ORCHID PARK SECONDARY SCHOOL
Preliminary Examination 2024

CANDIDATE NAME

CLASS | INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **20 August 2024**

Secondary 4 Express / 5 Normal (Academic) **2 hours 15 minutes**

90 Marks

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST
Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
Use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks of this paper is 90.

For Examiner's Use	
Total	

This document consists of **19** printed pages.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \triangle = \frac{1}{2} bc \sin A$$

1 Find the range of values of p for which the line $y = px - 5$ meets the curve $y = 3x^2 + 4x - 2$. [4]

2 Express $\frac{5x^2-6x+13}{(x-1)(x^2+3)}$ in partial fractions. [5]

3 Prove that $2\cot 2\theta = \operatorname{cosec}\theta \sec\theta - 2\tan\theta$. [5]

4 **(a)** Find $\frac{d}{dx}(5xe^{2x+1})$. [2]

(b) Hence find $\int xe^{2x+1}\ dx$. [4]

5 Solve the equation $1 + 3\sin^2\theta = 4\cos\theta$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. [6]

6 The function f is given by $f(x) = \frac{x^2}{x-2k}$, for $x > 2k$, where k is a positive constant.

(a) Find $f'(x)$. [2]

The function g, defined for $x > 2k$, has the property that $g'(x) = (x-2k)^2 f'(x)$.
g decreases for $k < x < 6$.

(b) Show that a possible value of k is 3. [4]

7 Prove that there are no values of k for which $kx^2 + 2x - 2k - 3$ is always positive. [6]

8 The line $y - x = 2$ intersects the curve $y^2 = 4(2x + 1)$ at two points.
Find the coordinates of these two points. [5]

9 A curve has equation $y = x^3 + mx - 15$. It has a stationary point A where $x = 2$.

(a) Show that the value of the constant m is -12. [2]

(b) Find the coordinates of the other stationary point B. [2]

It is given that P is a point on the curve where the **gradient is a minimum.**

(c) Find the coordinates of the point P. [3]

(d) Prove that the gradient is a minimum at P. [2]

10 The equation of a circle C is $x^2 + y^2 - 4x - 6y - 12 = 0$.

(a) Find the coordinates of the centre of C, and the radius of C. [4]

(b) Find the coordinates of the points at which the circle intersects the x-axis. [3]

(c) State an equation of the circle which is a reflection of C in the y-axis. [2]

(d) Explain whether the circle in **part (c)** lies entirely in the 2^{nd} quadrant. [1]

11 An open cylinder has radius r cm and total surface area A cm^2.
It is given that $\frac{dA}{dr} = 2\pi(r + k)$.

(a) Find an expression for A in terms of r. [2]

(b) Express the height of the cylinder in terms of k. [1]

The radius of the cylinder is increasing with the height remaining constant.
It is given that, at time t seconds, $\frac{d^2r}{dt^2} = \frac{5}{2t+1}$.

It is also known that initially, the radius was increasing at 3 cm/s.

(c) Find an expression for $\frac{dr}{dt}$. [2]

(d) Hence find the rate of increase of the total surface area of the cylinder after 4 seconds, given that the radius is 15 cm and $k = 10$ at this instant. [3]

12 A piece of wire, 100 cm in length, is divided into two parts.
One part is bent to form a square of side x cm, and the other square of side y cm.

(a) Express y in terms of x. [2]

(b) Find the total area, A cm^2, of the two squares, leaving your answer in the form $p(x+q)^2+r$, where p, q and r are consants. [4]

(c) Hence state the minimum total area of the two squares, and the value of x at which this occurs. [2]

(d) When $x = x_1$, the total area of the two squares is A_1, where $0 < x_1 < 12\frac{1}{2}$.
State another value of x, in terms of x_1, which also gives a total area of A_1. [1]

13 The perpendicular bisector of the line joining the points $A(3, 2h)$ and $B(-7, -10)$ passes through the point $X(h, 3)$, where h is a constant.

(a) Find the mid-point M of AB. [2]

(b) Find the gradient of AB. [2]

(c) Hence find the possible values of h. [7]

END OF PAPER

Paper-1

Answers

Question			Solution	Marks	Remarks
1			$y = px - 5$ ____________ (1) $y = 3x^2 + 4x - 2$ ______ (2) Sub (1) into (2): $3x^2 + 4x - 2 = px - 5$ $3x^2 + (4 - p)x + 3 = 0$ Line meets curve => Discriminant ≥ 0: $(4 - p)^2 - 4(3)(3) \geq 0$ $16 - 8p + p^2 - 36 \geq 0$ $p^2 - 8p - 20 \geq 0$ $(p - 10)(p + 2) \geq 0$ $p \leq -2 \text{ or } p \geq 10$	 M1 M1 M1 A1	
2			Let $\frac{5x^2-6x+13}{(x-1)(x^2+3)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+3}$ $5x^2 - 6x + 13 = A(x^2 + 3) + (Bx + C)(x - 1)$ Method 1: When $x = 1$, $12 = 4A$ $A = 3$ When $x = 0$, $13 = 3(3) + C(-1)$ $C = -4$ When $x = -1$, $24 = 3(4) + (-B - 4)(-2)$ $B = 2$ Hence, $\frac{5x^2-6x+13}{(x-1)(x^2+3)} = \frac{3}{x-1} + \frac{2x-4}{x^2+3}$	M1 M1 M1 M1 A1	
3			RHS $= cosec\,\theta \sec\theta - 2\tan\theta$ $= \frac{1}{\sin\theta}\left(\frac{1}{\cos\theta}\right) - \frac{2\sin\theta}{\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta} - \frac{2\sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{1 - 2\sin^2\theta}{\sin\theta\cos\theta}$ $= \frac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$ $= \frac{2\cos 2\theta}{\sin 2\theta} = 2\cot 2\theta = LHS$	 M1 M1 M2 A1	for changing either cosec, sec or tan correctly for common denominator 1 mark for $\cos 2\theta$, 1 mark for $\sin 2\theta$

4	(a)		$5e^{2x+1} + 10xe^{2x+1}$	B2	1 mark for each term
	(b)		$5xe^{2x+1} = \frac{5}{2}\, e^{2x+1} + 10\int x\, e^{2x+1}\, dx + c_1$ M1 M1 where c_1 is an arbitrary constant $5xe^{2x+1} - \frac{5}{2}e^{2x+1} - c_1 = 10\int x\, e^{2x+1}\, dx$ $\int x\, e^{2x+1}\, dx = \frac{1}{2}xe^{2x+1} - \frac{1}{4}e^{2x+1} + c$ where $c = -\frac{1}{10}c_1$	 M1 A1	
5			$1 + 3(1 - \cos^2\theta) = 4\cos\theta$ $-3\cos^2\theta - 4\cos\theta + 4 = 0$ Let $y = \cos\theta$ $-3y^2 - 4y + 4 = 0$ $3y^2 + 4y - 4 = 0$ $(3y - 2)(y + 2) = 0$ $y = \frac{2}{3}$ or $y = -2$ $\cos\theta = \frac{2}{3}$ *or* $\cos\theta = -2$ (*rej*) $\theta = 0.841$ *or* -0.841 (3 s.f.)	M1 M1 M2 A2	
6	(a)		$f'(x)$ $= \frac{(x-2k)(2x) - x^2(1)}{(x-2k)^2}$ $= \frac{x^2 - 4kx}{(x-2k)^2}$	M1 A1	

6	(b)		$g'(x)$ $= x^2 - 4kx$ $= x(x-4k)$ g decreases => $g'(x) < 0$ $x(x-4k) < 0$ $0 < x < 4k$ Since $x > 2k$, then $2k < x < 4k$______(1) But it is also given that $k < x < 6$, i.e. $2k < x < 12$___(2) By (1) and (2), $4k = 12$ $k = 3$	M1 M1 M1 A1	
7			For the function to be always positive, $k > 0$ (so that the graph is U-shaped)___(1) We also require discriminant < 0 (so that graph never cuts x-axis) $2^2 - 4(k)(-2k-3) < 0$ $2k^2 + 3k + 1 < 0$ $(k+1)(2k+1) < 0$ $-1 < k < -\frac{1}{2}$________________(2) But (1) and (2) cannot happen at the same time (*k* cannot be positive but yet also be between -1 and $-\frac{1}{2}$) ∴ There is no value of *k* for which the function is positive (proven)	M1 M1 M1 M1 M1 A1	

Qn			Solution	Marks	Remarks
8			$y - x = 2$ $y = x + 2$ ________________ (1) $y^2 = 4(2x + 1)$ ________________ (2) Sub (1) into (2): $(x + 2)^2 = 4(2x + 1)$ $x^2 + 4x + 4 = 8x + 4$ $x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0 \text{ or } x = 4$ Sub into (1): $y = 2 \text{ or } y = 6$ The two points are (0, 2) and (4, 6)	M1 M1 M1 M1 A1	
9	(a)		$\frac{dy}{dx} = 3x^2 + m$ When $x = 2, \frac{dy}{dx} = 0 \ (given)$: $0 = 3(2)^2 + m$ $m = -12$	M1 A1	
	(b)		$\frac{dy}{dx} = 3x^2 - 12$ For stationary points, $\frac{dy}{dx} = 0$ $3x^2 - 12 = 0$ $x^2 = 4$ $x = 2 \text{ or } -2$ When $x = -2$, $y = (-2)^3 - 12(-2) - 15$ $y = 1$ B is (-2, 1)	M1 A1	

	(c)		For gradient to be a min, $\frac{d^2y}{dx^2} = 0$ $6x = 0$ $x = 0$ When $x = 0$, $y = -15$ P is (0, -15)	M1 M1 A1	
	(d)		$\frac{d^3y}{dx^3} = 6$ Since $\frac{d^3y}{dx^3}$ is positive, the gradient is a minimum.	M1 A1	
Question			Solution	Marks	Remarks
10	(a)		Method 1: $(x-2)^2 + (y-3)^2 - 2^2 - 3^2 - 12 = 0$ $(x-2)^2 + (y-3)^2 = 25$ Method 2: $2g = -4$ $2f = -6$ $g = -2$ $f = -3$ $g^2 + f^2 - c = 4 + 9 + 12 = 25$ Centre = (2, 3) Radius = 5 units	 M1 M1 M1 M1 A1 A1	
	(b)		When $y = 0$, $x^2 - 4x - 12 = 0$ $(x-6)(x+2) = 0$ $x = 6$ or $x = -2$ The points are (6, 0) and (-2, 0)	M1 A2	
	(c)		Centre = (-2, 3), radius = 5 units Eqn: $(x+2)^2 + (y-3)^2 = 25$	M1 A1 /B2	
	(d)		Since the centre is only 2 units away from the y-axis and the radius is 5 units, the circle will cut the y-axis and does not lie entirely in the 2nd quadrant.	M1 A1	Accept any similar answer (e.g. centre is 3 units away from x-axis)

11	(a)		$A = \pi r^2 + 2\pi kr + c$ where c is an arbitrary constant $When\ r = 0, A = 0$: (when there is no radius, there is no area) $c = 0$ Hence, $A = \pi r^2 + 2\pi kr$	M1 A1	
	(b)		Curved surface area: $2\pi rh = 2\pi kr$ $h = k$	B1	
	(c)		$\frac{dr}{dt} = \frac{5}{2}\ln(2t+1) + c_1$ where c_1 is an arbitrary constant $When\ t = 0, \frac{dr}{dt} = 3$: $3 = c_1$ $\frac{dr}{dt} = \frac{5}{2}\ln(2t+1) + 3$	M1 A1	
	(d)		$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $\frac{dA}{dt} = 2\pi(r+k) \times [\frac{5}{2}\ln(2t+1) + 3]$ When $r = 15, t = 4, k = 10$: $\frac{dA}{dt} = 2\pi(15+10) \times \left(\frac{5}{2}\ln 9 + 3\right)$ $\frac{dA}{dt} =$	M1 M1 A1	
[redacted]					
12	(a)		$4x + 4y = 100$ $y = 25 - x$	M1 A1	
	(b)		A $= x^2 + y^2$ $= x^2 + (25-x)^2$ $= 2x^2 - 50x + 625$ $= 2\left[\left(x - \frac{25}{2}\right)^2 - \left(\frac{25}{2}\right)^2 + \frac{625}{2}\right]$ $= 2\left(x - \frac{25}{2}\right)^2 + \frac{625}{2}$	 M1 M1 M1 A1	

	(c)		Min area = $\frac{625}{2}$ when $x = \frac{25}{2}$	B1 B1	
	(d)		$25 - x_1$ (E.g. if $x_1 = 10$, then $25 - x_1 = 15$ will give the same area)	B1	
13	(a)		$\left(\frac{3-7}{2}, \frac{2h-10}{2}\right)$ $= (-2, h-5)$	M1 A1	
	(b)		$\frac{2h+10}{3+7}$ $= \frac{h+5}{5}$	M1 A1	
	(c)		Let the equation of the perpendicular bisector be $\frac{y-(h-5)}{x-(-2)} = -\frac{5}{h+5}$ Sub (h, 3): $\frac{3-(h-5)}{h-(-2)} = -\frac{5}{h+5}$ $h^2 - 8h - 50 = 0$ $h = \frac{8 \pm \sqrt{8^2 - 4(1)(-50)}}{2(1)}$ $h = 12.1 \text{ or } -4.12 \ (3 \ s.f.)$	 M2 M1 M1 M1 A2	1 mark for substituting M 1 mark for gradient