Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS
Paper 2

4049/02

20 August 2024 Tuesday 2 hours 15 min

2024 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use											
Qn	1	2	3	4	5	6	7	8	9	10	*Marks Deducted*
Marks											

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
/ 90

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

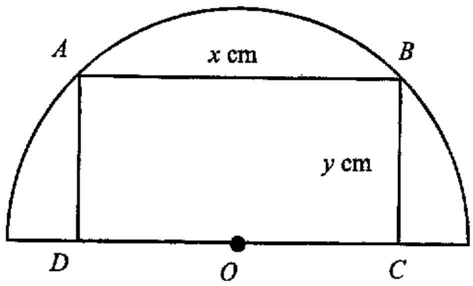
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Write down, and simplify, the first three terms in the expansion of $\left(3-\frac{2}{x}\right)^5$ in descending powers of x. [2]

(b) Given that there is no term independent of x in the expansion of $\left(5+ax^2\right)\left(3-\frac{2}{x}\right)^5$, hence find the value of the constant a. [3]

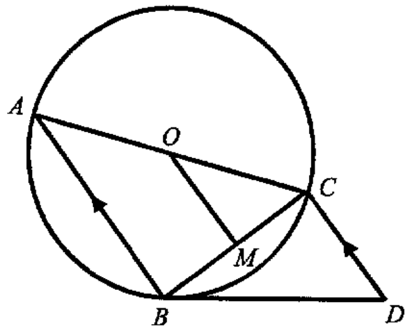
2 In the figure, $ABCD$ is a rectangle inscribed within a semicircle of radius 4 cm and centre O. It is given that $AB = x$ cm and $BC = y$ cm.



(a) Show that the area of the rectangle, A cm, is given by $A = \frac{1}{2}x\sqrt{64 - x^2}$. [2]

(b) Find the exact value of x for which A has a stationary value.
Give your answer in the form $k\sqrt{2}$, where k is an integer. [4]

3 The diagram shows a triangle ABC inscribed in the circle with centre O.
BD is a tangent to the circle at B and AB is parallel to CD. Point M is the midpoint of BC.



(a) Prove that triangles ABC and BCD are similar. [3]

(b) Prove that $ABMO$ is a trapezium. [2]

(c) Prove that $OM = \dfrac{BC^2}{2CD}$. [3]

4 Milk is poured into an empty cup and heated. The temperature, T_m °C, of the milk in the cup, t minutes after it is heated, is modelled by the formula, $T_m = 5(2)^t + 20$.

(a) State the initial temperature of the milk. [1]

Coffee is poured into another empty cup. The temperature, T_c °C, of the coffee in the cup, t minutes after it is poured, is modelled by the formula, $T_c = 60(2)^{-t} + 25$.

(b) Find the time taken for the temperature of the coffee to drop to 35°C. [3]

(c) Find the time taken for the milk and the coffee to reach the same temperature. [4]

5 It is given that $f(x) = 2x^3 - x^2y - 13xy^2 - 6y^3$.

(a) Show that $x - 3y$ is a factor of $f(x)$. [2]

(b) If $y = 1$, find an expression in fully factorised form for $f(x)$. [3]

(c) Hence solve the equation $2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$ and show that the solution may be written in the form $\ln\sqrt{p}$, where p is an integer. [3]

6 **(a)** Given that $\tan\theta = 2\operatorname{cosec}\theta$, show that $\cos^2\theta + 2\cos\theta - 1 = 0$. [3]

(b) Using **part (a)**, find the exact value of $\cos\theta$ in simplest form, given that $0° < \theta < 90°$. [3]

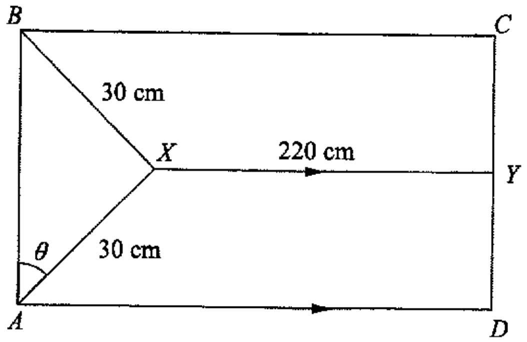
(c) Hence find the value of $\sec^2 \theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

7 **(a)** Prove that $(\sin 2x)(\cot x) - 1 = \cos 2x$. [2]

(b) Given that $y = (\sin 2x)(\cot x) - 1$, hence show that $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right) + 2y + 9\sin 2x = 0$ may be written in the form $\tan 2x = k$, where k is a constant to be found. [4]

(c) Solve $\tan 2x = -\sqrt{3}$ for $0 \leq x \leq 2\pi$, giving your answers in terms of π. [4]

8



The diagram shows a rectangular flag $ABCD$. XAB is a triangle with $AX = BX = 30$ cm and angle $XAB = \theta$ for $0 < \theta < 90°$. XY is parallel to AD and $XY = 220$ cm.

(a) Express the area of triangle XAB in the form $q\sin 2\theta$, where q is an integer. [2]

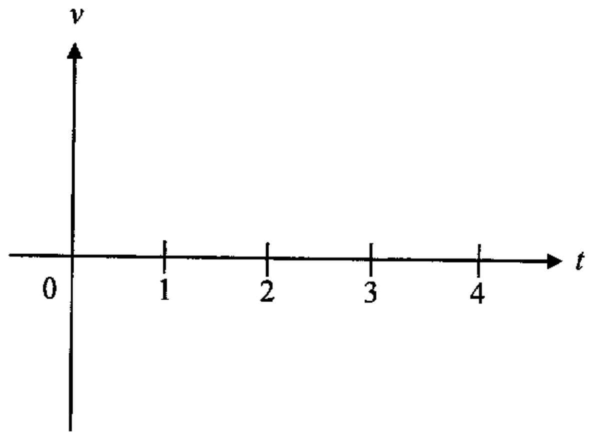
(b) Given that θ can vary, find the maximum possible area of triangle XAB and the value of θ at which this occurs. [2]

(c) Show that the perimeter, P cm, of the rectangular flag $ABCD$ can be expressed in the form $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

(d) By expressing P in the form $R\sin(\theta+\alpha)+c$, where $R > 0$ and $0 < \alpha < 90°$, explain if it is possible to have a flag with perimeter 550 cm. Show your working clearly. [5]

9 A particle moves in a straight line so that, t seconds after passing a fixed point O, its velocity, v metres per second, is given by $v = \pi\cos(\pi t) + \pi$.

(a) Sketch the velocity-time graph of the particle for $0 \le t \le 4$. [3]



(b) Determine how many times the particle is at instantaneous rest in the first 10 seconds. [1]

(c) Explain why the particle will never return to the origin O. [2]

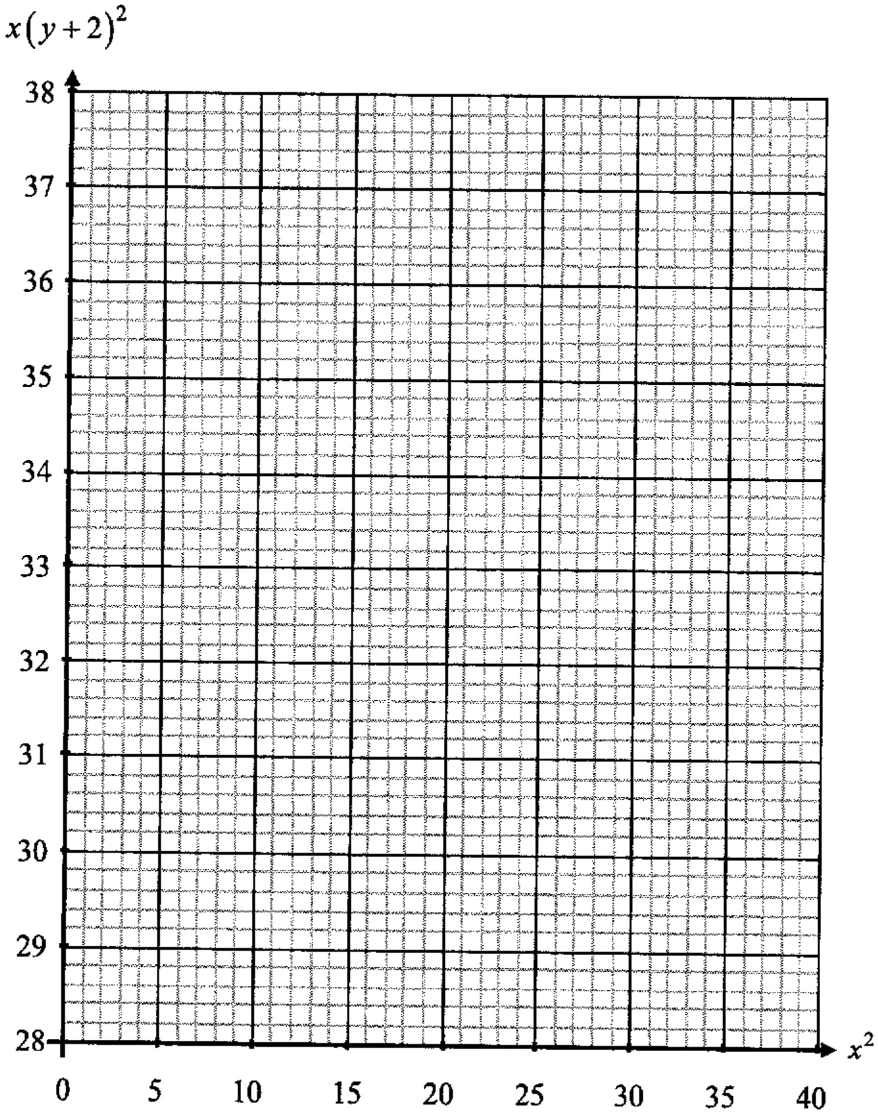
(d) Find an expression, in terms of t, for the displacement of the particle. [2]

(e) Calculate the average speed of the particle in the first 4 seconds. [3]

10 It is known that x and y are related by the equation $y=\sqrt{Ax+\frac{B}{x}}-2$, where A and B are positive constants. The following table shows the values of the variables, x and y.

x	2	3	4	5	6
y	1.92	1.26	0.881	0.646	0.490

(a) Plot $x(y+2)^2$ against x^2 and draw a straight line graph to illustrate the information. [3]



(b) Express the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$ in a form that will yield the straight line graph in **part (a)**. [2]

(c) Use your graph to estimate the value of A and of B. [2]

(d) Explain why the graph $y = \sqrt{Ax + \frac{B}{x}} - 2$ is undefined for $x \leq 0$. [2]

(e) By drawing a suitable line on your graph, estimate the value of x for which $y + 2 = \frac{6}{\sqrt{x}}$. Give your answer to 3 significant figures. [2]

END OF PAPER

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2024 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

MARK SCHEME

1 **(a)** Write down, and simplify, the first three terms in the expansion of $\left(3-\frac{2}{x}\right)^5$ in descending powers of x. [2]

$$\left(3-\frac{2}{x}\right)^5 = 3^5 + 5(3)^4\left(-\frac{2}{x}\right) + \binom{5}{2}(3)^3\left(-\frac{2}{x}\right)^2 + \ldots$$

$$\left(3-\frac{2}{x}\right)^5 = 243 - \frac{810}{x} + \frac{1080}{x^2} + \ldots$$

B2: Three correct terms

(B1: Two correct terms)

(b) Given that there is no term independent of x in the expansion of $\left(5+ax^2\right)\left(3-\frac{2}{x}\right)^5$, hence find the value of the constant a. [3]

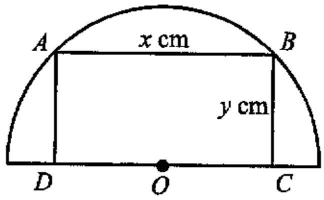
$$\left(5+ax^2\right)\left(3-\frac{2}{x}\right)^5 = \left(5+ax^2\right)\left(243 - \frac{810}{x} + \frac{1080}{x^2} + \ldots\right)$$

Term independent of $x = (5)(243) + \left(ax^2\right)\left(\frac{1080}{x^2}\right)$ — M1: Derive terms indep. of x

$\Rightarrow 1215 + 1080a = 0$ — M1: Equate terms to zero

$\Rightarrow a = -\frac{1215}{1080} = -1.125$ — A1: Accept $-1\frac{1}{8}$ or $-\frac{9}{8}$

2 In the figure, *ABCD* is a rectangle inscribed within a semicircle of radius 4 cm and centre *O*. It is given that $AB = x$ cm and $BC = y$ cm.



(a) Show that the area of the rectangle, A cm, is given by $A = \frac{1}{2}x\sqrt{64-x^2}$. [2]

$y^2 = 4^2 - \left(\frac{1}{2}x\right)^2$

$y = \sqrt{16 - \frac{1}{4}x^2}$ M1: Correct application of Pythagoras Theorem

$A = x\sqrt{16 - \frac{1}{4}x^2}$

$A = x\sqrt{\frac{1}{4}}\sqrt{64-x^2}$ M1: Factorise and simplify surd

$A = \frac{1}{2}x\sqrt{64-x^2}$ (a.g.)

(b) Find the exact value of x for which A has a stationary value.
Give your answer in the form $k\sqrt{2}$, where k is an integer. [4]

$\frac{dA}{dx} = \frac{1}{2}x \cdot \left[\frac{1}{2}(64-x^2)^{-\frac{1}{2}} \cdot (-2x)\right] + \sqrt{64-x^2} \cdot \left(\frac{1}{2}\right)$ M1, M1: Product rule

$\frac{dA}{dx} = -\frac{1}{2}x^2(64-x^2)^{-\frac{1}{2}} + \frac{1}{2}(64-x^2)^{\frac{1}{2}}$

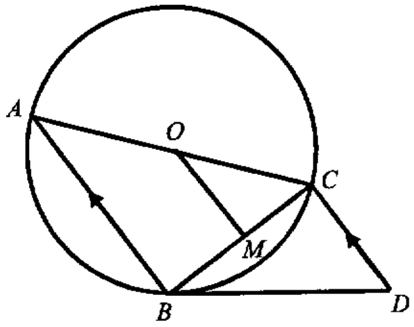
$\frac{dA}{dx} = \frac{1}{2}(64-x^2)^{-\frac{1}{2}}\left[-x^2 + (64-x^2)\right] = \frac{32-x^2}{\sqrt{64-x^2}}$

For stationary value, $\frac{dA}{dx} = \frac{32-x^2}{\sqrt{64-x^2}} = 0$ M1: Equate dA/dx to zero

$32 - x^2 = 0$

$x = \sqrt{32} = 4\sqrt{2}$ A1

3 The diagram shows a triangle ABC is inscribed in the circle with centre O.
BD is a tangent to the circle at B and AB is parallel to CD. Point M is the midpoint of BC.



(a) Prove that triangles ABC and BCD are similar. [3]

$\angle ABC = \angle BCD$ (*alt.* $\angle s$, $AB // CD$)	M1
$\angle BAC = \angle CBD$ (*alternate segment theorem*)	
Triangles ABC and BCD are similar. (*AA similarity*)	A1

(b) Prove that $ABMO$ is a trapezium. [2]

Since O and M are the midpoints of AC and BC respectively,	
$OM // AB$ (*midpoint theorem*)	M1
ABMO is a trapezium. (*one pair of parallel sides*)	A1

(c) Prove that $OM = \dfrac{BC^2}{2CD}$. [3]

$\dfrac{AB}{BC} = \dfrac{BC}{CD}$ (*corr. sides of similar* Δs)	M1
Since $AB = 2OM$ (*midpoint theorem*)	M1
$\Rightarrow \dfrac{2OM}{BC} = \dfrac{BC}{CD}$	
$\therefore OM = \dfrac{BC^2}{2CD}$	A1

***Penalise 1m per question for any missing or incorrect reasons.**

4 Milk is poured into an empty cup and heated. The temperature, T_m °C, of the milk in the cup, t minutes after it is heated, is modelled by the formula, $T_m = 5(2)^t + 20$.

(a) State the initial temperature of the milk. [1]

Initial temperature of milk = $5(2)^0 + 20 = 25$°C B1

Coffee is poured into another empty cup. The temperature, T_c °C, of the coffee in the cup, t minutes after it is poured, is modelled by the formula, $T_c = 60(2)^{-t} + 25$.

(b) Find the time taken for the temperature of the coffee to drop to 35°C. [3]

$60(2)^{-t} + 25 = 35$

$(2)^{-t} = \frac{35-25}{60} = \frac{1}{6}$ M1: Isolate $(2)^{-t}$

$\lg(2)^{-t} = \lg\left(\frac{1}{6}\right)$ M1: Take lg on both sides

$-t\lg(2) = \lg\left(\frac{1}{6}\right)$

$t = -\lg\left(\frac{1}{6}\right) \div \lg(2) = 2.5849$

$t \approx 2.58$ min ($3sf$) A1

(c) Find the time taken for the milk and the coffee to reach the same temperature. [4]

$5(2)^t + 20 = 60(2)^{-t} + 25$ M1: Equate T_m to T_c

$5(2)^{2t} + 20(2)^t = 60 + 25(2)^t$ M1: Multiply 2^t throughout / obtain quad. eqn.

$5(2)^{2t} - 5(2)^t - 60 = 0$

$(2)^{2t} - (2)^t - 12 = 0$

Let $u = (2)^t$, $u^2 - u - 12 = 0$

Let $u = (2)^t$, $5u + 20 = \frac{60}{u} + 25$

$5u^2 - 5u - 60 = 0$

$u^2 - u - 12 = 0$

$(u-4)(u+3) = 0$ M1: Solve quadratic equation

$u = 4$ *or* $u = -3$ (rejected)

$(2)^t = 4$

$t = 2$ min A1

5 It is given that $f(x) = 2x^3 - x^2y - 13xy^2 - 6y^3$.

(a) Show that $x - 3y$ is a factor of $f(x)$. [2]

$f(3y) = 2(3y)^3 - (3y)^2 y - 13(3y)y^2 - 6$

$f(3y) = 54y^3 - 9y^3 - 39y^3 - 6y^3 = 0$ M1: Sub. into f(x) & simplify

Since $f(3y) = 0$, by Factor Theorem, $x - 3y$ is a factor of $f(x)$. AG1

(b) If $y = 1$, find an expression in fully factorised form for $f(x)$. [3]

Let $f(x) = 2x^3 - x^2 - 13x - 6$

$= (x-3)\left[2x^2 + bx + 2\right]$

Comparing x^2 term:

$-1 = b + (-3)(2)$

$b = 5$

M1: Comparing coefficient (or long division)

$\Rightarrow f(x) = (x-3)\left[2x^2 + 5x + 2\right]$ A1

$\Rightarrow f(x) = (x-3)(2x+1)(x+2)$ A1

(c) Hence solve the equation $2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$ and show that the solution may be written in the form $\ln\sqrt{p}$, where p is an integer. [3]

Let $x = e^{2z}$,

we get $2e^{6z} - e^{4z} - 13e^{2z} - 6 = 0$

$\Rightarrow (e^{2z} - 3)(2e^{2z} + 1)(e^{2z} + 2) = 0$ M1: Sub. $x = e^{2z}$ into (b)

$\Rightarrow e^{2z} = 3$ *or* $2e^{2z} = -1$ (*rejected*) *or* $e^{2z} = -2$ (*rejected*) A1: Seen $e^{2z} = 3$

$\ln e^{2z} = \ln 3$

$2z = \ln 3$

$z = \frac{1}{2}\ln 3$

$\therefore z = \ln\sqrt{3}$ A1

6 (a) Given that $\tan\theta = 2\operatorname{cosec}\theta$, show that $\cos^2\theta + 2\cos\theta - 1 = 0$. [3]

$\tan\theta = 2\operatorname{cosec}\theta$	
$\frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$	M1: Seen either one
$\sin^2\theta = 2\cos\theta$	
$1-\cos^2\theta = 2\cos\theta$	M1: Apply Pythagorean identity
$\cos^2\theta + 2\cos\theta - 1 = 0$	AG1

(b) Using **part (a)**, find the exact value of $\cos\theta$ in simplest form, given that $0° < \theta < 90°$. [3]

$\cos^2\theta + 2\cos\theta - 1 = 0$	
$\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$	M1: Apply quadratic formula
$\cos\theta = \frac{-2 \pm \sqrt{8}}{2}$	
$\cos\theta = -1 \pm \sqrt{2}$	M1: Attempt to simplify
Since $0° < \theta < 90°$, $\cos\theta$ must be positive.	
$\therefore \cos\theta = -1 + \sqrt{2}$	A1

(c) Hence find the value of $\sec^2\theta$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

$\sec^2\theta = \frac{1}{\cos^2\theta}$	M1: Seen $\frac{1}{\cos^2\theta}$
$\sec^2\theta = \frac{1}{\left(-1+\sqrt{2}\right)^2}$	
$\sec^2\theta = \frac{1}{\left(\sqrt{2}\right)^2 - 2\left(\sqrt{2}\right)(1) + 1^2}$	M1: Expand the denominator
$\sec^2\theta = \frac{1}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$	M1: Rationalise the denominator
$\sec^2\theta = \frac{3+2\sqrt{2}}{3^2 - \left(2\sqrt{2}\right)^2}$	M1: Simplify the denominator
$\sec^2\theta = 3 + 2\sqrt{2}$	A1

7 **(a)** Prove that $(\sin 2x)(\cot x)-1=\cos 2x$. [2]

$LHS=(\sin 2x)(\cot x)-1$	
$=(2\sin x\cos x)\left(\frac{\cos x}{\sin x}\right)-1$	M1: Seen 2sinxcosx
$=2\cos^2 x-1$	M1
$=\cos 2x=RHS$	(a.g)

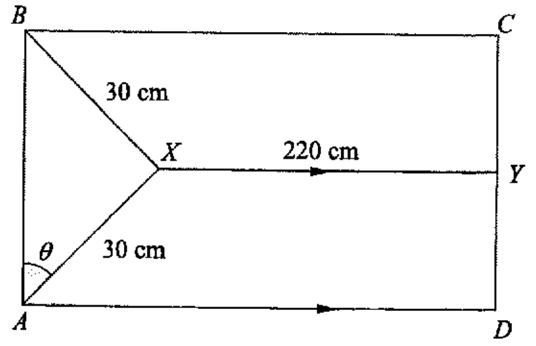
(b) Given that $y=(\sin 2x)(\cot x)-1$, hence show that $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)+2y+9\sin 2x=0$ may be written in the form $\tan 2x=k$, where k is a constant to be found. [4]

$y=(\sin 2x)(\cot x)-1=\cos 2x$	
$\frac{dy}{dx}=-2\sin 2x$	B1
$\frac{d^2y}{dx^2}=-4\cos 2x$	B1
$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)+2y+9\sin 2x=0$	
$-4\cos 2x+3(-2\sin 2x)+2\cos 2x+9\sin 2x=0$	M1: Correct substitution
$3\sin 2x=2\cos 2x$	
$\tan 2x=\frac{2}{3}$	
$\therefore k=\frac{2}{3}$	A1

(c) Solve $\tan 2x=-\sqrt{3}$ for $0\le x\le 2\pi$, giving your answers in terms of π. [4]

$\alpha=\tan^{-1}(\sqrt{3})=\frac{\pi}{3}$	M1: Find reference angle
$2x=\pi-\frac{\pi}{3},2\pi-\frac{\pi}{3},3\pi-\frac{\pi}{3},4\pi-\frac{\pi}{3}$	M1: Find angles in 2nd and 4th quadrants
$2x=\frac{2\pi}{3},\frac{5\pi}{3},\frac{8\pi}{3},\frac{11\pi}{3}$	
$x=\frac{\pi}{3},\frac{5\pi}{6},\frac{4\pi}{3},\frac{11\pi}{6}$	A2: One mark for each correct pair of angles

***Penalise 1m for answers not in terms of π.**



The diagram shows a rectangular flag *ABCD*. *XAB* is a triangle with $AX = BX = 30$ cm and angle $XAB = \theta$ for $0 < \theta < 90°$. *XY* is parallel to *AD* and $XY = 220$ cm.

(a) Express the area of triangle *XAB* in the form $q\sin 2\theta$, where q is an integer. [2]

Area of triangle $XAB = \frac{1}{2}(30)(30)\sin(180° - 2\theta)$	M1: Apply formula ½*bc*sin*A*
Area of triangle $XAB = 450\sin 2\theta$	A1

(b) Given that θ can vary, find the maximum possible area of triangle *XAB* and the value of θ at which this occurs. [2]

This occurs when $\sin 2\theta = 1$,	
Maximum area of triangle $XAB = 450$ cm²	B1: F.T.
Value of $\theta = 45°$	B1

(c) Show that the perimeter, P cm, of the rectangular flag *ABCD* can be expressed in the form $a\sin\theta + b\cos\theta + c$, where a, b and c are constants to be found. [3]

$AD = 30\sin\theta + 220$	
$AB = 2\times 30\cos\theta$	M1: Either *AD* or *AB*
Perimeter = $= 2[30\sin\theta + 220] + 2[2\times 30\cos\theta]$	M1: Attempt to find perimeter
$P = 60\sin\theta + 120\cos\theta + 440$	A1

(d) By expressing P in the form $R\sin(\theta + \alpha) + c$, where $R > 0$ and $0 < \alpha < 90°$, explain if it is possible to have a flag with perimeter 550 cm. Show your working clearly. [5]

$R=\sqrt{60^2+120^2}=\sqrt{18000}=60\sqrt{5}$	M1: Seen $R=\sqrt{a^2+b^2}$
$\alpha=\tan^{-1}\left(\frac{120}{60}\right)=63.434°$	M1: Seen $\alpha=\tan^{-1}\frac{b}{a}$
$P=60\sqrt{5}\sin(\theta+63.434°)+440$	

Method 1

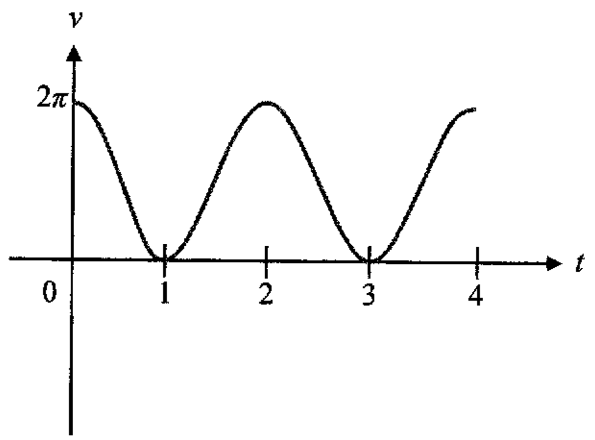
Let $60\sqrt{5}\sin(\theta+63.434°)+440=550$	
$\sin(\theta+63.434°)=\frac{550-440}{60\sqrt{5}}$	
Reference angle = $\sin^{-1}\left(\frac{11}{6\sqrt{5}}\right)=55.0739$	M1: Find reference angle
$\theta+63.434°=55.0739°$ *or* $180°-55.0739°$	M1: Find θ in 1st & 2nd quad
$\theta=-8.3601°$ (*rejected*) *or* $61.4921°$	
Yes, it is possible to have a flag with perimeter 550 cm when $\theta\approx61.5°$ (1dp)	A1

Method 2

Maximum $P=60\sqrt{5}+440=574$ cm	M1
When $\theta=90°$, *Minimum* $P=60\sqrt{5}\sin(90°+63.434°)+440=500$ cm	M1
Since $500<P\le574$, it is possible to have a flag with perimeter 550 cm.	A1

9 A particle moves in a straight line so that, t seconds after passing a fixed point O, its velocity, v metres per second, is given by $v=\pi\cos(\pi t)+\pi$.

(a) Sketch the velocity-time graph of the particle for $0 \le t \le 4$. [3]



B1: Correct shape B1: Correct max & min values B1: Correct period

(b) Determine how many times the particle is at instantaneous rest in the first 10 seconds. [1]

From the graph, the particle is at instantaneous rest when $v = 0$ at every odd second.

Therefore, there are **5 times** in the first 10 seconds. B1

(c) Explain why the particle will never return to the origin O. [2]

Since $v \ge 0$, the **velocity** of the particle is **never negative**, B1
hence the particle **does not change its direction of motion**. B1
Therefore, the particle will never return to the origin O. (a.g)

(d) Find an expression, in terms of t, for the displacement of the particle. [2]

$s = \int \pi \cos(\pi t) + \pi \, dt$

$s = \dfrac{\pi \sin(\pi t)}{\pi} + \pi t + c$ M1: Apply integration

When $t = 0$, $s = 0$, thus $c = 0$.

(e) Calculate the average speed of the particle in the first 4 seconds. [3]

When $t = 0, s = 0$. When $t = 4,\ s = \sin(4\pi) + 4\pi = 4\pi$	M1: Find displacement at $t = 4$
Average speed $= \dfrac{4\pi}{4}$	M1: Find average speed
Average speed $= \pi$ m/s	A1

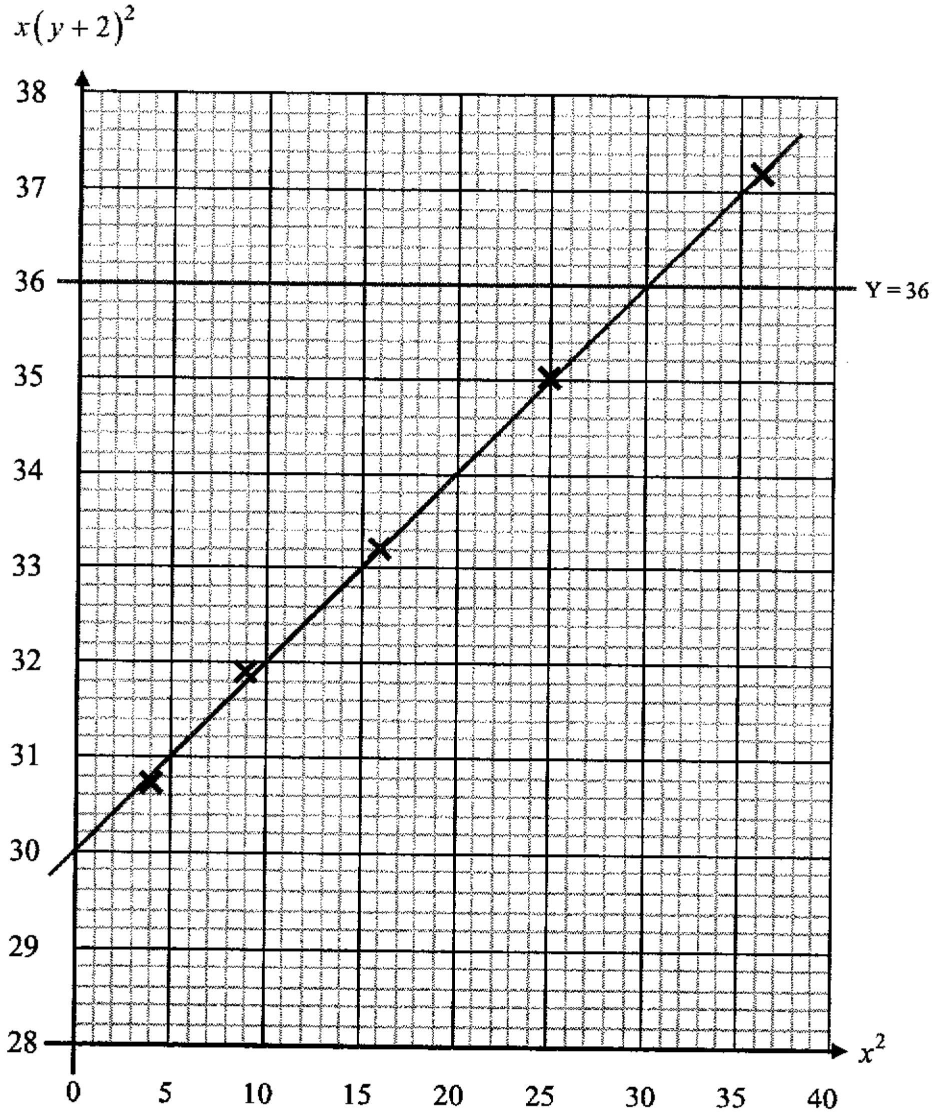
10 It is known that x and y are related by the equation $y = \sqrt{Ax + \frac{B}{x}} - 2$, where A and B are positive constants. The following table shows the values of the variables, x and y.

x	2	3	4	5	6
y	1.92	1.26	0.881	0.646	0.490

x^2	4	9	16	25	36
$x(y+2)^2$	30.7	31.9	33.2	35.0	37.2

(a) Plot $x(y+2)^2$ against x^2 and draw a straight line graph to illustrate the information. [3]

B2: All correct points plotted (B1: at least 3 correct) B1: Best fit line



(b) Express the equation $y=\sqrt{Ax+\frac{B}{x}}-2$ in a form that will yield the straight line graph in **part (a)**. [2]

$y=\sqrt{Ax+\frac{B}{x}}-2$

$y+2=\sqrt{Ax+\frac{B}{x}}$

$(y+2)^2=Ax+\frac{B}{x}$ M1: Taking square on both sides

$x(y+2)^2=Ax^2+B$ A1

(c) Use your graph to estimate the value of A and of B. [2]

$A=gradient=\frac{37.2-33.2}{36-16}=0.2$ B1: Accept +/- 0.01

$B=Y-\text{intercept}=30$ B1: Accept +/- 0.5

(d) Explain why the graph $y=\sqrt{Ax+\frac{B}{x}}-2$ is undefined for $x\le 0$. [2]

When $x=0$, $\frac{B}{x}$ **results in division by zero error.** B1

When $x<0$, since $A>0$ and $B>0$, $Ax+\frac{B}{x}<0$, **hence $\sqrt{Ax+\frac{B}{x}}$ has no real roots.** B1

Hence, $y=\sqrt{Ax+\frac{B}{x}}-2$ is undefined for $x\le 0$. (a.g.)

(e) By drawing a suitable line on your graph, estimate the value of x for which $y+2=\frac{6}{\sqrt{x}}$.

Give your answer to 3 significant figures. [2]

$y+2=\frac{6}{\sqrt{x}}$

$(y+2)^2=\frac{36}{x}$ B1: Draw $Y=36$ on the same axes

$x(y+2)^2=36$

From the graph, when $x(y+2)^2=36$, $x^2=30$

$\therefore x=\sqrt{30}=5.4772\approx 5.48\ (3sf)$ B1: Accept +/- 0.1