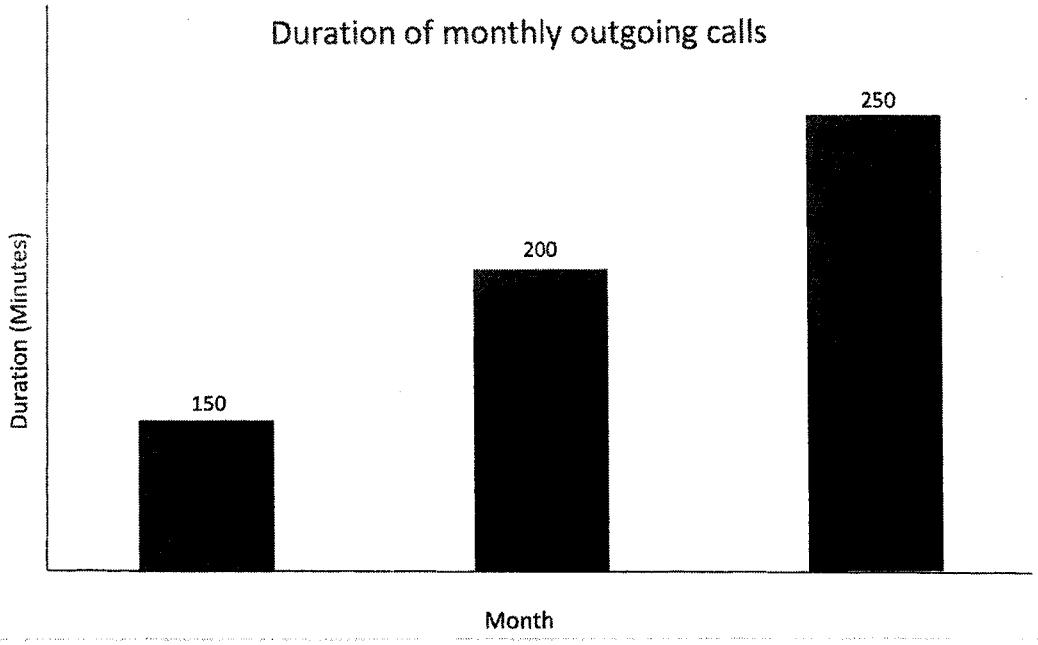
.....[2]

16 Peter draws this graph to show the duration of his monthly outgoing calls (in minutes) for the last three months.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer

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.....[2]

17 $\xi = \{\text{integer } x : 0 < x \leq 20\}$
$P = \{\text{perfect square}\}$
$Q = \{\text{even number which solves } 3x > 11\}$
$R = \{\text{multiple of 4}\}$

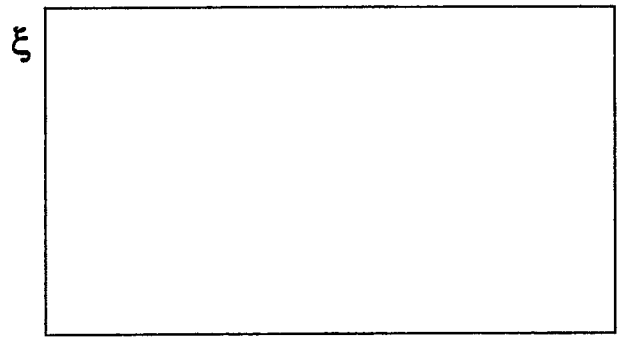
(a) (i) List all the elements in P.

Answer ..... [1]

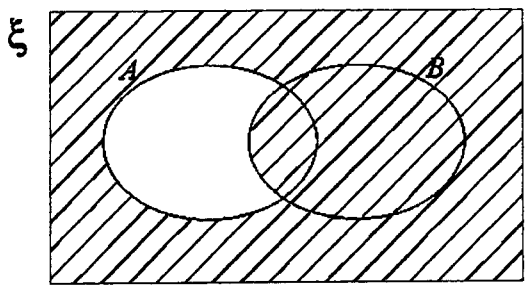
(ii) Find $\text{n}(P' \cap Q)$.

Answer ..... [1]

(iii) On the answer space provided, draw the Venn diagram to illustrate the relationship between sets P, Q and R. [2]

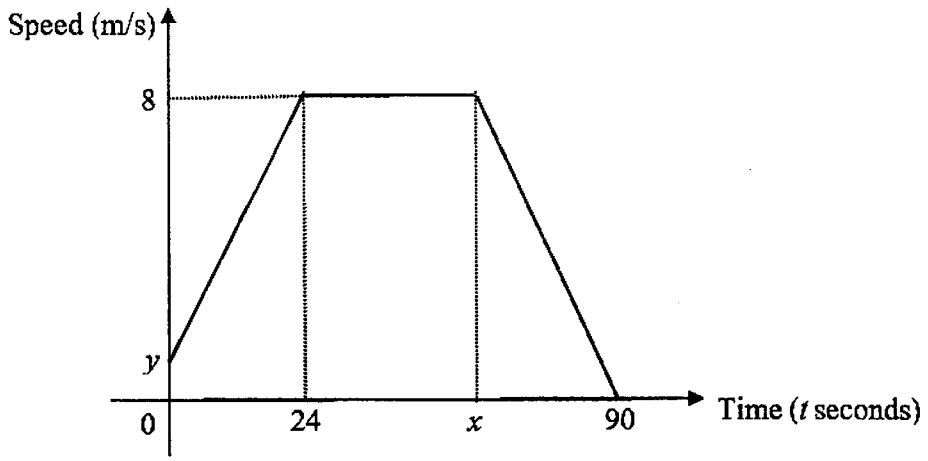


(b) Use set notation to describe the set shaded in the Venn diagram below.



Answer ..... [1]

18 The diagram shows the speed-time graph for a cyclist's journey for a period of 90 seconds.
The cyclist accelerates at 0.25 m/s^2 in the first 24 seconds.
He then travels at a constant speed of 8 m/s for a distance of 288 m.



(a) Find the values of x and y.

Answer $x =$ .....

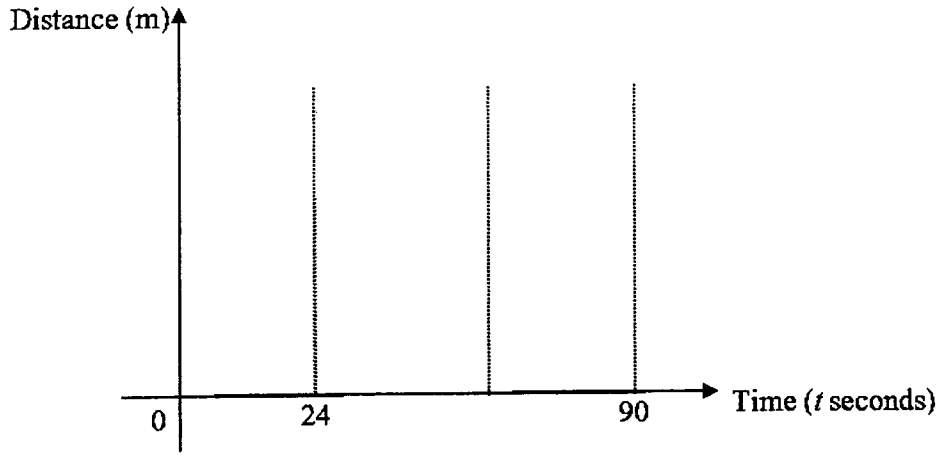
[2]

$y =$ .....

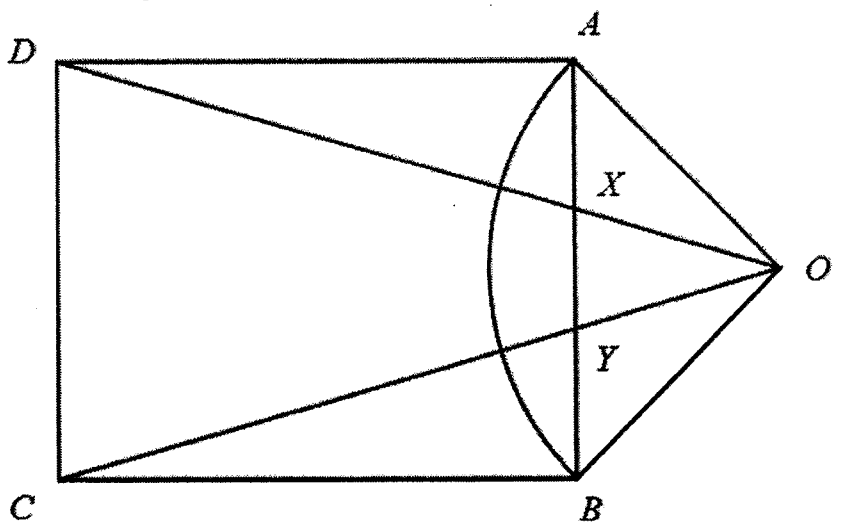
(b) On the grid provided, complete the distance-time graph of the journey from $t = 0$ to $t = 90$ seconds.

Answer

[3]



19 In the diagram, $ABCD$ is a rectangle.
OAB is a sector of a circle, centre O.
OXD and OYC are straight lines.



(a) Show that triangle OAD is congruent to triangle OBC.
Give a reason for each statement you make. [3]

Answer

(b) Show that triangle OXY is similar to triangle ODC.
Give a reason for each statement you make. [2]

Answer

(c) The area of triangle ODC is 36 times that of the area of triangle OXY.
Find the ratio of the area of quadrilateral $DCYX$ to area of triangle OAB.

Answer ..... [1]

20 The ages of 18 swimmers and 11 cyclists in a sports carnival race were recorded. The results are shown in the stem-and-leaf diagram.

Swimmers								Cyclists						
4	4	3	2	1	1	0	2	1	1	2	2	3	3	
			9	x	5	5	2	5	7					
					4	1	3							
				9	8	8	3	6	9					
					3	2	4	0						

Key (Swimmers)
1 | 2 means 21 years old

Key (cyclists)
2 | 3 means 23 years old

(a) Given that the median age of the swimmers is 26 years old, find the value of x.

Answer $x =$ ..... [1]

(b) Find the interquartile range of the cyclists' age.

Answer ..... [2]

(c) Make two comments comparing the ages of the swimmers and the cyclists.

Answer

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.....[2]

21 The plan of a triangular-shaped garden, PQR, is such that $QR = 9.5\text{cm}$ and $\angle PQR = 30°$. PQ has been drawn for you.

(a) Construct triangle PQR in the space provided. [2]

(b) (i) Construct the perpendicular bisector of R and P. [1]

(ii) Construct the bisector of angle QPR. [1]

(c) A bench S needs to be built inside the garden such that it is nearer to PR than to PQ and $SR \geq 3$ cm. Shade the region where S could be possibly built. [2]

P Q

22 Adam invested \$$P$ in a savings account X with interest compounded quarterly at the rate of 1.5% per annum.

Ben invested \$$P$ in a savings account Y, paying simple interest at the rate of x% per year.

At the end of 5 years, Ben made 10% more than Adam.

Mary would like to invest \$$P$ for 20 years.
She believes that savings account Y is better as Ben made more money than Adam.
Justify with clear mathematical working, whether Mary is correct. [4]

– End of Paper –

Name	( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2022

Subject : **Mathematics**
Paper : **4048/02**
Level : **Secondary Four Express**
Date : **18 August 2022**
Duration : **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** questions.
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Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

For Examiner's Use

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	**TOTAL**

This paper consists of **28** printed pages and **3** blank pages.

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** questions.

1 **(a)** Simplify $\frac{3a^2}{5b^3} \div \left(\frac{12c}{25ab}\right)^2$.

Answer ..... [3]

(b) n is a positive integer.

Show that, for all n, $(3n+2)^2 - (3n-2)^2$ is a multiple of 3.

Answer

[2]

(c) Solve the equation $\frac{2}{x-4}+\frac{7x}{3x-2}=1$.

Answer $x =$ ..... or ..... [4]

(d) Use the quadratic formula to solve the equation.

$$3x^2-7x-4=0$$

Answer $x =$ ..... or ..... [2]

2 A box of chocolate contains 7 dark chocolates and 5 milk chocolates.
Sufyan takes a chocolate, selected at random, from the box and eats it.
LeLe then takes a chocolate, selected at random, from the box.

(i) Draw a tree diagram to show the probabilities of the possible outcomes.

Answer

[2]

(ii) Find, as a fraction in its simplest form, the probability that

(a) Sufyan and LeLe both picked dark chocolates,

Answer ..... [1]

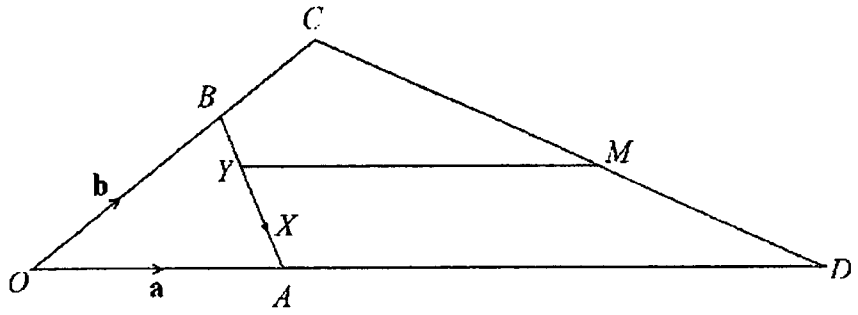
(b) LeLe picked a milk chocolate,

Answer ..... [2]

(c) at least one milk chocolate was chosen.

Answer ..... [2]

3



In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

$2\overrightarrow{OB} = 3\overrightarrow{BC}$, $\overrightarrow{OD} = 3\overrightarrow{OA}$, and $2\overrightarrow{AX} = \overrightarrow{XB}$.

(i) Express, as simply as possible, in terms of **a** and **b**,

(a) $\overrightarrow{BA}$,

Answer $\overrightarrow{BA} =$ ..... [1]

(b) $\overrightarrow{OX}$,

Answer $\overrightarrow{OX} =$ ..... [1]

(c) $\overrightarrow{CD}$.

Answer $\overrightarrow{CD} =$ ..... [1]

(ii) It is given that $CM : MD$ is $10 : 9$.

(a) Express $\overrightarrow{OM}$, as simply as possible, in terms of **a** and **b.**

Answer $\overrightarrow{OM} =$ ..... [1]

(b) Show that O, X and M are collinear.

Answer

[2]

(c) Given that MY is parallel to OD, express $\overrightarrow{MY}$ in terms of **a** and **b**.

Answer $\overrightarrow{MY}$ =..... [1]

(iii) Find the ratio of

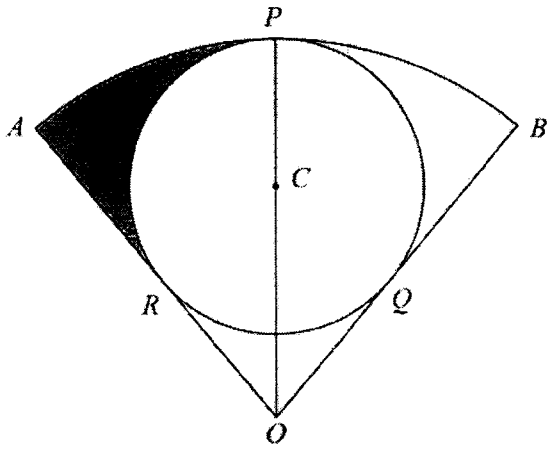
(a) $\dfrac{\text{area of } \Delta XMY}{\text{area of } \Delta XOA}$,

Answer ..... [1]

(b) $\dfrac{\text{area of } \Delta OBX}{\text{area of } \Delta OYA}$.

Answer ..... [2]

4



In the diagram, $OAPB$ is a sector of circle with centre O and radius 9 cm.
Angle $AOB = 1.4$ radians.
C is the centre of the circle enclosed inside the sector.
OCP is a straight line and the circle touches the sector at points P, Q and R.

(i) Show that the radius of the enclosed circle is 3.526 cm, correct to 3 decimal places.

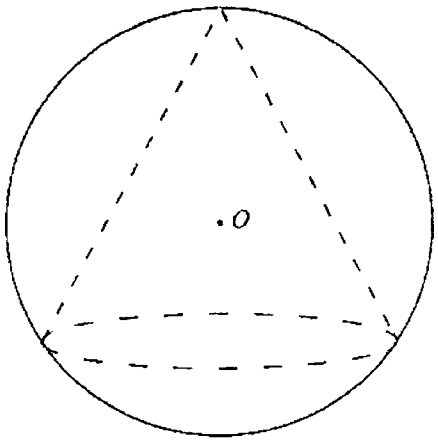
Answer

[5]

(ii) Calculate the area of the shaded region.

Answer ..... cm^2 [4]

5



The diagram shows a right circular cone cut from a solid steel sphere.
Point O is the centre of the sphere with radius 12 cm.

(a) Given the circumference of the base of the cone is 50 cm, show that the height of the cone is 21.0 cm, corrected to 3 significant figures.

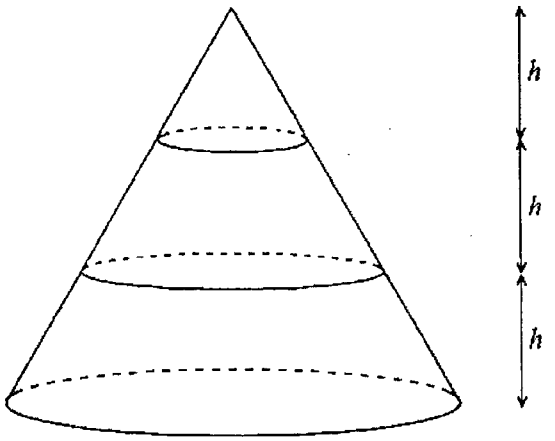
Answer

[3]

(b) Find the curved surface area of the cone.

Answer ..... cm^2 [2]

(c) After the cone is cut from the steel sphere, the remaining steel is melted down to form part of a solid right circular cone as shown in the following diagram.



The cone comprises 3 layers of equal heights, h cm.
The top and bottom layers are cast from the remaining steel.
The centre section is made from acrylic.

Find the volume of the acrylic used to make the centre layer of the solid cone.

Answer ..... cm^3 [4]

6 Plane A travels at an average speed of x km/h for 3 hours 20 minutes and then at an average speed of y km/h for 1 hour 10 minutes.
The plane travels a total distance of 3700 km.

(a) Write down an equation in x and y to represent this information and show that it simplifies to $20x + 7y = 22200$.

Answer

[1]

Plane B travels at an average speed of x km/h for 2 hours 30 minutes and then at an average speed of y km/h for 1 hour 50 minutes.
It travels 350 km lesser than Plane A.

(b) Write down an equation in x and y to represent this information.

Answer ..... [1]

(c) Solve these two equations to find the value of x and the value of y.

Answer

Answer $x =$ .....and $y =$ ..... [3]

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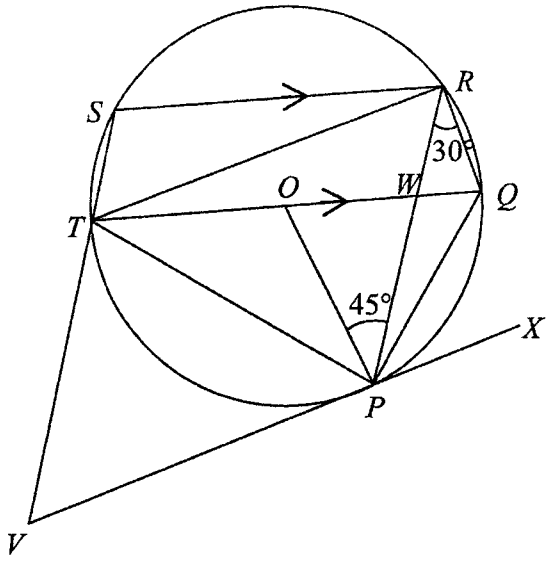
7 In the diagram, P, Q, R, S and T are points on the circle with centre O.
VX is a tangent to the circle at P.
Line SR is parallel to the diameter of the circle TQ.
W lies on TQ and PR.



It is given that $\angle OPR = 45°$ and $\angle PRQ = 30°$.

(a) **(i)** Find angle TOP.
Give a reason for each step of your working.

Answer Angle TOP = ... [2]

(ii) Find angle *TPV*.
Give a reason for each step of your working.

Answer Angle *TPV* = ... [2]

(iii) Find angle *TSR*.
Give a reason for each step of your working.

Answer Angle *TSR* = ... [1]

(b) Show that *SRWT* is a parallelogram.

Answer

[3]

8 The variables x and y are connected by the equation $y=\frac{1}{3}x\left(11-x^2\right)$.

Some corresponding values of x and y, correct to 1 decimal places, are given in the following table.

x	–4	–3	–2	–1	0	1	2	3
y	6.7	–2	–4.7	–3.3	0	3.3	p	2

(a) Calculate the value of p.

Answer $p=$ ..... [1]

(b) On the grid opposite, draw the graph of $y=\frac{1}{3}x\left(11-x^2\right)$ for $-4\le x\le 3$.

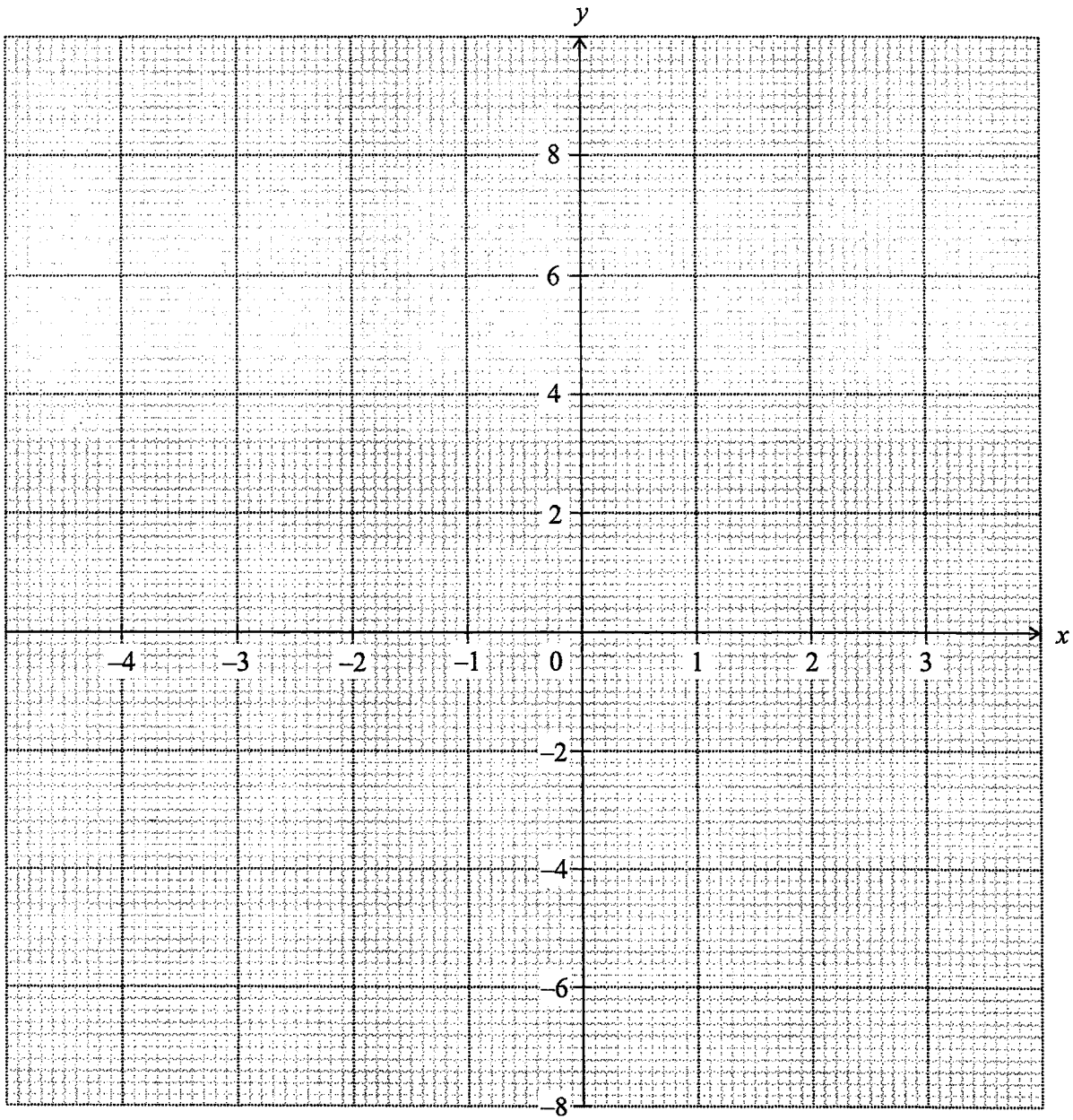
[3]

(c) Use your graph to find the solution of $\frac{1}{3}x\left(11-x^2\right)=3$ in the range $-4\le x\le 3$.

Answer $x=$ .....or.....or..... [2]

(d) By drawing a tangent, find the gradient of the curve at the point $(-1,\ -3.3)$.

Answer ..... [2]



(e) On the same axes, draw the graph of $5y+2x=7$ for $-4 \le x \le 3$.

[1]

(f) **(i)** Show that the points of intersection of the line and the curve give the solutions of the equation $5x^3-61x+21=0$.

Answer

[2]

(ii) Use your graphs to solve the equation $5x^3-61x+21=0$.

Answer $x =$ .....or ..... [1]

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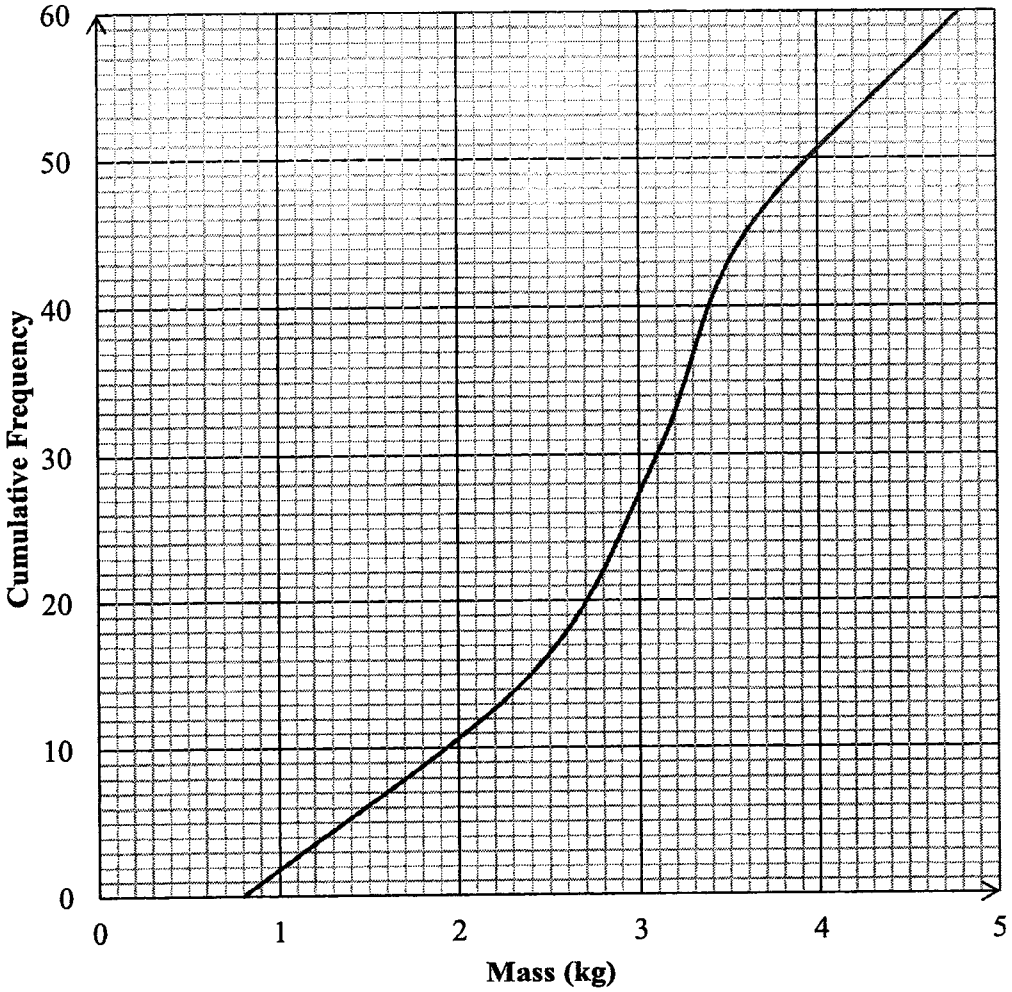
9 60 potato plants produce 5 to 10 potatoes each.
The mass of potatoes produced by each plant were measured.
The cumulative frequency curve below shows the distribution of the masses of the potatoes produced by each plant.



(i) Use the curve to estimate

(a) the median mass,

Answer ..... kg [1]

(b) the interquartile range.

Answer ..... kg [1]

(ii) It was stated that 20% of the potato plants were considered premium plants as they produced greater mass of potatoes.

Find the least mass of potatoes produced for the plant to be 'premium'.

Answer ..... kg [2]

(iii) The potatoes produced by another group of 60 plants have the same median but smaller interquartile range.

Describe how the cumulative frequency curve will differ from the given curve.

Answer .....

.....

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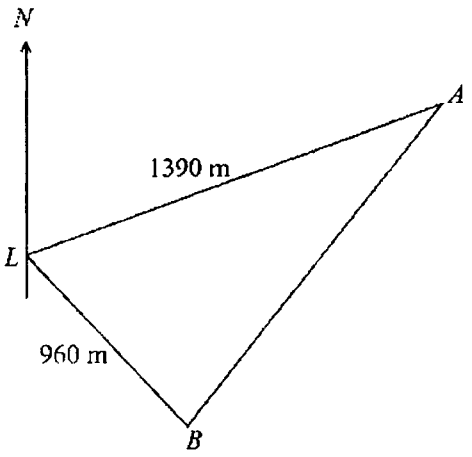
..... [1]

10



ABL is a park on horizontal ground.
A is 1390 m from *L* on a bearing of 076°.
B is 960 m from *L* on a bearing of 138°.

(a) (i) Find *AB*.

Answer ..... m [2]

(ii) Find angle *LAB*.

Answer ..... [2]

(iii) Find the area of triangle LAB.

Answer ..... m^2 [2]

(iv) Find the bearing of B from A.

Answer ..... ° [2]

(b) T is the top of a tower at L.
The greatest angle of depression from T to the path AB is 5.06°.

Calculate the height of the tower.

Answer ..... m [3]

(c) Jonah goes on a jog along the edge of the park at a speed of 8.5 km/h.
He starts from L towards A then to B before going back to L.

Calculate the time he takes to jog.
Give your answer in minutes and seconds, corrected to the nearest 10 seconds.

Answer ..... seconds [3]

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11 Mr Ho would like to replace all 60 units of air conditioners in his office building. He is considering between two brands of air conditioners.

Information about the two brands of air conditioners is shown in the table below.

Brand	Samsing	Potasonic
Price of each air conditioner unit (after GST)	S$1388	S$740
Power consumption per air conditioner unit	3 kW	3.5 kW
Servicing Frequency	Twice a year	Four times a year
Total cost for servicing 60 units	$2100 per servicing	
Warranty*	Two years	Three years
*Warranties cover servicing and maintenance of air conditioner units with free replacement of parts for the stated duration.		

Electricity Tariff from 1 July 2022 is shown in the table below.

SPgroup Empowering the Future of Energy	**Tariff (without GST)**	**Tariff with GST**
kWh* charge (¢ per kWh)	30.17	32.28
* kWh (kilowatt-hour) is a unit of energy equal to one kilowatt (kW) of power sustained for one hour		

The usage of air conditioner units in Mr Ho's office building is shown in the table below.

Days	Usage Time/unit	Number of units used
Monday	8 hours	50
Tuesday to Friday	6 hours each day	60
Saturday and Sunday	No usage	

(a) In view of public holidays, he estimates that the company operates for 51 weeks per year.

Find the usage of all air conditioner units, in hours, for a year.

Answer ..... h [2]

(b) Based on the usage of his office building, which air conditioner model will have a lower cost after 4 years of use?
Justify your decision with calculations.
Answer

[7]

(c) State one assumption you have made in the above calculations.

Answer .....

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