Calculator Model: ___________

NAME:	CLASS:	INDEX NO:

QUEENSWAY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXP / 5NA

Parent's Signature:

ADDITIONAL MATHEMATICS

4049/01

Paper 1

21 August 2024
2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction tape.

Answer **all** the questions.
Give non-exact numerical values correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in bracket [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = cos^2 A - sin^2 A = 2cos^2 A - 1 = 1 - 2sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Given that $f(x) = 3x^5 - 11x^3 + 30x^2 + 39 = (x-1)(x+3)Q(x) + ax + b$ for all values of x and that $Q(x)$ is a polynomial, find

(a) the values of a and of b. [5]

(b) the remainder when $f(x) - 3$ is divided by $x^2 + 2x - 3$. [2]

2 **(a)** Water is poured into a jar at a rate of 35 cm^3/s. The volume of water in the jar is V cm^3, where $V = 3\left(\frac{h^2}{4} + \frac{8\pi}{h^3}\right)$ and h is the height of water in the jar. Find the rate at which the height of the water is increasing when $h = 4\ cm$. [3]

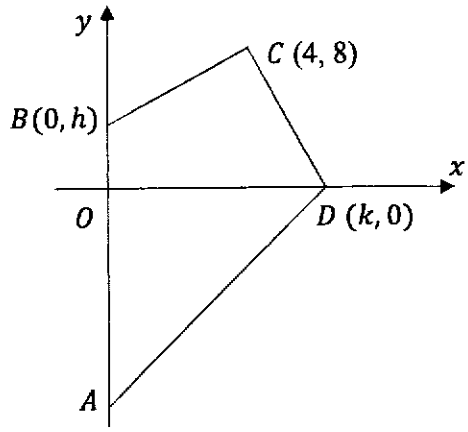
(b) **(i)** Find the set of values of x for which $y = \frac{2-5x}{e^x}$ is a decreasing function of x. [3]

(ii) Find the gradient of the curve $y = \frac{2-5x}{e^x}$ at the point where it cuts the x- axis. [2]

3 **(a)** Express $\frac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

3 **(b)** Hence, find $\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

4



The diagram shows a kite $ABCD$ in which $AB = AD$ and $BC = CD$, Point A and B lie on the y-axis and D lies on the x-axis. The coordinates of C is $(4, 8)$, B is $(0, h)$ and D is $(k, 0)$, where h and k are positive constants.

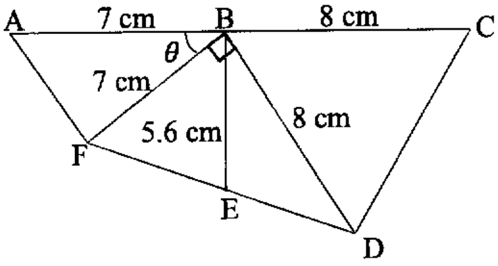
(a) Show that $h^2 - k^2 = 16h - 8k$. [2]

(b) It is now given that $h = 1$.

(i) Find the coordinates of A. [4]

(ii) Find the area of the kite. [2]

5



In the diagram, angle $ABF = \theta$, angle $FBD = 90°$. ABC is a straight line and BE is perpendicular to AC. $AB = BF = 7$ cm, $BC = BD = 8$ cm and $BE = 5.6$ cm.

(a) Show that the area Q cm^2 of the quadrilateral $ACDF$ is given by $Q = 51.6\cos\theta + 46.9\sin\theta$. [3]

(b) Express $Q = 51.6\cos\theta + 46.9\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R is a positive constant and α is acute. [2]

(c) State the maximum value of Q and find the corresponding value of θ. [2]

(d) State the maximum value of $\frac{1}{Q^2+3}$. [1]

6 An insect leaves its nest and flies along in a straight path. The velocity, v m/s, that it flies in time, t s, after it leaves the nest is given by $v = 4e^{-t} - \frac{1}{2}e^{2t}$.

(a) Find the acceleration when $t = 0.5$. [2]

(b) Find the value of t when the acceleration is maximum. [3]

(c) Show that the insect is instantaneously at rest when $t = \ln k$ where k is an integer. [3]

(d) Find the total distance travelled by the insect between $t = 0$ and $t = 3$. [4]

7 **(a)** Prove the identity $\frac{\cot A - \tan A}{\cot A + \tan A} = 2cos^2 A - 1$. [4]

(b) Hence, solve $\frac{\cot A - \tan A}{\cot A + \tan A} = \cos A$ for $-\pi < A < \pi$. [4]

8 A curve is given by $y = \tan x$.

(a) Find $\frac{dy}{dx}$. [1]

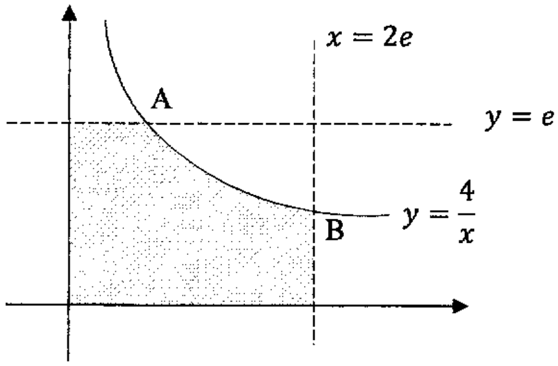
(b) Hence, find the equation of the tangent to the curve $y = \tan x$ at the point where $x = \frac{\pi}{6}$, leaving your answer in terms of π. [3]

(c) Find the value of $\frac{d^2y}{dx^2}$ when $x = \frac{\pi}{6}$. [2]

(d) A student concludes from part (c) that since the value of $\frac{d^2y}{dx^2} > 0$ at $x = \frac{\pi}{6}$, hence there is a minimum point at $x = \frac{\pi}{6}$. State with reason and mathematical working whether the above conclusion is correct. [2]

9 Evaluate $\int_0^{\frac{4}{3}}(3x^2 - 16x + 16)\,dx + \int_{\frac{4}{3}}^{4}(3x^2 - 16x + 16)\,dx$. Hence, state what could be deduced about the graph $y = 3x^2 - 16x + 16$ between $x = 0$ and $x = 4$? [4]

10 The diagram shows part of the curve $y = \frac{4}{x}$. The curve $y = \frac{4}{x}$ cuts the line $y = e$ at A and the line $x = 2e$ at B.



(a) Find the coordinates of A and of B. [2]

(b) Find the area bounded by the curve $y = \frac{4}{x}, y = e, x = 2e$, the y-axis and the x-axis, leaving your answer in the form $a + \ln b$, where a and b are integers. [4]

(c) Explain why the area of the shaded region is between 4 units^2 and $2e^2$ units^2. [2]

11 The table below shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	14.1	26.0	40.0	55.9	73.5	92.6

It is known that x and y are related by an equation of the form $y = A(1+x)^n$, where A and n are constants.

(a) Plot $\lg y$ against $\lg(1+x)$and draw a straight line graph on the grid given in the next page. [3]

(b) Use your graph to estimate the values of A and of n. [3]

(c) On the same diagram, draw a straight line representing the equation $y = \frac{16}{1+x}$. [2]

(d) Hence, find the value of x for which $A(1+x)^{n+1} = 16$. [2]

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2024 4E5N Prelim P1 Solutions

No.	Solution
1(a)	$f(x) = 3x^5 - 11x^3 + 30x^2 + 39 = (x-1)(x+3)Q(x) + ax + b$ When $x = 1$, $61 = a + b$ ----- eqn (1) When $x = -3$, $-123 = -3a + b$ ----- eqn (2) (1) – (2) $184 = 4a$ $a = 46$ $b = 15$
1(b)	$f(x) = (x-1)(x+3)Q(x) + 46x + 15$ $f(x) - 3 = (x-1)(x+3)Q(x) + 46x + 15 - 3$ Remainder = $46x + 15 - 3$ $= 46x + 12$
2(a)	$V = 3\left(\frac{h^2}{4} + \frac{8\pi}{h^3}\right)$ $\frac{dV}{dh} = \frac{3}{2}h - \frac{72\pi}{h^4}$ When $h = 4$, $\frac{dV}{dh} = 6 - \frac{9\pi}{32}$ $= \frac{192-9\pi}{32}$ $(or\ \ 5.1164)$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\frac{dh}{dt} = \frac{32}{192-9\pi} \times 35$ $= \frac{1120}{192-9\pi}\ cm/s$ $(or\ \ 6.84\ cm/s)$
2(bi)	$y = \frac{2-5x}{e^x}$ $\frac{dy}{dx} = \frac{e^x(-5)-(2-5x)e^x}{(e^x)^2}$ $= \frac{5x-7}{e^x}$ For decreasing function, $\frac{5x-7}{e^x} < 0$ $x < \frac{7}{5}$ $(or\ \ 1.4)$
2(bii)	When $y = 0$, $2 - 5x = 0$ $x = 0.4$ $\frac{dy}{dx} = -3.35\ (3s.f)$
3(a)	$\frac{1-3x-3x^2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $1 - 3x - 3x^2 = A(x+1)^2 + Bx(x+1) + Cx$ When $x = -1$, $C = -1$ When $x = 0$, $A = 1$ When $x = 1$, $B = -4$ $\frac{1-3x-3x^2}{x(x+1)^2} = \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2}$

3(b)	$\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx = \frac{1}{2}\int \frac{1-3x-3x^2}{x(x+1)^2}\,dx$ $= \frac{1}{2}\int \frac{1}{x} - \frac{4}{x+1} - \frac{1}{(x+1)^2}\,dx$ $= \frac{1}{2}\ln x - 2\ln(x+1) + \frac{1}{2(x+1)} + c$ [Accept $\ln\sqrt{x} - \ln(x+1)^2 + \frac{1}{2(x+1)} + c$]
4(a)	$4^2 + (h-8)^2 = (k-4)^2 + 8^2$ $16 + h^2 - 16h + 64 = k^2 - 8k + 16 + 64$ $h^2 - 16h = k^2 - 8k$ $h^2 - k^2 = 16h - 8k$ (shown)
4(bi)	When $h = 1$, $1 - k^2 = 16 - 8k$ $k^2 - 8k + 15 = 0$ $(k-5)(k-3) = 0$ $k = 5 \quad or\ k = 3\ (rejected\ based\ on\ diagram)$ Let $A(0, y)$ $(1-y)^2 = 5^2 + y^2$ $1 - 2y + y^2 = 25 + y^2$ $y = -12$ $\therefore A(0, -12)$
4(bii)	Area $= \frac{1}{2}\begin{vmatrix} 0 & 5 & 4 & 0 & 0 \\ -12 & 0 & 8 & 1 & -12 \end{vmatrix}$ $= \frac{1}{2}\lvert 44 - (-60)\rvert$ $= 52\ units^2$
5(a)	Area $= \frac{1}{2}(7)(7)\sin\theta + \frac{1}{2}(7)(5.6)\sin(90-\theta)$ $+ \frac{1}{2}(5.6)(8)\sin\theta + \frac{1}{2}(8)(8)\sin(90-\theta)$ $= \frac{49}{2}\sin\theta + \frac{98}{5}\cos\theta + \frac{112}{5}\sin\theta + 32\cos\theta$ $= 51.6\cos\theta + 46.9\sin\theta$ (shown)
5(b)	$Q = 51.6\cos\theta + 46.9\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{51.6^2 + 46.9^2}$ $= 69.729$ $\tan\alpha = \frac{46.9}{51.6}$ $\alpha = 42.268$ $\therefore 51.6\cos\theta + 46.9\sin\theta = 69.7\cos(\theta - 42.3°)$
5(c)	Max value of Q = 69.7 $\cos(\theta - 42.268°) = 1$ $\theta = 42.268$ Corresponding value= 42.3°
5(d)	maximum value of $\frac{1}{Q^2+3} = \frac{1}{0+3}$ $= \frac{1}{3}$
	:

6(a)	$v = 4e^{-t} - \frac{1}{2}e^{2t}$ $a = -4e^{-t} - e^{2t}$ When $t = 0.5, a = -5.14\ m/s^2$
6(b)	$\frac{da}{dt} = 4e^{-t} - 2e^{2t}$ When $\frac{da}{dt} = 0,\quad 4e^{-t} = 2e^{2t}$ $e^{3t} = 2$ $t = \frac{1}{3}\ln 2$ $\frac{d^2a}{dt^2} = -4e^{-t} - 4e^{2t}$ When $t = \frac{1}{3}\ln 2,\quad \frac{d^2a}{dt^2} < 0$ (max)
6(c)	When $v = 0$, $4e^{-t} = \frac{1}{2}e^{2t}$ $e^{3t} = 8$ $t = \frac{1}{3}\ln 8$ $= \ln 8^{\frac{1}{3}}$ $= \ln 2$ (shown)
6(d)	$s = -4e^{-t} - \frac{1}{4}e^{2t} + c$ When $t = 0, s = 0, \therefore c = \frac{17}{4}$ $s = -4e^{-t} - \frac{1}{4}e^{2t} + \frac{17}{4}$ When $t = 0, s = 0$ When $t = \ln 2, s = 1.25$ When $t = 3, s = -96.806$ Total distance travelled $= 1.25 + (96.806 + 1.25)$ $= 99.3m$
7(a)	$\frac{\cot A - \tan A}{\cot A + \tan A} = 2\cos^2 A - 1$ $LHS = \frac{\cot A - \tan A}{\cot A + \tan A}$ $= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}$ $= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$ $= \cos^2 A - \sin^2 A$ $= \cos^2 A + \cos^2 A - 1$ $= 2\cos^2 A - 1$ $= RHS$ Alternative $LHS = \frac{\cot A - \tan A}{\cot A + \tan A}$ $= \frac{\frac{1}{\tan A} - \tan A}{\frac{1}{\tan A} + \tan A}$ $= \frac{1 - \tan^2 A}{1 + \tan^2 A}$ $= \frac{1 - \tan^2 A}{\sec^2 A}$ $= \cos^2 A - \sin^2 A$ $= 2\cos^2 A - 1$ $= RHS$

7(b)	$\frac{\cot A-\tan A}{\cot A+\tan A}=\cos A \qquad -\pi<A<\pi$ $2cos^2A-1=\cos A$ $(2\cos A+1)(\cos A-1)=0$ $\cos A=-\frac{1}{2} \qquad or \quad \cos A=1$ $ref\ angle=\frac{\pi}{3} \quad or\ ref\ angle=0$ $A=\frac{2\pi}{3},\frac{-2\pi}{3} \qquad or \qquad A=0$ $A=\frac{-2\pi}{3},0,\frac{2\pi}{3}$
8(a)	$y=\tan x$ $\frac{dy}{dx}=sec^2x$
8(b)	When $x=\frac{\pi}{6}, \quad y=\frac{\sqrt{3}}{3}, \quad \frac{dy}{dx}=\frac{4}{3}$ $y=mx+c$ $\frac{\sqrt{3}}{3}=\frac{4}{3}\left(\frac{\pi}{6}\right)+c$ $c=\frac{\sqrt{3}}{3}-\frac{2\pi}{9}$ $y=\frac{4}{3}x+\frac{\sqrt{3}}{3}-\frac{2\pi}{9}$ Accept $\left(y=\frac{4}{3}x+\frac{3\sqrt{3}-2\pi}{9}\right)$ or $\left(9y=12x+3\sqrt{3}-2\pi\right)$
8(c)	$\frac{d^2y}{dx^2}=-2cos^{-3}x(-\sin x)$ $=\frac{2\sin x}{cos^3x}$ At $x=\frac{\pi}{6},\frac{d^2y}{dx^2}=1.5396$ $=1.54\ (3s.f)$
8(d)	$\frac{dy}{dx}=sec^2x$ When $\frac{dy}{dx}=0, \quad \frac{1}{cos^2x}=0$ Since $sec^2x=0$ is not defined, ∴ the above conclusion is wrong.
9	$\int_0^{\frac{4}{3}}(3x^2-16x+16)\,dx+\int_{\frac{4}{3}}^{4}(3x^2-16x+16)\,dx$ $=[x^3-8x^2+16x]_0^{\frac{4}{3}}+[x^3-8x^2+16x]_{\frac{4}{3}}^{4}$ $=\frac{256}{27}+\left(0-\frac{256}{27}\right)$ $=0$ The **area above the x-axis**, bounded from $x=0$ to $x=\frac{4}{3}$ is the **same** as the **area below the x-axis,** bounded from $x=\frac{4}{3}$ to $x=4$.

10(a)

When $y = e,\ e = \frac{4}{x}$

$x = \frac{4}{e}$

$\therefore A\left(\frac{4}{e}, e\right)$

When x= $2e,\ y = \frac{4}{2e}$

$y = \frac{2}{e}$

$\therefore B\left(2e, \frac{2}{e}\right)$

10(b)

$Area = \int_{\frac{4}{e}}^{2e} \frac{4}{x}\, dx \ + \left(\frac{4}{e}\right)(e)$

$= 4[\ln x]_{\frac{4}{e}}^{2e} + 4$

$= 4\left[\ln 2e - \ln\frac{4}{e}\right] + 4$

$= 4[\ln 2 + \ln e - \ln 4 + \ln e] + 4$

$= 4[\ln 2 - \ln 4] + 12$

$= 4[-\ln 2] + 12$

$= 12 - \ln 16$

10(c)

Area of rect from from y-axis to A = $e \times \frac{4}{e}$

$= 4\ units^2$

Area of whole rect= $2e \times e$

$= 2e^2\ units^2$

$\therefore 4 < area\ of\ shaded\ region < 2e^2$ (explained)

11(a)

Refer to graph.

$y = A(1+x)^n$

$\lg y = \lg A + n\lg(1+x)$

lg(1+x)	0.301	0.477	0.602	0.699	0.778	0.845
lg y	1.15	1.41	1.60	1.75	1.87	1.97

Correct points plotted

Straight line

Plot table

11(b)

From the graph,

$\lg A = 0.7$ [Accept $0.68 \le \lg A \le 0.72$]

$A = 5.01$

$n = grad = 1.51$ [Accept $1.49 \le n \le 1.51$]

11(c)

$y = \frac{16}{1+x}$

$\lg y = \lg 16 - \lg(1+x)$

Draw the line $\lg y = \lg 16 - \lg(1+x)$

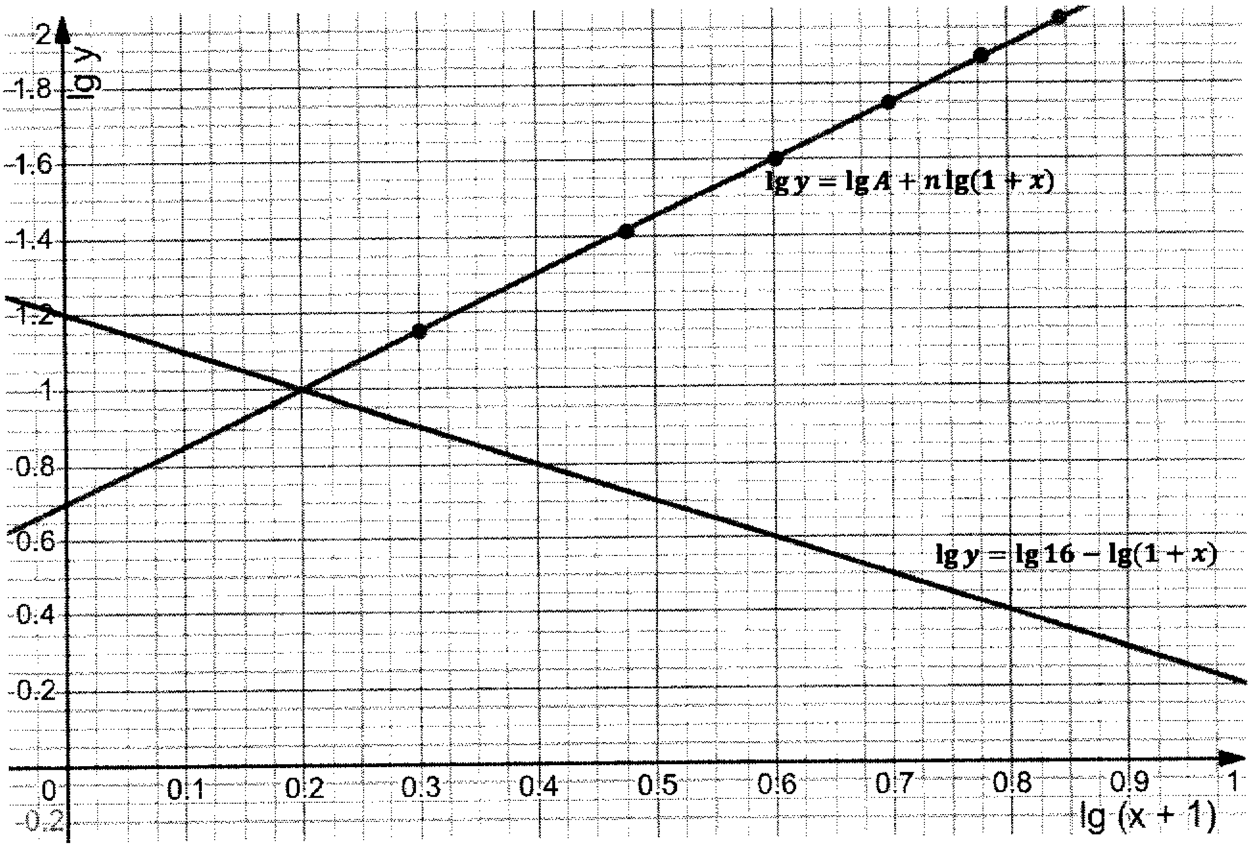
11(d)

$A(1+x)^{n+1} = 16$

$A(1+x)^n = \frac{16}{1+x}$

$\therefore \lg(x+1) = 0.2$

$x = 0.585$


lg y
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
lg (x + 1)
lg y = lg A + n lg(1 + x)
lg y = lg 16 − lg(1 + x)

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