CANBERRA SECONDARY SCHOOL

O

2024 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/02

26 Aug 2024
2 hours 15 minutes
1130h — 1345h

Name: ____________________________ (   ) **Class:** ________

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

This question paper consists of 22 printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions

1 A curve has an equation $y = 2x\sqrt{2x+1}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{ax+b}{\sqrt{2x+1}}$, where a and b are positive integers. [2]

(b) Hence, find $\int \dfrac{3x+1}{\sqrt{2x+1}}\,dx$. [2]

2 **A calculator must not be used in this question.**

(a) Show that $\sin 15^{\circ} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. [4]

(b) Hence, find $\sin^2 15^o$, giving your answer in the form $\frac{a-\sqrt{3}}{b}$, where a and b are positive integers. [2]

(c) By using part **(b)**, show that $\cos^2 15^o = \frac{2+\sqrt{3}}{4}$. [2]

3 **(a)** Prove that $\operatorname{cosec}2x + \cot 2x = \cot x$. [4]

(b) Hence, solve $\operatorname{cosec} x + \cot x = \sqrt{3}$ where $0 \le x \le 2\pi$. [3]

4 **(a)** Solve the equation $3^x - 3^{2-x} = 8$. [4]

(b) The curve $y = \log_5 (2x+5)$ intersects the x-axis at A and the y-axis at B.

(i) Find the coordinates of A and B. [3]

(ii) Explain why the graph does not exist for all values of $x \le -\frac{5}{2}$. [2]

5 A compound produced in the laboratory has a growth equation given by

$$w = \frac{600}{10 + 30e^{-0.5t}}$$

where w is the mass in grams and t is the time in hours after the compound was first produced.

Find

(ai) the initial mass of the compound, [2]

(aii) the mass of the compound after 5 hours 15 minutes, [2]

(aiii) the time when the mass is $30g$. [3]

(b) Explain if the mass will continue increasing indefinitely or if it will stabilise to a final mass. [2]

6 The polynomial $f(x) = Ax^3 - (2A+B)x^2 + 2x + B$, where A and B are constants, is exactly divisible by $(x-1)$.
When $f(x)$ is divided by x, the remainder obtained is 3.

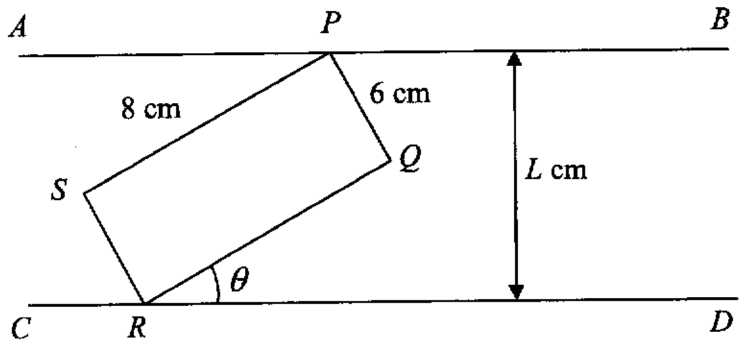
(a) Find A and B. [3]

(b) Factorise $f(x)$ completely. [4]

(c) Hence, or otherwise factorise

$$g(x) = A(x-2)^3 - (2A+B)(x-2)^2 + 2(x-2) + B.$$ [3]

7 The diagram below shows two horizontal lines AB and CD, where L is the vertical distance between the lines AB and CD.
A rectangle $PQRS$ touches lines AB and CD at P and R respectively.
It is given that $PQ = 6$ cm, $SP = 8$ cm and $\angle QRD = \theta°$.



(a) Show that $L = 8\sin\theta + 6\cos\theta$. [3]

(b) Express $8\sin\theta + 6\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your answer for α correct to 2 decimal places. [3]

Given that the angle θ can vary from $0° \leq \theta < 90°$

(c) Hence, state the maximum value of L and the corresponding value(s) of θ. [2]

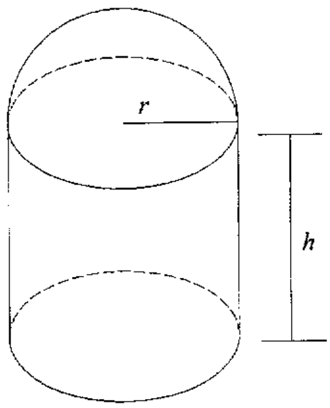
(d) State the minimum value of L and the corresponding value(s) of θ. [2]

8 A capsule is made of a cylinder with a hemisphere at one end of the cylinder.

The cylinder has a height, h cm and radius, r cm.

The hemisphere also has radius, r cm.

(Volume of sphere = $\frac{4}{3}\pi r^3$, Surface Area of sphere = $4\pi r^2$,
Volume of cylinder = $\pi r^2 h$, Curved surface area of cylinder = $2\pi rh$)



Given that the volume of the capsule is $100\,\pi cm^2$.

(a) Show that $h = \frac{100}{r^2} - \frac{2}{3}r$. [2]

(b) Show that the total surface area of the capsule is $A = \frac{5\pi}{3}r^2 + \frac{200\pi}{r}$. [2]

(c) Find the value of r for the stationary value of A.
Explain if the value of r gives the maximum or minimum value of A.

[6]

9 The equation of a circle C_1 is given by the equation $x^2+y^2-4x-8y=0$.

(a) Find the coordinates of the center and radius of C_1. [3]

Point A $(0,8)$ lies on the circle C_1. [3]

(b) Find the equation of tangent to the circle, C_1 at A.

The circle, C_1 is reflected about the y-axis.

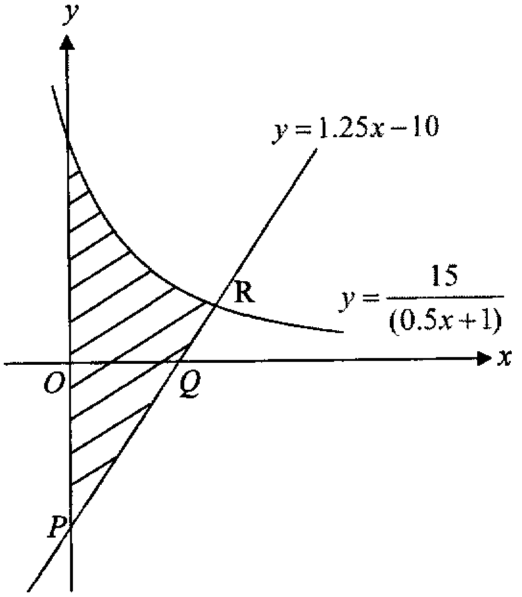
(c) Find the equation that represents the reflected circle C_2. [2]

A circle, C_3 has the equation $(x-5)^2+(y-8)^2=5$

(d) Explain if C_1 and C_3 intersect or do not intersect each other. [4]

10 The diagram shows part of the curve $y = \dfrac{15}{(0.5x+1)}$ and a straight line $y = 1.25x - 10$.

The straight line intersects the y-axis at P, the x-axis at Q and the curve at R.



(a) Find the coordinates of P and Q. [2]

(b) Find the coordinates of R. [4]

(c) Find the area of the shaded region bounded by the curve, the line $y = 1.25x - 10$ and the y-axis. [5]

- End of paper -

CANBERRA SECONDARY SCHOOL

Challenge, Conquer, Contribute

O

2024 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/02

26 Aug 2024
2 hours 15 minutes
1130h — 1345h

Name: ______________________________ (   ) **Class:** ________

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0, a \neq 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}.$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc \cos A.$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions

1 A curve has an equation $y = 2x\sqrt{2x+1}$.

(a) Show that $\frac{dy}{dx} = \frac{ax+b}{\sqrt{2x+1}}$, where a and b are positive integers. [2]

$u = 2x$ $\qquad v = (2x+1)^{\frac{1}{2}} = \sqrt{2x+1}$

$\frac{du}{dx} = 2$ $\qquad \frac{dv}{dx} = \frac{1}{2}(2x+1)^{-\frac{1}{2}}(2) = \frac{1}{\sqrt{2x+1}}$

$\frac{dy}{dx} = \frac{2x}{\sqrt{2x+1}} + 2\sqrt{2x+1}$ M1

$= \frac{2x+4x+2}{\sqrt{2x+1}} = \frac{6x+2}{\sqrt{2x+1}}$ A1

(b) Hence, find $\int \frac{3x+1}{\sqrt{2x+1}}\, dx$. [2]

$\int \frac{6x+2}{\sqrt{2x+1}}\, dx = 2x\sqrt{2x+1} + c$ M1

$2\int \frac{3x+1}{\sqrt{2x+1}}\, dx = 2x\sqrt{2x+1} + c$

$\int \frac{3x+1}{\sqrt{2x+1}}\, dx = x\sqrt{2x+1} + c$ A1

2 **A calculator must not be used in this question.**

(a) Show that $\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. [4]

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \qquad \text{M1}$$

$$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right) \qquad \text{M1}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \qquad \text{M1}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4} \qquad \text{A1}$$

(b) Hence, find $\sin^2 15^\circ$, giving your answer in the form $\frac{a-\sqrt{3}}{b}$, where a and b are positive integers. [2]

$\sin^2 15^o = \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2$ M1

$= \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)$

$= \frac{6-2\sqrt{12}+2}{16}$

$= \frac{8-2\sqrt{12}}{16}$

$= \frac{2-\sqrt{3}}{4}$ A1

(c) By using part **(b)**, show that $\cos^2 15^\circ = \frac{2+\sqrt{3}}{4}$. [2]

$\sin^2 15^o + \cos^2 15^o = 1$ M1

$\frac{2-\sqrt{3}}{4} + \cos^2 15^o = 1$

$\cos^2 15^o = 1 - \frac{2-\sqrt{3}}{4}$

$= \frac{4}{4} - \frac{2-\sqrt{3}}{4} = \frac{2+\sqrt{3}}{4}$ A1

3 **(a)** Prove that $\operatorname{cosec} 2x + \cot 2x = \cot x$. [4]

$LHS = \operatorname{cosec} 2x + \cot 2x$

$= \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$ M1

$= \dfrac{1 + \cos 2x}{\sin 2x}$

$= \dfrac{1 + 2\cos^2 x - 1}{2\sin x \cos x}$ M1

$= \dfrac{\cos x}{\sin x}$ M1

$= \cot x = RHS$ (Proved) A1

(b) Hence, solve $\cos ec x+\cot x=\sqrt{3}$ where $0 \le x \le 2\pi$. [3]

$$\cos ec x+\cot x=\sqrt{3}$$

$$\cot\frac{1}{2}x=\sqrt{3} \qquad \text{M1}$$

$$\tan\frac{1}{2}x=\frac{1}{\sqrt{3}}$$

$$\text{Basic angle }=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$=\frac{\pi}{6} \qquad \text{M1}$$

Tangent is positive, 1st and 3rd quadrant.

$$\frac{1}{2}x=\frac{\pi}{6},\frac{7\pi}{6}$$

$$x=\frac{\pi}{3} \qquad \text{A1}$$

4 **(a)** Solve the equation $3^x - 3^{2-x} = 8$. [4]

$3^x - 3^{2-x} = 8$

$$3^x - \frac{3^2}{3^x} = 8$$ M1

$3^{2x} - 8(3^x) - 9 = 0$

Let $3^x = X$

$X^2 - 8X - 9 = 0$

$(X+1)(X-9) = 0$ M1

$3^x = -1$ (NA) or $3^x = 9$ M1

$x = 2$ A1

(b) The curve $y = \log_5(2x+5)$ intersects the x-axis at A and the y-axis at B.

(i) Find the coordinates of A and B. [3]

Cuts x-axis, y-coordinate = 0

$\log_5(2x+5) = 0$

$2x + 5 = 1$ M1

$2x = -4$

$x = -2$ $A(-2, 0)$ A1

Cuts y-axis, x-coordinate = 0

$y = \log_5(2(0)+5)$

$y = \log_5 5 = 1$ $B(0, 1)$ B1

(ii) Explain why the graph does not exist for all values of $x \le -\frac{5}{2}$. [2]

$y = \log_5(2x+5)$ does not exist for $2x+5 \le 0$ M1

$2x \le -5$

$x \le -\frac{5}{2}$ A1

5 A compound produced in the laboratory has a growth equation given by
[Turn Over

$$w = \frac{600}{10 + 30e^{-0.5t}}$$

where w is the mass in grams and t is the time in hours after the compound was first produced.

Find

(ai) the initial mass of the compound, [2]

$$w = \frac{600}{10 + 30e^{-0.5t}}$$

Initial mass when $t = 0$ M1

$$w = \frac{600}{10 + 30e^{-0.5(0)}} = \frac{600}{10 + 30} = 15g$$ A1

(aii) the mass of the compound after 5 hours 15 minutes, [2]

5h 15 min = 5.25 h M1

$$w = \frac{600}{10 + 30e^{-0.5(5.25)}} = 49.3g$$ A1

(aiii) the time when the mass is $30g$. [3]

$$30=\frac{600}{10+30e^{-0.5t}}$$ M1

$$30\left(10+30e^{-0.5t}\right)=600$$

$$10+30e^{-0.5t}=20$$

$$30e^{-0.5t}=10$$

$$e^{-0.5t}=\frac{1}{3}$$ M1

$$-0.5t=\ln\left(\frac{1}{3}\right)$$

$$t=-2\ln\left(\frac{1}{3}\right)=2.20 \text{ hours or 2 h 12 minutes}$$ A1

(b) Explain if the mass will continue increasing indefinitely or if it will stabilise to a final mass. [2]

$t\to\infty$,

$30e^{-0.5t}\to 0$ M1

$$\therefore \frac{600}{10+30e^{-0.5t}}\to\frac{600}{10}=60g$$

Therefore, the compound will stabilize to a final mass of 60g. A1

6 The polynomial $f(x) = Ax^3 - (2A+B)x^2 + 2x + B$, where A and B are constants, is exactly divisible by $(x-1)$.
When $f(x)$ is divided by x, the remainder obtained is 3.

[Turn Over

(a) Find A and B. [3]

$f(x) = Ax^3 - (2A+B)x^2 + 2x + B$

$f(0) = A(0)^3 - (2A+B)(0)^2 + 2(0) + B = 3$ M1

Therefore $B = 3$ A1

$f(x) = Ax^3 - (2A+3)x^2 + 2x + 3$

$f(1) = A(1)^3 - (2A+3)(1)^2 + 2(1) + 3 = 0$

$A - (2A+3) + 2 + 3 = 0$

$A - 2A - 3 + 2 + 3 = 0$

$-A = -2$

$A = 2$ A1

(b) Factorise $f(x)$ completely. [4]

$f(x) = 2x^3 - 7x^2 + 2x + 3$

By long division or solving $2x^3 - 7x^2 + 2x + 3 = (Ax^2 + Bx + C)(x-1)$

$A = 2$ B1

$C = -3$ B1

$B = -5$ B1

$2x^3 - 7x^2 + 2x + 3 = (2x^2 - 5x - 3)(x-1)$

$= (2x+1)(x-3)(x-1)$ B1

(c) Hence, or otherwise factorise

$$g(x)=A(x-2)^3-(2A+B)(x-2)^2+2(x-2)+B.$$ [3]

$$g(x)=A(x-2)^3-(2A+B)(x-2)^2+2(x-2)+B$$

M1

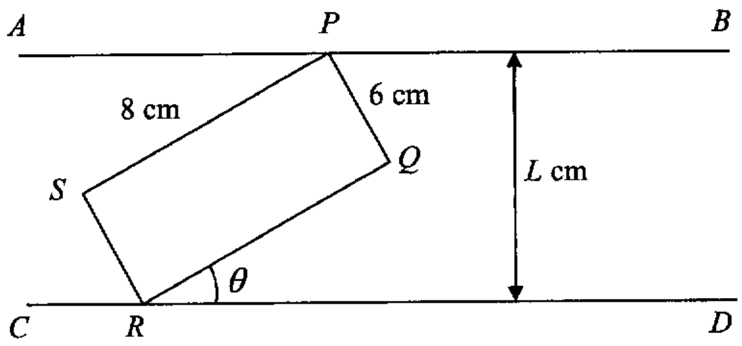
$$=2(x-2)^3-7(x-2)^2+2(x-2)+3$$

$$=(2(x-2)+1)(x-2-3)(x-2-1)$$ M1

$$=(2x-4+1)(x-5)(x-3)$$

$$=(2x-3)(x-5)(x-3)$$ A1

7 The diagram below shows two horizontal lines AB and CD, where L is the vertical distance between the lines AB and CD. **[Turn Over**
A rectangle $PQRS$ touches lines AB and CD at P and R respectively.
It is given that $PQ = 6$ cm, $SP = 8$ cm and $\angle QRD = \theta°$.



(a) Show that $L = 8\sin\theta + 6\cos\theta$. [3]

Draw a vertical line QX from Q to line AB,

$\angle XQP = \theta$ M1

$\cos\theta = \dfrac{XQ}{6}$

$XQ = 6\cos\theta$ A1

Draw a vertical line from QY to line CD.

$\sin\theta = \dfrac{QY}{6}$

$QY = 6\sin\theta$ A1

$L = QY + XQ = 8\sin\theta + 6\cos\theta$

(b) Express $8\sin\theta + 6\cos\theta$ in the form $R\sin(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give your answer for α correct to 2 decimal places. [3]

$8\sin\theta + 6\cos\theta = R\sin(Q+\alpha)$

$R\cos\alpha = 8$ $R\sin\alpha = 6$ M1

$R = \sqrt{8^2+6^2} = 10$ A1

$\alpha = \tan^{-1}\frac{6}{8} = 36.9^o$ A1

$8\sin\theta + 6\cos\theta = 10\sin(\theta + 36.9^o)$

Given that the angle θ can vary from $0° \le \theta < 90°$

(c) Hence, state the maximum value of L and the corresponding value(s) of θ. [2]

Maximum value of L = 10 cm B1

When $\sin(\theta + 36.9^o) = 1$

$\theta + 36.9^o = 90^o$

$\theta = 90^o - 36.9^o = 53.1^o$ B1

(d) State the minimum value of L and the corresponding value(s) of θ. [2]

Minimum value of L = 6 cm when $\theta = 0^o$ B2

8 A capsule is made of a cylinder with a hemisphere at one end of the cylinder.

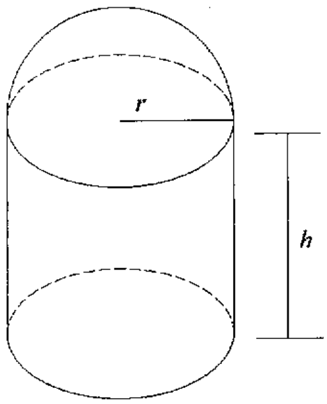
[Turn Over

The cylinder has a height, h cm and radius, r cm.

The hemisphere also has radius, r cm.

(Volume of sphere = $\frac{4}{3}\pi r^3$, Surface Area of sphere = $4\pi r^2$,

Volume of cylinder = $\pi r^2 h$, Curved surface area of cylinder = $2\pi rh$)



Given that the volume of the capsule is $100\,\pi cm^2$.

(a) Show that $h = \frac{100}{r^2} - \frac{2}{3}r$. [2]

Volume of capsule = $\pi r^2 h + \frac{2}{3}\pi r^3 = 100\pi$ M1

$$r^2 h + \frac{2}{3}r^3 = 100$$

$$h + \frac{2}{3}r = \frac{100}{r^2}$$

$$h = \frac{100}{r^2} - \frac{2}{3}r$$ A1

(b) Show that the total surface area of the capsule is $A=\frac{5\pi}{3}r^2+\frac{200\pi}{r}$. [2]

Total surface area of capsule,

$$A=2\pi r^2+2\pi rh+\pi r^2$$

$$=3\pi r^2+2\pi r\left(\frac{100}{r^2}-\frac{2}{3}r\right)$$ M1

$$=3\pi r^2+\frac{200\pi}{r}-\frac{4\pi}{3}r^2$$

$$=\frac{5\pi}{3}r^2+\frac{200\pi}{r}$$ A1

(c) Find the value of r for the stationary value of A.
Explain if the value of r gives the maximum or minimum value of A.

[6]

$$A=\frac{5\pi}{3}r^2+\frac{200\pi}{r}$$

$$\frac{dA}{dr}=\frac{10\pi}{3}r-\frac{200\pi}{r^2}=0$$ M1,M1

$$\frac{10\pi}{3}r=\frac{200\pi}{r^2}$$

$$r^3=60$$

$$r=\sqrt[3]{60}=3.91cm$$ A1

$$\frac{d^2A}{dr^2}=\frac{10\pi}{3}+\frac{400\pi}{r^3}$$ M1

When $r^3=60$

$$\frac{d^2A}{dr^2}=\frac{10\pi}{3}+\frac{400\pi}{60}>0$$ A1

Since $\frac{d^2A}{dr^2}>0$, Area is minimum (By Second Derivative Test). A1

9 The equation of a circle C_1 is given by the equation $x^2+y^2-4x-8y$ **[Turn Over**

(a) Find the coordinates of the center and radius of C_1. [3]

$x^2+y^2-4x-8y=0.$

$(x-2)^2+(y-4)^2-2^2-4^2=0$

$(x-2)^2+(y-4)^2=20$ M1

Center (2 , 4) radius = $\sqrt{20}$ or $2\sqrt{5}$ A1 , A1

Point A $(0,8)$ lies on the circle C_1. [3]

(b) Find the equation of tangent to the circle, C_1 at A.

$M_{OA}=\dfrac{8-4}{0-2}=-2$ M1

Perpendicular gradient = $\dfrac{1}{2}$

$8=\dfrac{1}{2}(0)+c$

$c=8$ M1

$y=\dfrac{1}{2}x+8$ A1

The circle, C_1 is reflected about the y-axis.

(c) Find the equation that represents the reflected circle C_2. [2]

Center (2 ,4) reflected about y-axis to be (-2 , 4) M1
Radius remains the same

$(x+2)^2+(y-4)^2=20$

Or $x^2+y^2+4x-8y=0$. A1

A circle, C_3 has the equation $(x-5)^2+(y-8)^2=5$

(d) Explain if C_1 and C_3 intersect or do not intersect each other. [4]

Center C_3 (5 , 8) radius = $\sqrt{5}$ M1

Distance between C_1 and C_3 = $\sqrt{(5-2)^2+(8-4)^2}=\sqrt{25}=5$ units M1

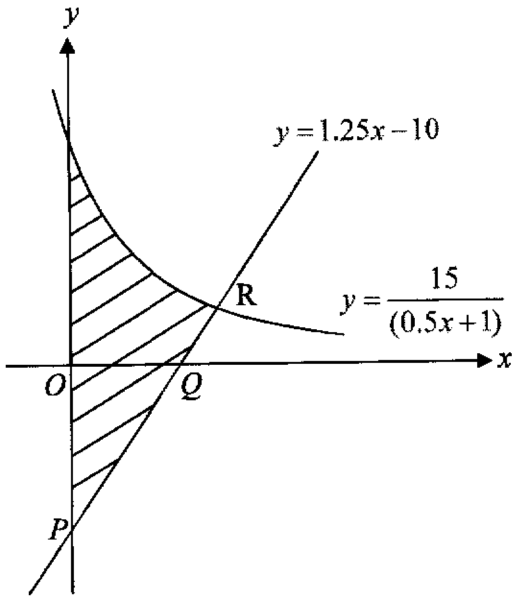
$R_1+R_3=2\sqrt{5}+\sqrt{5}=3\sqrt{5}$ M1

Since $R_1+R_3>$ Distance between C_1 and C_3, therefore the 2 circles intersect.
A1

10 The diagram shows part of the curve $y = \dfrac{15}{(0.5x+1)}$ and a straight line $y = 1.25x - 10$.

[Turn Over

The straight line intersects the *y*-axis at *P*, the *x*-axis at *Q* and the curve at *R*.



(a) Find the coordinates of *P* and *Q*. [2]

Cuts the y-axis at *P*, $x = 0$

$y = 1.25(0) - 10 = -10$

P(0 , -10) B1

Cuts the *x*-axis at *Q*, $y = 0$

$0 = 1.25x - 10$

$1.25x = 10$

$x = \dfrac{10}{1.25} = 8$

Q(8 , 0) B1

(b) Find the coordinates of R. [4]

$$\frac{15}{(0.5x+1)}=1.25x-10$$ M1

$$(1.25x-10)(0.5x+1)=15$$ M1

$$\frac{5x^2}{8}-\frac{15x}{4}-10=15$$

$$\frac{5x^2}{8}-\frac{15x}{4}-25=0$$

$$5x^2-30x-200=0$$

$$x^2-6x-40=0$$ M1

$$(x+4)(x-10)=0$$

$x = -4$ (rej) $x = 10$

$$y=1.25(10)-10=2.5$$

$R(10, 2.5)$ A1

(c) Find the area of the shaded region bounded by the curve, the line $y = 1.25x - 10$ and the y-axis. [5]

[Turn Over

Area of triangle $OPQ = \frac{1}{2} \times 8 \times 10 = 40 units^2$ M1

Area of QR $= \frac{1}{2} \times 2 \times 2.5 = 2.5 units^2$ M1

Area under curve $= \int_0^{10} \frac{15}{(0.5x+1)}\, dx$ M1

$= \left[30 \ln(0.5x+1)\right]_0^{10}$

$= [30 \ln 6] - [30 \ln 1] = 30 \ln 6$ M1

Shaded area $= 30 \ln 6 + 40 - 2.5 = 30 \ln 6 + 37.5 = 91.3 units^2$ A1

- End of paper -