KENT RIDGE SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2024

ADDITIONAL MATHEMATICS **4049/02**
PAPER 2

SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC

Monday 26 August 2024 **2 hour 15 minutes**

Name: ______________________________ (   ) **Class: Sec** __________

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.

Do not open this question paper until you are told to do so.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue, correction fluid or correction tape.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Total	/ 90

This Question Paper consists of 19 printed pages, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

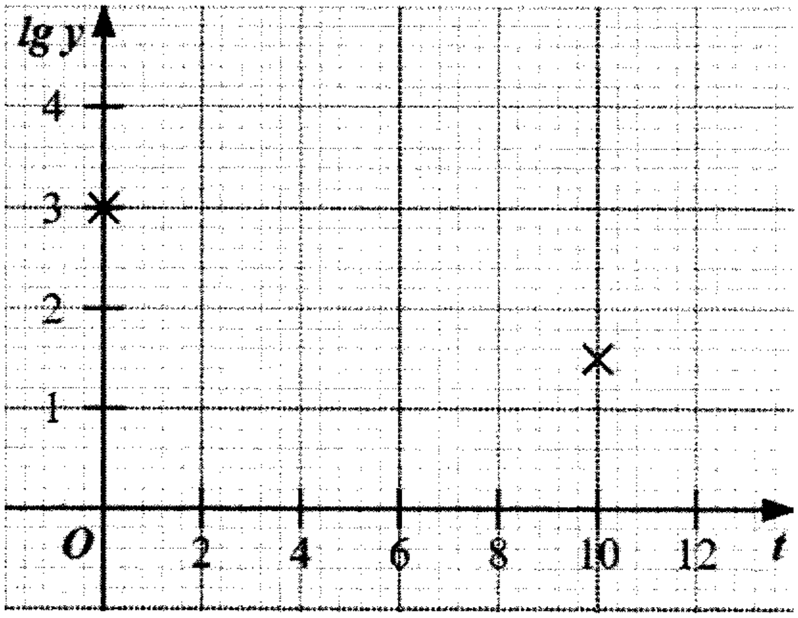
1 Express $\frac{4}{(x^2+1)(x+1)}$ in partial fractions. [5]

2 (a) Radiation intensity, R, varies inversely with the square of d, the distance from the source of radiation such that $R = \frac{k}{d^2}$, where k is a constant.

Values of R for different values of d have been collected and tabulated.
Explain how a straight-lined graph can be drawn to determine the formula connecting R and d. [4]

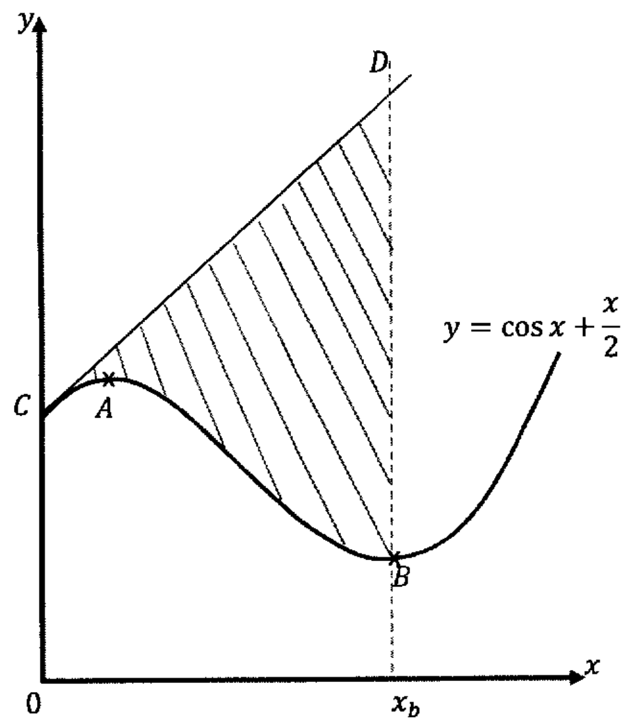
(b) The number of particles present in a room, t minutes after turning on the air filter is y. When corresponding values of $\lg y$ and t are plotted on a $\lg y$ against t axes, the points form a straight line that passes through $(0,3)$ and $(10,1.5)$ as drawn on the axes on the next page.

(i) Find y in terms of t. [4]



(ii) Use the graph to estimate the time taken for the number of particles in the room to be halved. [3]

3 (a) The diagram below shows the graph of the curve $y = \cos x + \frac{x}{2}$ for $x \geq 0$ radians.
The tangent to the curve when $x = 0$ at C, is drawn to D which is vertically above point B, the minimum point of the curve.
Points A and B are the first two stationary points of the curve.
Find x_b, the x coordinates of point B.
You do not need to show that it is a minimum point. [4]



(b) (i) Find the equation of CD. [2]

(ii) Find the area shaded that is bounded by the tangent to the curve $y = \cos x + \frac{x}{2}$ at $x = 0$, the curve and the line $x = x_b$. [5]

4 (a) $2y = 16x + k$ is a tangent to the curve $y = \frac{1}{2x} + 2kx$. Find the value of constant k. [4]

(b) Find the range of values of a such that $ax^2 + \sqrt{8}x + (a-1) < 0$ for all values of x. [4]

5 Given $y = e^{2x} \sin 3x$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find $\frac{d^2y}{dx^2}$. [2]

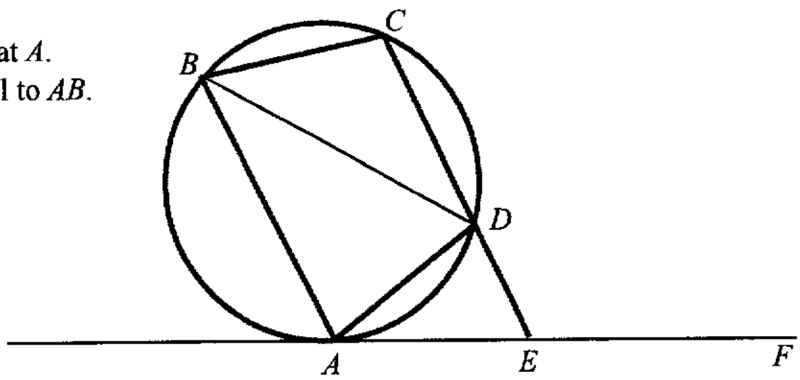
(c) Given that $\frac{dy}{dx} + \frac{d^2y}{dx^2} + ay = be^{2x}\cos 3x$, form 2 equations involving a and b and use them to find the value of a and of b. [4]

6 (a) Show that $\frac{d}{dx}\left(\frac{x-2}{\sqrt{3x+1}}\right) = \frac{3x+8}{2\sqrt{(3x+1)^3}}$. [4]

(b) Hence evaluate $\int_0^5 \frac{3x+7}{2\sqrt{(3x+1)^3}}\,dx$. [5]

For continuation of working for question 6 part (b)

7 AF is a tangent to the circle $ABCD$ at A.
E is on AF such that EDC is parallel to AB.



(a) Prove that triangle ABD and triangle DAE are similar. [3]

(b) Show that if triangle BCD and triangle DAB are similar, BD must be the diameter of the circle. [3]

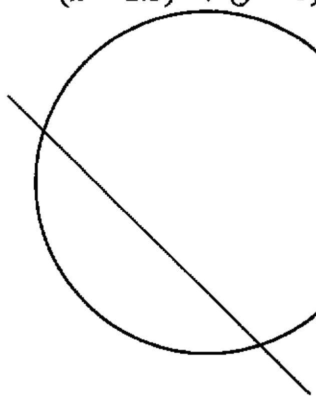
8 (a) Solve $3(3^{x+1}) = 10 - 3^{-x}$. [4]

(b) Given $\log_{100} x + \lg y = 3$, express y in terms of x. [4]

9 The chord AB of a circle C has equation $y = -\frac{1}{2}x + 10$, where the x coordinate of A is smaller than the x coordinate of B.

The circle C has equation $(x - 2.5)^2 + (y - 5)^2 = \frac{365}{4}$ with centre E.

(a) Find the coordinates of A. [4]



(b) State the centre of circle C, and use it to show that the perpendicular bisector of AB passes through the origin. [4]

(c) The chord AB is extended to cut the x-axis at point D. Show that the mid-point of AD lies inside circle C. [4]

10 A particle starts moving in a straight line when it is 6 metres from a fixed point O, such that its velocity, t seconds after the start of the motion is given by $v = 4e^{-2t} + t - 3$ m/s.

(a) Find the initial velocity of the particle. [2]

(b) Show that the minimum velocity is negative, and it happens when $t = \frac{1}{2}\ln 8$. [4]

(c) Using your answer from part (a) and part (b), explain if the particle changes its direction of motion. [2]

(d) Find the displacement of the particle from O, 2 seconds after the start of the motion. [4]

End of Paper

Kent Ridge Secondary School
Secondary 4 Express/5 Normal Academic Preliminary Examination 2024
Add Math Prelim 2024 P2 Mark scheme

Qn	Solutions	Marks	
1a	$\frac{4}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$ $\frac{4}{(x^2+1)(x+1)} = \frac{(Ax+B)(x+1)}{(x^2+1)(x+1)} + \frac{C(x^2+1)}{(x^2+1)(x+1)}$ $4 = (Ax+B)(x+1) + C(x^2+1)$ Sub x = -1 $4 = C((-1)^2+1)$ $C = 2$ Sub x = 0 $4 = (B)(1) + C(1)$ $B = 2$ Compare coef of x^2: $A + C = 0$ $A = -2$ $\frac{4}{(x^2+1)(x+1)} = \frac{2-2x}{x^2+1} + \frac{2}{x+1}$	A1 M1 M1 M1 A1	
2a	Plot points of **corresponding values of R and $\frac{1}{d^2}$** Draw best fit line through points and the **origin** Find **gradient** of the line gives the **value of *k***	B1 B1 B1 B1	
2bi	Gradient of line $= \frac{1.5}{-10} = -0.15$ Lg y intercept = 3 $\lg y = -0.15t + 3$ $y = 10^{-0.15t+3}$	M1 B1 M1 A1	
2bii	Initial number of particles : $\lg y = 3$ $y = 10^3 = 1000$ Find the point on the straight line when $\lg y = \lg 500 = 2.69$ The time taken is the t value of the point	 M1 M1 B1 – their t value (± 0.4)	
3a	$\frac{dy}{dx} = -\sin x + \frac{1}{2}$ $-\sin x + \frac{1}{2} = 0$ $\sin x = \frac{1}{2}$ $\alpha = \frac{\pi}{6}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{5\pi}{6}$	M1 A1 M1 A1	
3bi	Gradient of tangent at x = 0 $\frac{dy}{dx} = -\sin 0 + \frac{1}{2} = \frac{1}{2}$ $y = \cos 0 + 0 = 1$ Equation of tangent $y = \frac{1}{2}x + 1$	 M1 M1	

Qn	Solutions	Marks	
3bii	$Area = \int_0^{\frac{5\pi}{6}} \frac{1}{2}x + 1 - \cos x - \frac{x}{2}\, dx$ $= [x - \sin x]_0^{\frac{5\pi}{6}}$ $= \frac{5\pi}{6} - \sin\left(\frac{5\pi}{6}\right)$ $= \frac{5\pi}{6} - \frac{1}{2} = 2.12\ (3\ s.f.)$	M1 – mtd to find area trap under tangent M1 – definite integral of curve from 0 to x_b A1 – correct expr of their integrals A1 – correct sub of limits A1	
4a	$\frac{k}{2} + 8x = \frac{1}{2x} + 2kx$ $kx + 16x^2 = 1 + 4kx^2$ $(4k - 16)x^2 - kx + 1 = 0$ $k^2 - 4(4k - 16)(1) = 0$ $k^2 - 16k + 64 = 0$ $k = 8$	M1 M1 M1 A1	
4b	Discriminant: $\sqrt{8}^2 - 4a(a-1) < 0$ $8 - 4a^2 + 4a < 0$ $a^2 - a - 2 > 0$ $(a-2)(a+1) > 0$ Since $a < 0$, $a < -1$	M1 – expr for D M1 – condition for D B1 A1	
5a	$y = e^{2x}\sin 3x$ $\frac{dy}{dx} = 2e^{2x}\sin 3x + 3e^{2x}\cos 3x$	M1 either term seen A1 use of product rule and final ans	
5b	$\frac{d^2y}{dx^2} = 2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) + 3(2e^{2x}\cos 3x - 3e^{2x}\sin 3x$	M1 use of at one correct product rule of their dy/dx	
5c	$\frac{d^2y}{dx^2} = -5e^{2x}\sin 3x + 12e^{2x}\cos 3x$ $2e^{2x}\sin 3x + 3e^{2x}\cos 3x - 5e^{2x}\sin 3x + 12e^{2x}\cos 3x + ae^{2x}\sin 3x = be^{2x}\cos 3x$ $2 - 5 + a = 0$ $3 + 12 = b$ $a = 3, b = 15$	A1 M1 M1 M1 A1	

Qn	Solutions	Marks	
6	$\frac{d}{dx}\left(\frac{x-2}{\sqrt{3x+1}}\right)=\frac{\sqrt{3x+1}-\frac{3(x-2)}{2\sqrt{3x+1}}}{3x+1}$	M1 – quotient rule seen with positive sq root or product seen with negative sq root	
	$=\frac{\frac{2(3x+1)}{2\sqrt{3x+1}}-\frac{3(x-2)}{2\sqrt{3x+1}}}{3x+1}$	M1 – simplify with common denominator or taking out common factor	
	$=\frac{3x+8}{2\sqrt{3x+1}(3x+1)}$	M1 – all factors in denominator collected	
	$=\frac{3x+8}{2\sqrt{(3x+1)^3}}$	B1	
6b	$\int_{x_1}^{x_2}\frac{3x+8}{2\sqrt{(3x+1)^3}}dx=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}$	M1 – seen or implied	
	$\int_{x_1}^{x_2}\frac{3x+7}{2\sqrt{(3x+1)^3}}dx+\int_{x_1}^{x_2}\frac{1}{2\sqrt{(3x+1)^3}}dx=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}$		
	$\int_{x_1}^{x_2}\frac{3x+7}{2\sqrt{(3x+1)^3}}dx$	M1 – any equivalent form To show 7 = 8-1 or 8 = 7+ 1	
	$=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}-\int_{x_1}^{x_2}\frac{1}{2\sqrt{(3x+1)^3}}dx$		
	$=\left[\frac{x-2}{\sqrt{3x+1}}\right]_{x_1}^{x_2}-\frac{1}{2}\int_{x_1}^{x_2}(3x+1)^{-\frac{3}{2}}dx$	M1 – standard integral	
	$=\left[\frac{x-2}{\sqrt{3x+1}}\right]_0^5-\left[\frac{1}{2}\frac{(3x+1)^{-\frac{1}{2}}}{3\left(-\frac{1}{2}\right)}\right]_0^5$	M1 – show the correct limits substituted into a valid integral	
	$=\frac{3}{4}-\frac{-2}{1}-\left(-\frac{1}{3(4)}+\frac{1}{3(1)}\right)$		
	$=2\frac{1}{2}$	A1	
7a	$\angle DAE=\angle ABD$ (angles in alternate segment)	B1	
	$\angle ADE=\angle BAD$ (alternate angles of parallel lines)	B1	
	triangle ABD is similar to triangle DAE (AA similarity)	B1	
7b	$\angle BAD=\angle DCB$ (corresponding angles of similar triangles)	B1	
	$\angle BAD+\angle DCB=180°$ (angles in opposite segment)	B1	

Qn	Solutions	Marks	
	$\angle BAD = \angle DCB = 90°$ BD is diameter (angle in semicircle = 90°)	B1	
8a	$3(3^{x+1}) = 10 - 3^{-x}$ Let $u = 3^x$ $3(3u) = 10 - \frac{1}{u}$ $9u^2 - 10u + 1 = 0$ $3^x = \frac{1}{9}$ or $3^x = 1$ $x = -2$ or 0	M1 – breakdown 3^{x+1} M1 – general QE M1 – eqn in x A1	
8b	$\log_{100} x + \lg y = 3$ $\frac{\lg x}{\lg 100} + \lg y = 3$ $\frac{\lg x}{2} + \lg y = 3$ $\lg \sqrt{x} + \lg y = 3$ $\lg \sqrt{x}\, y = 3$ $\sqrt{x}y = 10^3$ $y = \frac{1000}{\sqrt{x}}$	M1 – change base M1 – step before simplifying to one log term M1 – one log term A1	
9a	$(x - 2.5)^2 + (-\frac{1}{2}x + 5)^2 = \frac{365}{4}$ $x^2 - 5x + 6.25 + \frac{1}{4}x^2 - 5x + 25 = \frac{365}{4}$ $\frac{5}{4}x^2 - 10x + 31.25 = \frac{365}{4}$ $5x^2 - 40x + 125 = 365$ $5x^2 - 40x - 240 = 0$ $x = 12, x = -4$ $A(-4, 12)$	M1v - substitution M1 – general QE A1 A1	
9b	centre of circle (2.5,5) $y = 2x + c$ Sub centre of circle (2.5,5) $5 = 2(2.5) + c$ $c = 0$	B1 M1 – grad ⊥ seen B1 A1	
9c	Sub y = 0 into AB $0 = -\frac{1}{2}x + 10$ $x = 20$ D(20,0) M, Mid point AD = (8,6) Distance ME² = $(8 - 2.5)^2 + (6 - 5)^2 = \frac{125}{4} < \frac{365}{4}$	M1 M1 M1 M1	
10a	Sub t = 0, v = 1	B1, B1	
10b	$a = -8e^{-2t} + 1 = 0$ $e^{-2t} = \frac{1}{8}$	M1 M1	

Qn	Solutions	Marks	
	$$-2t = \ln\frac{1}{8}$$ $$-2t = \ln 1 - \ln 8 = -\ln 8$$ $$t = \frac{1}{2}\ln 8$$ $$v = 4e^{-\ln 8} + \frac{1}{2}\ln 8 - 3 = -1.46$$	B1 A1	
10c	Since **velocity changes from positive to negative**, the particle **did change** its direction of motion	B1 B1	
10d	$$s = -2e^{-2t} + \frac{t^2}{2} - 3t + c$$ Sub t= 0, s = 6 $$6 = -2 + c$$ $$c = 8$$ Sub t= 2 $s = -2e^{-4} + 2 - 6 + 8 = 3.96$m	M1 integrate exp term M1 integrate power term M1 A1	