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GAN ENG SENG SCHOOL
Preliminary 1 Examination 2017

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
4047/01

Paper 1

9 May 2017

2 hours

Sec 4 Express/ 5 Normal (Academic)

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer all the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

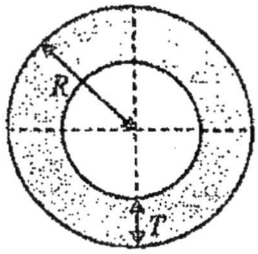
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

	For Examiner's Use
Total	80

Answer all questions.

1 A hollow copper pipe with an external radius, $R=\left(4\sqrt{3}-1\right)$cm has a thickness, $T=\sqrt{3}$ cm.

The volume of copper needed to make the pipe is $\left(521\sqrt{3}-108\right)\pi$ cm^3.



Find

(i) the cross sectional area of the pipe, in the form $\pi\left(a+b\sqrt{3}\right)$, where a and b are integers. [2]

(ii) the length of the pipe in the form $\left(c+d\sqrt{3}\right)$, where c and d are integers. [3]

2 Given that $\int_1^4 px^2\,dx=6$, where p is a constant,

(i) express $\int_1^4\left(px^2+\frac{3k}{2}\right)dx$ in terms of constant k, [2]

(ii) determine the value of p and find the value of $\int_2^8 px^2\,dx$. [3]

3 (i) Find the range of values of k for which the equation $x^2+6x+k=2kx-9$ has no real roots. [3]

(ii) Hence, deduce by giving a reason, whether the line $y=18x-9$ intersects the curve $y=(x+3)^2$. [2]

4 (a) Solve each of the following equations.

(i) $10^x=e^{3x+1}$. [2]

(ii) $\lg|2x+3|=0$. [2]

(b) Express $2\log_3 15-(\log_a 5)(\log_3 a)$, where $a>1$, as a single logarithm to base 3. [3]

5 The equation of a curve is $y = 7 - 5x + 6x^2 - 3x^3$ and $x + y = k$ is a tangent to the curve.

(i) Find the value of k. [4]

(ii) Show that $y = 7 - 5x + 6x^2 - 3x^3$ decreases as x increases. [2]

6 (i) Express $\dfrac{3x^2 - 6x + 2}{(x-1)^2(x-2)}$ in partial fractions. [5]

(ii) Hence find $\displaystyle\int \frac{9x^2 - 18x + 6}{(x-1)^2(x-2)}\,dx$. [2]

7 A piece of metal is heated to 100°C and allowed to cool to room temperature. The temperature of metal, θ after it has been cooled for t minutes is given by the equation $\theta = 26 + 74e^{-0.5t}$. Find

(i) its temperature at $t = 3$, [1]

(ii) the time needed to the nearest minute for the temperature to drop to 50°C, [2]

(iii) the rate at which θ is decreasing when $t = 5$, [2]

(iv) the expected room temperature. [1]

8 The number of hours of daylight in a city is given by $L(t) = -3.5\cos\left(\frac{\pi}{6}t\right) + 12.75$, where t is an integer ranging from 1 to 12 inclusive, which represents the month of January to December.

(i) Find the number of hours of daylight in the month of September. [1]

(ii) Find the month which has the highest number of hours of daylight. [3]

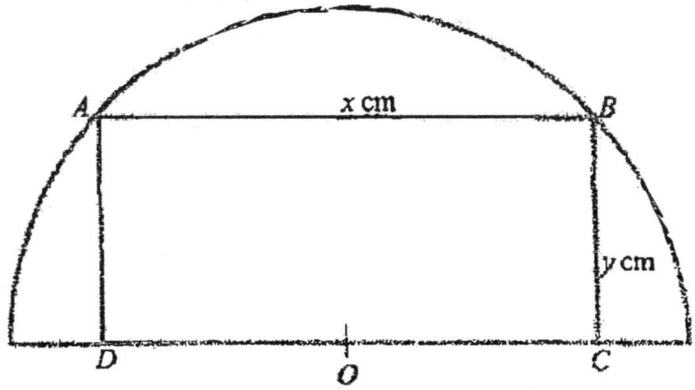
(iii) When should you plant in a garden if you want to do it during the month where there are 11 hours of daylight? [2]

9 (i) Given that $\sin(x + \alpha) = \lambda\cos(x - \alpha)$, show that $\tan x = \dfrac{\lambda - \tan\alpha}{1 - \lambda\tan\alpha}$. [4]

(ii) Hence solve, for $0 < x < 2\pi$, the equation [3]

$$\sin\left(x + \frac{\pi}{3}\right) = \frac{2\sqrt{3}}{3}\cos\left(x - \frac{\pi}{3}\right).$$

10



In the figure above, $ABCD$ is a rectangle with $AB = x$ cm and $BC = y$ cm. It fits inside a semicircle of radius 10 cm and centre O.

(i) Express y in terms of x. [1]

(ii) Show that A cm^2, the area of the rectangle, is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [1]

(iii) Given that x can vary, find the value of x for which the area of the rectangle is stationary. [4]

(iv) Explain why this value of x gives the rectangle the largest possible area. [1]

11 The equation of a curve is $y = 3x^2 \ln x$. The tangent to the curve at the point $x = e^2$ meets the x-axis at A and the y-axis at B.

(i) Show that the coordinates of B are $(0, -9e^4)$. [5]

(ii) Calculate the area of triangle AOB in terms of e. [2]

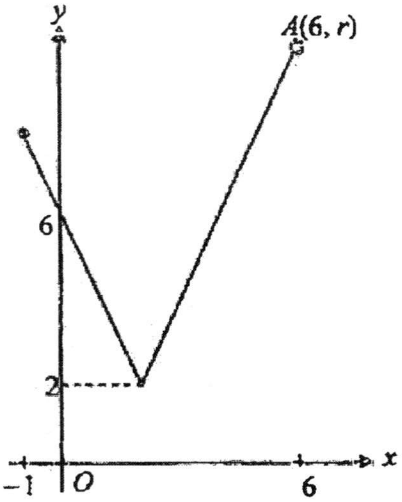
12 It is given that the graph of the function $f(x) = 1 - 3\sin nx$ between the interval $0 \le x \le \frac{\pi}{2}$, where n is a positive integer, intersects the x axis at 2 points.

(i) State the value of n. [1]

(ii) Sketch the graph of $f(x) = 1 - 3\sin nx$, given the value of n in (i) [2]

(iii) By sketching an additional linear graph on the same axes, find the number of solution that satisfy the equation $\sin nx - 1 = \frac{-x}{2\pi}$. [2]

13



The diagram above shows part of the graph of $y=|2x-p|+q$, where p and q are constants for $-1\leq x<6$. The y-intercept of the graph is 6 and its minimum y-value is 2. A point $A(6, r)$ lies on the graph.

(i) State the value of p and of q. [2]

(ii) Show that $r = 10$. [1]

(iii) A line, $y=mx+c$ is added onto the same axes in the diagram for $-1\leq x<6$.

(a) In the case where $m = 0$, write down the greatest integer value of c such that the line intersects the graph of $y=|2x-p|+q$ at exactly one point. [1]

(b) In the case where $c=1$, find the range of values of m such that the line intersects the graph of $y=|2x-p|+q$ at exactly two points. [3]

End-of-paper

Answer ALL questions

1. (i) Given that $2x+1$ is a factor of the expression $2x^3+5x^2+kx-6$, solve the equation $2x^3+5x^2+kx-6=0$, giving non-integer solutions in the form $a \pm b\sqrt{7}$. [6]

(ii) The roots of the equation $x^2-6x+3=0$ are $2\alpha+\beta$ and $\alpha+2\beta$

Find

(a) the value of $\alpha^3+\beta^3$, [2]

(b) the quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [5]

2. (i) Solve the equation

$$9^x - 5(3^{x+1}) + 50 = 0$$ [4]

(ii) Solve the simultaneous equations

$$64^x \times 8^y = 2^{x+1}$$

$$81^{5-x} \div 27^{y+1} = \frac{1}{729^y}$$ [5]

3. (i) The expansion of $(1+ax+bx^2)^8$ in ascending powers of x is given by $1-40x+748x^2+....$
Find the value of a and of b. [4]

(ii) Evaluate the term independent of x in the binomial expansion of $\left(x^2-\frac{1}{2x^6}\right)^{16}$. [2]

(iii) In the binomial expansion of $\left(x+\frac{k}{x}\right)^9$, where k is a positive constant, the coefficient of x and x^3 are equal.

(a) Find the value of k.

(b) Use the value of k found in part (a) to find the coefficient of x^3 in the expansion of $(1-3x^2)\left(x+\frac{k}{x}\right)^9$. [4]

4. A curve is such that $\frac{d^2y}{dx^2} = 6x-6$ and the gradient of the curve at $(2,-40)$ is -24.
Find the coordinates of the stationary points of the curve and determine their nature. [7]

5. One night when the street lights were switched on, Jovan was walking towards a lamppost at 1.2 m/s. The lighted lamp on the lamppost was 6 m above the ground and Jovan was 1.5 m tall.

(i) At what rate was the length of his shadow decreasing? [3]

(ii) At what rate was the distance from the end of his shadow to the lamppost decreasing? [3]

6. It is given that $f(x)$ is such that $f'(x) = \cos 2x - \sin x$.

Given that $f\left(\frac{\pi}{2}\right) = 1$, show that $f''(x) + 4f(x) = 3\cos x + 4$. [4]

7. (i) Given that $A = \cos^{-1} p$ and A is acute, calculate in terms of p,

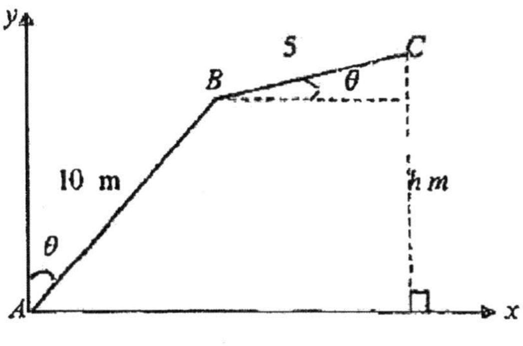
(a) $\sin 2A$ [2]

(b) $\cos 4A$ [2]

(c) $\sin \frac{A}{2}$ [2]

(ii) Prove that $2\operatorname{cosec}^2 2x + 2\cot 2x \operatorname{cosec} 2x = \operatorname{cosec}^2 x$. [3]

8.



The diagram shows an extended mobile crane made up of a movable boom AB and a movable jib BC. At a certain time, the crane is in a vertical plane. Ax is horizontal and Ay is vertical. The boom makes an angle of θ with the vertical and the jib makes an angle of θ with the horizontal. The lengths of AB and BC are 10 m and 5 m respectively. Given that C is h m above Ax,

(i) Find the values of the integers a and b for which $h = a\cos\theta + b\sin\theta$. [2]

Using the value of a and of b found in part (i),

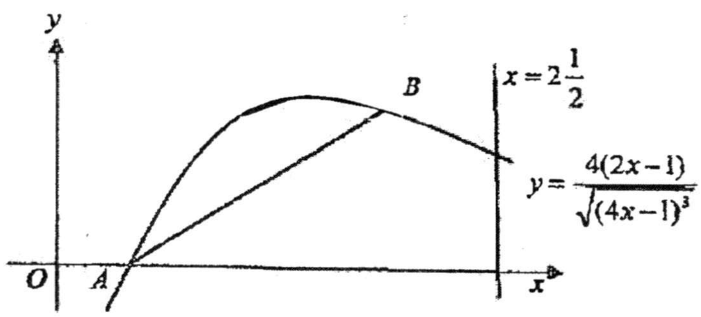
(ii) express h in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Hence state the maximum value of h and find the corresponding value of θ. [2]

(iv) Find the values of θ when $h = 10.5$ m. [2]

9. (i) Show that $\frac{d}{dx}\left(\frac{2x}{\sqrt{4x-1}}\right) = \frac{4x-2}{\sqrt{(4x-1)^3}}$. [2]

(ii)



The diagram shows the line $x = 2\frac{1}{2}$ and part of the curve $y = \frac{4(2x-1)}{\sqrt{(4x-1)^3}}$. The curve intersects the x-axis at A. The line through A with gradient 1 intersects the curve again at B.

(a) Verify that the y-coordinate of B is $\frac{3}{4}$. [5]

(b) Find the area enclosed by the curve $y = \frac{4(2x-1)}{\sqrt{(4x-1)^3}}$, the lines AB, $x = 2\frac{1}{2}$ and the x-axis, giving your answer correct to three decimal places. [4]

10. The equation of a circle with centre A is $x^2 + y^2 - 24x - 16y + 108 = 0$.

(i) Find the coordinates of A and the radius of the circle. [3]

(ii) Show that $y = -2$ is a tangent to the circle. [2]

A tangent to the circle at B passes through the point $P(-8, -2)$.

(iii) Find the coordinates of B. [4]

(iv) Find the equation of the tangent PB. [2]

11. The variables x and y are related by the equation $y = e^{-A}b^{x}$, where A and b are constants. The table below shows values of x and corresponding values of y.

x	5	10	15	20	25
y	0.14	1.06	8.02	60.9	462.5

(i) By drawing a straight line graph of ln y against x, estimate the value of A and of b. [6]

(ii) Use your graph to estimate the value of x when $y = 15$. [2]

(iii) On the same diagram, draw the line representing $y^3 = e^{-x}$ and hence find the value of x for which

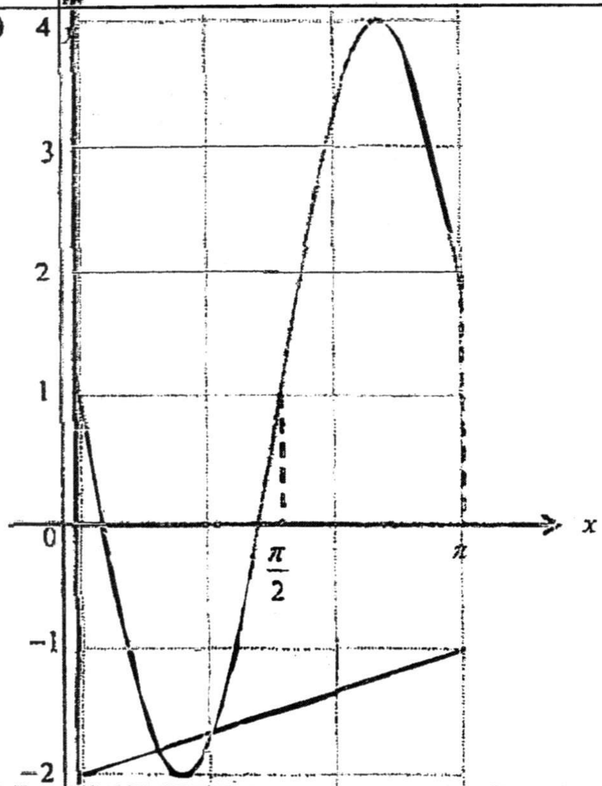
$$e^{A-\frac{x}{3}} = b^{x}.$$ [3]

END of PAPER

Answers

2017 Sec 4 Express 5 Normal (A) AM Paper 1

1	(i)	Cross-sectional area = $\left(21-2\sqrt{3}\right)\pi$ cm^2
	(ii)	$25\sqrt{3}+2$ cm
2	(i)	$6+\frac{9k}{2}$
	(ii)	48
3	(i)	$0<k<7$
	(ii)	The line $y=18x-9$ will intersect the curve.
4	(a)(i)	-1.43
	(ii)	$x=-1$ or $x=-2$
	(b)	$\log_3 45$
5	(i)	$k=6\frac{1}{9}$
6	(i)	$\frac{3x^2-6x+2}{(x-1)^2(x-2)}=\frac{1}{x-1}+\frac{1}{(x-1)^2}+\frac{2}{x-2}$
	(ii)	$\int\frac{9x^2-18x+6}{(x-1)^2(x-2)}dx=3\int\frac{3x^2-6x+2}{(x-1)^2(x-2)}dx$ $=3\ln(x-1)-\frac{3}{(x-1)}+6\ln(x-2)+c$, where c is a constant
7	(i)	42.5°C
	(ii)	3 minutes (nearest min)
	(iii)	The rate at which θ is decreasing = 3.04°C/min
	(iv)	The room temperature is 26°C
8	(i)	12.75 hours
	(ii)	June has the highest number of hours of daylight.
	(iii)	You should plant in February or October.
9	(ii)	$x=\frac{\pi}{6},\frac{7\pi}{6}$
10	(i)	$\left(\frac{x}{2}\right)^2+y^2=10^2$ $y=\sqrt{100-\frac{x^2}{4}}$
	(iii)	$\frac{dA}{dx}=\frac{-2x^2+400}{2\sqrt{400-{}^{2}}}$

		$x = 10\sqrt{2}$ or 14.1 cm
11	(ii)	Area of triangle Triangle AOB = $\frac{27}{10}e^{6}$ units^2
12	(i)	$m = 2$
	(ii)	
	(iii)	No of solutions = 2
13	(i)	$q = 2$ $p = 4$
	(iii)(a)	$c = 9$
	(b)	$\frac{1}{2} < m < \frac{3}{2}$

Marking Scheme

2017 Sec 4 Express 5 Normal (A) Paper 1

1	(i)	Cross-sectional area $= \pi(4\sqrt{3}-1)^2 - \pi(3\sqrt{3}-1)^2$ $= \pi(16(3) - 8\sqrt{3} + 1) - \pi(9(3) - 6\sqrt{3} + 1)$ $= (21 - 2\sqrt{3})\pi$ cm^2
	(ii)	Give that volume $= (521\sqrt{3} - 108)\pi$ $\pi(21-2\sqrt{3})(c+d\sqrt{3}) = (521\sqrt{3}-108)\pi$ $(c+d\sqrt{3}) = \frac{(521\sqrt{3}-108)\pi}{(21-2\sqrt{3})\pi}$ $= \frac{(521\sqrt{3}-108)\pi}{(21-2\sqrt{3})\pi} \times \frac{21+2\sqrt{3}}{21+2\sqrt{3}}$ $= \frac{10941\sqrt{3} + 3126 - 2268 - 216\sqrt{3}}{441-12}$ $= \frac{10725\sqrt{3} + 858}{429}$ $= 25\sqrt{3} + 2$ cm
2	(i)	$\int_1^4 \left(px^2 + \frac{3k}{2}\right)dx$ $= \int_1^4 px^2 dx + \frac{3k}{2}[x]_1^4$ $= 6 + \frac{9k}{2}$
	(ii)	$\int_2^8 px^2 dx$ $= \left[\frac{px^3}{3}\right]_2^8$ $= p\left[\frac{512}{3} - \frac{8}{3}\right]$ $= 168p$ Given $\int_1^4 px^2 dx = 6$ $\left[\frac{px^3}{3}\right]_1^4 = 6$ $p\left[\frac{64}{3} - \frac{1}{3}\right] = 6$ $\frac{63}{3}p = 6$ p

		$\int_2^8 px^2 dx = 168\left(\frac{2}{7}\right)$ $= 48$
	(ii)	$\int_1^4 \left(px^2 + \frac{3k}{2}\right) dx$ $= \int_1^4 px^2 dx + \frac{3k}{2}[x]_1^4$ $= 6 + \frac{9k}{2}$
3	(i)	$x^2 + 6x + k = 2kx - 9$ $x^2 + 6x + k - 2kx + 9 = 0$ ------------------ (1) $a = 1, b = 6 - 2k, c = k + 9$ Since equation has no real roots, $D < 0$ $(6-2k)^2 - 4(1)(k+9) < 0$ $36 - 24k + 4k^2 - 4k - 36 < 0$ $4k^2 - 28k < 0$ $4k(k-7) < 0$ $0 < k < 7$
	(ii)	$18x - 9 = (x+3)^2$ $18x - 9 = x^2 + 6x + 9$ $x^2 - 12x + 18 = 0$ ------------------ (2) Compare (1) and (20 $k = 9$ The line $y = 18x - 9$ will intersect the curve.
4	(a)(i)	$10^x = e^{3x+1}$ Take ln on both sides, $x \ln 10 = 3x + 1$ $x(\ln 10 - 3) = 1$ $x = \frac{1}{\ln 10 - 3}$ $= -1.43$
	(ii)	$\lg\|2x+3\| = 0$ $\|2x+3\| = 1$ $2x + 3 = 1$ or $2x + 3 = -1$ $x = -1$ or $x = -2$
	(b)	$2\log_3 15 - (\log_a 5)(\log_3 a)$ $= 2\log_3 15 - \left(\frac{\log_3 5}{\log_3 a}\right)(\log_3 a)$ $= 2\log_3 15$

		$=\log_3 15^2-\log_3 5$ $=\log_3 \frac{15^2}{5}$ $=\log_3 45$
5	(i)	$y=7-5x+6x^2-3x^3$ $\frac{dy}{dx}=-5+12x-9x^2$ $y+x=k$ $y=-x+k$ Gradient of tangent $=-1$ $-5+12x-9x^2=-1$ $9x^2-12x+4=0$ $(3x-2)^2=0$ $x=\frac{2}{3}$ At $x=\frac{2}{3}$, $y=7-5\left(\frac{2}{3}\right)+6\left(\frac{2}{3}\right)^2-3\left(\frac{2}{3}\right)^3$ $=\frac{49}{9}$ Sub $x=\frac{2}{3}$, $y=\frac{49}{9}$ into $y+x=k$ $k=\frac{49}{9}+\frac{2}{3}$ $=\frac{55}{9}$ $=6\frac{1}{9}$
	(ii)	$\frac{dy}{dx}=-5+12x-9x^2$ If y is a decreasing function, $\frac{dy}{dx}<0$ $-5+12x-9x^2$ $=-9\left(x^2-\frac{12}{9}x\right)-5$ $=-9\left[\left(x-\frac{6}{9}\right)^2-\frac{36}{81}\right]-5$ $=-9\left(x-\frac{2}{3}\right)^2+\frac{36}{9}-5$ $=-9\left(x-\frac{2}{3}\right)^2-1$

		Since $\left(x-\frac{2}{3}\right)^2>0$ $-9\left(x-\frac{2}{3}\right)^2<0$ $-9\left(x-\frac{2}{3}\right)^2-1<0$ Since $\frac{dy}{dx}<0$, y decreases as x increases for all real values of x.
6	(i)	$\frac{3x^2-6x+2}{(x-1)^2(x-2)}$ $=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$ $=\frac{A(x-1)(x-2)+B(x-2)+C(x-1)^2}{(x-1)^2(x-2)}$ $3x^2-6x+2\equiv A(x-1)(x-2)+B(x-2)+C(x-1)^2$ Sub $x=1$, $-B=3-6+2$ $B=1$ Sub $x=2$, $C=3(2)^2-6(2)+2$ $C=2$ Compare coefficient of x^2 $A+C=3$ $A+2=3$ $A=1$ $\frac{3x^2-6x+2}{(x-1)^2(x-2)}=\frac{1}{x-1}+\frac{1}{(x-1)^2}+\frac{2}{x-2}$
	(ii)	$\int\frac{9x^2-18x+6}{(x-1)^2(x-2)}dx=3\int\frac{3x^2-6x+2}{(x-1)^2(x-2)}dx$ $=3\ln(x-1)-\frac{3}{(x-1)}+6\ln(x-2)+c$, where c is a constant
7	(i)	At $t=3$, $\theta=26+74e^{-0.5(3)}$ $=42.5°C$
	(ii)	$26+74e^{-0.5(t)}=50$ $74e^{-0.5(t)}=24$ $e^{-0.5(t)}=\frac{24}{74}$

		$-0.5t = \ln\frac{24}{74}$ $t = 2.2520$ $= 3$ minutes (nearest min)
	(iii)	$\theta = 26 + 74e^{-0.5(t)}$ $\frac{d\theta}{dt} = 74(-0.5)e^{-0.5t}$ At $t = 5$, $\frac{d\theta}{dt} = 74(-0.5)e^{-0.5(5)}$ $= -3.0371$ $= -3.04\,^{\circ}\text{C/min}$ The rate at which θ is decreasing $= 3.04\,^{\circ}\text{C/min}$
	(iv)	As t becomes larger, $e^{-0.5t}$ approaches 0 $74e^{-0.5t}$ approaches 0 $\theta = 26 + 74e^{-0.5(t)}$ approaches 26 The room temperature is $26\,^{\circ}\text{C}$
8	(i)	$L(t) = -3.5\cos\left(\frac{\pi}{6}t\right) + 12.75$ At $t = 9$, $L(9) = -3.5\cos\left(\frac{\pi}{6}\right)(9) + 12.75$ $= 12.75$ hours
	(ii)	For greatest value of $L(t)$, $\cos\left(\frac{\pi}{6}t\right) = -1$ $\frac{\pi}{6}t = \pi, 3\pi$ $t = 6$ June has the highest number of hours of daylight.
	(iii)	$-3.5\cos\left(\frac{\pi}{6}t\right) + 12.75 = 11$ $-3.5\cos\left(\frac{\pi}{6}t\right) = 11 - 12.75$ $\cos\left(\frac{\pi}{6}t\right) = \frac{1}{2}$ $\frac{\pi}{6}t = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ $t = 2, 10$ You should plan

9	(i)	$\sin(x+\alpha)=\lambda\cos(x-\alpha)$ $\sin x\cos\alpha+\cos x\sin\alpha=\lambda[\cos x\cos\alpha+\sin x\sin\alpha]$ $\sin x\cos\alpha-\lambda\sin x\sin\alpha=\lambda\cos x\cos\alpha-\cos x\sin\alpha$ $\div\cos x$ throughout $\tan x\cos\alpha-\lambda\tan x\sin\alpha=\lambda\cos\alpha-\sin\alpha$ $\tan x[\cos\alpha-\lambda\sin\alpha]=\lambda\cos\alpha-\sin\alpha$ $\tan x=\frac{(\lambda\cos\alpha-\sin\alpha)\div\cos\alpha}{(\cos\alpha-\lambda\sin\alpha)\div\cos\alpha}$ $=\frac{\lambda-\tan\alpha}{1-\lambda\tan\alpha}$ (shown)
	(ii)	Sub $\alpha=\frac{\pi}{3}$ and $\lambda=\frac{2\sqrt{3}}{3}$ $\tan x=\frac{\frac{2\sqrt{3}}{3}-\tan\frac{\pi}{3}}{1-\frac{2\sqrt{3}}{3}\tan\frac{\pi}{3}}$ $=\frac{\frac{2\sqrt{3}}{3}-\sqrt{3}}{1-\left(\frac{2\sqrt{3}}{3}\right)(\sqrt{3})}$ $=\frac{\frac{-\sqrt{3}}{3}}{-1}$ $=\frac{\sqrt{3}}{3}$ or $\frac{1}{\sqrt{3}}$ $x=\frac{\pi}{6},\frac{7\pi}{6}$
10	(i)	$\left(\frac{x}{2}\right)^2+y^2=10^2$ $y=\sqrt{100-\frac{x^2}{4}}$
	(ii)	$A=xy$ $=x\sqrt{100-\frac{x^2}{4}}$ $=x\sqrt{\frac{400-x^2}{4}}$ $=\frac{x}{2}\sqrt{400-x^2}$
	(iii)	$\frac{dA}{dx}=\frac{x}{2}\times\frac{d}{dx}\sqrt{400-x^2}+\sqrt{400-x^2}\times\frac{d}{dx}\left(\frac{x}{2}\right)$ $=\frac{x}{2}\times\frac{1}{2}(400-x^2)^{\frac{-1}{2}}(-2x)+\frac{1}{2}\sqrt{400-x^2}$

$= \frac{-x^2}{2\sqrt{400-x^2}} + \frac{\sqrt{400-x^2}}{2}$

$= \frac{-x^2+400-x^2}{2\sqrt{400-x^2}}$

$= \frac{-2x^2+400}{2\sqrt{400-x^2}}$

Let $\frac{dA}{dx} = 0$

$\frac{-2x^2+400}{2\sqrt{400-x^2}} = 0$

$400-2x^2 = 0$

$x = 10\sqrt{2}$ or 14.14 cm

(iv)

x	14	$10\sqrt{2}$	14.2
$\frac{dA}{dx}$	0.280	0	-0.116
Sketch of gradient	/	—	\

Using <u>1st derivative test</u>, there is a change of sign of $\frac{dA}{dx}$ from positive to negative as x increases through $x = 10\sqrt{2}$, thus at $x = 10\sqrt{2}$, the area of the rectangle, A is the largest possible.

11 (i)

$y = 3x^2 \ln x$

$\frac{dy}{dx} = 3x^2\left(\frac{1}{x}\right) + 6x\ln x$

$= 3x + 6x\ln x$

At $x = e^2$

$\frac{dy}{dx} = 3e^2 + 6e^2 \ln e^2$

$= 3e^2 + 12e^2$

$= 15e^2$

At $x = e^2$

$y = 3(e^2)^2 \ln e^2$

$y = 6e^4$

Equation of tangent:

$y - 6e^4 = 15e^2(x - e^2)$

$y = 15e^2x - 9e^4$

Intersection with y-axis, let $x = 0$,

$y = 15e^2(0) - 9e^4$

$y = -9e^4$

B $(0, -9e^4)$

(ii)

Intersection with x-axis, let $y = 0$,

$15e^2x - 9e^4 = 0$

$15e^2x = 9e^4$

$x = \frac{3}{5}e^2$

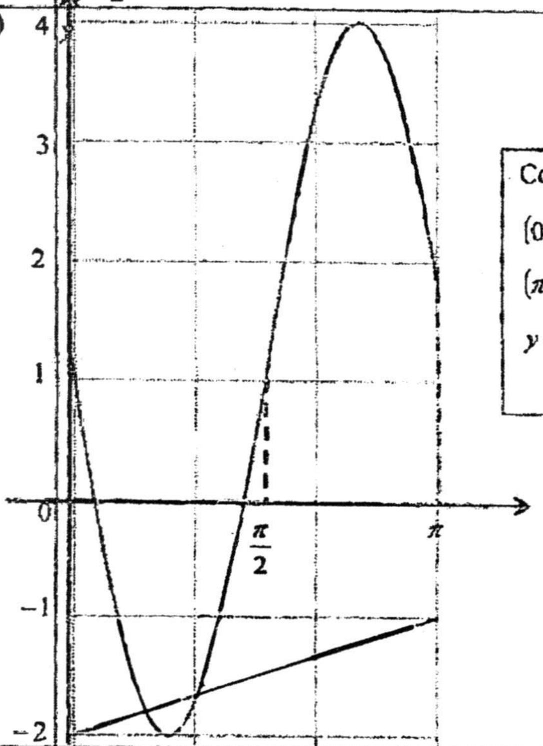
$B\left(\frac{3}{5}e^2, 0\right)$

Area of triangle Triangle AOB $= \frac{1}{2} \times \frac{3}{5}e^2 \times 9e^4$

$= \frac{27}{10}e^6$ units^2

12 (i) $a = 2$

(ii)



Correct Shape

$(0,1)$, $\left(\frac{\pi}{4},-2\right)$, $\left(\frac{\pi}{2},1\right)$, $\left(\frac{3\pi}{4},4\right)$, $(\pi,1)$ seen

$y = \frac{3x}{2\pi} - 2$ drawn

(iii)

$\sin nx - 1 = \frac{-x}{2\pi}$

$-3\sin nx + 3 = \frac{3x}{2\pi}$

$-3\sin nx + 1 = \frac{3x}{2\pi} - 2$

To draw $y = \frac{3x}{2\pi} - 2$

No of solutions = 2

13 (i) $q = 2$

		$p=4$
	(ii)	$y=\|2x-4\|+2$ Sub in $x=6$. $y=\|2(6)-4\|+2$ $y=10$
	(iii)(a)	$c=9$
	(b)	Consider $(0,1)$ and $(2,2)$ Gradient $=\dfrac{2-1}{2-0}$ $=\dfrac{1}{2}$ Consider $(0,1)$ and $(6,10)$ Gradient $=\dfrac{10-1}{6-0}$ $=\dfrac{3}{2}$ $\dfrac{1}{2}<m<\dfrac{3}{2}$

GAN ENG SENG SCHOOL

PRELIMINARY EXAMINATION 1 – 2017

SECONDARY 4 EXPRESS/SECONDARY 5 NORMAL

ADDITIONAL MATHEMATICS 2

1. (i)

Let $f(x) = 2x^3 + 5x^2 + kx - 6$

Since $2x + 1$ is a factor,

$$f(-\frac{1}{2}) = 2\left(-\frac{1}{2}\right)^3 + 5\left(-\frac{1}{2}\right)^2 - \frac{1}{2}k - 6 = 0$$

$$-\frac{1}{4} + \frac{5}{4} - \frac{1}{2}k - 6 = 0$$

$$-\frac{1}{2}k = 6 - 1$$

$$k = -10$$

$$\begin{array}{r} x^2 + 2x - 6 \\ 2x + 1 \overline{\smash{)}\, 2x^3 + 5x^2 - 10x - 6} \\ -\ \underline{(2x^3 + x^2)} \\ 4x^2 - 10x \\ -\ \underline{(4x^2 + 2x)} \\ -12x - 6 \\ -\ \underline{(12x - 6)} \\ 0 \end{array}$$

$$(2x + 1)(x^2 + 2x - 6) = 0$$

$$x = -\frac{1}{2} \text{ or } x^2 + 2x - 6 = 0$$

If $x^2 + 2x - 6 = 0$

$$x = \frac{-2 \pm \sqrt{(-2)^2 - (4)(1)(-6)}}{2}$$

$$= \frac{-2 \pm \sqrt{28}}{2}$$

$$= \frac{-2 \pm 2\sqrt{7}}{2}$$

$$= -1 \pm \sqrt{7}$$

$$x = -\frac{1}{2} \text{ or } x = -1 \pm \sqrt{7}$$

1. (ii)

$$x^2 - 6x + 3 = 0$$

Sum of roots $= 2\alpha + \beta + \alpha + 2\beta = 6$

$$3\alpha + 3\beta = 6$$
$$\alpha + \beta = 2$$

Product of roots $= (2\alpha + \beta)(\alpha + 2\beta)$

$$2\alpha^2 + 5\alpha\beta + 2\beta^2 = 3$$
$$2\alpha^2 + 2\beta^2 + 5\alpha\beta = 3$$
$$2(\alpha^2 + \beta^2) + 5\alpha\beta = 3$$
$$2[(\alpha + \beta)^2 - 2\alpha\beta] + 5\alpha\beta = 3$$
$$2(2^2) - 4\alpha\beta + 5\alpha\beta = 3$$
$$\alpha\beta = 3 - 8$$
$$\alpha\beta = -5$$

(a)

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$$
$$= 2[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta]$$
$$= 2[(\alpha + \beta)^2 - 3\alpha\beta]$$
$$= 2[2^2 - 3(-5)]$$
$$= 2[4 + 15]$$
$$= 38$$

(b)

Sum of roots $= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

$$= \frac{\alpha^3 + \beta^3}{\alpha\beta}$$
$$= \frac{38}{-5}$$

Product of roots $= \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha}$

$$= \alpha\beta$$
$$= -5$$

The quadratic equation is $x + \frac{38x}{5} - 5 = 0$

$$5x^2 + 38x - 25 = 0$$ Ans

2 (i)

$$9^x - 5(3^{x+1}) + 50 = 0$$
$$3^{2x} - 5(3^x \times 3) + 50 = 0$$
$$3^{2x} - 15(3^x) + 50 = 0$$

Let $u = 3^x$

$$u^2 - 15u + 50 = 0$$
$$(u - 10)(u - 5) = 0$$
$$u = 10 \text{ or } 5$$

If $u = 1$

$3^x =$

		$\lg 3^x = \lg 10$
		$x \lg 3 = 1$ $x = \dfrac{1}{\lg 3} = 2.0959$ $x \approx 2.10$ If $u = 5$ $3^x = 5$ $\lg 3^x = \lg 5$ $x \lg 3 = \lg 5$ $x = \dfrac{\lg 5}{\lg 3}$ $x \approx 1.464$ $x \approx 1.46$

	(ii)	$64^x \times 8^y = 2^{x+1}$ $2^{6x} \times 2^{3y} = 2^{x+1}$ $6x + 3y = x + 1$ $5x + 3y = 1$ .... (1) $81^{5-x} \div 27^{y+1} = \dfrac{1}{729^y}$ $3^{4(5-x)} \div 3^{3(y+1)} = \dfrac{1}{3^{6y}}$ $3^{20-4x-3y-3} = 3^{-6y}$ $20 - 4x - 3y - 3 = -6y$ $-4x - 3y + 6y = -17$ $4x - 3y = 17$ ....(2) Eqn (1) + Eqn (2) $9x = 18.$ $x = 2$ Ans If $x = 2$, $5(2) + 3y = 1$ $3y = 1 - 10$ $3y = -9$ $y = -3$ Ans

3.	(i)	$(1 + ax + bx^2)^8 = [1 + (ax + bx^2)]^8$ $= 1^8 + \binom{8}{1}(1^7)(ax + bx^2)^1 + \binom{8}{2}(1^6)(ax + bx^2)^2 + \ldots\ldots$ $= 1 + 8(ax + bx^2) + 28(a^2x^2 + \ldots) + \ldots$ $= 1 + 8ax + 8bx^2 + 28a^2x^2 + \ldots\ldots$ $= 1 - 40x + 748x^2 + \ldots$ Compare terms in x, $8ax = -40x$ $8a = -40$ $a = -\dfrac{40}{8}$ $a =$

		Compare terms in x^2 $8bx^2 + 28a^2x^2 = 748x^2$ $8b + 28a^2 = 748$ $8b = 748 - 28(-5)^2$ $b = \frac{748 - 28(-5)}{8}$ $b = 6$
3.	(ii)	$\left(x^2 - \frac{1}{2x^6}\right)^{16} = (x^2)^{16} + \binom{16}{1}(x^2)^{15}\left(-\frac{1}{2x^6}\right)^1 + \binom{16}{2}(x^2)^{14}\left(-\frac{1}{2x^6}\right)^3 +$ $\binom{16}{3}(x^2)^{13}\left(-\frac{1}{2x^6}\right)^4 + \binom{16}{4}(x^2)^{12}\left(-\frac{1}{2x^6}\right)^4 + \ldots\ldots\ldots$ Term independent of x is $113\frac{3}{4}$ Ans

	(iii)	(a)	$\left(x + \frac{k}{x}\right)^9 = x^9 + \binom{9}{1}(x^8)\left(\frac{k}{x}\right) + \binom{9}{2}(x^7)\left(\frac{k}{x}\right)^2 + \binom{9}{3}(x^6)\left(\frac{k}{x}\right)^3 + \binom{9}{4}(x^5)\left(\frac{k}{x}\right)^4$ $\binom{9}{3}k^3 = \binom{9}{4}k^4$ $\frac{9\cdot 8\cdot 7k^3}{1\cdot 2\cdot 3} = \frac{9\cdot 8\cdot 7\cdot 6k^4}{1\cdot 2\cdot 3\cdot 4}$ $1 = \frac{6k}{4}$ $k = \frac{2}{3}$
		(b)	$\left(x + \frac{k}{x}\right)^9 = \ldots\ldots\ldots \binom{9}{3}(x^6)\left(\frac{2}{3x}\right)^3 + \binom{9}{4}(x^5)\left(\frac{2}{3x}\right)^4 + \ldots\ldots$ $= \ldots\ldots\ldots + 84 \times \frac{8x^3}{27} + 126 \times \frac{16x}{81} + \ldots\ldots\ldots$ $(1 - 3x^2)\left(\ldots\ldots + \frac{224x^3}{9} + \frac{224x}{9} + \ldots\ldots\right)$ Term with $x^3 = \frac{224x^3}{9} - \frac{3 \times 224x^3}{9}$ Coefficient of $x^3 = -\frac{2 \times 224}{9} = -\frac{448}{9}$

4.	$\frac{d^2y}{dx^2} = 6x - 6$ $\frac{dy}{dx} = \int(6x-6)d$ $= 3x^2 - 6x + c$

At $x=2$, $\frac{dy}{dx}=-24$

$$-24=3\times2^2-6\times2+c$$
$$c=-24-12+12$$
$$c=-24$$

Therefore $\frac{dy}{dx}=3x^2-6x-24$

$$y=\int(3x^2-6x-24)dx$$

$y=x^3-\frac{6x^2}{2}-24x+p$ where p is a constant.

At $x=2, y=-40$

$$-40=2^3-3\times2^2-24\times2+p$$
$$p=-40-8+12+48$$
$$p=12$$

therefore $y=x^3-3x^2-24x+12$

When $\frac{dy}{dx}=0$, $3x^2-6x-24=0$

$$x^2-2x-8=0$$
$$(x-4)(x+2)=0$$
$$x=4 \text{ or } x=-2$$

At $x=4$, $y=4^3-3\times4^2-24\times4+12$

$$y=-68$$

At $x=4$, $\frac{d^2y}{dx^2}=6\times4-6=18>0$

Therefore $(4, -68)$ is a minimum point.

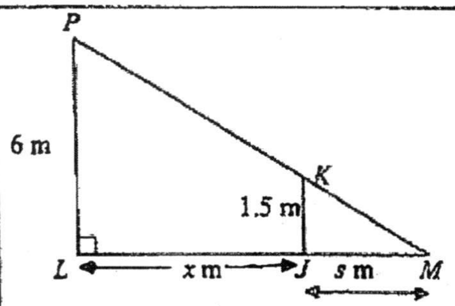
At $x=-2$, $\frac{d^2y}{dx^2}=6(-2)-6=-18<0$

At $x=-2$, $y=(-2)^3-3(-2)^2-24(-2)+12$

$$y=40$$

Therefore $(-2, 40)$ is a maximum point.

5. (i)



Let PM be the lamppost, JK be Jovan, JM be the shadow of s m.
Let LJ be the distance of Jovan from the lamppost of x m.

By similar triangles,

$$\frac{s}{s+x}=\frac{1.5}{6}$$

$$s=\frac{1}{4}(x+s)$$

$$s-\frac{1}{4}s=\frac{x}{4}$$

$$\frac{3}{4}s=\frac{x}{4}$$

$$s=\frac{x}{3}$$

$$\frac{ds}{dx}=\frac{1}{3}$$

By the chain rule, $\frac{ds}{dt}=\frac{ds}{dx}\times\frac{dx}{dt}$

$$\frac{ds}{dt}=\frac{1}{3}(-1.2)$$

$$=-0.4 \text{ m/s}$$

Shadow is decreasing at 0.4 m/s.

(ii) Let $LM=y$ m.
By similar triangles

$$\frac{y-x}{y}=\frac{1.5}{6}$$

$$\frac{y-x}{y}=\frac{1}{4}$$

$$4y-4x=y$$

$$4y-y=4x$$

$$3y=4x$$

$$y=\frac{4x}{3}$$

$$\frac{dy}{dx}=\frac{4}{3}$$

By the chain rule, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$$=\frac{4}{3}(-1.2)$$

Shadow is decr

At $x=2$, $\frac{dy}{dx}=-24$

$$-24=3\times2^2-6\times2+c$$
$$c=-24-12+12$$
$$c=-24$$

Therefore $\frac{dy}{dx}=3x^2-6x-24$

$$y=\int(3x^2-6x-24)dx$$

$y=x^3-\frac{6x^2}{2}-24x+p$ where p is a constant.

At $x=2, y=-40$

$$-40=2^3-3\times2^2-24\times2+p$$
$$p=-40-8+12+48$$
$$p=12$$

therefore $y=x^3-3x^2-24x+12$

When $\frac{dy}{dx}=0$, $3x^2-6x-24=0$

$$x^2-2x-8=0$$
$$(x-4)(x+2)=0$$
$$x=4 \text{ or } x=-2$$

At $x=4$, $y=4^3-3\times4^2-24\times4+12$

$$y=-68$$

At $x=4$, $\frac{d^2y}{dx^2}=6\times4-6=18>0$

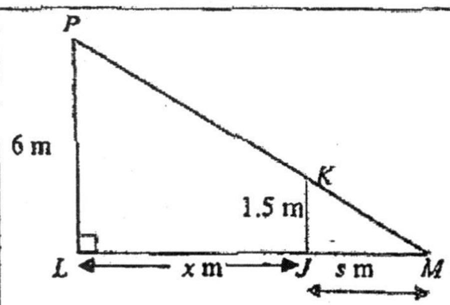
Therefore $(4, -68)$ is a minimum point.

At $x=-2$, $\frac{d^2y}{dx^2}=6(-2)-6=-18<0$

At $x=-2$, $y=(-2)^3-3(-2)^2-24(-2)+12$

$$y=40$$

Therefore $(-2, 40)$ is a maximum point.

5.	(i)		Let PM be the lamppost, JK be Jovan, JM be the shadow of s m. Let LJ be the distance of Jovan from the lamppost of x m.

By similar triangles,

$$\frac{s}{s+x} = \frac{1.5}{6}$$

$$s = \frac{1}{4}(x+s)$$

$$s - \frac{1}{4}s = \frac{x}{4}$$

$$\frac{3}{4}s = \frac{x}{4}$$

$$s = \frac{x}{3}$$

$$\frac{ds}{dx} = \frac{1}{3}$$

By the chain rule, $\frac{ds}{dt} = \frac{ds}{dx} \times \frac{dx}{dt}$

$$\frac{ds}{dt} = \frac{1}{3}(-1.2)$$

$$= -0.4 \text{ m/s}$$

Shadow is decreasing at 0.4 m/s.

(ii) Let $LM = y$ m.

By similar triangles

$$\frac{y-x}{y} = \frac{1.5}{6}$$

$$\frac{y-x}{y} = \frac{1}{4}$$

$$4y - 4x = y$$

$$4y - y = 4x$$

$$3y = 4x$$

$$y = \frac{4x}{3}$$

$$\frac{dy}{dx} = \frac{4}{3}$$

By the chain rule, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

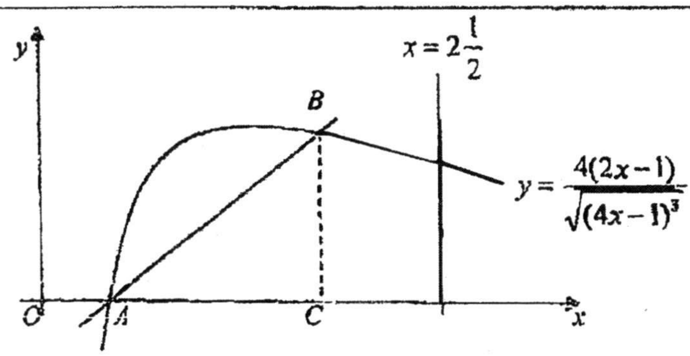
$$= \frac{4}{3}(-1.2)$$

Shadow is decreas

			$= \sqrt{125}\sin(\theta + 63.4^\circ)$

8.	(iii)	Max value of $h = \sqrt{125}$ m when $\sin(\theta + 63.4^\circ) = 1$. $\theta + 63.4^\circ = 90^\circ$ $\theta = 90^\circ - 63.4^\circ$ $= 26.6^\circ$
	(iv)	When $h = 10.5$m $\sqrt{125}\sin(\theta + 63.4^\circ) = 10.5$ $\sin(\theta + 63.4^\circ) = \dfrac{10.5}{\sqrt{125}}$ $\theta + 63.4^\circ = 69.9^\circ$ or 110.1° $\theta = 6.5^\circ$ or 46.7°

9.	(i)	$\frac{d}{dx}\left(\frac{2x}{\sqrt{4x-1}}\right) = \dfrac{(4x-1)^{\frac{1}{2}}\frac{d}{dx}(2x) - 2x\frac{d}{dx}\left(\sqrt{4x-1}\right)}{4x-1}$ $= \dfrac{2(4x-1)^{\frac{1}{2}} - 2x\left(\frac{1}{2}\right)(4x-1)^{\frac{-1}{2}}(4)}{4x-1}$ $= \dfrac{2(4x-1)^{\frac{1}{2}} - \frac{4x}{(4x-1)^{\frac{1}{2}}}}{4x-1}$ $= \dfrac{2(4x-1) - 4x}{(4x-1)^{\frac{3}{2}}}$ $= \dfrac{8x - 2 - 4x}{\sqrt{(4x-1)^3}}$ $= \dfrac{4x-2}{\sqrt{(4x-1)^2}}$
9.	(ii)	If $y = \dfrac{4(2x-1)}{\sqrt{(4x-1)^3}}$ meets $y = 0$ $4(2x-1) = 0$ $2x - 1 = 0$ $x = \frac{1}{2}$ $A\left(\frac{1}{2}, 0\right)$ Equation of AB is $y - 0 = 1(x - \frac{1}{2})$. $y = x - \frac{1}{2}$

		If $y = x - \frac{1}{2}$ meets $y = \frac{4(2x-1)}{\sqrt{(4x-1)^3}}$, $\frac{4(2x-1)}{\sqrt{(4x-1)^3}} = x - \frac{1}{2}$ $(2x-1)\left(\frac{4}{\sqrt{(4x-1)^3}} - \frac{1}{2}\right) = 0$ If $\frac{4}{\sqrt{(4x-1)^3}} = \frac{1}{2}$ $(4x-1)^{\frac{3}{2}} = 8$ $4x-1 = 8^{\frac{2}{3}}$ $4x-1 = 4$ $4x = 5$ $x = \frac{5}{4}$ If $x = \frac{5}{4}, y = \frac{5}{4} - \frac{1}{2} = \frac{3}{4}$ (proven)
	(ii)	(b)  Area of triangle ABC = $\frac{1}{2}\left(\frac{5}{4} - \frac{1}{2}\right) \times \frac{3}{4} = \frac{9}{32}$ units^2. Area enclosed by $x = 2\frac{1}{2}$, $x = \frac{5}{4}$, the curve and the x-axis $= \int_{\frac{5}{4}}^{2\frac{1}{2}} \frac{4(2x-1)}{(4x-1)^{\frac{3}{2}}} dx = 2\int_{\frac{5}{4}}^{2\frac{1}{2}} \frac{4x-2}{(4x-1)^{\frac{3}{2}}} dx$ $= 2\left[\frac{2x}{\sqrt{4x-1}}\right]_{1.25}^{2.5}$ $= 2\left[\frac{2 \times 2\frac{1}{2}}{\sqrt{4 \times 2.5 - 1}} - \frac{2 \times 1.25}{\sqrt{4 \times 1.25 - 1}}\right]$

$$= 2\left[\frac{5}{3} - \frac{\frac{5}{2}}{2}\right] = 2\times5\left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= \frac{10}{12} \text{ units}^2$$

Area enclosed by the curve, the line AB, $x = 2\frac{1}{2}$, and the x-axis $= \frac{9}{32} + \frac{10}{12}$

$$= 1\frac{11}{42} \quad \text{or} \quad 1.11 \text{ units}^2$$

10. (i)

$x^2 + y^2 - 24x - 16y + 108 = 0$

$(x^2 - 24x) + (y^2 - 16y) = -108$

$x^2 - 24x + 144 + y^2 - 16y + 64 = -108 + 144 + 64$

$(x - 12)^2 + (y - 8)^2 = 100$

Coordinates of A is (12, 8)

Radius of circle $= \sqrt{100} = 10$ units

(ii)

If $y = -2$

$x^2 + (-2)^2 - 24x - 16(-2) + 108 = 0$

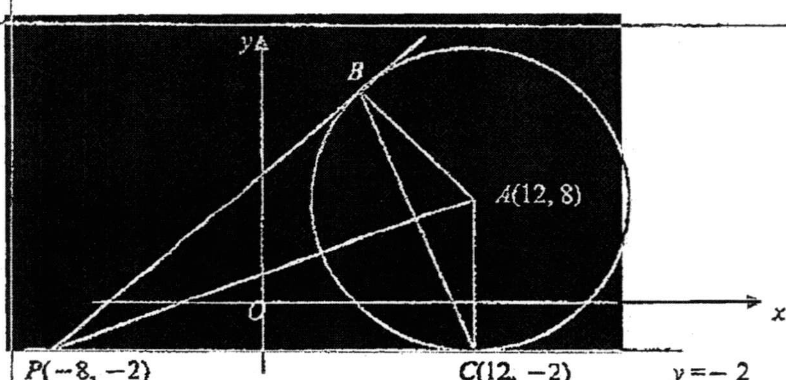
$x^2 - 24x + 144 = 0$

$(x - 12)^2 = 0$

$x = 12$

Since there is only point of contact, $y = -2$ is a tangent.

(iii)



Let C be at (12, −2).

Gradient of $AP = \frac{8+2}{12+8} = \frac{1}{2}$

Gradient of $BC = -2$.

Equation of BC is $y + 2 = -2(x - 12)$.

$y = -2x + 24 - 2$

$y = -2x + 22$

If $y = -2x + 22$ meets $x^2 + y^2 - 24x - 16y + 108 = 0$

$x^2 + (-2x + 22)^2 - 24x - 16(-2x + 22) + 108 = 0$

$x^2 + 4x^2 - 88x + 484 - 24x + 32x - 352 + 108 = 0$

0

$$x^2 - 16x + 48 = 0$$
$$(x-12)(x-4) = 0$$
$$x = 12 \text{ (rejected) or } x = 4$$

If $x = 4$, $y = -2\times 4 + 22$

$y = 14$

$B(4, 14)$ Ans

10. (iv)

Gradient of $BP = \frac{14+2}{4+8} = \frac{4}{3}$

Equation of tangent PB is $y - 14 = \frac{4}{3}(x-4)$

$$3y - 42 = 4x - 16$$
$$3y = 4x + 26 \text{ Ans}$$

11. (i)

$y = e^{-A}b^x$

$\ln y = \ln e^{-A} + \ln b^x$

$\ln y = -A + x\ln b$

x	5	10	15	20	25
$\ln y$	-1.97	0.058	2.08	4.11	6.14

(i)

From the graph, $-A = -4$

$A = 4$ (accept 4 ± 0.2)

$\ln b \approx \frac{4+2}{15}$

$b \approx 1.49$ (accept ± 0.1)

(ii)

If $y = 15$, $\ln 15 = 2.70$

From the graph $x \approx 17.2$

(iii)

$y^3 = e^{-x}$

$y = e^{\frac{-x}{3}}$

$\ln y = -\frac{x}{3}$

x	0	12
lny	0	-4

$e^{A-\frac{x}{3}} = b^x$

$e^{-\frac{x}{3}} = \frac{b^x}{e^A}$

$e^{-\frac{x}{3}} = e^{-A}b^x$

From the graph $x \approx 5.5$.