ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4E, 5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS

4047/01

PAPER 1

17 Sep 2019
2 hours

Candidates answer on the question paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page..
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
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The total number of marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

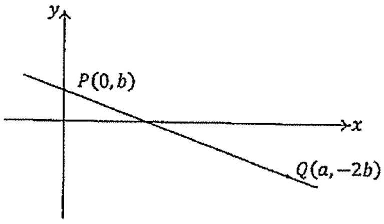
Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

1 The diagram shows a straight line passing through $P(0, b)$ and $Q(a, -2b)$

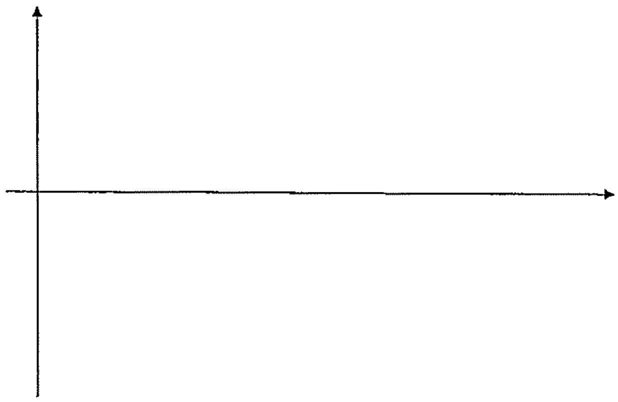


Given that the gradient of PQ is $-\frac{3}{7}$ and that the distance $PQ = 3\sqrt{58}$, find the value of a and of b. [5]

2 The equation of a curve is $f(x) = |3cos2x| - 2$

(i) State the minimum and maximum values of $f(x)$. [2]

(ii) Sketch the graph of $f(x) = |3cos2x| - 2$ for $0° \leq x \leq 180°$. [3]

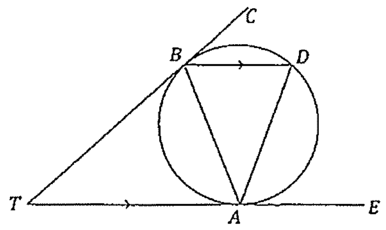


3 The expression $ax^3 - 4x^2 + bx + 6$ is exactly divisible by $x^2 - x - 2$.

(i) Find the value of a and of b. [4]

(ii) Using these values of a and of b, solve the equation $ax^3 - 4x^2 + bx + 6 = 0$. [2]

4



In the diagram, TE and TC are tangents to the circle at A and B respectively.
BD is parallel to TE.

(i) Prove that $AB = AD$. [3]

(ii) Prove that triangle TAB is similar to triangle ABD. [3]

5 The function f is defined by $f(x) = \frac{5-x^2}{x^2+3}$, $x > 0$

(i) Explain, with working, whether f is an increasing or decreasing function. [4]

(ii) A point P moves along the curve $y = f(x)$ in such a way that the y-coordinate of P is increasing at a rate of 0.2 units per second. Find the rate of change of the x-coordinate of P when $x = 4$. [2]

6 At m minutes after an oven is switched on, its temperature, T°C, is given by $T = 200 - 175e^{-km}$. The oven reached a temperature of 150°C after 16 minutes 15 seconds.

(i) State the initial temperature of the oven. [1]

(ii) Estimate the temperature after 10 minutes. [3]

(iii) "If the oven is switched on for a very long time, it will never exceed a certain temperature." Do you agree with the statement? Justify your answer with clear explanation. [2]

7 (i) Express $-2x^2 + 3x + 2$ in the form $a(x + b)^2 + c$ where a, b and c are constants. [2]

(ii) Use your answer from part (i) to explain why the curves with equations $y = -2x^2 + 3x + 2$ and $(x + 5)^2 + (y - 9)^2 = 30$ will not intersect. [4]

8 (i) Form a quadratic equation for which the sum of roots is 4 and the sum of the squares of the roots is 20. [3]

(ii) Given that the equation $(2m+3)x^2-(8m+8)x+8m+3=0$, find the value of m, for which

(a) one root is the negative of the other, [2]

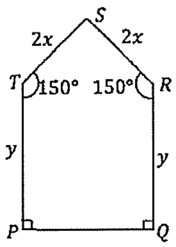
(b) one root is the reciprocal of the other. [2]

9 Without using a calculator,

(i) show that $sin105° = \frac{1+\sqrt{3}}{2\sqrt{2}}$, [2]

(ii) hence, express $1 + cot^2 105°$ in the form $a + b\sqrt{3}$, where a and b are integers. [5]

10



The figure shows a piece of iron bar of length 8 metres, bent to form a pentagonal window frame $PQRST$.
$PT = QR = y$ metres, $RS = ST = 2x$ metres and angle PTS = angle $QRS = 150°$.

(i) Show that $y = 4 - 3x$. [3]

(ii) Express the area, A, enclosed by the frame, in terms of x. [3]

10 (iii) Given that x can vary, find the exact value of x for which the area enclosed by the frame is a maximum. [2]

11 (i) Prove the identity $\frac{tanA-cotA}{tanA+cotA}+1 = 2sin^2A$ [3]

(ii) Hence, solve the equation $\frac{tanA-cotA}{tanA+cotA}+1 = 5sinAcosA$ for $0° < A < 360°$ [5]

12 A particle starts from rest, travels in a straight line so that t is the time in seconds after passing a fixed point O. Its velocity, $v\, m/s$, is given by $v = 6t - 2t^2$. The particle comes to instantaneous rest at A.

(i) Find the acceleration of the particle at A. [3]

(ii) Find the maximum velocity of the particle. [2]

12 (iii) Find the total distance travelled by the particle during the first 5 seconds. [5]

End of Paper

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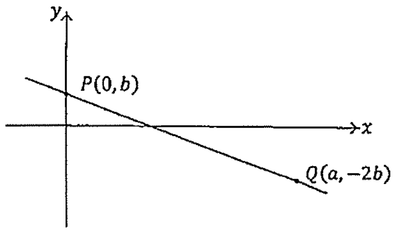


Given that the gradient of PQ is $-\frac{3}{7}$ and that the distance $PQ = 3\sqrt{58}$, find the value of a and of b. [5]

$Grad\ PQ = \frac{b-(-2b)}{0-a} = -\frac{3}{7}$ M1

$\frac{3b}{-a} = -\frac{3}{7}$

$a = 7b$ M1

$PQ = 3\sqrt{58}$

$\sqrt{(0-a)^2 + (b+2b)^2} = 3\sqrt{58}$ M1

$a^2 + 9b^2 = 9(58)$

$(7b)^2 + 9b^2 = 9(58)$

$58b^2 = 522$

$b^2 = 9$

$b = 3$ ($b > 0$ *as shown in diagram*) A1

$\therefore a = 21$ A1

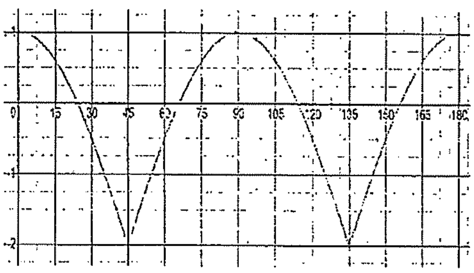
2 The equation of a curve is $f(x) = |3cos2x| - 2$

(i) State the minimum and maximum values of $f(x)$. [2]

$Min\ of\ f(x) = 1$ B1

$Max\ of\ f(x) = -2$ B1

(ii) Sketch the graph of $f(x) = |3cos2x| - 2$ for $0° \leq x \leq 180°$. [3]



One complete cycle B1

Starts and ends at f(x)=1 B1

Correct curvature B1

3 The expression $ax^3 - 4x^2 + bx + 6$ is exactly divisible by $x^2 - x - 2$.

(i) Find the value of a and of b. [4]

Let $f(x) = ax^3 - 4x^2 + bx + 6$

$$x^2 - x - 2 = (x-2)(x+1)$$

$$f(2) = 8a - 16 + 2b + 6 = 0 \quad \text{M1}$$

$$f(-1) = -a - 4 - b + 6 = 0 \quad \text{M1}$$

$$4a + b = 5$$

$$a + b = 2$$

$$3a = 3$$

$$a = 1 \quad \text{A1}$$

$$b = 1 \quad \text{A1}$$

(ii) Using these values of a and of b, solve the equation $ax^3 - 4x^2 + bx + 6 = 0$. [2]

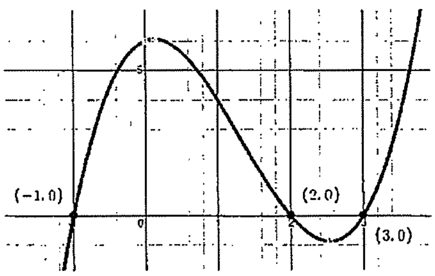
$$x^3 - 4x^2 + x + 6 = 0$$

By inspection, $(x-3)$ is the linear factor of $f(x)$

$$(x^2 - x - 2)(x-3) = 0 \quad \text{B1}$$

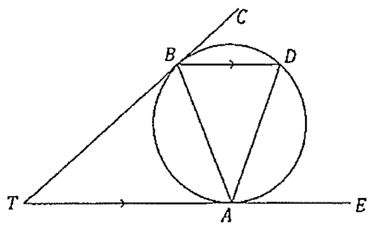
$$(x-2)(x+1)(x-3) = 0$$

$$x = -1, 2, 3 \quad \text{A1}$$



$y = x^3 - 4x^2 + x + 6 = (x-2)(x+1)(x-3)$

4



In the diagram, TE and TC are tangents to the circle at A and B respectively.

BD is parallel to TE.

(i) Prove that $AB = AD$. [3]

$$Let \measuredangle ABD = \theta$$

$\measuredangle TAB = \theta$ (alternate angles, $BD \parallel TE$) B1

$\measuredangle ADB = \measuredangle TAB = \theta$ (tangent − chord theorem) B1

Since $\measuredangle ADB = \measuredangle ABD = \theta$,

therefore, ΔABD is an isosceles triangle with $AB = AD$ B1

(ii) Prove that triangle TAB is similar to triangle ABD. [3]

In triangles TAB and ABD,

$\measuredangle TAB = \measuredangle ABD = \theta$ (alternate angles, $BD \parallel TE$) B1

$\measuredangle TBA = \measuredangle ADB = \theta$ (tangent − chord theorem) B1

$\measuredangle ATB = \measuredangle BAD$ (angle sum of triangles)

Since all 3 pairs of corresponding angles are equal, triangle TB is similar to triangle ABD. B1

5 The function f is defined by $f(x) = \frac{5-x^2}{x^2+3}$, $x > 0$

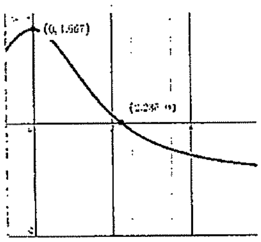
(i) Explain, with working, whether f is an increasing or decreasing function. [4]

$$f'(x) = \frac{(x^2+3)(-2x)-(5-x^2)(2x)}{(x^2+3)^2}$$

$$= \frac{-2x^3 - 6x - 10x + 2x^3}{(x^2+3)^2}$$

$$= \frac{-16x}{(x^2+3)^2}$$

Since $x > 0, -16x < 0$ and $(x^2+3)^2 > 0$, $f'(x) < 0$ and thus $f(x)$ is a decreasing function for $x > 0$.



$$f(x) = \frac{5-x^2}{x^2+3}$$

(ii) A point P moves along the curve $y = f(x)$ in such a way that the y-coordinate of P is increasing at a rate of 0.2 units per second. Find the rate of change of the x-coordinate of P when $x = 4$. [2]

$$\frac{dy}{dt} = +0.2$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$0.2 = \frac{-16(4)}{(4^2+3)^2} \times \frac{dx}{dt}$$ M1

$$\frac{dx}{dt} = -1.1281 = -1.13 \textit{units per sec}$$ A1

6 At m minutes after an oven is switched on, its temperature, T°C, is given by $T = 200 - 175e^{-km}$. The oven reached a temperature of 150°C after 16 minutes 15 seconds.

(i) State the initial temperature of the oven. [1]

When $m = 0, T = 200 - 175e^{-0}$

$= 25°C$ B1

(ii) Estimate the temperature after 10 minutes. [3]

$$150 = 200 - 175e^{-k(16.25)}$$

$$175e^{-k(16.25)} = 50$$

$$e^{-k(16.25)} = \frac{2}{7}$$ M1

$$-16.25k = \ln\frac{2}{7}$$

$$k = \frac{\ln\frac{2}{7}}{-16.25} \text{ or } 0.077093$$ A1

When $m = 10mins$, $T = 200 - 175e^{-k(10)}$

$= 119.04°C = 119°C\ (3\ sig\ fig)$ A1

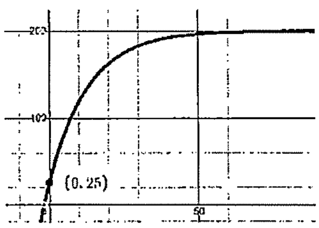
(iii) "If the oven is switched on for a very long time, it will never exceed a certain temperature." Do you agree with the statement? Justify your answer with clear explanation. [2]

$$T = 200 - 175e^{-km}$$

$$T = 200 - \frac{175}{e^{km}}$$

Yes, I agree with the statement.

Since $k > 0, m > 0$, and when m becomes large, $\frac{175}{e^{km}}$ becomes a very small positive value. Therefore, T will never exceed 200°C. B2



$$T = 200 - \frac{175}{e^{km}}$$

7 (i) Express $-2x^2+3x+2$ in the form $a(x+b)^2+c$ where a , b and c are constants. [2]

$$-2x^2+3x+2=-2\left[x^2-\frac{3}{2}x-1\right]$$

$$=-2\left[\left(x-\frac{3}{4}\right)^2-\frac{9}{16}-1\right]$$ $\left(x-\frac{3}{4}\right)^2$ seen B1

$$=-2\left[\left(x-\frac{3}{4}\right)^2-\frac{25}{16}\right]$$

$$=-2\left(x-\frac{3}{4}\right)^2+\frac{25}{8}$$ B1

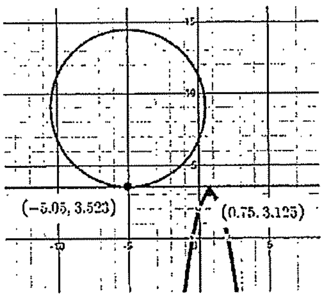
(ii) Use your answer from part (i) to explain why the curves with equations $y=-2x^2+3x+2$ and $(x+5)^2+(y-9)^2=30$ will not intersect. [4]

$y=-2x^2+3x+2=-2\left(x-\frac{3}{4}\right)^2+\frac{25}{8}$ has a maximum value at $y=\frac{25}{8}$ *or* 3.125 B1

$(x+5)^2+(y-9)^2=30$ is a circle with center $(-5,9)$ and radius $\sqrt{30}$. M1
for either centre or radius

The minimum value of $y=9-\sqrt{30}=3.522$ B1

Since maximum of $y=-2x^2+3x+2$ < minimum of $(x+5)^2+(y-9)^2=30$,
the 2 curves will not intersect. B1



8 (i) Form a quadratic equation for which the sum of roots is 4 and the sum of the squares of the roots is 20. [3]

$$\alpha + \beta = 4$$

$$\alpha^2 + \beta^2 = 20 \quad \text{M1}$$

$$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 20$$

$$(4)^2 - 2\alpha\beta = 20$$

$$\alpha\beta = -2 \quad \text{M1}$$

$$\therefore \textit{the quadratic equation is } x^2 - 4x - 2 = 0 \quad \text{A1}$$

(ii) Given that the equation $(2m+3)x^2 - (8m+8)x + 8m + 3 = 0$, find the value of m, for which

(a) one root is the negative of the other, [2]

Let the roots be α and $-\alpha$.

Sum of roots: $\alpha + (-\alpha) = \frac{8m+8}{2m+3}$ — attempt to form eqn — M1

$$\frac{8m+8}{2m+3} = 0$$

$$8m + 8 = 0$$

$$m = -1 \quad \text{A1}$$

(b) one root is the reciprocal of the other. [2]

Let the roots be α and $\frac{1}{\alpha}$.

Product of roots: $\alpha \times \frac{1}{\alpha} = \frac{8m+3}{2m+3}$ — attempt to form eqn — M1

$$8m + 3 = 2m + 3$$

$$6m = 0$$

$$m = 0 \quad \text{A1}$$

9 Without using a calculator,

(i) show that $sin105° = \frac{1+\sqrt{3}}{2\sqrt{2}}$, [2]

$$\sin 105° = \sin(45° + 60°)$$ M1

$$= \sin 45° \cos 60° + \cos 45° \sin 60°$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$$ A1

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$ AG

(ii) hence, express $1 + cot^2 105°$ in the form $a + b\sqrt{3}$, where a and b are integers. [5]

$$1 + cot^2 105° = cosec^2 105°$$ M1

$$= \frac{1}{sin^2 105°}$$

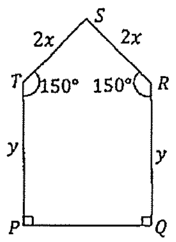
$$= \left(\frac{2\sqrt{2}}{1+\sqrt{3}}\right)^2$$ M1

$$= \frac{8}{4 + 2\sqrt{3}}$$ For correctly squaring $1+\sqrt{3}$ M1

$$= \frac{4}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$ For rationalising M1

$$= 8 - 4\sqrt{3}$$ A1

10



The figure shows a piece of iron bar of length 8 metres, bent to form a pentagonal window frame $PQRST$.
$PT = QR = y$ metres, $RS = ST = 2x$ metres and angle PTS = angle $QRS = 150°$.

(i) Show that $y = 4 - 3x$. [3]

PQRT is a rectangle

$\Rightarrow \measuredangle STR = \measuredangle SRT = 150° - 90° = 60°$ M1

$\therefore \measuredangle TSR = 180° - 2(60°) = 60°$

Since TSR is an equilateral triangle,

$TR = 2x$

$\Rightarrow PQ = 2x$ M1

$3(2x) + 2y = 8$ B1

$2y = 8 - 6x$

$y = 4 - 3x$ AG

(ii) Express the area, A, enclosed by the frame, in terms of x. [3]

$A = 2x(y) + \frac{1}{2}(2x)(2x)\sin 60°$ For ½(absin c) o.e. B1

$= 2x(4 - 3x) + 2x^2\left(\frac{\sqrt{3}}{2}\right)$ For substituting y B1

$= 8x - 6x^2 + \sqrt{3}x^2$ A1

10 (iii) Given that x can vary, find the exact value of x for which the area enclosed by the frame is a maximum. [2]

$$A = 8x - 6x^2 + \sqrt{3}x^2$$

$$\frac{dA}{dx} = 8 - 12x + 2\sqrt{3}x$$ M1

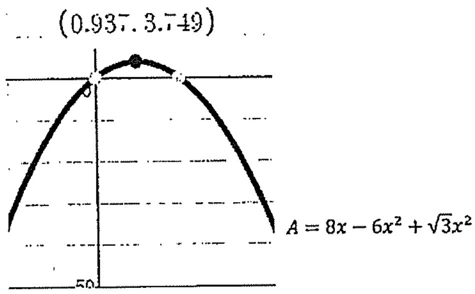
For max area, $\frac{dA}{dx} = 0$

$$8 - 12x + 2\sqrt{3}x = 0$$

$$12x - 2\sqrt{3}x = 8$$

$$x = \frac{8}{12 - 2\sqrt{3}}$$

$$= \frac{4}{6 - \sqrt{3}}$$ A1



11 (i) Prove the identity $\frac{tanA-cotA}{tanA+cotA}+1=2sin^2A$ [3]

$$LHS=\frac{tanA-cotA}{tanA+cotA}+1$$

$$=\frac{\left(\frac{\sin A}{\cos A}-\frac{\cos A}{\sin A}\right)}{\left(\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)}+1$$ For converting tan A or cot A B1

$$=\frac{sin^2A-cos^2A}{sin^2A+cos^2A}+1$$ M1

$$=(sin^2A-cos^2A)+(sin^2A+cos^2A)$$ For $sin^2A+cos^2A=1$ and leading to final answer. B1

$$=2sin^2A$$

$$=RHS$$

(ii) Hence, solve the equation $\frac{tanA-cotA}{tanA+cotA}+1=5sinAcosA$ for $0°<A<360°$ [5]

From (i), subs $\frac{tanA-cotA}{tanA+cotA}+1=2sin^2A$

$$2sin^2A=5\sin A\cos A$$ B1

$$\sin A(2\sin A-5\cos A)=0$$ M1

$$\sin A=0 \quad or \quad 2\sin A=5\cos A$$

$$A=180°(rejected) \quad or \quad \tan A=\frac{5}{2}$$

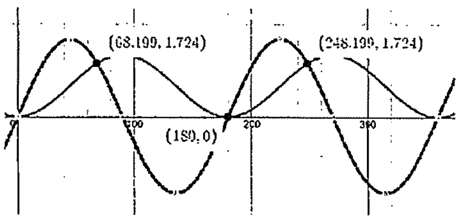
$$basic\ angle, \alpha=tan^{-1}\frac{5}{2}$$

$$=68.2°$$ M1

$$A=68.2°,\ 248.2°$$

Ans: $A=68.2°,\ 248.2°$ For 180°(rejected) A1

For 68.2°, 248.2° A1



12 A particle starts from rest, travels in a straight line so that t is the time in seconds after passing a fixed point O. Its velocity, $v\ m/s$, is given by $v = 6t - 2t^2$. The particle comes to instantaneous rest at A.

(i) Find the acceleration of the particle at A. [3]

$$at\ A,\ v = 0$$

$$\Rightarrow 2t(3 - t) = 0$$ For $v = 0$ M1

$$t = 0 \quad or \quad t = 3$$

$$a = \frac{dv}{dt}$$

$$a = 6 - 4t$$ M1

$$at\ A,\quad acceleration = 6 - 4(3) = -6\ m/s$$ A1

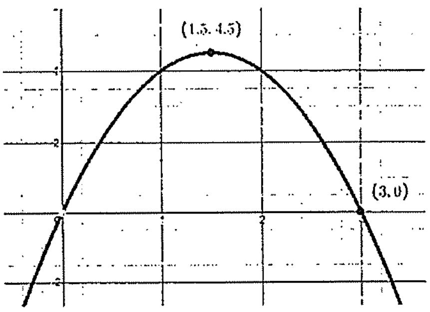
(ii) Find the maximum velocity of the particle. [2]

$$For\ \max velocity,\ \frac{dv}{dt} = 0$$

$$6 - 4t = 0$$ M1

$$t = \frac{3}{2}$$

$$\therefore \max velocity = 6\left(\frac{3}{2}\right) - 2\left(\frac{3}{2}\right)^2 = 4\frac{1}{2}\ m/s$$ A1



$v = 6t - 2t^2$.

12 (iii) Find the total distance travelled by the particle during the first 5 seconds. [5]

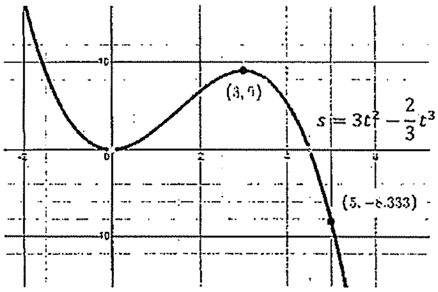
$$s = \int v\,dt$$
$$= \int 6t - 2t^2\,dt$$
$$= 3t^2 - \frac{2}{3}t^3 + c \qquad \text{M1}$$

$$when\ t = 0, s = 0 \quad \Rightarrow \quad c = 0$$
$$\therefore\ s = 3t^2 - \frac{2}{3}t^3 \qquad \text{B1}$$

$$when\ t = 3,\ s = 3(3)^2 - \frac{2}{3}(3)^3 = 9 \qquad \text{A1}$$

$$when\ t = 5,\ s = 3(5)^2 - \frac{2}{3}(5)^3 = -8\frac{1}{3} \qquad \text{A1}$$

$$Total\ distance\ = 9 + 9 + 8\frac{1}{3} = 26\frac{1}{3}\ m \qquad \text{A1}$$



End of Paper

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS

4047/02

PAPER 2

20 September 2019
2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

This question paper consists of 21 printed pages (including this cover page)

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions

1 The equation of a curve is $y = -x^2 + (k-2)x + 2k$, where k is a constant.

(i) Find the set of values of k for which the line $y = 2kx + 2k + 1$ lies entirely above the curve. [3]

(ii) State the values of k for which the line $y = 2kx + 2k + 1$ is a tangent to the curve. [1]

1 (iii) Explain why there is only one value of k for which y cannot be positive and state this value. [4]

2 (i) Find the first 3 terms in the expansion of $\left(2-\frac{x}{4}\right)^n$ in ascending powers of x, where n is a positive integer greater than 2. Give the terms in their simplest forms. [2]

(ii) In the expansion of $(4+x)^2\left(2-\frac{x}{4}\right)^n$, there is no term in x^2.

Find the value of n. [5]

[Turn over

3 (i) Express $\dfrac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)}$ as $a+\dfrac{bx^2+cx+d}{(x^2+2)(2x-1)}$

where a, b, c and d are integers. [2]

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [2]

3 Using the results from (i) and (ii) and expressing $\frac{bx^2+cx+d}{(x^2+2)(2x-1)}$ as partial fractions,

find

(iii) $\int \frac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)}\,dx$. [8]

4 (a) Variables x and y are related by the equation $px^2 + qy = x$, where p and q are constants. Y is plotted against X to obtain a straight line graph.

(i) If $X = x$, state Y in terms of x and/or y. [1]

(ii) Explain clearly how p and q can be obtained from the straight line graph drawn. [2]

(b) The number of snails in a colony is being studied. The number of snails in a colony after t weeks is given by $P = P_o e^{kt}$, where P_o and k are constants.

The table below gives some values of P and t.

t (weeks)	4	8	12	16	20
P	30	45	66	99	147

(i) On the grid on page 9, draw a suitable straight line graph to illustrate this data. [3]

Use your graph to estimate

(ii) the number of snails in the colony when the study began, [2]

(iii) the value of k. [2]

[Turn over

5 (a) Given that $\dfrac{\log_x y}{\log_y x} + \log_x y - 6 = 0$, express y in terms of x. [4]

(b) Solve the equation $9^{x+1} - 3^{x+1} = 2$. [3]

[Turn over

6 (i) On the same diagram, sketch the curve $y=x^{\frac{3}{2}}$ and $y=8x^{-\frac{3}{2}}$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [3]

7 A circle, C_1, has the equation $x^2+y^2-4x+6y=12$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

The equation of the tangent to C_1 at the point P is $4y-3x+43=0$.

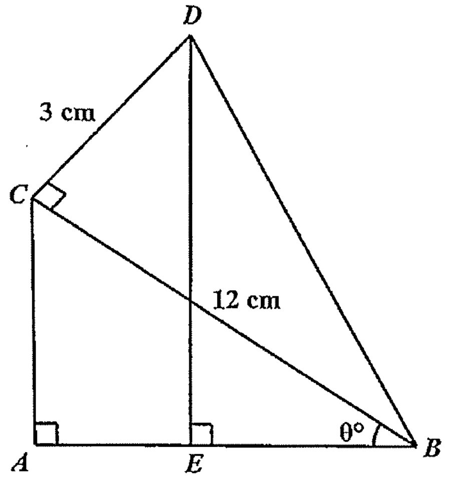
Hence, using (i), find

(ii) the coordinates of the point P on C_1. [5]

7 A second circle, C_2, passes through the points (19, 8) and (−9, 12) and has a radius of 20 units.

(iii) Find the two possible centres of circle, C_2. [6]

8



The diagram shows two triangles, ABC and DCB. It is given that $BC = 12$ cm, $DC = 3$ cm, angle CAB = angle $DCB = 90°$ and angle $ABC = \theta°$, where θ varies.

A perpendicular is dropped from D to meet AB at E.

(i) Show that the perimeter of triangle DEB, P can be expressed as

$\sqrt{153} + 9\sin\theta + 15\cos\theta$. [4]

8 (ii) Express P in the form $\sqrt{153}+R\cos(\theta-\alpha)$, where $R>0$ and α is an acute angle. [3]

(iii) Find the value of θ for which $P=27$. [3]

9 (a) Show that $\dfrac{1+\sin 4A-\cos 4A}{1+\sin 4A+\cos 4A}=\tan 2A$ [3]

(b) The temperature, T, in degrees Celsius, of a metal plate undergoing a chemical process after x seconds, is given by $T=-\dfrac{25}{9}x^2+50x+\dfrac{55}{3}$.

Determine when the metal plate starts cooling. [3]

[Turn over

10 A cubic curve has the stationary points at $(1,\ -7)$ and $\left(-\frac{7}{3},\ \alpha\right)$, where α is a constant.

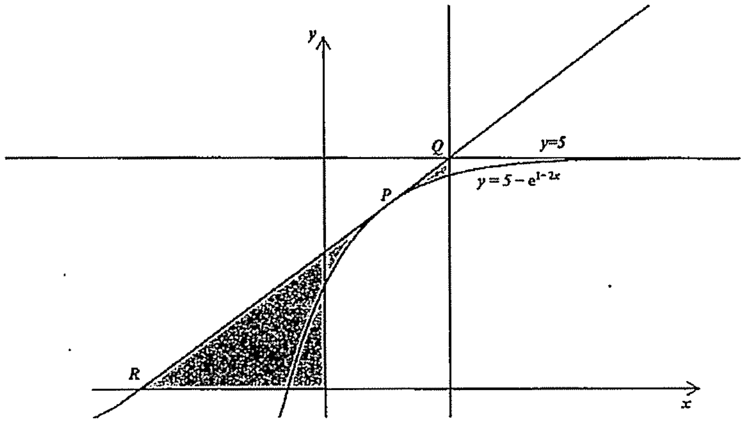
(i) Explain why $\frac{dy}{dx} = k(x-1)(3x+7)$, where k is a constant. [3]

10 (ii) Given that the curve passes through the point $(-1,\ 17)$, determine its equation. [6]

[Turn over

10 (iii) Determine the nature of the two stationary points. [3]

11



The diagram shows part of the curve $y=5-e^{1-2x}$ passing through the point P where $x=k$, where k is a constant. The tangent to the curve at P meets the line $y=5$ at $Q(1,\ 5)$ and x-axis at $R\left(-\frac{3}{2},0\right)$. A vertical line is drawn at Q.

(i) Find the value of k. [4]

11 (ii) Find the total area of the shaded region. [5]

End of paper

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2019
SECONDARY 4E/5N

Candidate's Name	Class	Register Number
MARKING SCHEME		

ADDITIONAL MATHEMATICS 4047/02

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Answer all the questions

1 The equation of a curve is $y=-x^2+(k-2)x+2k$, where k is a constant.

(i) Find the set of values of k for which the line $y=2kx+2k+1$ lies entirely above the curve. [3]

For $2kx+2k+1>-x^2+(k-2)x+2k$

$x^2+(k+2)x+1<0$ M1

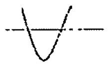
Discriminant <0

$x^2+(k+2)x+1<0$

$(k+2)^2-4(1)(1)<0$ M1

$(k+2-2)(k+2+2)<0$

$k(k+4)<0$



$-4<k<0$ A1

(ii) State the values of k for which the line $y=2kx+2k+1$ is a tangent to the curve. [1]

$k=-4$ or $k=0$ A1

1 (iii) Explain why there is only one value of k for which y cannot be positive and state this value. [4]

Discriminant = $(k-2)^2-4(-1)(2k)$

$= k^2-4k+4+8k$

$= k^2+4k+4$

$= (k+2)^2$ **B1**

When $k=-2$, Discriminant $=0$

$y=0$ is a tangent to the curve.

Since coefficient of x^2 is negative, the maximum value of $y=0$.

$\therefore y$ cannot be positive. **B1**

For any other values of k, $(k+2)^2>0$

Discriminant >0

The curve intersects the x- axis at two points,

Maximum value of $y>0$

y can be positive. **B1**

Ans: $k=-2$

2 (i) Find the first 3 terms in the expansion of $\left(2-\frac{x}{4}\right)^n$ in ascending powers of x,

where n is a positive integer greater than 2. Give the terms in their simplest forms. [2]

$$\left(2-\frac{x}{4}\right)^n = 2^n + \binom{n}{1}(2)^{n-1}\left(-\frac{x}{4}\right) + \binom{n}{2}(2)^{n-2}\left(-\frac{x}{4}\right)^2 + ...$$

$$= 2^n - \frac{n\times 2^n x}{8} + \frac{n(n-1)}{2}\times 2^{n-2}\times\frac{x^2}{16} + ...$$

$$= 2^n - \frac{n\times 2^n x}{8} + \frac{n(n-1)}{128}\times 2^n \times x^2 + ...$$

B1 for any 2 terms correct

B2 for all 3 terms correct

(ii) In the expansion of $(4+x)^2\left(2-\frac{x}{4}\right)^n$, there is no term in x^2.

Find the value of n. [5]

$$\left(16+8x+x^2\right)\left(2^n - \frac{n\times 2^n x}{8} + \frac{n(n-1)}{128}\times 2^n \times x^2 + ...\right)$$

expansion of $(4+x)^2$ B1

Term in x^2,

$$= 16\times\frac{n(n-1)}{128}\times 2^n \times x^2 + 8x\left(-\frac{n\times 2^n x}{8}\right) + x^2\times 2^n$$ M1-adding of 3 product

coefficient of $x^2 = \frac{n(n-1)}{8}\times 2^n - n\times 2^n + 2^n = 0$

$$2^n\left(\frac{n(n-1)}{8} - n + 1\right) = 0$$ M1 (Factorisation)

$2^n \neq 0$, $n(n-1) - 8n + 8 = 0$

$n^2 - 9n + 8 = 0$ B1

$(n-1)(n-8) = 0$

$n = 1$(rejected) or $n = 8$ A1

[Turn over

3 (i) Express $\dfrac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)}$ as $a+\dfrac{bx^2+cx+d}{(x^2+2)(2x-1)}$

where a, b, c and d are integers. [2]

$$\begin{array}{r}2\\ 2x^3-x^2+4x-2\;\big/\;\;4x^3+21x^2-4x-6\\ -\;(\;4x^3-2x^2+8x-4\;)\\ \hline 23x^2-12x-2\end{array}$$

$$\frac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)} = 2+\frac{23x^2-12x-2}{(x^2+2)(2x-1)}$$ M1, A1

(ii) Differentiate $\ln(x^2+2)$ with respect to x. [2]

$$\frac{d}{dx}\ln(x^2+2)$$
$$= \frac{2x}{x^2+2} \qquad \frac{1}{x^2+2}\text{ seen}$$ B1, A1

[Turn over

3 Using the results from (i) and (ii) and expressing $\frac{bx^2+cx+d}{(x^2+2)(2x-1)}$ as partial fractions,

find

(iii) $\int \frac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)}\,dx$. [8]

$$\int \frac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)}\,dx = \int 2+\frac{23x^2-12x-2}{(x^2+2)(2x-1)}\,dx$$

Let $\frac{23x^2-12x-2}{(x^2+2)(2x-1)} = \frac{A}{2x-1}+\frac{Bx+C}{(x^2+2)}$ M1

$23x^2-12x-2 = A(x^2+2)+(2x-1)(Bx+C)$ M1

Let $x=\frac{1}{2}$,

$\frac{23}{4}-\frac{12}{2}-2 = A\left(\frac{1}{4}+2\right)$

$-\frac{9}{4} = A\left(\frac{9}{4}\right)$

$A=-1$ A1

Comparing the coefficient of x^2 and constant,

$23 = A+2B$

$B=12$ A1

$-2 = 2A-C$

$C=0$ A1

$$\int \frac{4x^3+21x^2-4x-6}{(x^2+2)(2x-1)}\,dx = 2x+\int -\frac{1}{2x-1}+\frac{12x}{(x^2+2)}\,dx$$

$$= 2x-\frac{\ln(2x-1)}{2}+6\int\frac{2x}{(x^2+2)}\,dx$$

$$= 2x-\frac{\ln(2x-1)}{2}+6\ln(x^2+2)+c$$ A1

B1 B1

[Turn over

4 (a) Variables x and y are related by the equation $px^2 + qy = x$, where p and q are constants. Y is plotted against X to obtain a straight line graph.

(i) If $X = x$, state Y in terms of x and/or y. [1]

(ii) Explain clearly how p and q can be obtained from the straight line graph drawn. [2]

(i) $qy = -px^2 + x$

$\frac{y}{x} = -\frac{p}{q}x + \frac{1}{q}$

$Y = \frac{y}{x}$ A1

(ii) Y-intercept $= \frac{1}{q}$ B1

gradient of graph $= -\frac{p}{q}$ B1

(b) The number of snails in a colony is being studied. The number of snails in a colony after t weeks is given by $P = P_o e^{kt}$, where P_o and k are constants

The table below gives some values of P and t.

t (weeks)	4	8	12	16	20
P	30	45	66	99	147

(i) On the grid on page 9, draw a suitable straight line graph to illustrate this data. [3]

$\ln P = \ln P_o + kt$ M1

Plot $\ln P$ against t. Straight line graph drawn B2

t	4	8	12	16	20
$\ln P$	3.40	3.81	4.19	4.60	4.99

Use your graph to estimate

(ii) the number of snails in the colony when the study began, [2]

From graph, $\ln P_o = 3$ M1

$P_o = e^3 = 20$ Number of snails = 20 A1

(iii) the value of k. [2]

Gradient $= \frac{4.60 - 3.40}{16 - 4}$ M1

k = gradient = 0. 1 A1

5 (a) Given that $\dfrac{\log_x y}{\log_y x} + \log_x y - 6 = 0$, express y in terms of x. [4]

$\dfrac{\log_x y}{\log_y x} + \log_x y - 6 = 0$

$\log_x y \div \dfrac{\log_x x}{\log_x y} + \log_x y - 6 = 0$ M1 change of base

$(\log_x y)^2 + \log_x y - 6 = 0$

$(\log_x y + 3)(\log_x y - 2) = 0$ Any method to solve quadratic eqn M1

$\log_x y = -3$ or $\log_x y = 2$

$y = x^{-3}$ or $y = x^2$ A1, A1

(b) Solve the equation $9^{x+1} - 3^{x+1} = 2$. [3]

Let $u = 3^x$

$9u^2 - 3u - 2 = 0$ either $3^{2x} \times 9 = 9u^2$ seen or $3^{x+1} = 3u$ seen B1

$(3u+1)(3u-2) = 0$

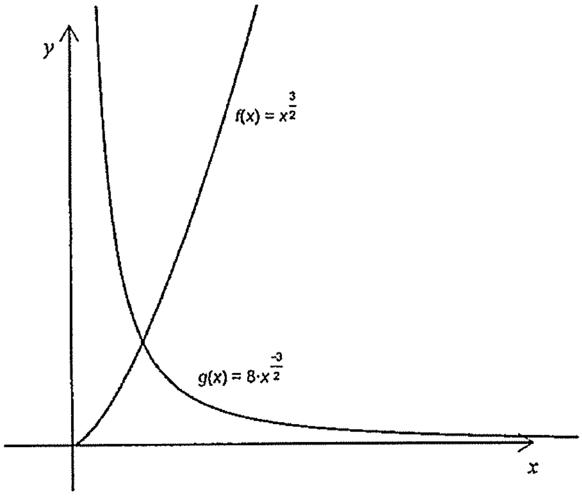
$u = -\dfrac{1}{3}$ or $u = \dfrac{2}{3}$ M1

$3^x = -\dfrac{1}{3}$ (rejected) or $3^x = \dfrac{2}{3}$

$x \lg 3 = \lg \dfrac{2}{3}$

$x = \dfrac{\lg \frac{2}{3}}{\lg 3} = -0.369$ (to 3 sf) A1

6 (i) On the same diagram, sketch the curve $y = x^{\frac{3}{2}}$ and $y = 8x^{-\frac{3}{2}}$. [2]



(ii) Find the coordinates of the point of intersection of the two curves. [3]

For points of intersection,

$x^{\frac{3}{2}} = 8x^{-\frac{3}{2}}$

$x^{\frac{3}{2}} = \dfrac{8}{x^{\frac{3}{2}}}$

$x^3 = 8$ M1

$x = 2$ A1

$y = 2^{\frac{3}{2}}$

Point of intersection = $(2,\ 2.83)$ A1

[Turn over

7 A circle, C_1, has the equation $x^2+y^2-4x+6y=12$.

(i) Find the coordinates of the centre and the radius of C_1. [3]

centre = (2, −3)

$r=\sqrt{2^2+(-3)^2-(-12)}$ M1

$=\sqrt{25}$

$=5$ A1

The equation of the tangent to C_1 at the point P is $4y-3x+43=0$.

Hence, using (i), find

(ii) the coordinates of the point P on C_1. [5]

Gradient of normal at $P=-\frac{4}{3}$ M1

$C=(2,-3)$

Equation of the normal at P is $y+3=-\frac{4}{3}(x-2)$

$y=-\frac{4}{3}x-\frac{1}{3}$ ——(1) B1

For P, substitute (1) into $4y-3x+43=0$

$4\left(-\frac{4}{3}x-\frac{1}{3}\right)-3x+43=0$ M1

$-\frac{25}{3}x=-\frac{125}{3}$

$x=5$ A1

$y=-\frac{4}{3}\times 5-\frac{1}{3}=-7$

$P=(5,-7)$ A1

7 A second circle, C_2, passes through the points (19, 8) and (−9, 12) and has a radius of 20 units.

(iii) Find the two possible centres of circle, C_2. [6]

For perpendicular bisector of $(19,\ 8)$ *and* $(-9,\ 12)$

gradient of perpendicular bisector $= -1 \div \left(\frac{12-8}{-28}\right)$ M1

$= 7$

mid point $= (5,10)$

Equation of perpendicular bisector is

$y - 10 = 7(x-5)$

$y = 7x - 25$ B1

Let centre $= (x,\ 7x-25)$

radius $= 20$

$\sqrt{(x+9)^2 + (7x-37)^2} = 20$ form an eqn in one unknown M1

$(x+9)^2 + (7x-37)^2 = 400$

$x^2 + 18x + 81 + 49x^2 - 2(7x)(37) + 1369 = 400$

$50x^2 - 500x + 1050 = 0$

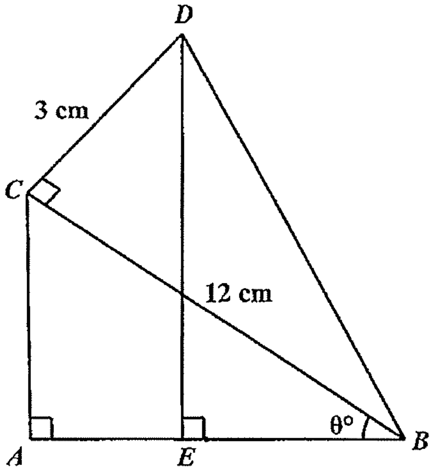
$x^2 - 10x + 21 = 0$

$(x-3)(x-7) = 0$ any method seen to solve quadratic eqn M1

$x = 3$ or $x = 7$

Centre $= (3,\ -4)$ or centre $= (7,\ 24)$ A1, A1

8



The diagram shows two triangles, ABC and DCB. It is given that $BC = 12$ cm, $DC = 3$ cm, angle CAB = angle $DCB = 90°$ and angle $ABC = \theta°$, where θ varies.

A perpendicular is dropped from D to meet AB at E.

$\sin\theta = \frac{AC}{12}$ $\quad\cos\theta = \frac{AB}{12}$

$AC = 12\sin\theta$ or $AB = 12\cos\theta$ B1 for correct AC or AB

Let the perpendicular from C to DE intersects at F.

angle $FCB = \theta$ (alternate angles)

angle $CDF = 180° - (90° - \theta) - 90°$

$= \theta$

$\cos\theta = \frac{DF}{3}$

$DF = 3\cos\theta$

$\sin\theta = \frac{CF}{3}$ [4]

$CF = 3\sin\theta$ B1 for correct DF or CF

Perimeter of triangle DFB, $P = DE + EB + DB$

$= 3\cos\theta + 12\sin\theta + 12\cos\theta - 3\sin\theta + \sqrt{3^2 + 12^2}$

(B1) or (B1) (B1)

$= \sqrt{153} + 9\sin\theta + 15\cos\theta$

8 (ii) Express P in the form $\sqrt{153}+R\cos(\theta-\alpha)$, where $R>0$ and α is an acute angle. [3]

$P = \sqrt{153}+9\sin\theta+15\cos\theta$

$= \sqrt{153}+15\cos\theta+9\sin\theta$

$= \sqrt{153}+R\cos(\theta-\alpha)$

$R=\sqrt{15^2+9^2}=\sqrt{306}$ A1

$\alpha = \tan^{-1}\dfrac{9}{15}$ M1

$= 30.964^\circ$

$P = \sqrt{153}+\sqrt{306}\cos(\theta-31.0^\circ)$ A1

(iii) Find the value of θ for which $P = 27$. [3]

$\sqrt{153}+\sqrt{306}\cos(\theta-30.964^\circ)=27$

$\sqrt{306}\cos(\theta-30.964^\circ) = \dfrac{27-\sqrt{153}}{\sqrt{306}}$ M1

$\theta-30.964^\circ = 33.240^\circ$ M1

$\theta = 64.2^\circ$ A1

9 (a) Show that $\dfrac{1+\sin 4A-\cos 4A}{1+\sin 4A+\cos 4A}=\tan 2A$ [3]

$$\text{LHS} = \frac{1+\sin 4A-\cos 4A}{1+\sin 4A+\cos 4A}$$

$$= \frac{1+2\sin 2A\cos 2A-(1-2\sin^2 2A)}{1+2\sin 2A\cos 2A+(2\cos^2 2A-1)}$$ use of double angle identity B1

$$= \frac{2\sin 2A\cos 2A+2\sin^2 2A}{2\sin 2A\cos 2A+2\cos^2 2A}$$ B1

$$= \frac{2\sin 2A(\cos 2A+\sin 2A)}{2\cos 2A(\sin 2A+\cos 2A)}$$ M1 Factorisation

$$= \frac{\sin 2A}{\cos 2A}$$

$$= \tan 2A$$

(b) The temperature, T, in degrees Celsius, of a metal plate undergoing a chemical process after x seconds, is given by $T=-\dfrac{25}{9}x^2+50x+\dfrac{55}{3}$.

Determine when the metal plate starts cooling. [3]

$\dfrac{dT}{dx}=-\dfrac{50}{9}x+50$ M1 correct differentiation

For cooling to start,

$\dfrac{dT}{dx}<0$ B1

$-\dfrac{50}{9}x+50<0$

$x>9$ A1

[Turn over

10 A cubic curve has the stationary points at $(1, -7)$ and $\left(-\frac{7}{3}, \alpha\right)$, where α is a constant.

(i) Explain why $\frac{dy}{dx} = k(x-1)(3x+7)$, where k is a constant. [3]

Since stationary points occur at $x = 1$ or $x = -\frac{7}{3}$

$\frac{dy}{dx} = 0$ when $x = 1$ or $x = -\frac{7}{3}$ B1

$x - 1 = 0$ or $3x + 7 = 0$
$(x - 1)(3x + 7) = 0$ either award B1

Since the curve is cubic, $\frac{dy}{dx}$ is quadratic,
Thus a cubic curve has at most 2 stationary points. B1

$\frac{dy}{dx} = k(x-1)(3x+7)$, k is a constant

[Turn over

10 (ii) Given that the curve passes through the point $(-1,\ 17)$, determine its equation. [6]

$y = \int k\left(3x^2 + 4x - 7\right)\mathrm{d}x$ M1

$= k\left(x^3 + 2x^2 - 7x\right) + c$ B1 correct integration

substitute $x = 1$, $y = -7$

$-7 = -4k + c$ (1) B1

substitute $x = -1$, $y = 17$

$17 = 8k + c$ (2) B1

(2) − (1), $24 = 12k$

$k = 2$, $c = 1$ either ans for k or c A1

Equation is $y = 2x^3 + 4x^2 - 14x + 1$ A1

10 (iii) Determine the nature of the two stationary points. [3]

$$\frac{d^2y}{dx^2} = \frac{d}{dx}2(x-1)(3x+7)$$
$$= 2(x-1)(3) + 2(3x+7)$$
$$= 12x + 8 \qquad \text{M1}$$

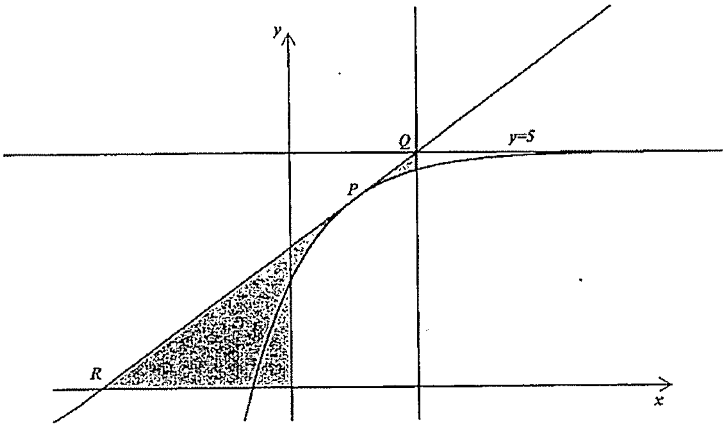
when $x = 1$,

$\frac{d^2y}{dx^2} = 20 > 0$, $(1, -7)$ is a minimum point. B1

when $x = -\frac{7}{3}$

$\frac{d^2y}{dx^2} = -20 < 0$, $\left(-\frac{7}{3}, \alpha\right)$ is a maximum point B1

11



The diagram shows part of the curve $y = 5 - e^{1-2x}$ passing through the point P where $x = k$, where k is a constant. The tangent to the curve at P meets the line $y = 5$ at $Q(1, 5)$ and x-axis at $R\left(-\frac{3}{2}, 0\right)$. A vertical line is drawn at Q.

(i) Find the value of k. [4]

gradient of tangent at $P = \frac{5}{2.5}$

$= 2$ B1

$\frac{dy}{dx} = -(-2)e^{1-2x}$ B1

when $x = k$, $\frac{dy}{dx} = 2$,

$2 = 2e^{1-2k}$ M1

$1 = e^{1-2k}$

$1 - 2k = 0$

$k = \frac{1}{2}$ A1

11 (ii) Find the total area of the shaded region. [5]

Total area $= \frac{1}{2}\times 5\times\left(1-\left(-\frac{3}{2}\right)\right) - \int_0^1 5-e^{1-2x}\,dx$ M1

$= \frac{1}{2}\times 5\times\frac{5}{2} - \left[5x-\frac{e^{1-2x}}{-2}\right]_0^1$ A1, A1

$= \frac{25}{4} - \left(5+\frac{1}{2}e^{-1} - \left(\frac{1}{2}e\right)\right)$ M1 correct evaluation of definite integral

$= \frac{5}{4} - \frac{1}{2e} + \left(\frac{1}{2}e\right)$

$= 2.43$ (to 3 sf) A1

End of paper

[Turn over