Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS
Paper 2

4047/02

28 August 2019 Wednesday 2 hrs 30 min

2019 SECONDARY FOUR EXPRESS PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Write your answers on the spaces provided below each question.
Give non exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Qn	1	2	3	4	5	6	7	8	9	10	11	*Marks Deducted*
Marks												

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks
/ 100

Setter: Mr Tan Lip Sing
Vetter: Mdm Manju Manoharan

This question paper consists of 19 printed pages and 1 blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C.$$

Answer all questions in the space provided

1. Find the value of the constant k for which $y=(1+x)e^{3x}$ is a solution of the equation

$$\frac{d^2y}{dx^2}-6\frac{dy}{dx}=ky.$$ [6]

2. (i) Given that $y=\tan^3 x$, show that $\frac{dy}{dx}=3\sec^4 x-3\sec^2 x$. [3]

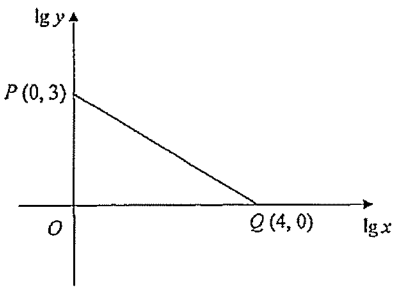
(ii) Hence, find the exact value of $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$. [4]

3. (a) Find the coefficient of x^2 in the expansion of $(8-9x)\left(1+\frac{x}{3}\right)^7$. [3]

(b) In the expansion of $(3+2x)^n$, the coefficient of x^3 is twice that of x^4. Find the value of n. [5]

4. (a) It is known that the variables x and y are related by the equation $ay^2 + \frac{b}{x} = 1$, where a and b are constants. When a graph of y^2 against $\frac{1}{x}$ is drawn, a straight line is obtained. Given that the line has a gradient of 3 and passes through $(0, 2)$, find the value of a and of b. [4]

(b) The diagram below shows part of a straight line obtained when $\lg y$ is plotted against $\lg x$. The coordinates of P and Q are $(0,3)$ and $(4,0)$ respectively.



(i) Express y in terms of x. [3]

(ii) Find the value of y when $\lg x = 0$. [2]

5. Solve the following equations.

(i) $\log_2(1-x)^2 - \log_2 x = 3 + \log_2 2x$ [4]

(ii) $\log_3 x - 12\log_x 3 = 4$ [5]

6. (a) Given that $4x^2 + kx + 9 > 0$ for all values of x, find the value(s) of k. [3]

(b) The line $y = 4x + k$ is a tangent to the curve $y = x^2 + 2x + 3$ at the point P.

(i) Find the value of k. [3]

(ii) Hence find the coordinates of P. [2]

(iii) The straight line L meets the curve at one point only. Given that L is not a tangent to the curve, what can be deduced about L? [1]

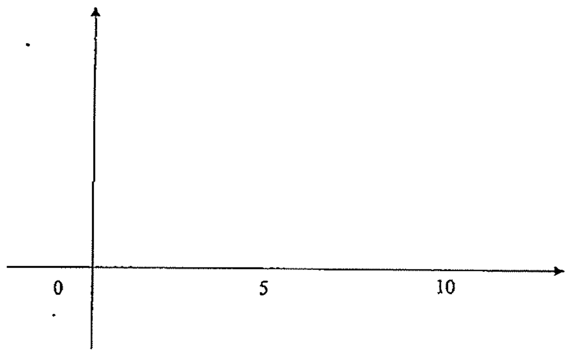
7. (a) The population P of parrots in a certain region can be modelled by $P = P_o e^{kt}$ for $0 \le t \le 10$, where P_o and k are constants, and t is the number of years after the beginning of 2010. At the beginning of 2010, the population was 600. At the beginning of 2015, the population was 840.

(i) Find the value of P_o. [1]

(ii) Find the value of k, correct to 3 decimal places. [2]

(iii) Find the year when the population reaches 1000. [3]

(iv) Sketch the graph of P against t for $0 \leq t \leq 10$. [1]



(b) A basic formula for calculating the magnitude of an earthquake on the Richter scale M is given by

$$M = \lg \frac{I}{I_o},$$

where I is the intensity of seismic waves and I_o is a reference constant.

The earthquake that occurred in Japan on 11 March 2011 had a magnitude of 9.0 on the Richter scale. The foreshock on 9 March 2011 had a magnitude of 7.2 on the Richter scale.

How many times stronger was the intensity of the seismic waves during the earthquake compared to that of the foreshock? Give your answer correct to the nearest integer. [3]

8. (a) The variables x and y are related by the equation $y = \frac{5}{2(x-1)^2}$, where $x \neq 1$.

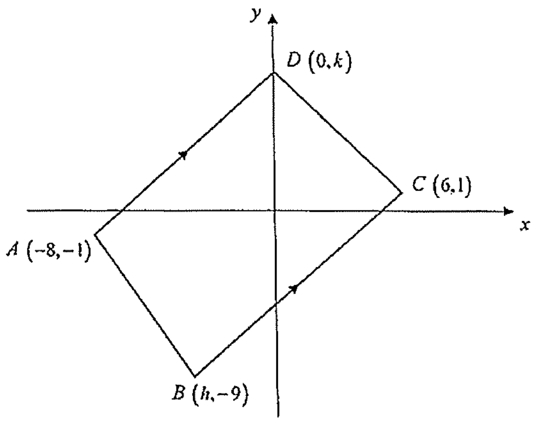
Given that x is decreasing at the rate of 0.2 units per second, find the value of x when y increases at the rate of 8 units per second. [3]

(b) The equation of a curve is $y = -x^4 - 4x^3 + 16x + 10$.

(i) Find the stationary point(s) of the curve. [4]

(ii) Determine whether each stationary point is a maximum point, minimum point or point of inflexion. [3]

9. The diagram shows a trapezium $ABCD$, where A is $(-8,-1)$, B is $(h,-9)$ and C is $(6,1)$. Point D $(0,k)$ lies on the y-axis such that AD is parallel to BC and the y-axis bisects angle ADC.



(i) Express the gradients of AD and CD in terms of k. [2]

(ii) Hence show that $k=7$. [2]

(iii) Find the value of h. [2]

(iv) Show that the angle ABC is not $90°$. [2]

(v) Find the area of the trapezium $ABCD$. [2]

10. (a) It is given that $\tan x = -\frac{3}{4}$ and $\cos y = -\frac{1}{\sqrt{5}}$, where x and y are in the same quadrant.

(i) State which quadrant x and y are in. [1]

(ii) Without using a calculator, find the value of

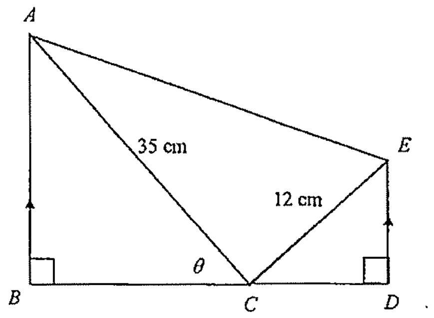
(a) $\cos x \operatorname{cosec} y$ [2]

(b) $\cot x + \tan y$ [2]

(b) If $\sin(A+B) = 5\sin(A-B)$, show that $\tan A = \frac{3}{2}\tan B$. [3]

(c) Prove that $\dfrac{1-\tan^2 x}{\sec^2 x} = \cos 2x$ [3]

11. The diagram shows a trapezium $ABCDE$.



AB is parallel to DE. $AC = 35\,\text{cm}$, $CE = 12\,\text{cm}$, $\angle ACE = 90°$ and $\angle ACB = \theta$, where θ is an acute angle in degrees.

(i) Show that the perimeter P of $ABCDE$ is given by $P = 37 + 47\sin\theta + 47\cos\theta$. [3]

(ii) Express P in the form $37 + R\sin(\theta + \alpha)$, where $R > 0$ and α is acute. [3]

(iii) Determine the maximum value of P and the corresponding value of θ. [2]

(iv) Explain, with proper working, if it is possible for the perimeter of $ABCDE$ to be 70 cm. [3]

END OF PAPER

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MARKING SCHEME

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1. Find the value of the constant k for which $y=(1+x)e^{3x}$ is a solution of the equation

$\frac{d^2y}{dx^2}-6\frac{dy}{dx}=ky$. [6]

	$y=(1+x)e^{3x}$	
	$\frac{dy}{dx}=(1+x)3e^{3x}+e^{3x}$	M1, M1
	$=e^{3x}(3+3x+1)$	
	$=e^{3x}(3x+4)$	A1
	$\frac{d^2y}{dx^2}=3e^{3x}+3e^{3x}(3x+4)$	
	$=3e^{3x}(1+3x+4)$	
	$=3e^{3x}(3x+5)$	A1
	$\frac{d^2y}{dx^2}-6\frac{dy}{dx}=ky$	
	$3e^{3x}(3x+5)-6e^{3x}(3x+4)=k(1+x)e^{3x}$	
	$3(3x+5)-6(3x+4)=k(1+x)$	M1 (Simplify equation)
	$9x+15-18x-24=k(1+x)$	
	$-9x-9=k(1+x)$	
	$-9(x+1)=k(1+x)$	
	$\therefore k=-9$	A1

2. (i) Given that $y=\tan^3 x$, show that $\frac{dy}{dx}=3\sec^4 x-3\sec^2 x$. [3]

	$y=\tan^3 x$	
	$\frac{dy}{dx}=3\tan^2 x\ \sec^2 x$	M1
	$=3(\sec^2 x-1)\sec^2 x$	M1
	$=3\sec^4 x-3\sec^2 x$ (*shown*)	A1

(ii) Hence, find the exact value of $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$. [4]

	$\int_0^{\frac{\pi}{4}} (3\sec^4 x - 3\sec^2 x)\, dx = \left[\tan^3 x\right]_0^{\frac{\pi}{4}}$	M1
	$\int_0^{\frac{\pi}{4}} 3\sec^4 x\, dx - \int_0^{\frac{\pi}{4}} 3\sec^2 x\, dx = \left[\tan^3 x\right]_0^{\frac{\pi}{4}}$	
	$3\int_0^{\frac{\pi}{4}} \sec^4 x\, dx - 3\left[\tan x\right]_0^{\frac{\pi}{4}} = \left[\tan^3 x\right]_0^{\frac{\pi}{4}}$	M1 (Integrate correctly)
	$3\int_0^{\frac{\pi}{4}} \sec^4 x\, dx - 3\left[1-0\right] = \left[1-0\right]$	M1 (Substitute values correctly)
	$3\int_0^{\frac{\pi}{4}} \sec^4 x\, dx = 4$	
	$\int_0^{\frac{\pi}{4}} \sec^4 x\, dx = \frac{4}{3}$	A1

3. (a) Find the coefficient of x^2 in the expansion of $(8-9x)\left(1+\frac{x}{3}\right)^7$. [3]

	$(8-9x)\left(1+\frac{x}{3}\right)^7$	
	$=(8-9x)\left[\ldots\ldots + \binom{7}{1}\left(\frac{x}{3}\right) + \binom{7}{2}\left(\frac{x}{3}\right)^2 + \ldots\ldots\right]$	
	$=(8-9x)\left[\ldots\ldots + \frac{7}{3}x + \frac{7}{3}x^2 + \ldots\ldots\right]$	M1
	Coefficient of x^2 $= 8\left(\frac{7}{3}\right) + (-9)\left(\frac{7}{3}\right)$	M1
	$= \frac{56}{3} - 21$	
	$= -\frac{7}{3}$	A1

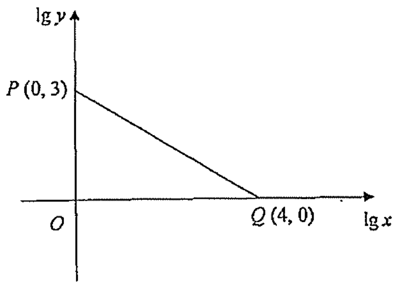
(b) In the expansion of $(3+2x)^n$, the coefficient of x^3 is twice that of x^4. Find the value of n. [5]

	$(3+2x)^n = \ldots\ldots+\binom{n}{3}3^{n-3}(2x)^3+\binom{n}{4}3^{n-4}(2x)^4+\ldots\ldots$	M1, M1
	$=\ldots\ldots+\binom{n}{3}3^{n-3}(8x^3)+\binom{n}{4}3^{n-4}(16x^4)+\ldots\ldots$	
	$\binom{n}{3}3^{n-3}(8)=2\binom{n}{4}3^{n-4}(16)$	M1
	$\frac{n(n-1)(n-2)}{6}(3^{n-3})(8)=\frac{n(n-1)(n-2)(n-3)}{24}(3^{n-4})(32)$	M1
	$3=n-3$ $n=6$	A1

4. (a) It is known that the variables x and y are related by the equation $ay^2+\frac{b}{x}=1$, where a and b are constants. When a graph of y^2 against $\frac{1}{x}$ is drawn, a straight line is obtained. Given that the line has a gradient of 3 and passes through $(0,2)$, find the value of a and of b. [4]

	$ay^2+\frac{b}{x}=1$	
	$y^2=-\frac{b}{a}\left(\frac{1}{x}\right)+\frac{1}{a}$	M1
	Since the line passes through (0, 2), $\frac{1}{a}=2$	
	$a=\frac{1}{2}$	A1 or B2
	Since the gradient is 3, $-\frac{b}{a}=3$	M1
	$-b=\frac{3}{2}$	
	$b=-\frac{3}{2}$	A1

(b) The diagram below shows part of a straight line obtained when $\lg y$ is plotted against $\lg x$. The coordinates of P and Q are $(0,3)$ and $(4,0)$ respectively.



(i) Express y in terms of x. [3]

	Gradient $=-\frac{3}{4}$	
	$\lg y=-\frac{3}{4}\lg x+3$	M1
	$=\lg x^{-\frac{3}{4}}+\lg 10^3$	M1
	$=\lg 1000x^{-\frac{3}{4}}$	
	$y=1000x^{-\frac{3}{4}}$	A1

(ii) Find the value of y when $\lg x=0$. [2]

	When $\lg x=0$	
	$\lg y=3$	M1
	$y=10^3=1000$	A1

5. Solve the following equations.

(i) $\log_2(1-x)^2 - \log_2 x = 3 + \log_2 2x$ [4]

	$\log_2 \frac{(1-x)^2}{x} = 3 + \log_2 2x$	M1 (Apply Quotient Law)
	$= \log_2 2^3 + \log_2 2x$ $= \log_2 8(2x)$	M1 (Apply Power or Product Law)
	$\frac{(1-x)^2}{x} = 8(2x)$	M1
	$(1-x)^2 = 16x^2$ $1-2x+x^2 = 16x^2$ $15x^2+2x-1=0$ $(3x+1)(5x-1)=0$ $3x+1=0$ or $5x-1=0$	
	$x=-\frac{1}{3}$ (reject) or $x=\frac{1}{5}$ (accept)	A1

(ii) $\log_3 x - 12\log_x 3 = 4$ [5]

	$\log_3 x - \frac{12}{\log_3 x} = 4$	M1 (Apply Change of base Law)
	Let $y = \log_3 x$	
	$y - \frac{12}{y} = 4$ $y^2 - 12 = 4y$ $y^2-4y-12=0$ $(y-6)(y+2)=0$	M1(Form QE)
	$y=6$ or $y=-2$	M1
	$\log_3 x = 6$; $\log_3 x = -2$ $x=3^6=729$; $x=3^{-2}=\frac{1}{9}$	A1, A1

6. (a) Given that $4x^2+kx+9>0$ for all values of x, find the value(s) of k. [3]

	$4x^2+kx+9>0$	
	$b^2-4ac<0$	
	$k^2-4(4)(19)<0$	M1
	$k^2-144<0$	
	$(k-12)(k+12)<0$	M1
	$-12<k<12$	A1

(b) The line $y=4x+k$ is a tangent to the curve $y=x^2+2x+3$ at the point P.

(i) Find the value of k. [3]

	$y=4x+k$ ……….(1)	
	$y=x^2+2x+3$ ……….(2)	
	(1) = (2): $4x+k=x^2+2x+3$	M1
	$x^2-2x+3-k=0$ ……..(3)	
	Since the line is tangent to the curve,	
	$b^2-4ac=0$	
	$(-2)^2-4(1)(3-k)=0$	M1
	$4-4(3-k)=0$	
	$4-12+4k=0$	
	$4k=8$	
	$k=2$	A1

(ii) Hence find the coordinates of P. [2]

	Substitute $k-2$ into equation (3):	
	$x^2-2x+3-2=0$	M1
	$x^2-2x+1=0$	
	$(x-1)^2=0$	
	$x=1$	
	(1): $y=4x+k=4(1)+2=6$	
	Coordinates of $P=(1, 6)$	A1

(iii) The straight line L meets the curve at one point only. Given that L is not a tangent to the curve, what can be deduced about L? [1]

	L is a vertical line.	B1

7. (a) The population P of parrots in a certain region can be modelled by $P = P_o e^{kt}$ for $0 \le t \le 10$, where P_o and k are constants, and t is the number of years after the beginning of 2010. At the beginning of 2010, the population was 600. At the beginning of 2015, the population was 840.

(i) Find the value of P_o. [1]

	$P_o = 600$	B1

(ii) Find the value of k, correct to 3 decimal places. [2]

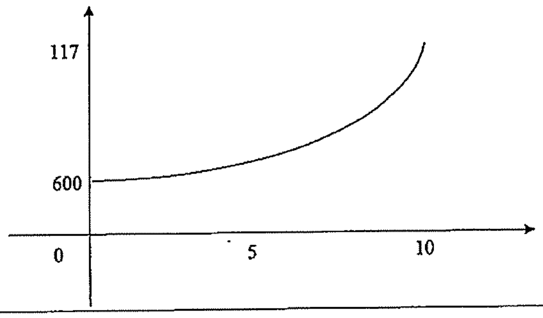
	$840 = 600e^{5k}$ $\frac{840}{600} = e^{5k}$ $\ln\frac{840}{600} = 5k\ln e$ $k = 0.067$ (3 d.p.)	M1 A1

(iii) Find the year when the population reaches 1000. [3]

	$1000 = 600e^{0.06729t}$ $\frac{10}{6} = e^{0.06729t}$ $\ln\frac{10}{6} = 0.06729t\ln e$ $t = 7.59$ years The year is 2017.	M1 M1 A1 (accept 2018)

[1]

(iv) Sketch the graph of P against t for $0 \le t \le 10$.



1 mark
Sketch graph and indicate values on vertical axis.

(b) A basic formula for calculating the magnitude of an earthquake on the Richter scale M is given by $M = \lg \frac{I}{I_o}$, where I is the intensity of seismic waves and I_o is a reference constant. The earthquake that occurred in Japan on 11 March 2011 had a magnitude of 9.0 on the Richter scale. The foreshock on 9 March 2011 had a magnitude of 7.2 on the Richter scale. How many times stronger was the intensity of the seismic waves during the earthquake compared to that of the foreshock? Give your answer correct to the nearest integer. [3]

	Let I_o and I_o be the intensity of the earthquake and foreshock respectively. Earthquake: $9.0 = \lg \frac{I_a}{I_o}$ ....(1) Foreshock : $7.2 = \lg \frac{I_b}{I_o}$ ....(2) (1): $\frac{I_a}{I_o} = 10^{9.0}$ ....(3) (2): $\frac{I_b}{I_o} = 10^{7.2}$ ....(4) $\frac{(3)}{(4)}: \frac{I_a}{I_b} = \frac{10^{9.0}}{10^{7.2}}$ $= 10^{1.8}$ $= 63.0957$ ≈ 63 times	M1 Show either (1) or (2) M1 A1

8. (a) The variables x and y are related by the equation $y=\dfrac{5}{2(x-1)^2}$, where $x \neq 1$.

Given that x is decreasing at the rate of 0.2 units per second, find the value of x when y increases at the rate of 8 units per second. [3]

	$\frac{dy}{dx}=\frac{5}{2}(-2)(x-1)^{-3} = \frac{-5}{(x-1)^3}$	M1 (correctly done)
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$	
	$8=\frac{-5}{(x-1)^3}\times(-0.2)$ $=\frac{1}{(x-1)^3}$	M1 (Correct values for $\frac{dy}{dt}$ & $\frac{dx}{dt}$)
	$(x-1)^3=\frac{1}{8}$ $x-1=\frac{1}{2}$ $x=\frac{3}{2}$	A1

(b) The equation of a curve is $y=-x^4-4x^3+16x+10$.

(i) Find the stationary point(s) of the curve. [4]

	$\frac{dy}{dx}=-4x^3-12x^2+16$	M1
	Let $\frac{dy}{dx}=0$ $-4x^3-12x^2+16=0$ $x^3+3x^2-4=0$	
	$(x+2)^2(x-1)=0$	M1 (Factorize)
	$x=-2$ or $x=1$	A1
	When $x=-2$, $y=-(-2)^4-4(-2)^3+16(-2)+10=-6$ When $x=1$, $y=-(1)^4-4(1)^3+16(1)+10=21$ The coordinates of the stationary points are	
	$(-2,-6)$ and $(1,21)$	A1

(ii) Determine whether each stationary point is a maximum point, minimum point or point of inflexion. [3]

<table>
<tr>
<td></td>
<td>
For point $(-2,-6)$

x	-2^-	-2	-2^+
$\frac{dy}{dx}$	$+ve$	0	$+ve$

$(-2,-6)$ is a point inflexion.

For point $(1,21)$

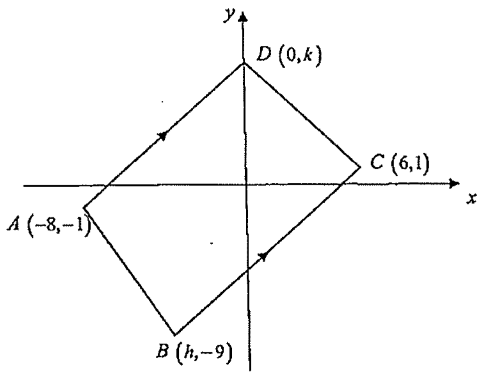
x	1^-	1	1^+
$\frac{dy}{dx}$	$+ve$	0	$-ve$

$(1,21)$ is a maximum point.
</td>
<td>
M1
Show either workings to determine the nature of the stationary points.

A1

A1
</td>
</tr>
</table>

9. The diagram shows a trapezium $ABCD$, where A is $(-8,-1)$, B is $(h,-9)$ and C is $(6,1)$. Point D $(0,k)$ lies on the y-axis such that AD is parallel to BC and the y-axis bisects angle ADC.



(i) Express the gradients of AD and CD in terms of k. [2]

	Gradient $AD = \frac{k-(-1)}{0-(-8)} = \frac{k+1}{8}$ Gradient $CD = \frac{k-1}{0-6} = -\frac{k-1}{6}$ or $\frac{1-k}{6}$ or $\frac{k-1}{-6}$	B1 B1

(ii) Hence show that $k=7$. [2]

	Since the y-axis bisects angle ADC, Gradient $AD = -$Gradient CD $\frac{k+1}{8} = -\left(\frac{k-1}{-6}\right)$ $6(k+1)=8(k-1)$ $6k+6=8k-8$ $2k=14$ $k=7$ (*Shown*)	M1 A1

(iii) Find the value of h. [2]

	Gradient $BC = \frac{1-(-9)}{6-h} = \frac{10}{6-h}$ Since AD is parallel to BC, Gradient $AD =$ Gradient BC $\frac{k+1}{8} = \frac{10}{6-h}$ $1 = \frac{10}{6-h}$ $6-h=10$ $h=-4$	M1 A1

(iv) Show that the angle ABC is not $90°$. [2]

	Gradient $AB = \frac{-1-(-9)}{-8-h} = \frac{8}{-4} = -2$ Gradient $BC = \frac{10}{6-h} = \frac{10}{10} = 1$ Gradient $AB \times$ Gradient $BC = (-2)(1) = -2 \neq -1$ Therefore, angle ABC is not $90°$. (shown)	M1 A1

(v) Find the area of the trapezium *ABCD*. [2]

	$A(-8,-1),\ B(-4,-9),\ C(6,1),\ D(0,7)$	
	Area of trapezium *ABCD*	
	$=\frac{1}{2}\begin{vmatrix} -8 & -4 & 6 & 0 & -8 \\ -1 & -9 & 1 & 7 & -1 \end{vmatrix}$	
	$=\frac{1}{2}[(72-4+42+0)-(4-54+0-56)]$	M1
	$=\frac{1}{2}[110-(-106)]$	
	$=\frac{1}{2}(216)$	
	$=108\ \text{units}^2$	A1

10. (a) It is given that $\tan x=-\frac{3}{4}$ and $\cos y=-\frac{1}{\sqrt{5}}$, where x and y are in the same quadrant.

(i) State which quadrant x and y are in. [1]

	x and y are in the second quadrant.	B1

(ii) **Without using a calculator**, find the value of

(a) $\cos x \operatorname{cosec} y$ [2]

(b) $\cot x+\tan y$ [2]

(a)	$\cos x \operatorname{cosec} y=-\frac{4}{5}\times\frac{1}{\left(\frac{2}{\sqrt{5}}\right)}$	M1
	$=-\frac{4}{5}\times\frac{\sqrt{5}}{2}$	
	$=-\frac{2\sqrt{5}}{5}$	A1
(b)	$\cot x+\tan y=\frac{1}{\left(-\frac{3}{4}\right)}\times(-2)$	M1
	$=-\frac{4}{3}-2$	
	$=-\frac{10}{3}$	A1

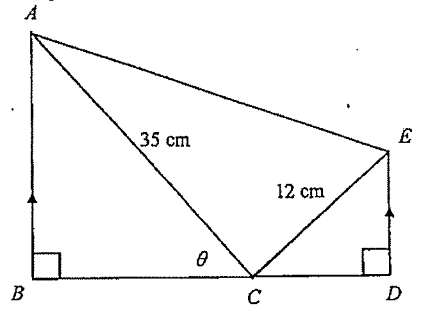
(b) If $\sin(A+B)=5\sin(A-B)$, show that $\tan A=\frac{3}{2}\tan B$. [3]

	$\sin(A+B)=5\sin(A-B)$	
	$\sin A\cos B+\cos A\sin B=5(\sin A\cos B-\cos A\sin B)$	M1
	$6\cos A\sin B=4\sin A\cos B$	
	$6\left(\frac{\sin B}{\cos B}\right)=4\left(\frac{\sin A}{\cos A}\right)$	M1
	$6\tan B=4\tan A$	
	$\tan A=\frac{3}{2}\tan B$ (*Shown*)	A1

(c) Prove that $\frac{1-\tan^2 x}{\sec^2 x}=\cos 2x$ [3]

	$LHS=\frac{1-\tan^2 x}{\sec^2 x}$	
	$=\frac{1-(\sec^2 x-1)}{\sec^2 x}$	M1
	$=\frac{2-\sec^2 x}{\sec^2 x}$	
	$=\frac{2}{\sec^2 x}-1$	
	$=2\cos^2 x-1$	M1
	$=\cos 2x$	A1
	$=RHS$	
	Alternative method	
	$LHS=\frac{1-\tan^2 x}{\sec^2 x}$	
	$=\cos^2 x\left(1-\frac{\sin^2 x}{\cos^2 x}\right)$	M1
	$=\cos^2 x-\sin^2 x$	M1
	$=\cos 2x$	A1
	$=RHS$	

11. The diagram shows a trapezium *ABCDE*.



AB is parallel to *DE*. $AC = 35$ cm, $CE = 12$ cm, $\angle ACE = 90°$ and $\angle ACB = \theta$, where θ is an acute angle in degrees.

(i) Show that the perimeter *P* of *ABCDE* is given by $P = 37 + 47\sin\theta + 47\cos\theta$. [3]

	$AB = 35\sin\theta$ $BC = 35\cos\theta$ $AE = \sqrt{35^2 + 12^2} = 37$ $CD = 12\sin\theta$ $DE = 12\cos\theta$	M1 (At least 1 out of 3 correct) M2 (At least 2 out of 3 correct)
	$P = 35\sin\theta + 35\cos\theta + 37 + 12\sin\theta + 12\cos\theta$ $P = 37 + 47\sin\theta + 47\cos\theta$ *(Shown)*	A1

(ii) Express *P* in the form $37 + R\sin(\theta + \alpha)$, where $R > 0$ and α is acute. [3]

	$P = 37 + 47\sin\theta + 47\cos\theta$ $P = 37 + R\sin(\theta + \alpha)$	
	$R = \sqrt{47^2 + 47^2} = 47\sqrt{2}$ *or* $\sqrt{4418}$ *or* 66.468	M1
	$\alpha = \tan^{-1}\left(\frac{47}{47}\right) = 45°$	M1
	$P = 37 + 47\sqrt{2}\sin(\theta + 45°)$ *or* $37 + 66.5\sin(\theta + 45°)$	A1

(iii) Determine the maximum value of P and the corresponding value of θ. [2]

	Maximum value of $P = 37 + 47\sqrt{2} = 103.468 \approx 104$ (3 s.f.)	B1
	For P to be maximum,	
	$\sin(\theta + 45°) = 1$	
	$\theta + 45° = 90°$	
	$\theta = 45°$	B1

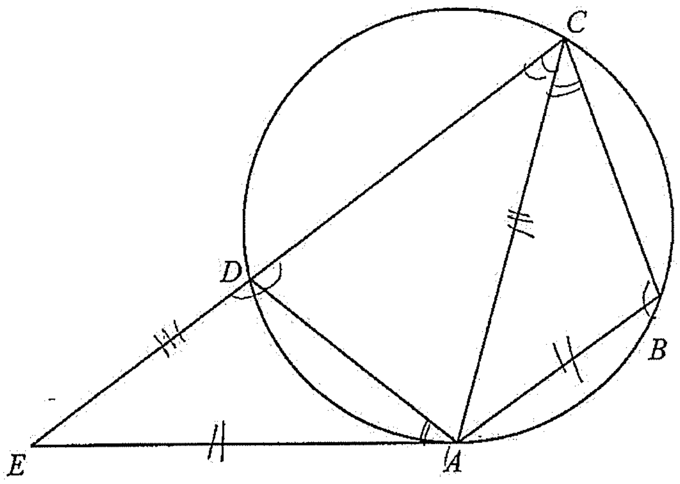
(iv) Explain, with proper working, if it is possible for the perimeter of *ABCDE* to be 70 cm. [3]

	$P = 37 + 47\sqrt{2}\sin(\theta + 45°)$	
	$70 = 37 + 47\sqrt{2}\sin(\theta + 45°)$	
	$\sin(\theta + 45°) = \dfrac{70 - 37}{47\sqrt{2}} = 0.496479$	
	$\theta + 45° = 29.767°, 180° - 29.767°$ $= 29.767°, 150.233°$	M1
	$\theta = -15.233°, 105.233°$	M1
	Since θ is acute, it is not possible for the perimeter of *ABCDE* to be 70 cm.	A1

1 The variables x and y are such that when x^2y is plotted against x, a straight line passing through (3, 9) and (5, 13) is obtained. Express y in terms of x. [3]

2 Given that $\left(\sqrt{15}+\sqrt{5}\right)\left(\sqrt{20}+\sqrt{60}\right)=a\left(b+\sqrt{3}\right)$, where a and b are integers, find the value of a and of b. [4]

3



In the diagram, $ABCD$ is a quadrilateral and AC bisects angle BCD. Chord CD is produced to meet the tangent at A at E.

(i) Prove that triangle ABC is similar to triangle EDA. [2]

(ii) Show that $AB \times AE = AC \times DE$. [2]

4 The equation of a circle is $x^2 - 2x + y^2 - 4y = 20$.

(i) Find the radius and the coordinates of the centre of the circle. [3]

(ii) Explain why a line that passes through the point (2, 3) cannot be a tangent to the circle. [2]

5 Express $\frac{9+x^4}{x^3+3x}$ in partial fractions. [6]

6 Given that the roots of the quadratic equation $-5x^2+3x+1=0$ are $\alpha-1$ and $\beta-1$, find the quadratic equation whose roots are $\dfrac{3}{\alpha}$ and $\dfrac{3}{\beta}$. [7]

7 (i) Show that $\dfrac{\cos\theta + \sin\theta}{\sin\theta - \cos\theta} = \dfrac{-\cos 2\theta}{1 - \sin 2\theta}$. [3]

(ii) Hence, find, for $-\pi \le \theta \le \pi$, the values of θ in radians which satisfy the equation $-2\cos 2\theta = 1 - \sin 2\theta$. [5]

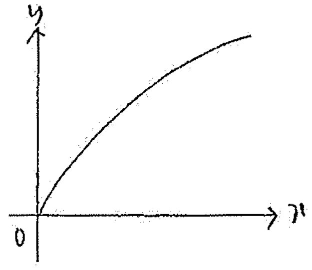
8 A particle starts from rest at a fixed point O and moves in a straight line with velocity v ms^{-1} given by $v = 2t^2 - 6t$, where t is the time in seconds after leaving O.

(i) Calculate the acceleration of the particle when $t = 5$. [2]

(ii) Find the value(s) of t when the particle is at instantaneous rest. [2]

(iii) Calculate the total distance travelled by the particle when it returns to O. [3]

9 **(i)** Sketch the graph of $y = 2\sqrt{x}$. [1]



(ii) Find the coordinates of the point of intersection of the curve $y = 2\sqrt{x}$ and the line $2y - x = 4$. [4]

(iii) Determine, with explanation, whether the tangent to the graph of $y = 2\sqrt{x}$ at the point of intersection is perpendicular to the line $2y - x = 4$. [4]

10 A rectangular box of height h cm has a horizontal rectangular base of sides x cm and $3x$ cm. The total surface area is 10750 cm^2.

(i) Express h in terms of x. [3]

(ii) Show that the volume, V cm^3, of the box is given by $V = \frac{16125}{4}x - \frac{9}{4}x^3$. [2]

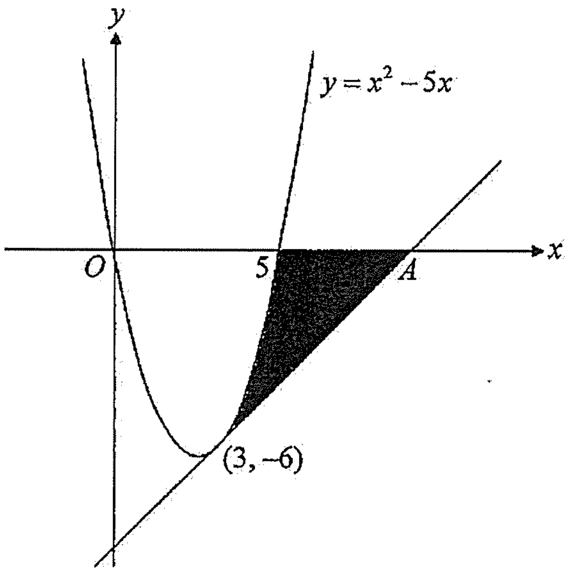
(iii) Given that x can vary, find, to the nearest whole number, the dimensions of the rectangular box that make V a maximum.
(You are not required to show that V is a maximum.) [4]

11 A curve is such that $\frac{d^2y}{dx^2} = 6x + a$, where a is a constant and it has stationary points at $(-3, 32)$ and $(1, 0)$.

(i) Find the value of a. [5]

(ii) Find the equation of the curve. [4]

12



The diagram shows the curve $y = x^2 - 5x$. The tangent to the curve at the point (3, –6) crosses the x-axis at the point A.

(i) Find the coordinates of the point A. [5]

(ii) Find the area of the shaded region. [4]

End of Paper

Solutions		
1	Gradient $= \frac{13-9}{5-3}$ $= 2$ $Y - Y_1 = m(X - X_1)$ $x^2y - 9 = 2(x-3)$ $x^2y - 9 = 2x - 6$ $x^2y = 2x + 3$ $y = \frac{2x+3}{x^2}$ (or $y = \frac{2}{x} + \frac{3}{x^2}$)	
2	$\left(\sqrt{15}+\sqrt{5}\right)\left(\sqrt{20}+\sqrt{60}\right) = a\left(b+\sqrt{3}\right)$ $\sqrt{300}+\sqrt{900}+\sqrt{100}+\sqrt{300} = ab + a\sqrt{3}$ $10\sqrt{3}+30+10+10\sqrt{3} = ab + a\sqrt{3}$ $40 + 20\sqrt{3} = ab + a\sqrt{3}$ Comparing coefficients, $\therefore a = 20$ $ab = 40$ $20b = 40$ $\therefore b = 2$	
3(i)	In triangles ABC and EDA, $\angle DAE = \angle ACD$ (Alternate Segment Theorem) $= \angle BCA$ (AC bisects $\angle BCD$) $\angle ADE = 180° - \angle ADC$ (adj. $\angle$s on a str. line) $= 180° - (180° - \angle CBA)$ ($\angle$s in the opp. segment) $= \angle CBA$ $\therefore$ triangle ABC is similar to triangle EDA.	

(ii)	$\frac{AB}{ED} = \frac{AC}{EA}$ $AB \times EA = AC \times ED$ $AB \times AE = AC \times DE$ (shown)	
4(i)	**Method 1:** $x^2 - 2x + y^2 - 4y = 20$ $x^2 - 2x + \left(\frac{-2}{2}\right)^2 - \left(\frac{-2}{2}\right)^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 - \left(\frac{-4}{2}\right)^2 = 20$ $(x-1)^2 + (y-2)^2 = 5^2$ Radius of the circle = 5 units Coordinates of the centre of the circle = (1, 2) **Method 2:** $x^2 + y^2 - 2x - 4y - 20 = 0$ $2g = -2$ and $2f = -4$ $-g = 1$ $\quad -f = 2$ Coordinates of the centre of the circle = (1, 2) Radius of the circle $= \sqrt{f^2 + g^2 - c}$ $= \sqrt{(-2)^2 + (-1)^2 - (-20)}$ $= \sqrt{25}$ $= 5$ units	
(ii)	Distance between point (2, 3) and centre of circle $= \sqrt{(2-1)^2 + (3-2)^2}$ $= \sqrt{2}$ units Since the point (2, 3) lies inside the circle as the distance between point (2, 3) and centre of circle is less than the radius, thus a line that passes through the point (2, 3) cannot be a tangent to the circle. **OR** $\because \sqrt{2} \neq 5$, the point (2, 3) is not on the circumference	

	of the circle. Thus, a line that passes through the point (2, 3) cannot be a tangent to the circle.	
5	By long division, $x^3 + 3x \overline{)\, x^4 \qquad + 9}$ quotient x $\quad x^4 + 3x^2$ $\quad -3x^2 + 9$ $\frac{9+x^4}{x^3+3x} = x + \frac{9-3x^2}{x^3+3x}$ $= x + \frac{9-3x^2}{x\left(x^2+3\right)}$ Let $\frac{9+x^4}{x^3+3x} = x + \frac{A}{x} + \frac{Bx+C}{x^2+3}$ Multiplying throughout by the denominator, $9 + x^4 = x\left(x^3+3x\right) + A\left(x^2+3\right) + (Bx+C)x$ Let $x = 0$, $9 = 3A$ $A = 3$ Let $x = 1$, $9 + 1 = 4 + 3(4) + B + C$ $B = -6 - C$ --- (1) Let $x = -1$, $9 + 1 = (-1)(-4) + 3(4) + (-1)(-B+C)$ $B = -6 + C$ --- (2) (1) = (2): $\quad -6 - C = -6 + C$ $C = 0$ Put $C = 0$ into (1): $\quad B = -6 - 0$ $B = -6$ $\frac{9+x^4}{x^3+3x} = x + \frac{3}{x} - \frac{6x}{x^2+3}$	
6	Sum of roots $= -\frac{b}{a}$	

	$\alpha - 1 + \beta - 1 = -\frac{3}{-5}$ $\alpha + \beta = \frac{13}{5}$ Product of roots $= \frac{c}{a}$ $(\alpha-1)(\beta-1) = \frac{1}{-5}$ $\alpha\beta - \alpha - \beta + 1 = -\frac{1}{5}$ $\alpha\beta - (\alpha+\beta) = -\frac{6}{5}$ $\alpha\beta - \frac{13}{5} = -\frac{6}{5}$ $\alpha\beta = \frac{7}{5}$ New sum of roots $= \frac{3}{\alpha} + \frac{3}{\beta}$ $= \frac{3\beta + 3\alpha}{\alpha\beta}$ $= \frac{3(\alpha+\beta)}{\alpha\beta}$ $= \frac{3\left(\frac{13}{5}\right)}{\frac{7}{5}}$ $= \frac{39}{7}$ New product of roots $= \frac{3}{\alpha}\left(\frac{3}{\beta}\right)$ $= \frac{9}{\alpha\beta}$ $= \frac{9}{\left(\frac{7}{5}\right)}$ $= \frac{45}{7}$	

Quadratic equation whose roots are $\frac{3}{\alpha}$ and $\frac{3}{\beta}$ is

$x^2-\frac{39}{7}x+\frac{45}{7}=0$ (or $7x^2-39x+45=0$)

7(i) **Method 1:**

$$\text{L.H.S}=\frac{\cos\theta+\sin\theta}{\sin\theta-\cos\theta}$$
$$=\frac{\cos\theta+\sin\theta}{\sin\theta-\cos\theta}\times\frac{\sin\theta-\cos\theta}{\sin\theta-\cos\theta}$$
$$=\frac{\sin\theta\cos\theta-\cos^2\theta+\sin^2\theta-\sin\theta\cos\theta}{\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta}$$
$$=\frac{-\left(\cos^2\theta-\sin^2\theta\right)}{1-2\sin\theta\cos\theta}$$
$$=\frac{-\cos2\theta}{1-\sin2\theta}$$
$=$ R.H.S (shown)

Method 2:

$$\text{R.H.S}=\frac{-\cos2\theta}{1-\sin2\theta}$$
$$=\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta}$$
$$=\frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{(\sin\theta-\cos\theta)^2}$$
$$=\frac{\cos\theta+\sin\theta}{\sin\theta-\cos\theta}$$
$=$ L.H.S (shown)

(ii)

$-2\cos2\theta=1-\sin2\theta$

$\frac{-\cos2\theta}{1-\sin2\theta}=\frac{1}{2}$

$\frac{\cos\theta+\sin\theta}{\sin\theta-\cos\theta}=\frac{1}{2}$

$2\cos\theta+2\sin\theta=\sin\theta-\cos\theta$

$3\cos\theta=-\sin\theta$

$\tan\theta=-3$

Basic angle, $\alpha=\tan^{-1}3$

$=1.249045772$

$\therefore\ \theta=-1.249045772,\ \pi-1.249045772$

$\approx-1.25,\ 1.89$ (correct to 3 s.f.)

8(i) $v=2t^2-6t$

$a=\frac{dv}{dt}$

$=4t-6$

When $t=5$, $a=4(5)-6$

$=14\ \text{ms}^{-2}$

(ii) When the particle is at instantaneous rest, $v=0$.

$2t^2-6t=0$

$2t(t-3)=0$

$t=0$ or 3

(iii) **Method 1:**

$s=\int\left(2t^2-6t\right)dt$

$=\frac{2t^3}{3}-3t^2+c$

When $t=0$ and $s=0$, $c=0$

$s=\frac{2t^3}{3}-3t^2$

When $t=3$, $s=-9$

Method 2:

$s=\int_0^3\left(2t^2-6t\right)dt$

$=\left[\frac{2t^3}{3}-\frac{6t^2}{2}\right]_0^3$

$=\left[\frac{2(3)^3}{3}-\frac{6(3)^2}{2}\right]-0$

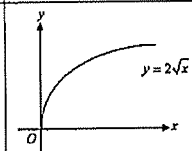
$=-9$ m

Total distance travelled by the particle when it returns to O

$=9\times2$

$=18$ m

9(i)



(ii) $y=2\sqrt{x}$ --- (1)

$2y-x=4$ --- (2)

Method 1:

Put (1) into (2): $2\left(2\sqrt{x}\right)-x=4$

$x-4\sqrt{x}+4=0$

$\left(\sqrt{x}-2\right)^2=0$

$x=4$

Put $x=4$ into (1): $y=2\sqrt{4}$

$=4$

Method 2:

Put (1) into (2): $2\left(2\sqrt{x}\right)-x=4$

$x+4=4\sqrt{x}$

$x^2+8x+16=16x$

$x^2-8x+16=0$

$(x-4)^2=0$

$x=4$

Put $x=4$ into (1): $y=2\sqrt{4}$

$=4$

Coordinates of the point of intersection are (4, 4).

(iii) $\frac{dy}{dx}=x^{-\frac{1}{2}}$

At (4, 4), $\frac{dy}{dx}=4^{-\frac{1}{2}}$

$=\frac{1}{2}$

$2y-x=4$

$y=\frac{1}{2}x+2$

Since the product of the gradient of the tangent to the graph of $y=2\sqrt{x}$ and the gradient of the line $2y-x=4$ is not equal to -1 (or $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\neq-1$), thus the tangent to the graph of $y=2\sqrt{x}$ at the point of intersection is not perpendicular to the line $2y-x=4$.

10(i) Total surface area of the box = 10750 cm^2

$2(x)(3x)+2(x)(h)+2(3x)(h)=10750$

$6x^2+8xh=10750$

$h=\frac{10750-6x^2}{8x}$

$=\frac{5375-3x^2}{4x}$

(ii) Volume of the box, $V=x(3x)(h)$

$=3x^2\left(\frac{5375-3x^2}{4x}\right)$

	$= \frac{3x^2}{4x}(5375-3x^2)$ $= \frac{16125}{4}x - \frac{9}{4}x^3 \text{ cm}^3$ (shown)	
(iii)	$\frac{dV}{dx} = \frac{16125}{4} - \frac{27}{4}x^2$ For stationary value, $\frac{dV}{dx} = 0$ $\frac{16125}{4} - \frac{27}{4}x^2 = 0$ $x^2 = \frac{5375}{9}$ $x = 24.4381305$ or -24.4381305 (rejected) Height of the box $= \frac{10750-6(24.4381305)^2}{8(24.4381305)}$ $= 36.65719574$ cm Dimensions of the rectangular box are 73 cm by 24 cm by 37 cm. (correct to nearest whole number)	
11(i)	$\frac{d^2y}{dx^2} = 6x + a$ $\frac{dy}{dx} = \int (6x+a)dx$ $= \frac{6x^2}{2} + ax + c$ $= 3x^2 + ax + c$ At $(-3, 32)$, $3(-3)^2 + a(-3) + c = 0$ $c = 3a - 27$ --- (1) At $(1, 0)$, $3(1)^2 + a(1) + c = 0$ $c = -a - 3$ --- (2) (1) = (2): $3a - 27 = -a - 3$ $\therefore a = 6$	

(ii)	Put $a = 6$ into (2): $c = -6 - 3$ $= -9$ $\frac{dy}{dx} = 3x^2 + 6x - 9$ $y = \int (3x^2 + 6x - 9)dx$ $= \frac{3x^3}{3} + \frac{6x^2}{2} - 9x + d$ $= x^3 + 3x^2 - 9x + d$ At $(1, 0)$, $0 = 1^3 + 3(1)^2 - 9(1) + d$ $d = 5$ Equation of the curve is $y = x^3 + 3x^2 - 9x + 5$.	
12(i)	$y = x^2 - 5x$ $\frac{dy}{dx} = 2x - 5$ At $(3, -6)$, $\frac{dy}{dx} = 2(3) - 5$ $= 1$ Equation of tangent is $y + 6 = (1)(x - 3)$ $y = x - 9$ When $y = 0$, $0 = x - 9$ $x = 9$ Coordinates of point A are $(9, 0)$.	
(ii)	Area of shaded region $= \frac{1}{2} \times 6 \times 6 - \left\| \int_3^5 (x^2 - 5x)dx \right\|$ $= 18 - \left\| \left[\frac{x^3}{3} - \frac{5x^2}{2} \right]_3^5 \right\|$ $= 18 - \left\| \left[\frac{5^3}{3} - \frac{5(5)^2}{2} \right] - \left[\frac{3^3}{3} - \frac{5(3)^2}{2} \right] \right\|$	

	$= 18 - \left\| -\frac{125}{6} - \left(-\frac{27}{2}\right) \right\|$ $= 18 - \left\| -\frac{22}{3} \right\|$ $= 10\frac{2}{3}$ units^2	