BP-447

NAME		INDEX NO.		CLASS	

NORTHLAND SECONDARY SCHOOL
PRELIMINARY EXAMINATION
Secondary 4 Express / 5 Normal Academic

ADDITIONAL MATHEMATICS

4049/01

Paper 1

27 August 2024

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

For Examiner's Use
90

This document consists of **22** printed pages.

[Turn over

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Express each of $2x^2+4x-1$ and $-x^2+x-8$ in the form $a(x+b)^2+c$, where a, b and c are constants. [4]

(b) Use your answers from part **(a)** to explain why the curves with equations $y=2x^2+4x-1$ and $y=-x^2+x-8$ will not intersect. [2]

2 Use the substitution $u = e^{2x}$ to show that the solution to the equation $9e^{2x} + 14 = 8e^{-2x}$ can be expressed in the form $e^x = k$, where k is a constant to be found. [3]

3 A right circular cylinder tank has a radius of $(\sqrt{5}-\sqrt{3})$ m and a volume of $(26\sqrt{3}-20\sqrt{5})\pi$ m^3. **Without using a calculator**, express the height of the cylinder in the form $(\sqrt{a}-\sqrt{b})$ m, where a and b are integers. [6]

[Turn over

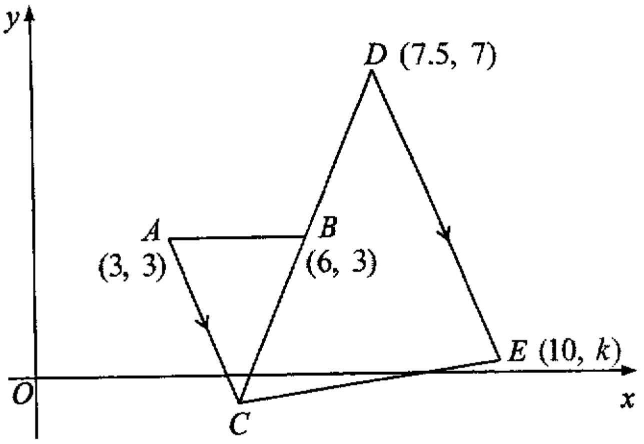
4 $f(x)$ is such that $f''(x) = 3\sin x - 4\cos 2x$. Given that $f\left(\frac{\pi}{6}\right) = 8$ and $f'(\pi) = 9$, find $f(x)$. [6]

5 **(a)** **(i)** State, in terms of π, the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$. [1]

(ii) Explain why the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ cannot be $-\frac{\pi}{4}$. [1]

(b) The acute angles A and B are such that $2\sin(A+B)=1-2\sin(A-B)$ and $\cos B=\frac{1}{3}$. Without using a calculator, find the exact value of $\tan A$. [4]

6



The diagram shows an isosceles triangle ABC in which $AC = BC$. Point A is $(3,\ 3)$, B is $(6,\ 3)$ and C lies below the x-axis. The line CB is extended to the point D with coordinates $(7.5,\ 7)$. A line is drawn from D, parallel to AC, to the point E $(10,\ k)$ and C is joined to E.

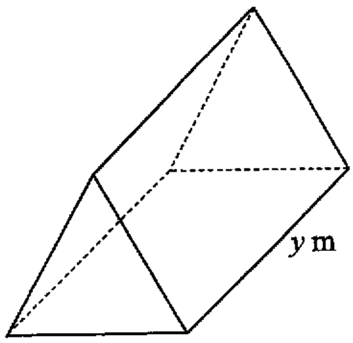
(a) Given that the area of the triangle ABC is 6 square units, find the value of k. [5]

(b) Using this value of k, find the area of triangle CDE. [2]

7 **(a)** A particle moves along the curve $y = \tan x$ in such a way that the y-coordinate of the particle is decreasing at a rate of 0.12 units per second. Find the rate of change of the x-coordinate of the particle at the instant when $y = 1$. [4]

(b) When an object is subject to different pressures, the pressure, P pascals, and the area, A square centimetres, of the object in contact with the surface are related by the formula $PA = k$, where k is a constant. Given that when $P = 120$, $A = 2$, find the rate at which A is changing with respect to P when $A = 2$. [4]

8



The diagram shows a triangular prism with an equilateral triangle as its cross-section. The total length of the 9 edges is 48 cm.

(a) Show that the total volume, V cm^3, of the prism is given by

$$V = \frac{\sqrt{3}x^2(8-x)}{2}.$$ [4]

(b) Given that x can vary, find the stationary value of V. [4]

9 **(a)** **(i)** Factorise x^3+1 and x^3-1. [2]

(ii) Hence write 6^6-1 as a product of four integers. [2]

(b) A polynomial, P, is $2x^3-3x^2-11x+6$, where k is a constant.

(i) Find the remainder when P is divided by $2x+1$. [2]

The quadratic expression $2x^2 + ax + 3$ is a factor of P.

(ii) Find the value of a and hence factorise P completely. [4]

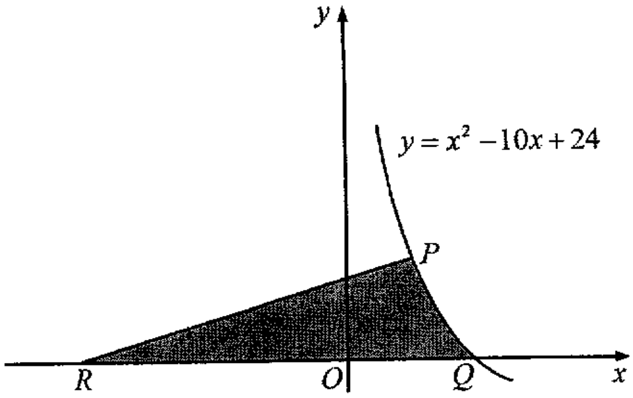
10 **(a)** **(i)** Write down the general term in the binomial expansion of $\left(x+\frac{1}{2x^3}\right)^{12}$. [1]

(ii) Write down the power of x in this general term. [1]

(iii) Hence, or otherwise, determine the term independent of x in the binomial expansion of $\left(x+\frac{1}{2x^3}\right)^{12}$. [2]

(b) The first three terms in the expansion of $(1+ax)\left(1+\frac{x}{2}\right)^n$, in ascending powers of x, are $1+x-5x^2$. Find the value of each of the constants a and n. [6]

11



The diagram shows part of the curve $y = x^2 - 10x + 24$ cutting the x-axis at Q. The point P lies on the curve and the gradient of the tangent to the curve at P is -4. The normal to the curve at P meets the x-axis at R. Find the area of the shaded region bounded by the curve, the line PR and the x-axis. [10]

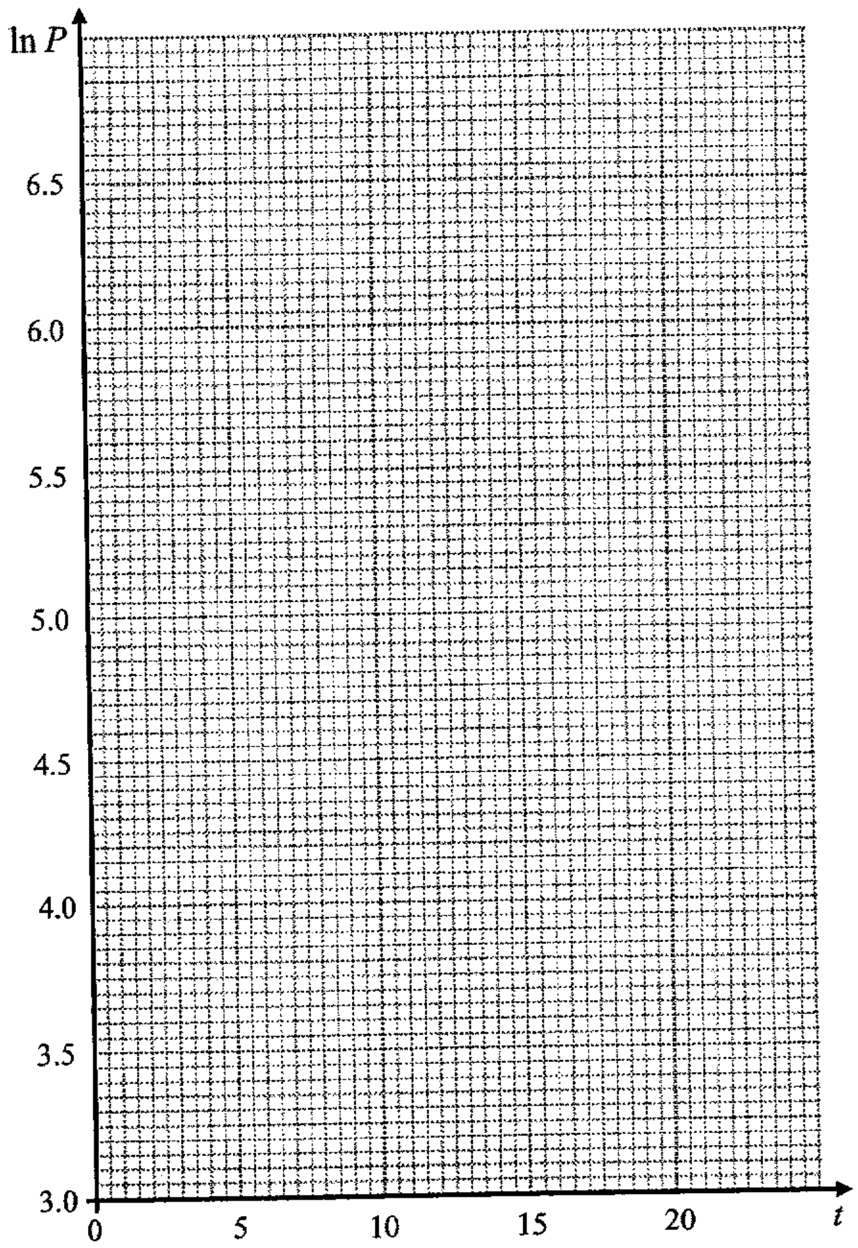
Continuation of working space for question **11**.

12 **(a)** Since 1995, the population of a small town has been steadily decreasing due to a widespread disease. The table shows the estimated population, P, in thousands, in 2000, 2005, 2010 and 2015.

Year	2000	2005	2010	2015
t (years)	5	10	15	20
P (thousands)	260	168	109	70

A demographer believed that these figures can be modelled by the formula $P = ab^t$, where a and b are constants.

(i) On the grid below, plot $\ln P$ against t and draw a straight line graph. [2]



(ii) Estimate the population of the small town in 1995. [2]

(iii) Find the year in which the population first drops below 200 000. [2]

(b) A formula for working out the displacement, s, for a moving vehicle travelling with an acceleration for a certain amount of time, t, is $s = ut + \frac{1}{2}at^2$, where a and u are constants. Values of s for different values of t have been collected. Explain clearly how a straight line graph can be drawn using the recorded data, and state how the values of a and u could be obtained from the line. [4]

Mark Scheme for 2024 S4E5N Add Math Prelim **Paper 1**				
1a	$2x^2+4x-1$			
	$2[(x+1)^2-1]-1$			
	$2(x+1)^2-3$	B2,1		−1 for each error
	$-x^2+x-8$			
	$-(x^2-x)-8$			
	$-\left[\left(x-\frac{1}{2}\right)^2-\frac{1}{4}\right]-8$			
	$-\left(x-\frac{1}{2}\right)^2-\frac{31}{4}$	B2,1		−1 for each error Accept $-(x-0.5)^2-7.75$
1b	Minimum point $(-1,-3)$, Maximum point $(0.5,-7.75)$	B1√		Only need the y-values
	max of $y=-x^2+x-8$ < min of $y=2x^2+4x-1$ → curves will not intersect	B1		Correct argument. Diagram may be used as long as all necessary working is shown
			[6]	
2a	Let $u=e^{2x}\rightarrow\ 9u+14=8u^{-1}$			
	$9u^2+14u-8=0$	M1		Form quadratic equation using substitution
	$(9u-4)(u+2)=0$ $\rightarrow u=\frac{4}{9}$ or $u=-2$ (impossible)	M1		Solving the quadratic and finding u
	$e^{2x}=\frac{4}{9}\rightarrow e^x=\frac{2}{3}$	A1		Do not accept 0.666…
			[3]	
3	$V=\pi r^2h$			
	$(26\sqrt{3}-20\sqrt{5})\pi=\pi(\sqrt{5}-\sqrt{3})^2h$	M1		Use of $V=\pi r^2h$ formula
	$(\sqrt{5}-\sqrt{3})^2=8-2\sqrt{15}$	B1		For correctly squaring $\sqrt{5}-\sqrt{3}$
	$h=\frac{(26\sqrt{3}-20\sqrt{5})}{8-2\sqrt{15}}$ or $h=\frac{(13\sqrt{3}-10\sqrt{5})}{4-\sqrt{15}}$			
	$\frac{(26\sqrt{3}-20\sqrt{5})}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}$	M1		For rationalising denominator
	$\frac{208\sqrt{3}+52\sqrt{45}-160\sqrt{5}-40\sqrt{75}}{64-4(15)}$	DM1		Depend on previous method. Correct expansion of numerator
	$\frac{8\sqrt{3}-4\sqrt{5}}{4}$			
	$2\sqrt{3}-\sqrt{5}=\sqrt{12}-\sqrt{5}$	A1 A1		A1 unsimplified answer A1 correct simplified
			[6]	
4	$f'(x)=-3\cos x-2\sin 2x\ \ (+c)$	B2,1		−1 for each error
	$9=-3\cos(\pi)-2\sin(2\pi)+c$	M1		Use gradient of 9 with $x=\pi$
	$9=3-0+c\ \rightarrow\ c=6$			
	$f(x)=-3\sin x+\cos 2x+6x\ \ \ (+d)$	B2,1√		−1 for each error. √ from $f'(x)$
	$8=-3\sin(\frac{\pi}{6})+\cos 2(\frac{\pi}{6})+6(\frac{\pi}{6})+d$			

	$8=-\frac{3}{2}+\frac{1}{2}+\pi+c \rightarrow c=9-\pi$			
	$f(x)=-3\sin x+\cos 2x+6x+9-\pi$	A1		Use $f(x)$ of 8 with $x=\frac{\pi}{3}$, c.a.o.
			[6]	
5ai	$-\dfrac{\pi}{3}$	B1		
5aii	Principal value for $\cos^{-1}x$ lies between 0 and π inclusive	B1		Accept $0\le\cos^{-1}x\le\pi$ and $0\le\theta\le\pi$
5b	$2\sin(A+B)=1-2\sin(A-B)$			
	$2(\sin A\cos B+\cos A\sin B)$ $=1-2(\sin A\cos B-\cos A\sin B)$	B1		Use of addition formula
	$4\sin A\cos B=1$			
	$4(\sin A)(\frac{1}{3})=1 \rightarrow \sin A=\frac{3}{4}$	M1		For $\sin A=k$
	$\sqrt{4^2-3^2}$	M1		For finding adjacent
	$\tan A=\dfrac{3}{\sqrt{7}}$	A1		c.a.o.
			[6]	
6a	$x-$coord of $C=4.5$	B1		
	$\frac{1}{2}\times(6-3)\times h=6 \rightarrow h=4$			
	$y-$coord of $C=3-4=-1$	M1		For $3-$height or use of shoelace
	$m_{AC}=\dfrac{3-(-1)}{3-4.5}=-\dfrac{8}{3}$	M1		For finding gradient of AC
	$m_{DE}=\dfrac{k-7}{10-7.5}=-\dfrac{8}{3}$	M1		For $m_{DE}=m_{AC}$
	$k=\dfrac{1}{3}$	A1		
6b	$\dfrac{1}{2}\begin{vmatrix}4.5 & 10 & 7.5 & 4.5\\ -1 & \frac{1}{3} & 7 & -1\end{vmatrix}$			
	$\dfrac{1}{2}\left\lvert\frac{1}{3}(4.5)+10(7)-1(7.5)-(-10)-\frac{1}{3}(7.5)-7(4.5)\right\rvert$	M1		Correct use of shoelace method
	20 units^2	A1		
			[7]	
7a	$\frac{dy}{dx}=\sec^2 x$	B1		
	$y=1 \rightarrow x=\frac{\pi}{4}$	M1		For finding x value
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$			
	$-0.12=\sec^2(\frac{\pi}{4})\times\frac{dx}{dt}$	M1		Chain rule used correctly. Ignore $-$ in $\frac{dy}{dt}$
	$\frac{dx}{dt}=-0.06$ units/second	A1		a.e.f.
7b	$k=(120)(2)=240$	B1		
	$PA=k \rightarrow A=kP^{-1}$	M1		Make A the subject
	$\frac{dA}{dP}=-kP^{-2}$	A1		
	$\frac{dA}{dP}=-240(120)^{-2}=-\dfrac{1}{60}$	A1		Accept $-0.0166...$
			[8]	
8a	$6x+3y=48$	B1		

	Perpendicular height $=\sqrt{x^2-\left(\frac{1}{2}x\right)^2}$ or Area of triangle $=\frac{1}{2}(x)(x)\sin 60°$	M1		For finding perpendicular height or area of equilateral triangle
	$V=\frac{1}{2}\left(\frac{\sqrt{3}}{2}x\right)(x)(y)$ or $\left(\frac{\sqrt{3}}{4}x^2\right)y$	M1		For cross section area $\times$ height
	$V=\left(\frac{\sqrt{3}}{4}x^2\right)(16-2x)=\frac{\sqrt{3}x^2(8-x)}{2}$	A1		Answer was given – so all working must be correct
8b	$V=4\sqrt{3}x^2-\frac{1}{2}\sqrt{3}x^3$			
	$\frac{dV}{dx}=8\sqrt{3}x-\frac{3}{2}\sqrt{3}x^2$	B1		
	$8\sqrt{3}x-\frac{3}{2}\sqrt{3}x^2=0$	M1		Sets $\frac{dV}{dx}$ to 0 and solves
	$\sqrt{3}x(8-\frac{3}{2}x)=0$			
	$x=0$ (rejected) or $x=\frac{16}{3}$	A1		a.e.f.
	$V=65.7$	A1		Accept 65.68 …
			[8]	
9ai	$(x+1)(x^2-x+1)$	B1		
	$(x-1)(x^2+x+1)$	B1		
9aii	$6^6-1=(6^3)^2-1^2$			
	$(6^3+1)(6^3-1)$	M1		For $a^2-b^2=(a+b)(a-b)$
	$(6+1)(6^2-6+1)(6-1)(6^2+6+1)$			
	$(7)(31)(5)(43)$	A1		SR1 for $(6^2)^3-1^3=(6^2-1)((6^2)^2+(6^2)+1)$ $=(35)(1333)$ but A0 thereafter
9bi	$2(-\frac{1}{2})^3-3(-\frac{1}{2})^2-11(-\frac{1}{2})+6$	M1		Use of remainder theorem or long division
	10.5	A1		
9bii	$2x^3-3x^2-11x+6=(Ax+B)(2x^2+ax+3)$ or $Ax+B\overline{)2x^3\ \ -3x^2\ \ -11x\ \ +6}$ with quotient $2x^2\ \ +ax\ \ +3$	M1		Sets up $P(x)=(Ax+B)$(factor) or long division with factor as dividend
	$2x^3-3x^2-11x+6=(x+2)(2x^2+ax+3)$	B1		For $(Ax+B)=(x+2)$
	$-3x^2=ax^2+4x^2$ or $-11x=3x+2ax$			
	$a=-7$	A1		
	$(x+2)(2x-1)(x-3)$	A1		
			[10]	
10ai	$T_{r+1}=\binom{12}{r}x^{12-r}\left(\frac{1}{2x^3}\right)^r$	B1		i.s.w.
10aii	$x^{12-r}\times(\frac{1}{x^3})^r=x^{12-4r}$			
	power of x $=12-4r$	B1		Accept x^{12-4r}

10aiii	$12-4r=0$ or $x^{12}+\binom{12}{1}x^{11}\left(\frac{1}{2x^3}\right)+\binom{12}{2}x^{10}\left(\frac{1}{2x^3}\right)^2+\ldots$	M1		Sets power to 0 or Correct expansion of first 3 terms
	$T_{3+1}=\binom{12}{3}x^{12-3}\left(\frac{1}{2x^3}\right)^3=\frac{55}{2}$	A1		a.e.f.
10b	$(1+ax)(1+\frac{x}{2})^n$			
	$(1+ax)(1+n(\frac{x}{2})+\frac{n(n-1)}{2}(\frac{x}{2})^2+\ldots)$	B2		B1 for each $\binom{n}{1}=n$ and $\binom{n}{2}=\frac{n(n-1)}{2}$
	$(1+ax)(1+\frac{n}{2}x+\frac{n(n-1)}{8}x^2+\ldots)$			
	$1+\frac{n}{2}x+\frac{n(n-1)}{8}x^2+ax+\frac{an}{2}x^2+\ldots$			
	$1+\frac{n}{2}x+ax+\frac{n(n-1)}{8}x^2+\frac{an}{2}x^2+\ldots$			
	$\frac{1}{2}n+a=1$	M1		Equate coeff. of x terms to 1
	$\frac{n(n-1)}{8}+\frac{an}{2}=-5$	M1		Equate coeff. of x^2 terms to -5
	$\frac{n(n-1)}{8}+\frac{4(1-\frac{1}{2}n)n}{8}=-5$			
	$n(n-1)+4n(1-\frac{1}{2}n)=-40$			
	$n^2-n+4n-2n^2=-40$			
	$n^2-3n-40=0$			$4a^2-2a-42=0$
	$n=8$ or $n=-5$ (rejected)	A1		$a=\frac{7}{2}$ (rejected) or $a=-3$
	$a=1-\frac{1}{2}(8)=-3$	A1		$n=8$
			[10]	
11	$\frac{dy}{dx}=2x-10$	B1		
	$2x-10=-4 \rightarrow x=3$	M1		For finding x-coord of P
	$x=3 \rightarrow y=3^2-10(3)+24=3$			
	$m_{PR}=\frac{1}{4}$	B1		
	$3=\frac{1}{4}(3)+c$ $c=\frac{9}{4}$ $y=\frac{1}{4}x+\frac{9}{4}$ $y=0 \rightarrow x=-9$ or $\frac{3-0}{3-x}=\frac{1}{4} \rightarrow x=-9$	M1		For finding x-coord of R
	$\frac{1}{2}(9+3)(3)=18$ or $\int_{-9}^{3}\frac{1}{4}x+\frac{9}{4}\,dx$	M1		For finding area of triangle. Answer 18 may be implied
	$x^2-10x+24=0 \rightarrow x=4$	B1		For finding x-coord of Q
	Integrate curve $\rightarrow \left[\frac{1}{3}x^3-5x^2+24x\right]$	M1 A1		Knowing to integrate. A1 might be implied
	Limits 3 to 4 $\rightarrow$ $\left[\frac{1}{3}(4)^3-5(4)^2+24(4)\right]-\left[\frac{1}{3}(3)^3-5(3)^2+24(3)\right]$ $=1\frac{1}{3}$	M1		Uses definite integral method on antiderivative
	$18+1\frac{1}{3}=19\frac{1}{3}$	A1		
			[10]	

12ai	Plot all points correctly (allow ± 1 mm)	M1		
	Straight line drawn through all points	A1		Dep. on method
12aii	Read off at $t = 0$ (allow ± 0.005)	M1		
	$e^6 = 403 \rightarrow 403$ thousands	A1		Accept 403 000
12aiii	Read off at $\ln 200 = 5.30 \rightarrow 1995 + 8 = 2003$	M1 A1		c.a.o.
12b	$s = ut + \frac{1}{2}at^2$			
	$\frac{s}{t} = \frac{1}{2}at + u \rightarrow$ Plot graph of $\frac{s}{t}$ against t	M1 A1		Divide throughout by t
	$\frac{1}{2}a$ value obtained from gradient of graph	DB1		Dep. on method
	u value obtained from y-intercept of graph	DB1		Dep. on method
	Alternative Answer:			
	$\frac{s}{t^2} = \frac{u}{t} + \frac{1}{2}a \rightarrow$ Plot graph of $\frac{s}{t^2}$ against $\frac{1}{t}$	M1 A1		Divide throughout by t^2
	$\frac{1}{2}a$ value obtained from y-intercept of graph	DB1		Dep. on method
	u value obtained from gradient of graph	DB1		Dep. on method
			[10]