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TAMPINES SECONDARY SCHOOL

Secondary Four Express / Five Normal Academic
Preliminary Examination 2024

NAME

CLASS | **REGISTER NUMBER**

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **9 September 2024**

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

For Examiner's Use

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The expression $2x^3+3x^2-8x+3$ leaves a remainder of p when divided by $(x+1)$.

(a) Find the value of p. [2]

(b) Solve the equation $2x^3+3x^2-8x+3=0$. [3]

(c) Hence, solve the equation $2(x-1)^3+3(x-1)^2-8x+11=0$. [2]

2 Peter buys a new car. After t months, its value \$$C$ is given by $C = 120000e^{-at}$, where a is a constant.

(a) Find the value of the car when Peter bought it. [1]

(b) The value of the car after 12 months is expected to be \$90000.

(i) Show that $a = 0.02397$, correct to 4 significant figures. [3]

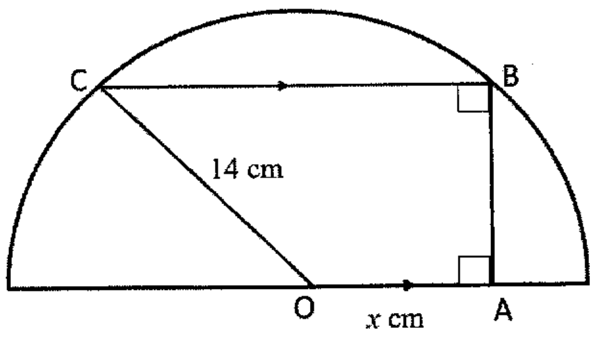
(ii) Calculate the age of the car, to the nearest month, when its expected value will be \$70000. [2]

(iii) After 5 years, a car dealer offers to pay Peter \$29000 for your car. Based on the equation above, would you agree to sell it? Explain your answer. [2]

3 **(a)** Without using a calculator, show that $\cos 75° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. [3]

(b) Prove that $\dfrac{\tan x + \cot x}{\sec x + \operatorname{cosec} x} = \dfrac{1}{\cos x + \sin x}$. Hence solve the equation $\dfrac{\tan x + \cot x}{\sec x + \operatorname{cosec} x} = \dfrac{3}{4\cos x - 3\sin x}$ for $0 \le x \le 2\pi$. [6]

4



The diagram shows a trapezium *OABC* inscribed in a semi-circle, centre *O* and radius 14 cm. *CB* is parallel to the diameter, and *AB* is perpendicular to both *CB* and *OA*. Given that $OA = x$ cm,

(a) show that the area, A cm^2, of the trapezium is given by $A = \frac{3x}{2}\left(\sqrt{196 - x^2}\right)$. [2]

Given that x may vary,

(b) find the value of x for which A has a stationary value and determine whether this value of A is a maximum or a minimum, [5]

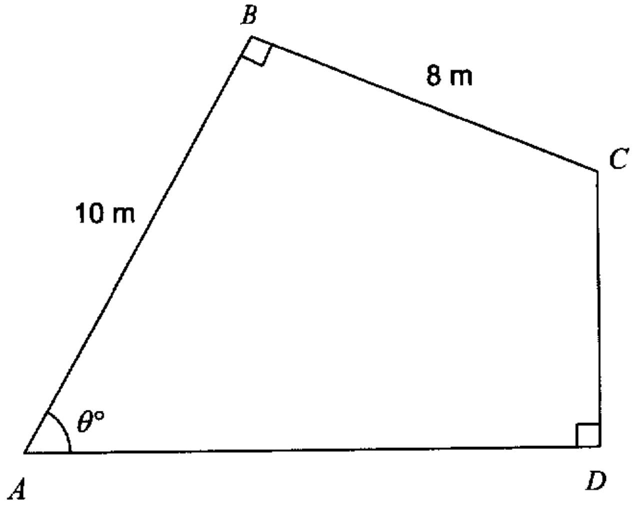
(c) find this stationary value of A. [2]

5 **(a)** By using a suitable substitution, solve the equation $2^{3+2x} + 2^{5+x} = 2^x + 4$. [5]

(b) Given that $x = \log_2 a$ and $y = \log_2 b$, express $\log_a \sqrt{ab^3}$ in terms of x and y. [3]

(c) Solve $\log_5 50 + 4\log_{25} y = \log_5(2y+4) + 2$. [4]

6 The diagram shows a plot of garden $ABCD$ in which $AB = 10$ m and $BC = 8$ m, $angle\ ABC = angle\ ADC = 90°$ and $angle\ BAD = \theta°$.
The garden will be used to grow sunflower plants.



(a) Show that the length $AD = 10\cos\theta + 8\sin\theta$. [2]

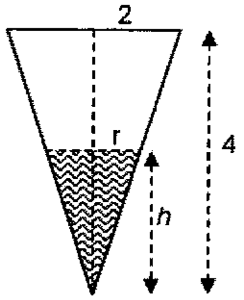
(b) Express $AD = 10\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R is a positive constant and α is acute. [3]

(c) Hence, find the values of θ such that $AD = 12$ m. [3]

7 **(a)** Show that $\frac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\} = (8x-1)(3x-1)^{\frac{2}{3}}$. [5]

(b) Hence find $\int x(3x-1)^{\frac{2}{3}}\,dx$, giving your answer in the form $(ax+b)(3x-1)^{\frac{5}{3}}+c$, where a and b are constant to be found and c is a constant of integration which cannot be found. [5]

8



The diagram shows a vertical cross-section of an inverted conical water tank of height 4 m and base radius 2 m. When the base radius of the water surface is r m, the height of the water level is h m and the volume of the water in the tank is V m^3.

(a) Show that

(i) $r = \frac{h}{2}$, [2]

(ii) $V = \frac{\pi}{12}h^3$. [2]

(b) When $t = 0$, the tank is full and there is a leak in the water tank. If the volume of water in the tank is decreasing at a rate of 2 m^3 per minute, find the rate at which the water level is decreasing when the water is 2 m deep. Correct your answer to 3 significant figures. [5]

9 The equation of the circle, C_1 is $x^2+y^2+2x-6y-6=0$.

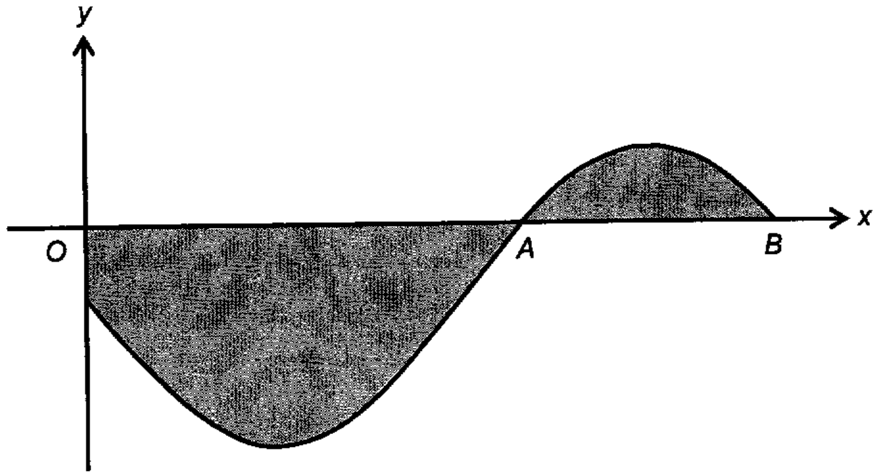
(a) Find the coordinates of the centre and radius of the circle. [4]

(b) Find the value of k, where $k>0$, if $A(-1,k)$ is a point on the circle, C_1. [3]

(c) **(i)** Given that the circle C_2 is a reflection of circle C_1 in the x-axis, state the coordinates of the centre of the circle, C_2? [1]

(ii) Determine whether the point $P(5,\ 0)$ is inside, outside or on the circle C_2. Explain. [2]

10 The diagram shows part of the curve $y = 2\sin(2x + \pi) - 1$, meeting the x-axis at the points A and B.



(a) Show the exact value of x-coordinate of A is $\frac{7\pi}{12}$. [3]

(b) State the exact value of x-coordinate of B. [1]

(c) Find the total area of the shaded regions. [4]

End of Paper 2

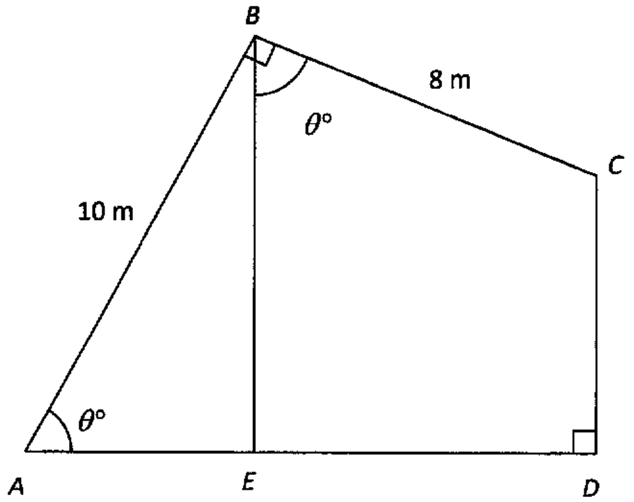
Marking Scheme for 2024 4E5N A.Math Prelim Paper 2

Qn				
1	**(a)**	Let $f(x) = 2x^3 + 3x^2 - 8x + 3$. By the Remainder Theorem, $f(-1) = p$ $p = 2(-1)^3 + 3(-1)^2 - 8(-1) + 3$ $p = -2 + 3 + 8 + 3$ $p = 12$	M1 A1	[2]
	(b)	$(2x-1)$ is a factor of $f(x)$ [or (x+3) or (x-1) is a factor] $2x^3 + 3x^2 - 8x + 3 = 0$ $(2x-1)(x^2 + 2x - 3) = 0$ $(2x-1)(x+3)(x-1) = 0$ $2x - 1 = 0$ xo $x + 3 = 0$ or $x - 1 = 0$ $2x = 1$ $x = -3$ $x = 1$ $x = \frac{1}{2}$ $x = -3, \frac{1}{2}$ or 1	M1 M1 [another quad expression] A1 [3 ans]	[3]
	(c)	$2(x-1)^3 + 3(x-1)^2 - 8x + 11 = 0$ $2(x-1)^3 + 3(x-1)^2 - 8(x-1) + 3 = 0$ Let $y = x - 1$. $2y^3 + 3y^2 - 8y + 3 = 0$ $y = -3, \frac{1}{2}$ or 1 $x - 1 = -3, \frac{1}{2}$ or 1 $x = -2, \frac{3}{2}$ or 2 Hence, $x = -2, \frac{3}{2}$ or 2.	M1 A1	[2]
2	**(a)**	$120 000	B1	[1]

	(b)	(i) Value after 1 year $= 120000e^{-12a}$ $90000 = 120000e^{-12a}$ $e^{-12a} = 0.75$ $-12a = \ln 0.75$ $a = \frac{\ln 0.75}{-12} = 0.0239735$ $= 0.02397 shown$	M1 M1 must show 0.0239735	[2]
		(ii) $70000 = 120000e^{-at}$ $\frac{7}{12} = e^{-at}$ $-0.02397t = \ln\frac{7}{12}$ $t = 22.49$ ≈ 23 months	M1 M1 A1	[3]
		(iii) 5 years = 60 months Value after 5 years $= 120000e^{-60(0.02397)}$ $= \$28482.55$ Yes, since car dealer is paying more ($29000). or No, there is not much difference, so I would rather use the car.	M1 A1	[2]
3	(a)	$\cos 75° = \cos(45° + 30°)$ $= \cos 45° \cos 30° - \sin 45° \sin 30°$ $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$ (Shown)	M1[use 45+30] M1 [any one -surd form]] M1 [simplify to desired ans]	[3]
	(b)	$\text{LHS} = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} + \frac{1}{\sin x}}$ $= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \times \frac{\sin x \cos x}{\cos x + \sin x}$ $= \frac{1}{\cos x + \sin x}$ $= \text{RHS}$	M1 use identities M1 [same denominator] M1 simplify	[6]

		$\frac{1}{\cos x+\sin x}=\frac{3}{4\cos x-3\sin x}$ $3\cos x+3\sin x=4\cos x-3\sin x$ $6\sin x=\cos x$ $\tan x=\frac{1}{6}$ $\alpha=0.165148677$ $x=0.165,\ 3.31$	M1 cross multiply M1 [change to tan] A1	
4	(a)	$AB=\sqrt{196-x^2},\ BC=2x$ $A=\frac{1}{2}(x+2x)\left(\sqrt{196-x^2}\right)$ $=\frac{3x}{2}\left(\sqrt{196-x^2}\right)$	B1 [$AB=\sqrt{196-x^2}$] M1 formula with h=3x	[2]
	(b)	$\frac{dA}{dx}=\frac{3x}{2}\left(-\frac{2x}{2\sqrt{196-x^2}}\right)+\frac{3\sqrt{196-x^2}}{2}$ $=\frac{3}{2}\left(-\frac{x^2}{\sqrt{196-x^2}}+\sqrt{196-x^2}\right)$ $=\frac{3}{2}\left(\frac{196-x^2-x^2}{\sqrt{196-x^2}}\right)$ $=\frac{3(98-x^2)}{\sqrt{196-x^2}}$ $\frac{dA}{dx}=0$ $\frac{3(98-x^2)}{\sqrt{196-x^2}}=0$ $x=\sqrt{98}$ $x\approx 9.90$ $\frac{d^2A}{dx_2}=\frac{3\sqrt{196-x^2}(-2x)-3(98-x^2)(\frac{1}{2})(196-x^2)^{-\frac{1}{2}}(-2x)}{196-x^2}$ $\frac{d^2A}{dx^2}<0$, *A* is max. [or Using first derivative test]	M1 M1 A1 M1 A1	[5]

	(c)	$A=\frac{3\sqrt{98}}{2}\left(\sqrt{196-98}\right)$ $=147\text{ cm}^2$	M1 A1	[2]
5	(a)	$2^{3+2x}+2^{5+x}=2^x+4$ $2^3\times(2^x)^2+2^5\times2^x=2^x+4$ $8(2^x)^2+32(2^x)=2^x+4$ Let $y=2^x$. $8y^2+32y=y+4$ $8y^2+31y-4=0$ $(8y-1)(y+4)=0$ $8y-1=0$ or $y+4=0$ $8y=1$ $y=-4$ $y=\frac{1}{8}$ $2^x=-4$ $2^x=\frac{1}{8}$ $2^x=2^{-3}$ $x=-3$ Since $2^x>0$, $x=-3$.	M1 [$8(2^x)^2$ seen] M1 [$32(2^x)$ seen] M1 factorise A1 for both y A1	[5]
	(b)	$\log_a\sqrt{ab^3}=\frac{\log_2\sqrt{ab^3}}{\log_2 a}$ $=\frac{\frac{1}{2}[\log_2(ab^3)]}{\log_2 a}$ $=\frac{\log_2(a)+3\log_2(b)}{2(\log_2 a)}$ $=\frac{x+3y}{2x}$	M1 [change base 2] M1 [$\log_2(a)+3\log_2(b)$] A1	[3]

	(c)	$\log_5 50+4\log_{25} y=\log_5(2y+4)+2$ $\log_5 50+4\dfrac{\log_5 y}{\log_5 25}=\log_5(2y+4)+\log_5 5^2$ $\log_5 50+2\log_5 y=\log_5 25(2y+4)$ $\log_5 50y^2=\log_5 25(2y+4)$ $50y^2=50y+100$ $y^2-y-2=0$ $(y-2)(y+1)=0$ $y=2$ or -1 (reject)	M1[$\dfrac{\log_5 y}{\log_5 25}$] M1[$2=\log_5 5^2$] M1 use product law A1	[4]
6	(a)	 $AE=10\cos\theta$ $\sin\theta=\dfrac{ED}{8}$ $ED=8\sin\theta$ $AD=10\cos\theta+8\sin\theta$ (shown)	Use trigo ratio M1 M1	[2]
	(b)	$10\cos\theta+8\sin\theta=R\cos(\theta-\alpha)$ $R=\sqrt{8^2+10^2}$ $=\sqrt{164}$ or 12.8 $\alpha=\tan^{-1}\left(\dfrac{8}{10}\right)$ $=38.66$ $\approx 38.7^o$ $10\cos\theta+8\sin\theta=\sqrt{164}\cos(\theta-38.7)$	M1 M1 A1	[3]

	(c)	$\sqrt{164}\cos(\theta-38.7)=12$ $\cos(\theta-38.7)=\dfrac{12}{\sqrt{164}}$ basic $\angle=20.4$ $\theta-38.7=20.4,\ -20.4$ $\theta=59.1,\ 18.3$ (1 decimal place)	M1 M1 A1 both	[3]
7	(a)	Show that $\dfrac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\}=(8x-1)(3x-1)^{\frac{2}{3}}$. $\dfrac{d}{dx}\{x(3x-1)^{\frac{5}{3}}\}$ $=x\left(\dfrac{5}{3}\right)(3x-1)^{2/3}(3)+(3x-1)^{5/3}$ $=5x(3x-1)^{2/3}+(3x-1)^{5/3}$ $=(3x-1)^{2/3}[5x+3x-1]$ $=(3x-1)^{2/3}[8x-1]$	B1 for $x\left(\dfrac{5}{3}\right)(3x-1)^{2/3}$ seen B1 for (3) seen B1 for $(3x-1)^{5/3}$ seen M1 for factorization A1 for showing clear working leading to the desired expression	[5]
	(b)	$\int(8x-1)(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx-\int(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+\left(\dfrac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}(3)}\right)+c$ $\int 8x(3x-1)^{\frac{2}{3}}\,dx=x(3x-1)^{\frac{5}{3}}+\left(\dfrac{(3x-1)^{\frac{5}{3}}}{5}\right)+c$ $\int x(3x-1)^{\frac{2}{3}}\,dx=\dfrac{1}{8}x(3x-1)^{\frac{5}{3}}+\dfrac{1}{40}(3x-1)^{\frac{5}{3}}+c$ $=\left(\dfrac{1}{8}x+\dfrac{1}{40}\right)(3x-1)^{\frac{5}{3}}+c$	M1 M1 M1 $\left(\dfrac{(3x-1)^{\frac{5}{3}}}{\frac{5}{3}(3)}\right)$ A1 [1/8 seen] A1 [1/40 seen]	[5]

8	(a)	(i) From the diagram, by similar triangles, $\frac{r}{h}=\frac{2}{4}$ $4r=2h$ $r=\frac{h}{2}$ (shown)	M1 M1	[2]
		(ii) From (a)(i), $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $=\frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$ $=\frac{\pi}{12}h^3$ (shown)	M1 M1	[2]
	(b)	$V=\frac{\pi}{12}h^3$ $\frac{dV}{dh}=\frac{\pi}{4}h^2$ Given: $\frac{dV}{dt}=-2\text{m}^3/\text{min}$ When $h=2$, $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $-2=\frac{\pi}{4}(2)^2\times\frac{dh}{dt}$ $-2=\pi\times\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{-2}{\pi}$ $=-0.637\text{m/min}$ (correct to 3 sig. fig.)	M1 B1 M1 M1 A1	[5]
9	(a)	$x^2+y^2+2x-6y-6=0$ Radius $=\sqrt{1^2+(-3)^2-(-6)}$ $=\sqrt{16}$ $=4\text{ cm}$ $\therefore$ Centre is $(-1,3)$	M1A1 M1A1 [or A2]	[4]

	(b)	$(-1)^2+k^2+2(-1)-6k-6=0$ $k^2-6k-7=0$ $(k-7)(k+1)=0$ $k=7$ or $k=-1$ (NA)	M1 M1 A1	[3]
	(c)	(i) Coordinates of *centre* C_2 = (-1, -3).	B1	[1]
		(ii) $\sqrt{(5+1)^2+(3+0)^2}$ $=\sqrt{45}=6.71>4=radius$ P(5,0) lies outside the circle C_2.	M1 A1	[2]
10	(a)	$2\sin(2x+\pi)-1=0$ $\sin(2x+\pi)=\frac{1}{2}$ $\alpha=\sin^{-1}\frac{1}{2}$ $=\frac{\pi}{6}$ $2x+\pi=\frac{\pi}{6},\ \pi-\frac{\pi}{6},\ \frac{\pi}{6}+2\pi,\ \pi-\frac{\pi}{6}+2\pi$ $x=-\frac{5\pi}{12},\ -\frac{\pi}{12},\ \frac{7\pi}{12},\ \frac{11\pi}{12}$ x coordinate of $A=\frac{7\pi}{12}$	M1 M1 A1	[3]
	(b)	x coordinate of $B=\frac{11\pi}{12}$	B1	[1]
	(c)	Shaded area $=-\int_0^{\frac{7\pi}{12}} 2\sin(2x+\pi)-1\,dx+\int_{\frac{7\pi}{12}}^{\frac{11\pi}{12}} 2\sin(2x+\pi)-1\,dx$ $=-\left[-\cos(2x+\pi)-x\right]_0^{\frac{7\pi}{12}}+\left[-\cos(2x+\pi)-x\right]_{\frac{7\pi}{12}}^{\frac{11\pi}{12}}$ $=-\left[-2.69862-1\right]+\left[-2.01377-(-2.69862)\right]$ $=3.69862+0.684853$ $=4.38\text{ unit}^2$	M1 M1 M1 A1	[4]

Total = 90 m