PRELIMINARY EXAMINATION 2016

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **2 hours**

Additional Materials: Answer paper
Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1. The equation of a curve is given by $f(x) = 2x^3 - 12x - 5$. Find the range of values of x for which $f(x)$ is an increasing function. [3]

2. (i) Given that $(3k-5)x^2 + (k-5)x - 2 = 0$ has no real roots, what condition must apply to the constant k? [3]

   (ii) From your results in part (i), determine if $y = (3k-5)x^2 + (k-5)x - 2$ has a minimum or maximum point. [2]

3. A sky diver jumps from a certain height above the ground. The downward velocity, $v\ m/s$, of the sky diver at time t seconds is given by $v = 30(1 - e^{-0.2t})$.

   (i) Find the initial velocity of the sky diver. [1]

   (ii) Find the velocity of the sky diver after 5 seconds. [1]

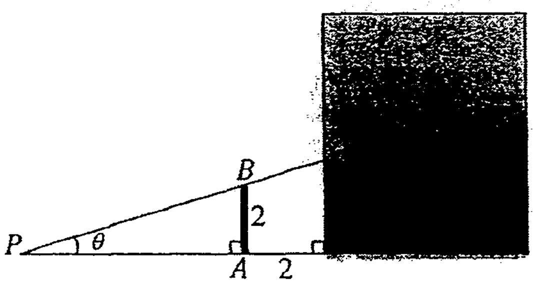
   (iii) Showing your working clearly, explain why the velocity experienced by the sky diver will not exceed 30 m/s. [2]

4. (i) Find the values of $\log_4 x$ that will satisfy the equation [3]
$$2(\log_4 x)^2 = \log_4 x + 6.$$

   (ii) Sketch the graph of $y = \log_4 x$ and indicate clearly on your graph the location of the values of $\log_4 x$ found in part (i). [2]

   Hence, show that the product of the two roots of the equation
$$2(\log_4 x)^2 = \log_4 x + 6$$
   is positive. [1]

5. A vertical wall AB is 2 m high and 2 m away from a warehouse. PQ is a ramp resting on the wall AB and just touching the ground at P and the warehouse at Q. The ramp PQ is of length L metres and makes an angle θ with the horizontal.



(i) Show that the length of the ramp, L, is given by

$$L = \frac{2}{sin\theta} + \frac{2}{cos\theta}$$ [1]

(ii) Hence, show that $\frac{dL}{d\theta} = \frac{2sin^3\theta - 2cos^3\theta}{sin^2\theta cos^2\theta}$ [2]

(iii) Given that θ can vary, find the shortest possible length of the ramp. [5]

6 (i) Sketch the curve $y^2 = 9x$ for $0 \leq y \leq 12$. [2]

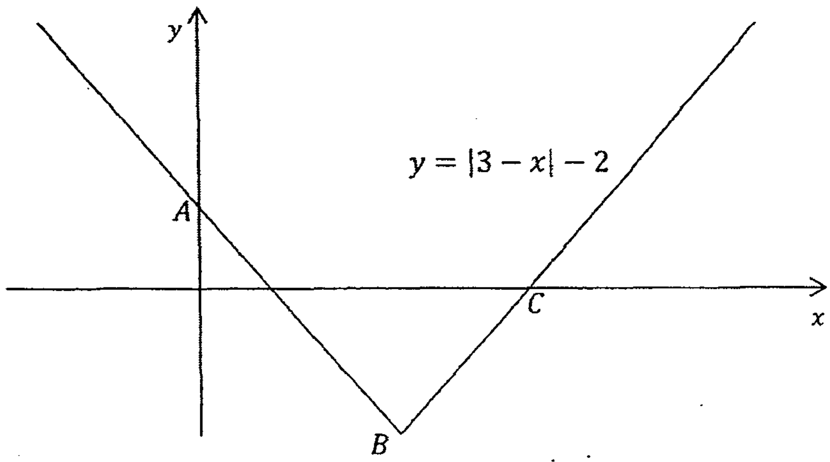
The line $4y - 3x = 9$ intersects the curve $y^2 = 9x$ at two points P and Q.

(ii) Find the coordinates of the midpoint of PQ. [6]

7 (i) Given that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{2}$, prove that $tan\,A + 5\,tan\,B = 0$. [3]

(ii) Hence, solve the equation $2\sin(2\theta - 30°) = 3\sin(2\theta + 30°)$ for $0° \leq \theta \leq 360°$. [5]

8 The diagram shows part of the graph of $y = |3 - x| - 2$.



(i) Find the coordinates of A, B and C. [4]

A line QR of gradient 1 cuts the y-axis at $(0, p)$.

(ii) State the number of intersection(s) of the line QR and $y = |3 - x| - 2$ when
(a) $p = 2$ [1]
(b) $p = -6$ [1]

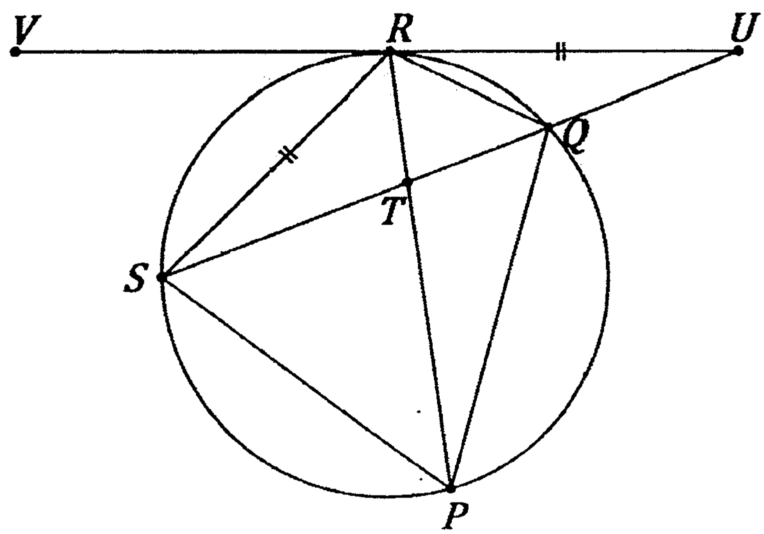
(iii) Determine the set of values of p for which the line QR intersects $y = |3 - x| - 2$ at only one point. [1]

9 A particle travelling in a straight line, passes a fixed point O on the line with a velocity of $9m/s$. The acceleration, $a\ m/s^2$, of the particle t *seconds* after passing through O is given by $a = 8 - 2t$.

(i) Show that the particle comes to instantaneous rest when $t = 9$. [3]

(ii) Find the average speed of the particle for the journey from $t = 0$ to $t = 12$. [5]

10 The diagram shows a circle passing through the points P, Q, R and S. SQU is a straight line that cuts RP at the point T. VRU is a tangent to the circle at R such that $SR = RU$.



Prove that

(i) angle $SPT = 2 \times$ angle QPT, [4]

(ii) triangle QRU is similar to triangle RSU, [2]

(iii) $QR \times SU = (RS)^2$ [2]

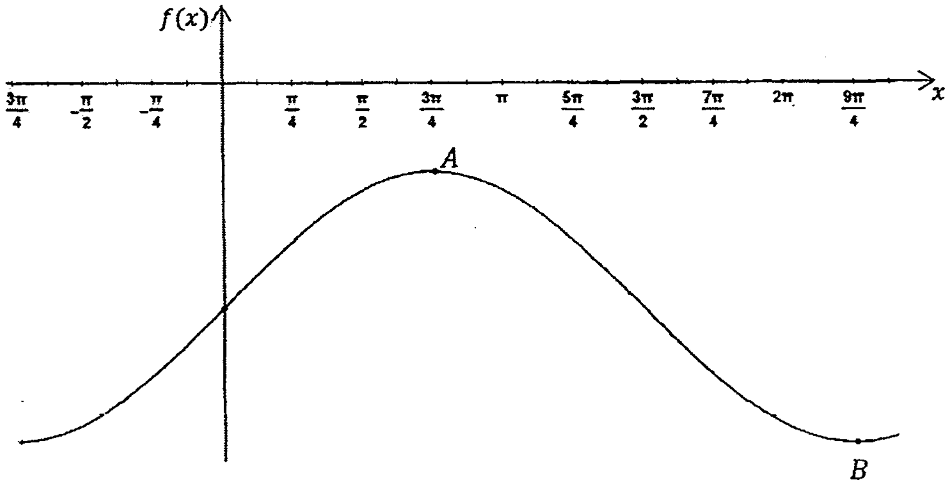
11 A container has a capacity of 960 cm^3 and is initially completely filled with water. The volume, $V\ cm^3$, of water in the container is given by $V = h^2 + 2h$ where $h\ cm$ is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water level in the container decreases at a rate of $\frac{3t}{2}\ cm/s$.

(i) Find the initial height of the water level in the container. [3]

(ii) Show that the height, h, can be expressed as $-\frac{3t^2}{4} + c$, where c is a constant. [2]

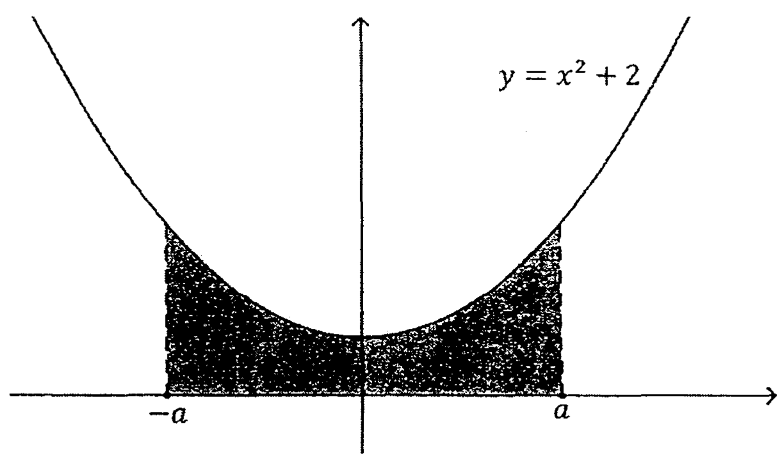
(iii) Find the rate of change of volume when $t = 4$. [3]

12 **(a)** The diagram below shows part of the curve $f(x) = 3\sin(px) - q$.



The coordinates of the turning points are $A(\frac{3\pi}{4}, -2)$ and $B(\frac{9\pi}{4}, -8)$.
Find the values of p and q. [2]

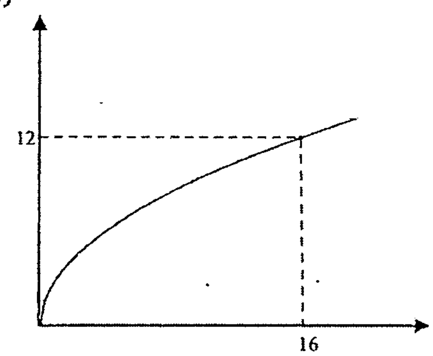
(b) The diagram below shows the graph of $y = x^2+2$. The shaded region from $x = a$ to $x = -a$ has an area of $6a\ units^2$. Find the exact value of a. [5]



END OF PAPER

Answer key:

1. $x < -\sqrt{2}$ or $x > \sqrt{2}$
2. (i) $-15 < k < 1$; (ii) maximum
3. (i) $0\ m/s$ (ii) $19.0\ m/s$
4. (i) $-\frac{3}{2}, 2$
5. (iii) $\frac{\pi}{4}$, 5.66m
6. (i)



(ii) (5,6)

7(ii) 54.6°, 144.6°, 234.6°, 324.6

8(i) (5,0) (ii)(a) 1 (ii)(b) 0 (iii) $p > -5$

9(i) $v = 8t - t^2 + 9$
(ii) $s = 4t^2 - \frac{t^3}{3} + 9t$; $18\ m/s$

11(i) $30cm$ (ii) $h = -\frac{3t^2}{4} + 30$ (iii) $-228\ cm^3/s$

12(a) $p = \frac{2}{3}$; $q = 5$ (b) $a = \sqrt{3}$

2016 ZHSS PRELIM ADD MATHS PAPER 1 MARKING SCHEME

1 $f(x) = 2x^3 - 12x - 5$

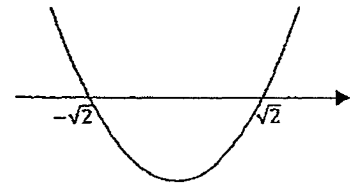
$f'(x) = 6x^2 - 12$

For increasing functions, $f'(x) > 0$

$6x^2 - 12 > 0$

$x^2 - 2 > 0$

$(x + \sqrt{2})(x - \sqrt{2}) > 0$



$\therefore$ the range of values of x is $x < -\sqrt{2}$ or $x > \sqrt{2}$.

2(i) $(3k - 5)x^2 + (k - 5)x - 2 = 0$

No real roots $\Rightarrow$ $discriminant < 0$

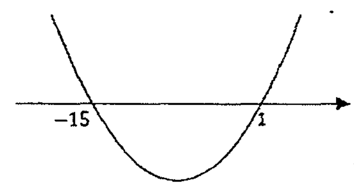
$(k - 5)^2 - 4(3k - 5)(-2) < 0$

$k^2 - 10k + 25 + 24k - 40 < 0$

$k^2 + 14k - 15 < 0$

$(k + 15)(k - 1) < 0$

$-15 < k < 1$



2(ii) coeff of $x^2 = 3k - 5$

From above, $-15 < k < 1$

$-45 < 3k < 3$

$-50 < 3k - 5 < -2$

Since coeff of $x^2 < 0$, the function has a maximum point.

Alternative method:

$y' = 2(3k - 5)x + (k - 5)$

$y'' = 2(3k - 5) = 6k - 10$

From (i), since $-15 < k < 1$, $6k - 10 < 0$

$\Rightarrow$ $y'' < 0 \ \forall x$

$\therefore$ $y = (3k - 5)x^2 + (k - 5)x - 2$ has a max point.

3 $v = 30(1 - e^{-0.2t})$

i) initial velocity, $v = 30(1 - e^{0}) = 0\ m/s$

ii) when $t = 5, v = 30(1 - e^{-1}) = 30\left(1 - \frac{1}{e}\right)$ or $19.0\ m/s$

iii) since $t \geq 0,\ 0 < e^{-0.2t} \leq 1$

$\Rightarrow \max(1 - e^{-0.2t}) < 1$

$\Rightarrow 30(1 - e^{-0.2t}) < 30$

$\therefore$ the velocity will never exceed 30 m/s.

4i) $2(\log_4 x)^2 = (\log_4 x) + 6$

Let $y = \log_4 x$

$2y^2 = y + 6$

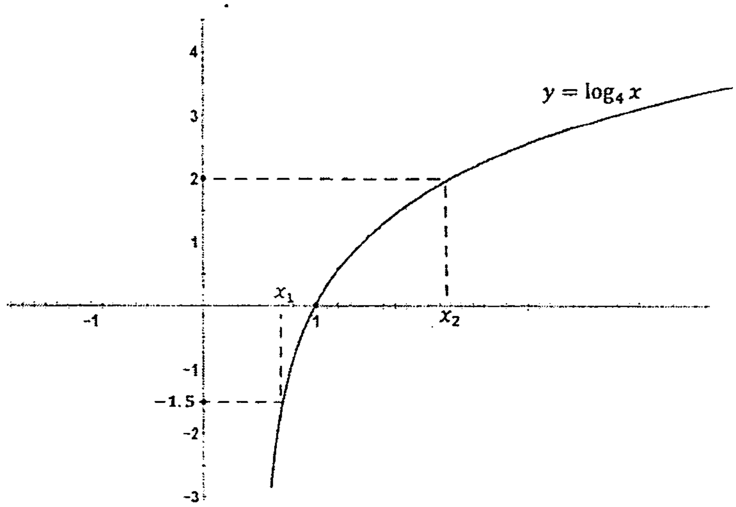
$2y^2 - y - 6 = 0$

$(2y + 3)(y - 2) = 0$

$y = -\frac{3}{2}$ or $y = 2$

$\therefore \log_4 x = -\frac{3}{2}$ or $\log_4 x = 2$

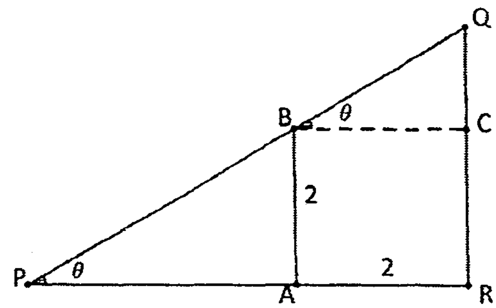
4ii)



From the graph, when $y = -\frac{3}{2}$ and $y = 2$, the x values are both positive.

$\therefore$ the product of the two roots of $2(\log_4 x)^2 = (\log_4 x) + 6$ is positive.

5i) $L = PB + BQ$



$\sin\theta = \frac{2}{PB} \quad \Rightarrow PB = \frac{2}{\sin\theta}$

$\cos\theta = \frac{2}{BQ} \quad \Rightarrow BQ = \frac{2}{\cos\theta}$

$\therefore L = \frac{2}{\sin\theta} + \frac{2}{\cos\theta}$ [AG]

5ii) $\frac{dL}{d\theta} = \frac{-2\cos\theta}{sin^2\,\theta} + \frac{2\sin\theta}{cos^2\,\theta}$

$= \frac{2sin^3\theta - 2cos^3\theta}{sin^2\theta cos^2\theta}$ [AG]

5iii) For max/min, $\frac{dL}{d\theta} = 0$

$2sin^3\theta - 2cos^3\theta = 0$

$sin^3\theta = cos^3\theta$

$tan^3\theta = 1$

$tan\,\theta = 1$

$\theta = \frac{\pi}{4} \qquad 0 < \theta < \frac{\pi}{2}$

Uisng 1st derivative test,

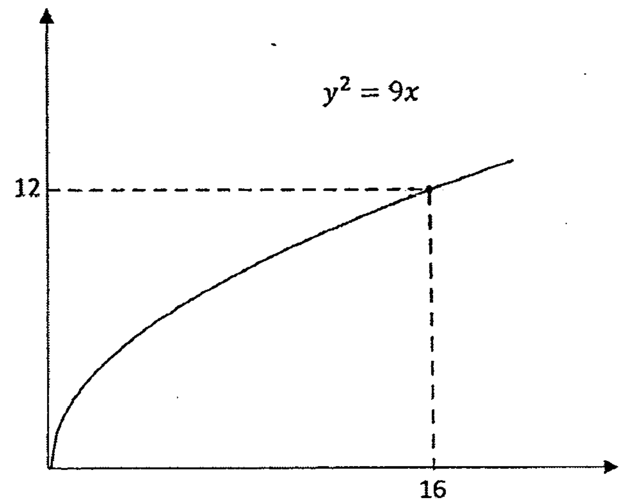
	$\frac{\pi}{4}^{-}$	$\frac{\pi}{4}$	$\frac{\pi}{4}^{+}$
$\frac{dL}{d\theta}$	− ╲	0 ―	+ ╱

$\therefore$ shortest possible length of the ramp

$= \frac{2}{\sin\frac{\pi}{4}} + \frac{2}{\cos\frac{\pi}{4}}$

$= 5.66\ m$ [5.6568]

6 i)



6ii) $4y - 3x = 9$

Subs $y = \frac{3x+9}{4}$ into $y^2 = 9x$

$\left(\frac{3x+9}{4}\right)^2 = 9x$

$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$x = 1 \quad or \quad x = 9$

x-coord of midpoint of PQ $= \frac{1+9}{2} = 5$

y-coord of midpoint of PQ $= \frac{3(5)+9}{4} = 6$

$\therefore$ coords of midpoint of PQ are (5,6)

7i) $\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{2}$

$\frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{3}{2}$

$2(\sin A \cos B - \cos A \sin B) = 3(\sin A \cos B + \cos A \sin B)$

$\sin A \cos B + 5\cos A \sin B = 0$

Divide throughout by $\cos A \cos B$,

$\therefore \tan A + 5\tan B = 0$ [AG]

7ii) $2\sin(2\theta - 30°) = 3\sin(2\theta + 30°)$ can be written as

$\frac{\sin(2\theta - 30°)}{\sin(2\theta + 30°)} = \frac{3}{2}$

Compare with (i) and let $A = 2\theta$ and $B = 30°$,

$\therefore \tan 2\theta + 5\tan 30° = 0$ using result from(i)

$$\tan 2\theta = -5\left(\frac{1}{\sqrt{3}}\right)$$

base angle, $\alpha = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right) = 70.893°$

$2\theta = 109.106°,\ 289.106°,\ 469.106°,\ 649.106°$

$$\therefore \theta = 54.6°,\ 144.6°,\ 234.6°,\ 324.6°$$

8i) $y = |3 - x| - 2$

At $A, x = 0, y = 3 - 2 = 1$

$\therefore A(0,1)$

At $B, \min|3 - x| = 0 \Rightarrow x = 3, y = -2$

$\therefore B(3, -2)$

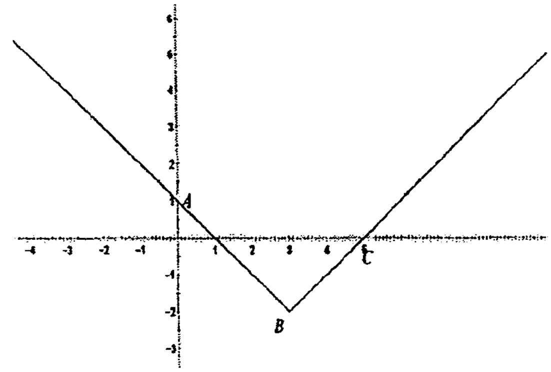
At $C, y = 0, \quad |3 - x| - 2 = 0$

$|3 - x| = 2$

$3 - x = 2 \quad or \quad 3 - x = -2$

$x = 1 \quad or \quad x = 5$

$\therefore C(5,0)$

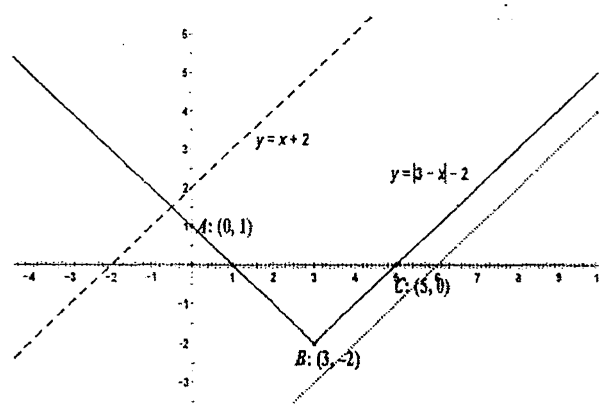


8ii) line $QR: y = x + p$

a) When $p = 2$,
$no. of\ intersections = 1$

b) When $p = -6$,
$no. of\ intersections = 0$

8iii) set of values of p for which no. of intersections is 1, is $p > -5$



9) $t = 0s,\ \ v = 9\text{m/s},\ \ a = 8 - 2t$

i) $v = \int a\, dt$

$= \int (8 - 2t)dt$

$= 8t - t^2 + c$

When $t = 0, v = 9$

$8t - t^2 + c = 9$

$c = 9$

$\therefore v = 8t - t^2 + 9$

At instantaneous rest, $v = 0$,

$\therefore 8t - t^2 + 9 = 0$

$t^2 - 8t - 9 = 0$

$(t + 1)(t - 9) = 0$

$t = -1\ (reject)\quad or\quad t = 9s\ \ [AG]$

9ii) $s = \int v\, dt$

$= \int (8t - t^2 + 9)dt$

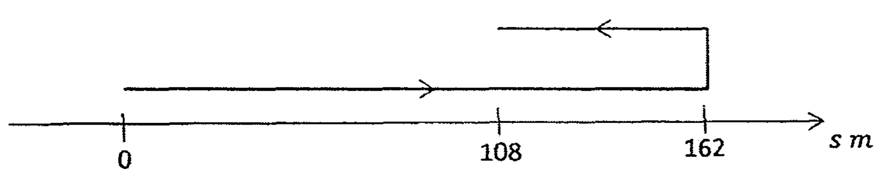
$= 4t^2 - \frac{t^3}{3} + 9t + c$

When $t = 0, s = 0 \ \Rightarrow c = 0$

$\therefore s = 4t^2 - \frac{t^3}{3} + 9t$

At instantaneous rest, $v = 0,\ \ t = 9,\ \ s = 162m$

$t = 12,\ \ s = 108m$



Total distance = $162 + (162 - 108) = 216m$

$\therefore$ average speed= $\frac{216m}{12s} = 18\ m/s$

10) Let $\measuredangle RSU = x$

then $\measuredangle RUS = x$ (base $\measuredangle s$, isos Δ)

$\measuredangle QPT = \measuredangle RSQ$

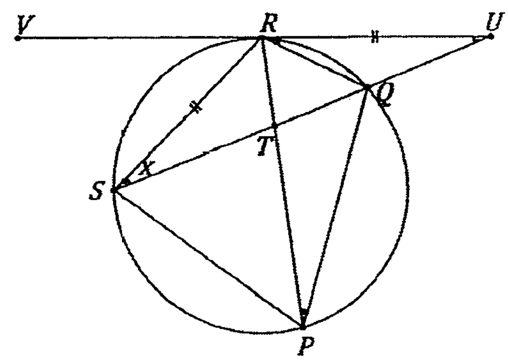
$= x$ ($\measuredangle s$ in the same segment)

$\measuredangle SRV = 2x$ (ext $\measuredangle$ of ΔSRU)

$\measuredangle SPT = \measuredangle SRV$ (alt segment thm)

$= 2x$

$\therefore \measuredangle SPT = 2 \times \measuredangle QPT$ [AG]



10ii) From (i), $\measuredangle QUR = \measuredangle RUS$ (common $\measuredangle$)

$\measuredangle QRU = \measuredangle RSU$ (alt segment thm)

$\measuredangle RQU = \measuredangle SRU$ ($\measuredangle$ sum of Δ)

$\therefore \Delta QRU$ is similar to ΔRSU (AAA similarity)

10iii) Using ratio of corresponding sides of similar $\Delta s QRU$ & RSU,

$$\frac{QR}{RS} = \frac{RU}{SU}$$

$QR \times SU = RU \times RS$

$QR \times SU = (RU)^2$ [AG] ($\because RU = RS$ given)

11) Given: $Vol = 960cm^3 \ at \ t = 0;$ $V = h^2 + 2h;$ $\frac{dh}{dt} = -\frac{3t}{2} \ cm/s$

11i) $h^2 + 2h = 960$

$h^2 + 2h - 960 = 0$

$(h + 32)(h - 30) = 0$

$h = 30 \quad or \quad h = -32 \ (rejected)$

$\therefore$ *initial height of water is 30cm.*

11ii) $\frac{dh}{dt} = -\frac{3t}{2}$

$h = -\frac{3t^2}{4} + c$

$when \quad t = 0, \ h = 30$

$\Rightarrow c = 30$

$$\therefore h = -\frac{3t^2}{4} + 30$$

11iii) $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$= (2h + 2) \times \left(-\frac{3t}{2}\right)$

$= \left[2\left(-\frac{3t^2}{4} + 30\right) + 2\right] \times \left(-\frac{3t}{2}\right)$

$when \ t = 4, \ \ rate \ of \ change \ of \ vol$

$$= \left.\frac{dV}{dt}\right|_{t=4}$$

$$= -228 \ cm^3/s$$

12a) $f(x) = 3\sin(px) - q$

$$-q = \frac{-2 + (-8)}{2}$$

$$= -5$$

$$\therefore q = 5$$

$$period = \frac{2\pi}{p}$$

From the graph, $period = \left(\frac{9\pi}{4} - \frac{3\pi}{4}\right) \times 2 = 3\pi$

$$\frac{2\pi}{p} = 3\pi$$

$$p = \frac{2}{3}$$

12b) Since graph of $y = x^2 + 2$ is symmetrical about the x-axis,

$$\int_0^a y\,dx = \frac{6a}{2}$$

$$\int_0^a (x^2 + 2)\,dx = \frac{6a}{2}$$

$$\left[\frac{x^3}{3} + 2x\right]_0^a = 3a$$

$$\frac{a^3}{3} + 2a = 3a$$

$$a^3 + 6a - 9a = 0$$

$$a^3 - 3a = 0$$

$$a(a^2 - 3) = 0$$

$$a = 0(rejected),\ a^2 = 3$$

$$\therefore a = \sqrt{3}\ \ since\ a > 0$$

2016 Preliminary Examination

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS — 4047/02

Paper 2

15 Sept 2016

2 hours 30 minutes

Additional Materials: Answer Paper
Graph paper(2 sheets)

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Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 (i) Sketch the graph of $y = 2x^{\frac{5}{2}}$ for $x > 0$. [1]

(ii) On the same diagram, sketch the graph of $y = 16x^{-\frac{1}{2}}$ for $x > 0$. [1]

(iii) Calculate the x-coordinate of the point of intersection of your graphs. [2]

2 (a) A polynomial $f(x)$ has a remainder of -2 when divided by $(2x + 1)$. Showing your method clearly,

(i) find the remainder when $f(x) - 1$ is divided by $(2x + 1)$, [2]

(ii) find in terms of $f(x)$, a polynomial which is completely divisible by $(2x + 1)$. [2]

(b) A polynomial $g(x)$ can be expressed as $g(x) = (x^2 - x - 2)P(x) + ax + b$, where $P(x)$ is a polynomials in x. Given that $g(x)$ leaves a remainder of -7 when divided by $(x - 2)$ and a remainder of -19 when divided by $(x + 1)$

(i) Find the value of a and of b. [5]

(ii) Find the remainder when $g(x)$ is divided by $(x - 2)(x + 1)$. [1]

3 **Do not use a calculator in this question.**

(a) (i) Simplify $(2 - \sqrt{5})^2$. [1]

(ii) Given that $x = \frac{1}{2-\sqrt{5}}$, find the exact value of $x^2 + x - 2$ [3]

(b) The volume of a cuboid with a square base is $19 + 11\sqrt{3}$ cm^3. The height of the cuboid is $\sqrt{3} + 1$ cm and the length of each side of the square base is $a + \sqrt{b}$, where a and b are integers. Find the values of a and of b. [6]

4 (a) The roots of the quadratic equation $2x^2 + 5x - 1 = 0$ are $\tan A$ and $\tan B$.

(i) Find the value of $\tan(A + B)$. [3]

(ii) Find the value of $sec^2(A + B)$. [2]

(b) (i) Show that $\frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x} = 4\, sec^2 3x$. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{12}} \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}\, dx$. [2]

5 A curve has the equation $y = 3x^2 e^{-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of these stationary points. [6]

6 (a) Find in ascending powers of x, the first four terms in the expansion of $(1 + x - x^2)^9$. [4]

(b) (i) Find the term independent of x in the expansion of $\left(2x^2 - \frac{1}{2x}\right)^{12}$. [3]

(ii) Determine the constant term in the expansion of $(3 + 4x^3)\left(2x^2 - \frac{1}{2x}\right)^{12}$. [4]

7 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{(2x-5)^2}$.

The equation of the tangent to the curve at the point $(3, -1)$ is $y - 2x + 7 = 0$.

(i) Find an expression for $\frac{dy}{dx}$. [4]

(ii) Find the equation of the curve. [5]

8 The table shows experimental values of the variables x and y.

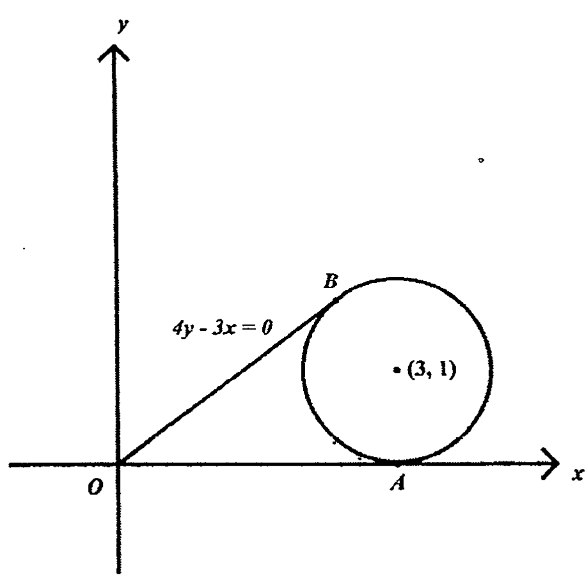
x	1	2	3	4	5
y	0.4	0.6	1.6	3.4	6

It is known that x and y are related by the equation of the form $p(x+y) = pq + qx^2$.

(i) Plot $x + y$ against x^2, draw the straight line graph and use it to estimate the value of p and q. [6]

(ii) Using your values of p and q, find the values of x for which $p(x^2 - 2q) = 2qx^2$. [2]

9 **(a)**



The circle with centre $C(3, 1)$ touches the x-axis at A. The line $4y - 3x = 0$ touches the circle at B.

Find the coordinates of B. [5]

(b) The equation of another circle is $(x-4)^2 + (y+1)^2 = 4$.

The line $y = mx$ is a tangent to the circle. Find the possible exact values of m. [4]

10 **(a)** **(i)** Express $\frac{2x^3+x^2}{x^2+x-2}$ in the form of $ax+b+\frac{cx+d}{x^2+x-2}$. [2]

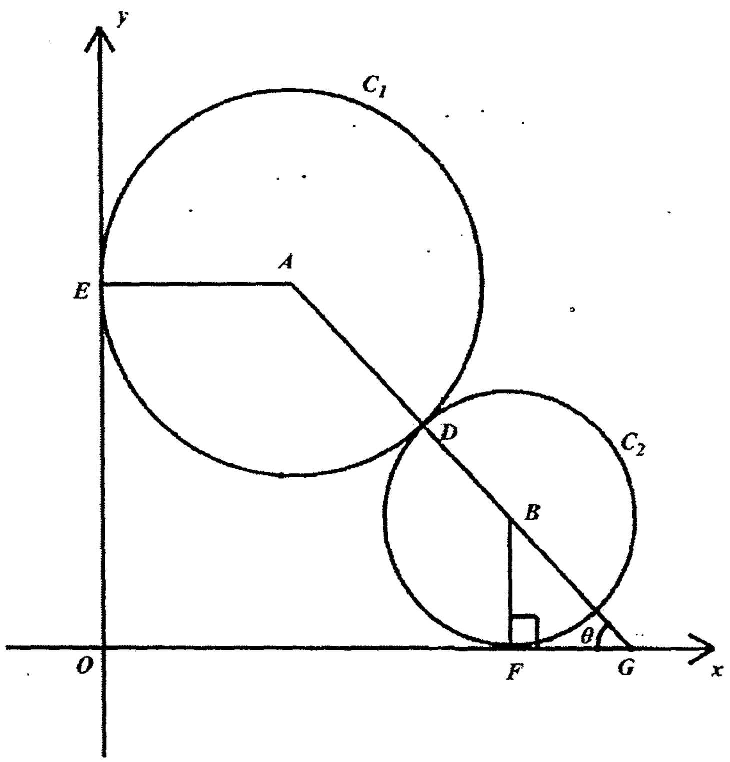
(ii) Using the values of c and d found in (i), express $\frac{cx+d}{x^2+x-2}$ as a sum of two partial fractions. [3]

(b) A curve has the equation $y=\frac{x-1}{\sqrt{4x+1}}$.

(i) Differentiate y with respect to x. [3]

(ii) Using the result in part **b(i)**, determine $\int \frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\,dx$. [2]

11.



The diagram shows two circles, C_1 and C_2 with centres A and B respectively. The two circles touch each other at D. C_1 has radius 3 units and touches the y-axis at E. C_2 has radius 2 units and touches the x-axis at F. The lines AB produced meets the x-axis at G and angle $BGO=\theta$ radians.

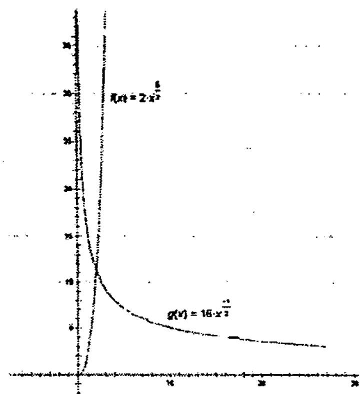
(i) Show with clear explanations, that $OE=5\sin\theta+2$ and $OF=5\cos\theta+3$. [2]

(ii) Show that $EF^2=38+20\sin\theta+30\cos\theta$. [2]

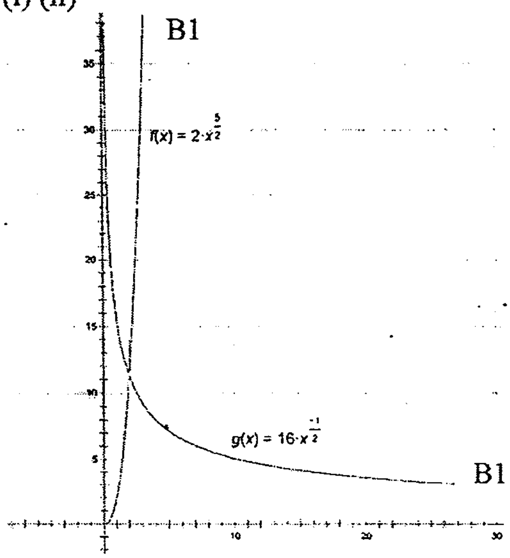
(iii) Express EF^2 in the form $38+R\cos(\theta-\alpha)$, where $R>0$ and α is an acute angle. [3]

(iv) Given that $EF^2=65$, find the value of θ. [2]

END OF PAPER

	Answer Key		
1	(i) (ii) 		
iii	$x = 2$	8i	$p = 2.5, q = 1$
2i	Remainder $= -3$	ii	$x = \pm\sqrt{10}$ or $x = \pm 3.16$
ii	A polynomial $= f(x) + 2$, any multiple of $f(x) + 2$	9a	$B(\frac{12}{5}, \frac{9}{5})$
2bi	$a = 4$, $b = -15$	9b	$m = \frac{-2 \pm\sqrt{13}}{6}$
ii	Remainder $= 4x - 15$	10ai	$2x - 1 + \frac{5x - 2}{x^2 + x - 2}$
3ai	$9 - 4\sqrt{5}$	aii	$\frac{5x - 2}{x^2 + x - 2} = \frac{4}{x + 2} + \frac{1}{x - 1}$
aii	$5 + 3\sqrt{5}$	bi	$\frac{2x + 3}{(4x + 1)^{\frac{3}{2}}}$
3b	$a = 2$ and $b = 3$	ii)	$\int \frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}} \, dx = \frac{2(x-1)}{\sqrt{4x+1}} + c'$
4ai)	$-\frac{5}{3}$	11iii	$EF^2 = 38 + 10\sqrt{13} cos\,(\theta - 0.58800)$
4aii)	$\frac{34}{9}$	11iv	$\theta = 1.31$
4bii	$\frac{4}{3}$		
5ai	$3xe^{-x}(2 - x)$, $(0, 0)$ and $(2, \frac{12}{e^2})$		
5ii	$(2, \frac{12}{e^2})$ is a maximum point $(0, 0)$ is a minimum point		
6a	$1 + 9x + 27x^2 + 12x^3 + \cdots)$		
bi)	$\frac{495}{16}$		
bii	$\frac{1265}{16}$		
7i	$\frac{dy}{dx} = -\frac{3}{(2x - 5)} + 5$		
ii	$y = -\frac{3\ln(2x - 5)}{2} + 5x - 16$		

1	(i) Sketch the graph of $y = 2x^{\frac{5}{2}}$ for $x > 0$.	[1]
	(ii) On the same diagram, sketch the graph of $y = 16x^{-\frac{1}{2}}$ for $x > 0$.	[1]
	(iii) Calculate the x-coordinate of the point of intersection of your graphs.	[2]

1 [2]	(i) (ii)  B1 B1	
(iii)	$2x^{\frac{5}{2}} = 16x^{-\frac{1}{2}}$	M1 equating with attempt to solve
[2]	$x^3 = 8$	
	$x = 2$	A1

2 (a) A polynomial $f(x)$ has a remainder of -2 when divided by $(2x + 1)$. Showing your method clearly,

(i) find the remainder when $f(x) - 1$ is divided by $(2x + 1)$, [2]

(ii) find in terms of $f(x)$, a polynomial which is completely divisible by $(2x + 1)$. [2]

2(a) (i)	Let $f(x) = (2x + 1))Q(x) - 2$	
[2]	$f(x) - 1 = (2x + 1))Q(x) - 2 - 1$	M1
	Remainder $= -3$	B1
(ii)	$f(x) + 2 = (2x + 1))Q(x) - 2 + 2$	M1
[2]	A polynomial = $f(x) + 2$, any multiple of $f(x) + 2$	B1

(b) A polynomial $g(x)$ can be expressed as $g(x) = (x^2 - x - 2)P(x) + ax + b$, where $P(x)$ is a polynomials in x. Given that $g(x)$ leaves a remainder of -7 when divided by $(x-2)$ and a remainder of -19 when divided by $(x+1)$

(i) Find the value of a and of b. [5]

(ii) Find the remainder when $g(x)$ is divided by $(x-2)(x+1)$. [1]

2(b) (i)	$g(x) = (x^2 - x - 2)P(x) + ax + b,$	
[5]	$= (x-2)(x+1)P(x) + ax + b,$	$(x-2)(x+1)$ seen or
	Substituting $x = -1$ or 2	$(-1)^2 - (-1) - 2$ seen or
	$g(2) = 2a + b = -7$	$2^2 - 2 - 2$ seen B1
	$2a + b = -7$ ....(1)	B1
	$g(-1) = -a + b = -19$ ....(2)	B1
2(b) (i)	(1) − (2), $3a = 12$	
	$a = 4$	A1
	$b = -15$	A1
(b) (ii) [1]	Remainder $= 4x - 15$	A1

3 Do not use a calculator in this question.

(a) **(i)** Simplify $(2 - \sqrt{5})^2$. [1]

(ii) Given that $x = \frac{1}{2-\sqrt{5}}$, find the exact value of $x^2 + x - 2$ [3]

3(a) (i)	$(2 - \sqrt{5})^2 = 4 - 4\sqrt{5} + 5$	
[1]	$= 9 - 4\sqrt{5}$	A1
(ii)	$x^2 + x - 2 = \dfrac{1}{9 - 4\sqrt{5}} + \dfrac{1}{2 - \sqrt{5}} - 2$	B1
[3]	$= \frac{9+4\sqrt{5}}{81-80} + \frac{2+\sqrt{5}}{-1} - 2$	Rationalising the denominator M1
	$= 5 + 3\sqrt{5}$	A1

(b) The volume of a cuboid with a square base is $19 + 11\sqrt{3}$ cm^3. The height of the cuboid is $\sqrt{3} + 1$ cm and the length of each side of the square base is $a + \sqrt{b}$, where a and b are integers. Find the values of a and of b. [6]

3(b)	Area $= \frac{19+11\sqrt{3}}{\sqrt{3}+1}$	M1
[6]	$= \frac{19+11\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$	
	$= \frac{19\sqrt{3}+33-19-11\sqrt{3}}{2}$	
	$(a+\sqrt{b})^2 = \frac{14+8\sqrt{3}}{2}$	B1
	$a^2 + b + 2a\sqrt{b} = 7 + 4\sqrt{3}$	
	$a^2 + b = 7$---------(1) $2a\sqrt{b} = 4\sqrt{3}$ $a\sqrt{b} = 2\sqrt{3}$ $a^2b = 12$ ---------(2)	Equating rational and irrational parts M1 Do not accept $a\sqrt{b} = 2\sqrt{3}$. $a = 2, b = 3$
	From (1), $a^2 = 7 - b$	
	$(7-b)b = 12$	
	$0 = b^2 - 7b + 12$	M1 obtain a quadratic equation
	$(b-4)(b-3) = 0$	
	$b = 3$ or $b = 4$	Obtain either both b's or both a's
	when $b = 4$, $a^2 = 7 - 4 = 3$ (rejected)	
	when $b = 3$, $a^2 = 7 - 3 = 4$	
	$a = 2$ or $a = -2$ (rejected)	
	$a = 2$ and $b = 3$	A1 [given provided M1 has been awarded]

4 **(a)** The roots of the quadratic equation $2x^2 + 5x - 1 = 0$ are $\tan A$ and $\tan B$.

(i) Find the value of $\tan(A+B)$. [3]

(ii) Find the value of $sec^2(A+B)$. [2]

4(a) (i)	$\tan A + \tan B = -\frac{5}{2}$	Either one B1
	$\tan A \tan B = -\frac{1}{2}$	
	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$	
	$= \frac{-\frac{5}{2}}{1+\frac{1}{2}}$	B1
	$= -\frac{5}{3}$	A1

4 (a) (ii)	$sec^2(A+B) = 1 + tan^2(A+B)$	
[2]	$= 1 + \frac{25}{9}$	M1
	$= \frac{34}{9}$	A1

(b) (i) Show that $\frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x} = 4\,sec^2 3x$. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{12}} \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}\,dx$. [2]

4(b) (i)	LHS = $\frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}$	
[2]	$= \frac{2(1+\sin 3x)+2(1-\sin 3x)}{(1-sin^2 3x)}$	B1
	$= \frac{4}{cos^2 3x}$	B1
	$= 4\,sec^2 3x$ (Shown)	
(ii)	$\int_0^{\frac{\pi}{12}} \frac{2}{1-\sin 3x} + \frac{2}{1+\sin 3x}\,dx$	
[2]	$= \int_0^{\frac{\pi}{12}} 4\,sec^2 3x\,dx$	
	$= \left[\frac{4}{3}\tan 3x\right]_0^{\frac{\pi}{12}}$	B1
	$= \frac{4}{3}$	A1

5 A curve has the equation $y = 3x^2e^{-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and obtain the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of these stationary points. [6]

5(i) [5]	$\frac{dy}{dx} = 6xe^{-x} + 3x^2(-e^{-x})$	Product rule M1, B1
	$= 3xe^{-x}(2-x)$	
	For stationary points, $\frac{dy}{dx} = 0$	M1
	$3xe^{-x}(2-x) = 0$	
	$e^{-x} \neq 0,\ x = 0$ or $x = 2$	A1[2 values of x]
	(0, 0) and $(2, \frac{12}{e^2})$	Both points A1

					Remarks
5(ii) [6]	$\frac{d^2y}{dx^2} = 6e^{-x} - 6xe^{-x} + 6x(-e^{-x}) + 3x^2(e^{-x})$				Award M1 if there is at most 1 wrong term
	$= 6e^{-x} - 12xe^{-x} + 3x^2(e^{-x})$				A1
	$= 3e^{-x}(2 - 4x + x^2)$				
	when $x = 0$, $\frac{d^2y}{dx^2} = 6 > 0$				B1
	(0, 0) is a minimum point				A1
	when $x = 2$, $\frac{d^2y}{dx^2} = -\frac{6}{e^2} < 0$				B1
	$(2, \frac{12}{e^2})$ is a maximum point				A1
OR	Using $\frac{dy}{dx}$,				
[6]	For (0, 0)				
	x	0^-	0	0^+	
	$\frac{dy}{dx}$	< 0	0	> 0	
	Sketch of tangent	\	—	/	B2
	(0, 0) is a minimum point				A1
	For $(2, \frac{12}{e^2})$				
	x	2^-	2	2^+	
	$\frac{dy}{dx}$	> 0	0	< 0	
	Sketch of tangent	/	—	\	B2
	$(2, \frac{12}{e^2})$ is a maximum point				A1

6 (a) Find in ascending powers of x, the first four terms in the expansion of $(1+x-x^2)^9$. [4]

6(a)	$(1+x-x^2)^9$	
[4]	$=1+\binom{9}{1}(x-x^2)+\binom{9}{2}(x-x^2)^2+\binom{9}{3}(x-x^2)^3+\ldots$	B1
	$=1+9x-9x^2+36(x^2-2x^3+x^4)+84(x^3+\cdots)$	
	$=1+9x+27x^2+12x^3+\cdots)$	A3 deduct 1 mark for every wrong term

(b) (i) Find the term independent of x in the expansion of $\left(2x^2-\frac{1}{2x}\right)^{12}$. [3]

(ii) Determine the constant term in the expansion of $(3+4x^3)\left(2x^2-\frac{1}{2x}\right)^{12}$. [4]

6(b) (i)	$(r+1)^{th}$ term $=\binom{12}{r}(2x^2)^{12-r}\left(-\frac{1}{2x}\right)^r$	M1
[3]	For term independent of x	
	$x^0=x^{2(12-r)}\times x^{-r}$	
	$0=24-3r$	
	$r=8$	B1
	Term independent of $x=\binom{12}{8}(2x^2)^{12-8}\left(-\frac{1}{2x}\right)^8$	
	$=\binom{12}{8}(2)^4\left(-\frac{1}{2}\right)^8$	
	$=\binom{12}{8}\left(\frac{1}{2}\right)^4$	
	$=\frac{495}{16}$	A1
6(b) (ii)	For x^{-3}, $-3=24-3r$	
[4]	$r=9$	M1
	Term in $x^{-3}=\binom{12}{9}(2x^2)^3\left(-\frac{1}{2x}\right)^9$	
	$=-\binom{12}{9}\left(\frac{1}{2^6}\right)x^{-3}$	
	$=-\frac{220}{64}x^{-3}$	B1
	Constant $=3\times\frac{495}{16}+4\times\left(-\frac{220}{64}\right)$	M1
	$=\frac{1265}{16}$	A1

7 A curve is such that $\frac{d^2y}{dx^2} = \frac{6}{(2x-5)^2}$.

The equation of the tangent to the curve at the point $(3, -1)$ is $y - 2x + 7 = 0$.

(i) Find an expression for $\frac{dy}{dx}$. [4]

(ii) Find the equation of the curve. [5]

7(i)	$\frac{dy}{dx} = \int 6(2x-5)^{-2}\,dx$	M1 attempt to integrate
[4]	$= \frac{6(2x-5)^{-1}}{(-1)(2)} + c$	B1
	$= -\frac{3}{(2x-5)} + c$	
	when $x = 3$, $\frac{dy}{dx} = 2$	
	$2 = -3 + c$	
	$c = 5$	M1 attempt to find c
	$\frac{dy}{dx} = -\frac{3}{(2x-5)} + 5$	A1
(ii)	$y = \int -\frac{3}{(2x-5)} + 5\;dx$	M1 attempt to find y by integrating $\frac{dy}{dx}$.
[5]	$= -\frac{3\ln(2x-5)}{2} + 5x + d$	B1
	substituting $x = 3$ and $y = -1$	
	$-1 = -\frac{3}{2}\ln 1 + 15 + d$	M1 attempt to find d.
	$d = -16$	B1
	$y = -\frac{3\ln(2x-5)}{2} + 5x - 16$	A1

8 The table shows experimental values of the variables x and y.

x	1	2	3	4	5
y	0.4	0.6	1.6	3.4	6

It is known that x and y are related by the equation of the form $p(x+y) = pq + qx^2$.

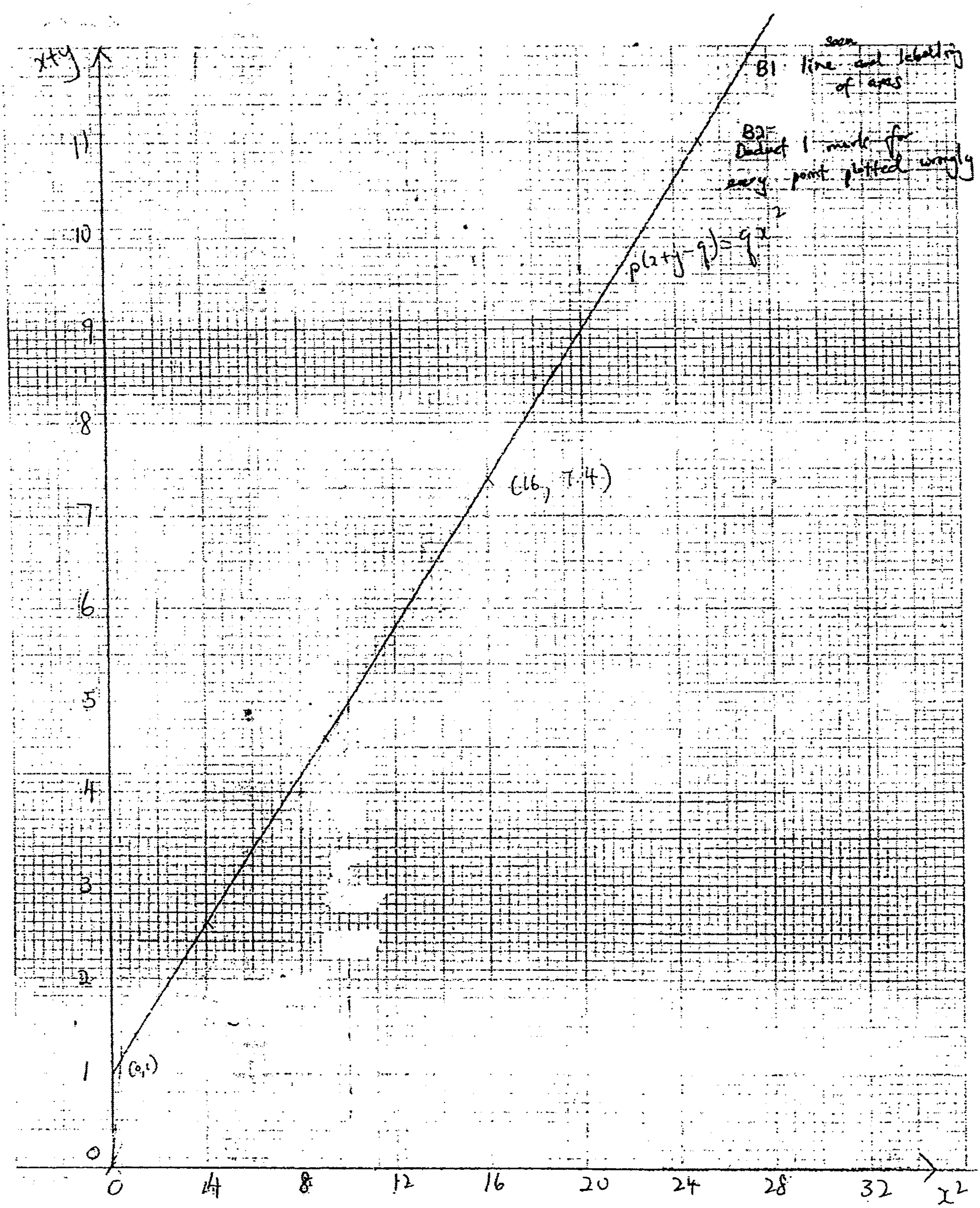
(i) Plot $x + y$ against x^2, draw the straight line graph and use it to estimate the value of p and q. [6]

(ii) Using your values of p and q, find the values of x for which $p(x^2 - 2q) = 2qx^2$. [2]

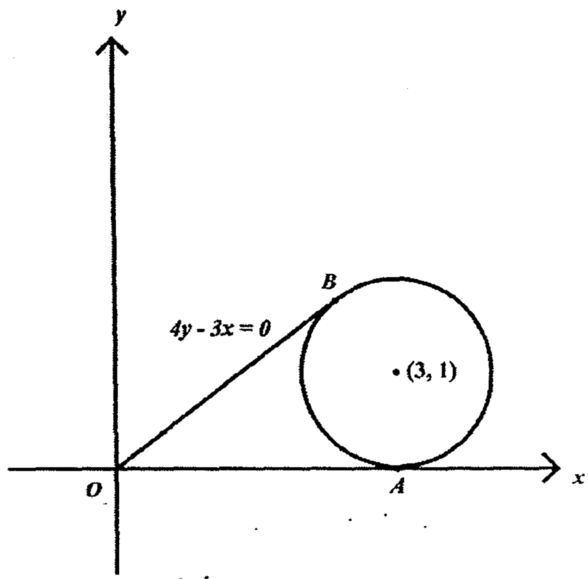
(i)	x^2: 1, 4, 9, 16, 25	
[6]	$x+y$: 1.4, 2.6, 4.6, 7.4, 11	
	$p(x+y) = pq + qx^2$	
	$x + y - q = \frac{q}{p}x^2$	
	$x + y = q + \frac{q}{p}x^2$ ----------(1)	Award B1 either for (1) or (2)
	gradient = $\frac{q}{p}$, $x+y$-intercept = q ------(2)	
	From graph, $x+y$-intercept = 1	
	$q = 1$	A1
	gradient $= \frac{7.4-1}{16} = 0.4$	
	$\frac{q}{p} = 0.4$	
	$\frac{1}{p} = 0.4$	
	$p = 2.5$	A1
	On graph paper	
	Straight line drawn with correct labelling of axes	B1
	All 5 points correctly plotted	B2 deduct 1 mark for every point plotted wrongly

8(ii)	$\frac{5}{2}(x^2 - 2) = 2x^2$	M1 FT for their answers in (i)
[2]	$\frac{1}{2}x^2 = 5$	
	$x^2 = 10$	
	$x = \pm\sqrt{10}$ or $x = \pm 3.16$	A1

Q 8


x+y
B1 seen line and labelling of axes
B2F Deduct 1 mark for every point plotted wrongly
$p(x+y-q)=qx^2$
(16, 7.4)
(0,1)
0
1
2
3
4
5
6
7
8
9
10
11
0
4
8
12
16
20
24
28
32
x^2

9 (a)



The circle with centre $C(3, 1)$ touches the x-axis at A. The line $4y - 3x = 0$ touches the circle at B.

Find the coordinates of B. [5]

9(a)	Equation of tangent at B is $y = \frac{3}{4}x$.	
[5]	Gradient of normal at B is $-\frac{4}{3}$	M1
	Equation of normal at B is $y - 1 = -\frac{4}{3}(x - 3)$	
	$y = -\frac{4}{3}x + 5$	B1
	For point of intersection B,	
	$\frac{3}{4}x = -\frac{4}{3}x + 5$	M1
	$\frac{25x}{12} = 5$	
	$x = \frac{12}{5}$	B1 for correct x or y
	$y = \frac{9}{5}$	
	$B(\frac{12}{5}, \frac{9}{5})$	A1

(b) The equation of another circle is $(x-4)^2 + (y+1)^2 = 4$.

The line $y = mx$ is a tangent to the circle. Find the possible exact values of m. [4]

9(b)	For points of intersection,	
[4]	substitute $y = mx$ into $(x-4)^2 + (y+1)^2 = 4$	
	$(x-4)^2 + (mx+1)^2 = 4$	M1
	$x^2 - 8x + 16 + m^2x^2 + 2mx + 1 = 4$	
	$x^2(1+m^2) + x(2m-8) + 13 = 0$	
	For line to be a tangent to the circle, Discriminant = 0	
	$(2m-8)^2 - 4(1+m^2)13 = 0$	M1
	$4m^2 - 32m + 64 - 52 - 52m^2 = 0$	
	$0 = 48m^2 + 32m - 12$	
	$0 = 12m^2 + 8m - 3$	
	$m = \dfrac{-8 \pm \sqrt{64 - 4(12)(-3)}}{2(12)}$	
	$m = \dfrac{-8 \pm 4\sqrt{13}}{24}$	
	$m = \frac{-2 \pm \sqrt{13}}{6}$ also accept $m = -\frac{1}{3} \pm \frac{1}{6}\sqrt{13}$	A1, A1 Deduct 1 mark if answers are not in the lowest terms

10 **(a)** **(i)** Express $\frac{2x^3+x^2}{x^2+x-2}$ in the form of $ax+b+\frac{cx+d}{x^2+x-2}$. [2]

(ii) Using the values of c and d found in (i), express $\frac{cx+d}{x^2+x-2}$ as a sum of two partial fractions. [3]

10(a) (i)	By long division	M1
[2]	$\frac{2x^3+x^2}{x^2+x-2}=2x-1+\frac{5x-2}{x^2+x-2}$	A1
(ii) [3]	$\frac{5x-2}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$	
	$5x-2=A(x-1)+B(x+2)$	M1
	Let $x=1$, $3=3B$	
	$B=1$	A1 for either
	Comparing coefficient of x, $A+B=5$	A or B correct
	$A=4$	
	$\frac{5x-2}{x^2+x-2}=\frac{4}{x+2}+\frac{1}{x-1}$	A1

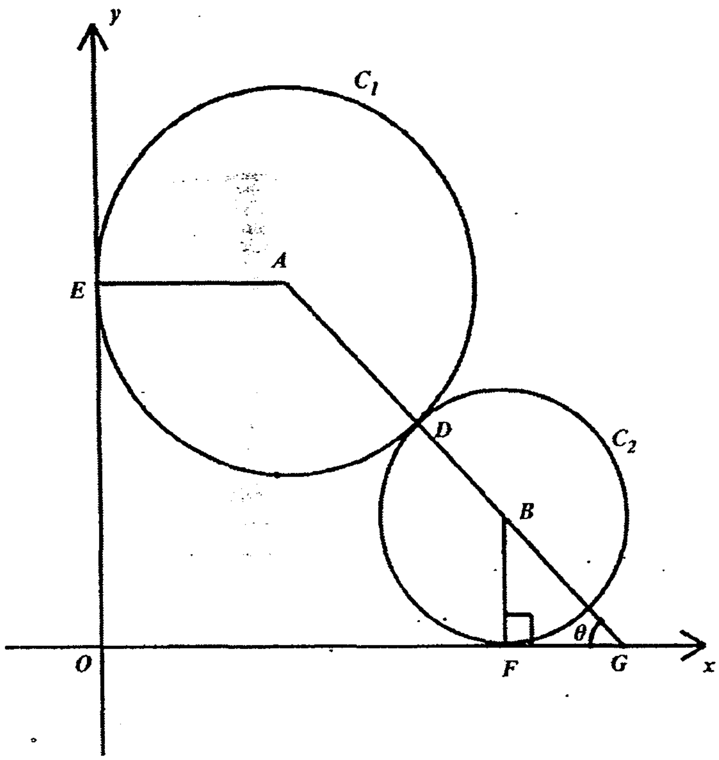
(b) A curve has the equation $y=\frac{x-1}{\sqrt{4x+1}}$.

(i) Differentiate y with respect to x. [3]

(ii) Using the result in part **b(i)**, determine $\int\frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\,dx$. [2]

(b) (i) [3]	$\frac{dy}{dx}=\frac{(4x+1)^{\frac{1}{2}}(1)-(x-1)\times\frac{1}{2}(4x+1)^{-\frac{1}{2}}\times 4}{(4x+1)}$	M1 quotient rule M1 chain rule
	$=\frac{(4x+1)^{-\frac{1}{2}}[4x+1-2(x-1)]}{(4x+1)}$	
	$=\frac{2x+3}{(4x+1)^{\frac{3}{2}}}$	A1
(ii)	$\int\frac{2x+3}{(4x+1)^{\frac{3}{2}}}\,dx=\frac{x-1}{\sqrt{4x+1}}+c$	M1 Reverse differentiation)
	$\int\frac{2(2x+3)}{(4x+1)^{\frac{3}{2}}}\,dx=\frac{2(x-1)}{\sqrt{4x+1}}+c'$	A1

11.



The diagram shows two circles, C_1 and C_2 with centres A and B respectively. The two circles touch each other at D. C_1 has radius 3 units and touches the y-axis at E. C_2 has radius 2 units and touches the x-axis at F. The lines AB produced meets the x-axis at G and angle $BGO = \theta$ radians.

(i) Show with clear explanations, that $OE = 5\sin\theta + 2$ and $OF = 5\cos\theta + 3$. [2]

(ii) Show that $EF^2 = 38 + 20\sin\theta + 30\cos\theta$. [2]

(iii) Express EF^2 in the form $38 + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [3]

(iv) Given that $EF^2 = 65$, find the value of θ. [2]

(i)	$AB = 3 + 2 = 5\text{cm}$	
[2]	$OE = AB\sin\theta + BF = 5\sin\theta + 2$	B1
	$OF = AB\cos\theta + AE = 5\cos\theta + 3$	B1

11(ii)	$EF^2 = (5\sin\theta + 2)^2 + (5\cos\theta + 3)^2$	M1
[2]	$= 25sin^2\theta + 20sin\theta + 4 + 25cos^2\theta + 30\cos\theta + 9$	
	$= 25(sin^2\theta + cos^2\theta) + 20sin\theta + 30\cos\theta + 13$	B1
	$= 38 + 20\sin\theta + 30\cos\theta$ (AG)	

11(iii)	$EF^2 = 38 + Rcos\ (\theta - \alpha)$	
[3]	$R = \sqrt{30^2 + 20^2} = 10\sqrt{13}$	B1
	$\alpha = tan^{-1}\left(\frac{20}{30}\right) = 0.58800$	B1
	$EF^2 = 38 + 10\sqrt{13}cos\ (\theta - 0.58800)$	A1

11(iv)	$EF^2 = 65$	
[2]	$65 = 38 + 10\sqrt{13}cos\ (\theta - 0.58800)$	
	$\frac{27}{10\sqrt{13}} = cos\ (\theta - 0.58800)$	M1
	$\theta - 0.58800 = 0.72448$	
	$\theta = 1.31$ (to 3 sig fig)	A1