JC 2 PRELIMINARY EXAMINATION
in preparation for General Certificate of Education Advanced Level
Higher 1

CANDIDATE NAME	
CIVICS GROUP	
INDEX NUMBER	

Mathematics **8864/01**

Paper 1 **22 August 2016**

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

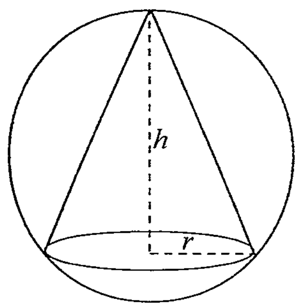
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Innova Junior College

Section A: Pure Mathematics [35 marks]

1 Find, algebraically, the set of values of k for which the equation $x^2 - 10x + k^2 = 4$ has real roots. [3]

2 [It is given that a cone of radius r and height h has volume $\frac{1}{3}\pi r^2 h$.]



A toy company wants to design a toy with a swirling right circular cone with base radius r cm and height h cm inscribed in a sphere of radius 3.5 cm as shown in the diagram. The volume of the right circular cone is V cm^3.

(i) Show that $r = \sqrt{(7h - h^2)}$. [1]

(ii) Find an expression for V in terms of h. [1]

(iii) Use differentiation to find the maximum value of V, justifying that this value is a maximum. [4]

3 **Do not use a graphic calculator in answering this question.**

The equation of a curve is $y = \ln\left(\frac{3x-7}{x+1}\right)$. The curve cuts the x-axis at the point A.

(i) Find the x-coordinate of A. [2]

(ii) Show that $\frac{dy}{dx}$ can be expressed as $\frac{k}{(3x-7)(x+1)}$, where k is an integer to be determined. [3]

(iii) Find the equation of the normal at A. [3]

4 The curve C_1 has equation $y=\dfrac{1}{2x-5}$. The curve C_2 has equation $y=\sqrt{(x-2)}$.

(i) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [3]

(ii) State the coordinates of the point of intersection of C_1 and C_2. [1]

(iii) Find the area bounded by C_1, C_2 and the line $x=5$. [2]

(iv) Find the value of the positive constant a such that the area of the region bounded by C_2, the x-axis and the line $x=a$ is $\dfrac{2}{3}$ units^2. [3]

5 **(a)** Find $\displaystyle\int \frac{2}{(4+3x)^6}\,\mathrm{d}x$. [2]

(b) The gradient of a curve $y=\mathrm{f}(x)$ at any point (x,y) is given by $\mathrm{f}'(x)=3\mathrm{e}^{-x}-2\mathrm{e}^{2x-1}$.

(i) Find the exact x-coordinate of the stationary point on the curve. [3]

(ii) Given that the curve intersects the y-axis at the point where $y=1$, find the equation of the curve, giving your answer in exact form. [4]

Section B: Statistics [60 marks]

6 A large condominium estate has 1050 residents. Its management committee wishes to carry out a survey involving 2% of its residents to find out their opinions of the activities it has organised for the last 2 years.

Describe how the sample could be chosen using systematic sampling. [2]

Name a more appropriate sampling method, and explain how it can be carried out to provide the representative sample that the committee wants. [2]

7 The mean and variance of a binomial random variable X are denoted by μ and σ^2 respectively. Given that $\mu = 12$ and $\sigma^2 = 4.8$, find $P(\mu - 2\sigma < X < \mu + 2\sigma)$. [4]

8 A company supplies rice to a hypermarket in big bags. The mass of rice in a bag has mean 10 kg and standard deviation 0.5 kg. A large random sample of n bags of rice is taken.

(i) Find the least value of n such that the estimated probability that the mean mass of a bag of rice from this sample is less than 9.9 kg is at most 0.04. [4]

(ii) State, giving a reason, whether it is necessary to assume the mass of a bag of rice follows a normal distribution. [1]

9 The brisk walking speeds, w ms^{-1}, of 6 adult males of different ages, t years, in a particular city are given in the table below.

t	20	30	40	50	60	70
w	2.49	2.41	2.38	2.14	1.97	2.03

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Calculate the product moment correlation coefficient. [1]

(iii) **(a)** Find the equation of the regression line of w on t in the form $w = mt + c$, giving the values of m and c correct to 4 significant figures. Sketch this line on your scatter diagram. [2]

(b) Give an interpretation, in context, of the value of m. [1]

(iv) Find $\bar{t}$ and $\bar{w}$, and mark the point $(\bar{t}, \bar{w})$ on your scatter diagram. [2]

(v) Use the equation of your regression line to calculate an estimate of the brisk walking speed of a 55-year-old adult male. Comment on the reliability of your estimate. [2]

10 A factory manufactures paperweights consisting of glass mounted on a wooden base. The volume of glass, in cm^3, in a randomly chosen paperweight has a normal distribution with mean 58.6 and standard deviation 2.1. The volume of the wooden base, in cm^3, has an independent normal distribution with mean 38.4 and standard deviation 3.3.

(i) Find the probability that two paperweights selected at random have volumes of glass which differ by at least $0.06\ cm^3$. [3]

(ii) Find the probability that the total volume of three randomly chosen paperweights is more than $300\ cm^3$. [3]

The glass weighs 4.4 grams per cm^3 and the wood weighs 0.88 grams per cm^3.

(iii) Find the probability that the total weight of a randomly chosen paperweight is between 280 and 290 grams. [3]

11 In a particular city, 85% of the people watch the movie *Star Wars* and 55% of the people watch the movie *Aliens*. 60% of those who do not watch the movie *Star Wars* watch the movie *Aliens*.

A person is selected at random from the city.

(i) Show that the probability that the person selected watches the movie *Aliens* only is 0.09. [1]

(ii) Find the probability that the person selected watches either the movie *Star Wars* or the movie *Aliens* (or both). [2]

(iii) Find the probability that the person selected does not watch the movie *Aliens* given that he does not watch the movie *Star Wars*. [3]

(iv) State, with a reason, whether or not the events "the person selected watches the movie *Star Wars*" and "the person selected watches the movie *Aliens*" are mutually exclusive. [2]

12 In Alpha Junior College, 68% of its students enjoy watching movies.

(i) Find the probability that, in a random sample of 20 students, more than 13 of them enjoy watching movies. [2]

(ii) The probability that there is at least one student who enjoys watching movies out of a random sample of n students is greater than 0.98. Find the least value of n. [4]

(iii) Use a suitable approximation to find the probability that, out of 120 randomly chosen students, more than 75 but less than 85 of them enjoy watching movies. You should state the mean and variance of the distribution used in the approximation. [4]

13 The "walking age" of toddlers is a measure of when toddlers take their first steps.

(a) The walking ages, in months, of a random sample of 90 toddlers taken from City A are summarised by

$$\sum x = 970.02, \qquad \sum x^2 = 11\,326.$$

(i) Calculate unbiased estimates of the population mean and variance. [2]

The mean walking age of a toddler is claimed to be 11.2 months.

(ii) Test at the 5% level of significance, whether the sample indicates that this claim is incorrect. [4]

(b) A podiatric researcher measures the walking ages of a random sample of 20 toddlers from City B. She carries out a test, at the 5% significance level, of whether the mean walking age of a toddler in this city is less than 11.2 months. She assumes that the walking ages of toddlers are normally distributed with standard deviation 10.3 months.

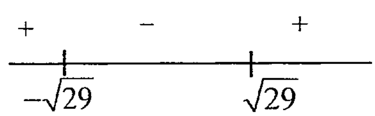
(i) State appropriate hypotheses for the test. [1]

The sample mean walking age is k months.

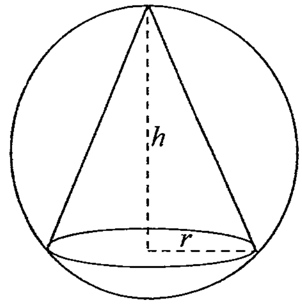
(ii) Find the set of values of k for which the null hypothesis would not be rejected. [3]

2016 IJC Preliminary Examinations – H1 Math

1. Find, algebraically, the set of values of k for which the equation $x^2 - 10x + k^2 = 4$ has real roots. [3]

1	$x^2 - 10x + k^2 = 4$ To have real roots, $D \geq 0$ or $b^2 - 4ac \geq 0$ $(-10)^2 - 4(1)(k^2 - 4) \geq 0$ $100 - 4k^2 + 16 \geq 0$ $k^2 \leq 29$ $(k + \sqrt{29})(k - \sqrt{29}) \leq 0$  $\therefore \{k \in \mathbb{R} : -\sqrt{29} \leq k \leq \sqrt{29}\}$

2 [It is given that a cone of radius r and height h has volume $\frac{1}{3}\pi r^2 h$.]

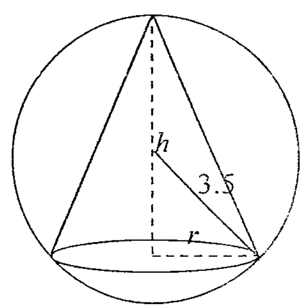


A toy company wants to design a toy with a swirling right circular cone with base radius r cm and height h cm inscribed in a sphere of radius 3.5 cm as shown in the diagram. The volume of the right circular cone is V cm^3.

(i) Show that $r = \sqrt{(7h - h^2)}$. [1]

(ii) Find an expression for V in terms of h. [1]

(iii) Use differentiation to find the maximum value of V, justifying that this value is a maximum. [4]

2	(i) Using Pythagoras Theorem, $r = \sqrt{3.5^2 - (h - 3.5)^2}$ $= \sqrt{\frac{49}{4} - \left(h^2 - 7h + \frac{49}{4}\right)}$ $= \sqrt{7h - h^2}$ $(\because r > 0)$  (ii) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(7h - h^2\right)h$ $= \frac{1}{3}\pi\left(7h^2 - h^3\right)$

(iii)

$$V = \frac{1}{3}\pi\left(7h^2 - h^3\right)$$

$$\frac{dV}{dh} = \frac{1}{3}\pi\left[14h - 3h^2\right]$$

$$= \frac{1}{3}\pi h\left(14 - 3h\right)$$

$$\frac{dV}{dh} = 0 \Rightarrow h = 0 \left(\text{N.A.} \because h \neq 0\right) \text{ or } h = \frac{14}{3}$$

h	$\frac{14}{3}^-$	$\frac{14}{3}$	$\frac{14}{3}^+$
$\frac{dV}{dh}$	+	0	−
Slope	/	—	\

$\therefore$ Vol is max when $h = \frac{14}{3}$ cm.

Max volume $= \frac{1}{3}\pi\left(\frac{14}{3}\right)^2\left(7 - \frac{14}{3}\right) = \frac{1372}{81}\pi$ cm^3

$= 53.2$ cm^3 (3 s.f.)

3 **Do not use a graphic calculator in answering this question.**

The equation of a curve is $y=\ln\left(\frac{3x-7}{x+1}\right)$. The curve cuts the x-axis at the point A.

(i) Find the x-coordinate of A. [2]

(ii) Show that $\frac{dy}{dx}$ can be expressed as $\frac{k}{(3x-7)(x+1)}$, where k is an integer to be determined. [3]

(iii) Find the equation of the normal at A. [3]

3(i)	$y=\ln\left(\frac{3x-7}{x+1}\right)$. When $y=0$, $\ln\left(\frac{3x-7}{x+1}\right)=0=\ln 1$ $\frac{3x-7}{x+1}=1$ $3x-7=x+1$ $x=4$
(ii)	$y=\ln\left(\frac{3x-7}{x+1}\right)=\ln(3x-7)-\ln(x+1)$ $\therefore\ \frac{dy}{dx}=\frac{3}{3x-7}-\frac{1}{x+1}$ $=\frac{3(x+1)-(3x-7)}{(3x-7)(x+1)}$ $=\frac{10}{(3x-7)(x+1)}$ $\therefore\ k=10$.
(iii)	At point $A(4,0)$, $\frac{dy}{dx}=\frac{10}{(12-7)(5)}=\frac{2}{5}$ Gradient of the normal at $A=-\frac{5}{2}$ Equation of the normal at A is: $y-0=-\frac{5}{2}(x-4)$ $y=-\frac{5}{2}x+10$.

4. The curve C_1 has equation $y=\dfrac{1}{2x-5}$. The curve C_2 has equation $y=\sqrt{(x-2)}$.

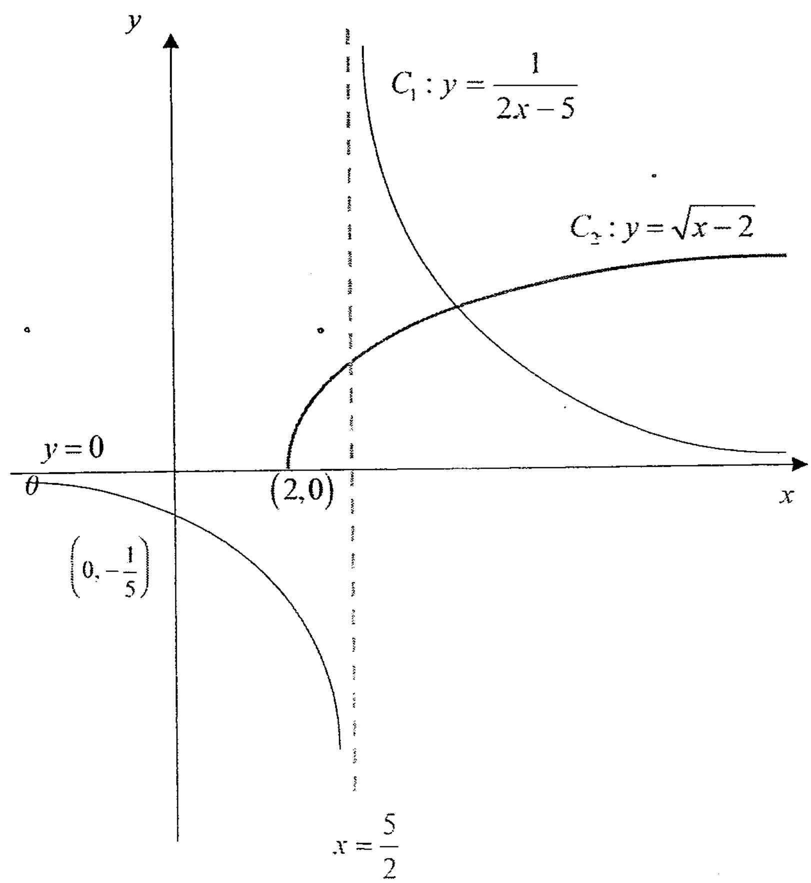
(i) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any points of intersection with the axes and the equation of any asymptotes. [3]

(ii) State the coordinates of the point of intersection of C_1 and C_2. [1]

(iii) Find the area bounded by C_1, C_2 and the line $x=5$. [2]

(iv) Find the value of the positive constant a such that the area of the region bounded by C_2, the x-axis and the line $x=a$ is $\dfrac{2}{3}$ units^2. [3]

(i)



(ii)
Using GC.
coordinates is (3.1)

(iii)

Area

$=\int_3^5 \sqrt{x-2}-\dfrac{1}{2x-5}\,\mathrm{d}x$

$=1.9927$

$=1.99\text{ units}^2$ (to 3 s.f.)

(iv)

$\int_2^a \sqrt{x-2}\ \mathrm{d}x=\dfrac{2}{3}$.........(*)

$\dfrac{2}{3}\left[(x-2)^{\frac{3}{2}}\right]_2^a=\dfrac{2}{3}$

$\left[(x-2)^{\frac{3}{2}}\right]_2^a=1$

$(a-2)^{\frac{3}{2}}-(2-2)^{\frac{3}{2}}=1$

$(a-2)^{\frac{3}{2}}=1$

$\therefore a=3$

5 (a) Find $\int \frac{2}{(4+3x)^6}\,dx$. [2]

(b) The gradient of a curve $y=f(x)$ at any point (x,y) is given by $f'(x)=3e^{-x}-2e^{2x-1}$.

(i) Find the exact x-coordinate of the stationary point on the curve. [3]

(ii) Given that the curve intersects the y-axis at the point where $y=1$, find the equation of the curve, giving your answer in exact form. [4]

5	(a) $\int \frac{2}{(4+3x)^6}\,dx$ $=2\int (4+3x)^{-6}\,dx$ $=2\left[\frac{(4+3x)^{-5}}{(3)(-5)}\right]+C$ $=-\frac{2}{15(4+3x)^5}+C$ (b)(i) At stationary point, $\frac{dy}{dx}=0$ $3e^{-x}-2e^{2x-1}=0$ $3e^{-x}=2e^{2x-1}$ $\frac{3}{2}=\frac{e^{2x-1}}{e^{-x}}$ $\frac{3}{2}=e^{2x-1+x}$ $\frac{3}{2}=e^{3x-1}$ ....(*) $\ln\frac{3}{2}=3x-1$ $\therefore x=\frac{1}{3}\left(\ln\frac{3}{2}+1\right)$

(b)(ii)

$$y = \int 3e^{-x} - 2e^{2x-1}\, dx$$
$$= 3\int e^{-x}\, dx - 2\int e^{2x-1}\, dx$$
$$= 3\left(\frac{e^{-x}}{-1}\right) - 2\left(\frac{e^{2x-1}}{2}\right) + C$$
$$= -3e^{-x} - e^{2x-1} + C$$

When the curve intersects y-axis, $x = 0$

$\therefore$ At $(0,1)$:

$$1 = -3e^{0} - e^{-1} + C$$
$$1 = -3 - \frac{1}{e} + C$$
$$C = 4 + \frac{1}{e}$$

Equation of curve:

$$y = -3e^{-x} - e^{2x-1} + 4 + \frac{1}{e}$$

6 A large condominium estate has 1050 residents. Its management committee wishes to carry out a survey involving 2% of its residents to find out their opinions of the activities it has organised for the last 2 years.

Describe how the sample could be chosen using systematic sampling. [2]

Name a more appropriate sampling method, and explain how it can be carried out to provide the representative sample that the committee wants. [2]

6	Obtain a list of the residents and arrange them in some order and number them from 1 to 1050 $\textit{Sample size} = 2\% \times 1050 = 21$ $k = \frac{1050}{21} = 50$ or $k = \frac{100\%}{2\%} = 50$ **<u>Randomly</u>** <u>select a number 1 to 50</u> inclusive and select the first resident attached to this number. Select every 50th resident thereafter to obtain a sample of 21 residents for the survey. A more appropriate sampling method is **stratified sampling**. This can be done by selecting **random** samples by grouping the residents into different **non-overlapping age group strata**. Draw random samples separately from each age group, with the sample size proportional to the relative size of each age group. These random samples are put together to form a stratified sample of the residents. This method is able to provide a more representative sample of that the committee wants.

7 The mean and variance of a binomial random variable X are denoted by μ and σ^2 respectively. Given that $\mu = 12$ and $\sigma^2 = 4.8$, find $\mathrm{P}(\mu - 2\sigma < X < \mu + 2\sigma)$. [4]

7	$X \sim \mathrm{B}(n, p)$. When $\mu = 12$ and $\sigma^2 = 4.8$, $np = 12$ -------- (1) $np(1-p) = 4.8$ --- (2) $\frac{(2)}{(1)}$: $\frac{np(1-p)}{np} = \frac{4.8}{12}$ $1 - p = 0.4$ $\therefore p = 0.6$ Substitute $p = 0.6$ into (1): $n(0.6) = 12$ $\therefore n = 20$ Thus, $X \sim \mathrm{B}(20, 0.6)$ $\mathrm{P}(\mu - 2\sigma < X < \mu + 2\sigma)$ $= \mathrm{P}(12 - 2\sqrt{4.8} < X < 12 + 2\sqrt{4.8})$ $= \mathrm{P}(7.6182 < X < 16.382)$ $= \mathrm{P}(8 \le X \le 16)$ $= \mathrm{P}(X \le 16) - \mathrm{P}(X \le 7)$ $= 0.963$ (to 3 s.f.)

8 A company supplies rice to a hypermarket in big bags. The mass of rice in a bag has mean 10 kg and standard deviation 0.5 kg. A large random sample of n bags of rice is taken.

(i) Find the least value of n such that the estimated probability that the mean mass of a bag of rice from this sample is less than 9.9 kg is at most 0.04. [4]

(ii) State, giving a reason, whether it is necessary to assume the mass of a bag of rice follows a normal distribution. [1]

8	
	Let X kg be the r.v. denoting the mass of rice in a bag. (i) $\overline{X} \sim \mathrm{N}\left(10, \frac{0.5^2}{n}\right)$ approximately by Central Limit Theorem, since n is large. $\mathrm{P}\left(\overline{X} < 9.9\right) \le 0.04$ $\mathrm{P}\left(Z < \frac{9.9-10}{\frac{0.5}{\sqrt{n}}}\right) \le 0.04$ $\frac{-0.1}{\frac{0.5}{\sqrt{n}}} \le -1.75069$ $\frac{0.1\sqrt{n}}{0.5} \ge 1.75069$ i.e. $\sqrt{n} \ge \frac{(0.5 \times 1.75069)}{0.1}$ $n \ge 76.62289$ Hence least value of n is 77. **<u>Alternative Method</u>** $\overline{X} \sim \mathrm{N}\left(10, \frac{0.5^2}{n}\right)$ approximately by Central Limit Theorem, since n is large. From GC,

n	$\mathrm{P}\left(\overline{X} < 9.9\right)$
76	0.04062 (> 0.04)
77	0.03963(< 0.04)
78	0.03867 (< 0.04)

Hence least value of n is 77.

(ii) Not necessary, since sample size n is large, Central Limit Theorem can be applied.

9 The brisk walking speeds, w ms^{-1}, of 6 adult males of different ages, t years, in a particular city are given in the table below.

t	20	30	40	50	60	70
w	2.49	2.41	2.38	2.14	1.97	2.03

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Calculate the product moment correlation coefficient. [1]

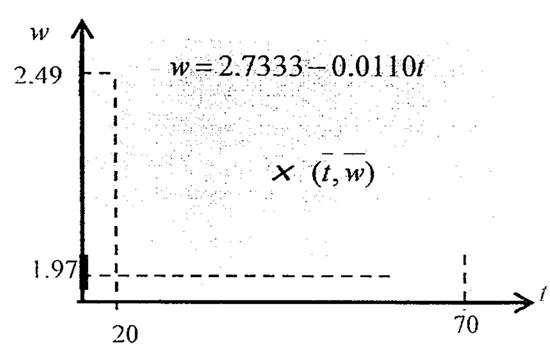
(iii) **(a)** Find the equation of the regression line of w on t in the form $w = mt + c$, giving the values of m and c correct to 4 significant figures. Sketch this line on your scatter diagram. [2]

(b) Give an interpretation, in context, of the value of m. [1]

(iv) Find $\bar{t}$ and $\bar{w}$, and mark the point $(\bar{t}, \bar{w})$ on your scatter diagram. [2]

(v) Use the equation of your regression line to calculate an estimate of the brisk walking speed of a 55-year-old adult male. Comment on the reliability of your estimate. [2]

9(i)



(ii) $r = -0.94582 = -0.946$ (3 s.f.)

(iii)(a)
$w = 2.73295 - 0.0110286t$

$w = 2.733 - 0.01103t$ (4 s.f.)

$\therefore m = -0.01103 \quad c = 2.733$ (4 s.f.)

(iii)(b)
For every year an adult male gets older, his brisk walking speed decreases by 0.011 m/s.

(iv) From GC, $\bar{t} = 45, \bar{w} = 2.23667 \approx 2.24$

Point $(\bar{t}, \bar{w}) = (45, 2.24)$ marked correctly on the scatter diagram and must lie on the regression line.

(v)
When $t = 55$,

$$w = 2.73295 - 0.01103(55)$$
$$= 2.1263 = 2.13 \ (3 \text{ s.f.})$$

Estimated brisk walking speed 2.13 metres per second.

Since $r = -0.94582$ is close to -1 and t = 55 lies within the data range of $20 \leq t \leq 70$, the estimate is reliable.

10 A factory manufactures paperweights consisting of glass mounted on a wooden base. The volume of glass, in cm^3, in a randomly chosen paperweight has a normal distribution with mean 58.6 and standard deviation 2.1. The volume of the wooden base, in cm^3, has an independent normal distribution with mean 38.4 and standard deviation 3.3.

(i) Find the probability that two paperweights selected at random have volumes of glass which differ by at least $0.06\ cm^3$. [3]

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The glass weighs 4.4 grams per cm^3 and the wood weighs 0.88 grams per cm^3.

(iii) Find the probability that the total mass of a randomly chosen paperweight is between 280 and 290 grams. [3]

10(i)	Let $X\ cm^3$ and $Y\ cm^3$ be the volume of glass and wooden base in a paperweight respectively. Then $X \sim N(58.6, 2.1^2)$, $Y \sim N(38.4, 3.3^2)$. Let $X_1 - X_2 \sim N(0, 2.1^2 + 2.1^2)$ i.e., $X_1 - X_2 \sim N(0, 8.82)$ Similarly, $X_2 - X_1 \sim N(0, 8.82)$ Required probability $= P(X_1 - X_2 \geq 0.06 \text{ or } X_2 - X_1 \geq 0.06)$ $= 0.49194 + 0.49194$ $= 0.984$ (3 s.f.)
(ii)	Let $T = X + Y \sim N(58.6+38.4, 2.1^2+3.3^2)$ i.e., $T \sim N(97, 15.3)$ Total volume of 3 paperweights, $S = T_1 + T_2 + T_3 \sim N(3\times97, 3\times15.3)$ i.e. $S \sim N(291, 45.9)$ Required probability $= P(S > 300) = 0.0920$ (3 s.f.)
(iii)	Let W grams be the weight of a paperweight. Then $W = 4.4X + 0.88Y$ $E(W) = (4.4)(58.6) + 0.88(38.4) = 291.632$ $Var(W) = 4.4^2(2.1^2) + 0.88^2(3.3^2) = 93.811$ Then $W \sim N(291.632, 93.811)$ $P(280 < W < 290) = 0.318$ (3 s.f.)

11 In a particular city, 85% of the people watch the movie *Star Wars* and 55% of the people watch the movie *Aliens*. 60% of those who do not watch the movie *Star Wars* watch the movie *Aliens*.

A person is selected at random from the city.

(i) Show that the probability that the person selected watches the movie *Aliens* only is 0.09. [1]

(ii) Find the probability that the person selected watches either the movie *Star Wars* or the movie *Aliens* (or both). [2]

(iii) Find the probability that the person selected does not watch the movie *Aliens* given that he does not watch the movie *Star Wars*. [3]

(iv) State, with a reason, whether or not the events "the person selected watches the movie *Star Wars*" and "the person selected watches the movie *Aliens*" are mutually exclusive. [2]

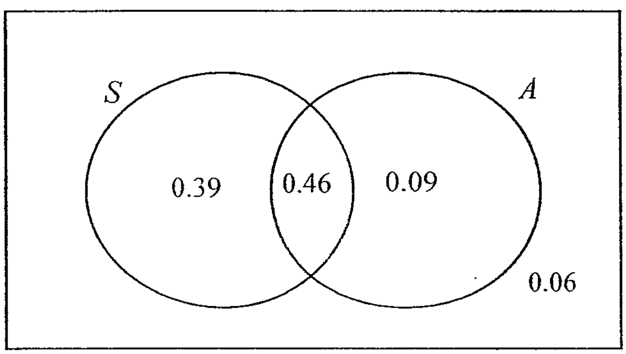
11

Let S and A denote the events "the person selected watches the movie Star Wars" and "the person selected watches the movie Aliens" respectively.

(i)

P(the person selected watches the movie *Aliens* only)

$= \mathrm{P}(S' \cap A) = 0.6 \times (1 - 0.85) = 0.09$ (shown)



11(ii)

Required probability $= \mathrm{P}(S \cup A)$

$$= \mathrm{P}(S) + \mathrm{P}(S' \cap A)$$
$$= 0.85 + 0.09$$
$$= 0.94$$

	11(iii) P(does not watch *Aliens* | does not watch *Star Wars*) $=\dfrac{P(A'\cap S')}{P(S')}$ $=\dfrac{1-P(A\cup S)}{1-0.85}$ $=\dfrac{1-0.94}{0.15}$ $=0.4$ Alternative method: P(does not watch *Aliens* | does not watch *Star Wars*) $=\dfrac{P(A'\cap S')}{P(S')}$ $=\dfrac{0.06}{0.09+0.06}$ (using Venn Diagram) $=0.4$
	(iv) $P(S\cap A)=0.55-0.09=0.46$ Since $P(S\cap A)\neq 0$, therefore events S and A are not mutually exclusive.

12 In Alpha Junior College, 68% of its students enjoy watching movies.

(i) Find the probability that, in a random sample of 20 students, more than 13 of them enjoy watching movies. [2]

(ii) The probability that there is at least one student who enjoys watching movies out of a random sample of n students is greater than 0.98. Find the least value of n. [4]

(iii) Use a suitable approximation to find the probability that, out of 120 randomly chosen students, more than 75 but less than 85 of them enjoy watching movies. You should state the mean and variance of the distribution used in the approximation. [4]

12	12(i) Let X be the number of students who enjoy watching movies, out of 20 students. $X \sim B(20, 0.68)$ $P(X > 13) = 1 - P(X \leq 13)$ $= 0.53072$ $= 0.531$ (to 3 s.f.) 12(ii) Let Y be the number of students who enjoy watching movies, out of n students. $Y \sim B(n, 0.68)$ $P(Y \geq 1) > 0.98$ $1 - P(Y < 1) > 0.98$ $1 - P(Y = 0) > 0.98$ $P(Y = 0) < 0.02$ Using GC, when $n = 3$, $P(Y = 0) = 0.03277 (> 0.02)$ when $n = 4$, $P(Y = 0) = 0.01049 (< 0.02)$ when $n = 5$, $P(Y = 0) = 0.00336 (< 0.02)$ $\therefore$ least value of $n = 4$

	Alternative method

$\mathrm{P}(Y \geq 1) > 0.98$

$1 - \mathrm{P}(Y < 1) > 0.98$

$1 - \mathrm{P}(Y = 0) > 0.98$

$\mathrm{P}(Y = 0) < 0.02$

$\binom{n}{0} 0.68^0 (1 - 0.68)^{n-0} < 0.02$

$(1 - 0.68)^n < 0.02$

$0.32^n < 0.02$

$n \ln 0.32 < \ln 0.02$

$n > \dfrac{\ln 0.02}{\ln 0.32}$

$n > 3.43$

$\therefore$ least value of $n = 4$

12(iii)

Let W be the number of students who enjoy watching movies, out of 120 students.

$W \sim \mathrm{B}(120, 0.68)$

Since $n = 120$ is large,

$np = 120 \times 0.68 = 81.6 > 5$ and

$nq = 120 \times 0.32 = 38.4 > 5$

$W \sim \mathrm{N}(81.6, 26.112)$ approximately

$\mathrm{P}(75 < W < 85)$

$= \mathrm{P}(75.5 < W < 84.5)$ (with continuity correction)

$= 0.599$ (to 3 s.f.)

13 The "walking age" of toddlers is a measure of when toddlers take their first steps.

(a) The walking ages, in months, of a random sample of 90 toddlers taken from City A are summarised by

$$\sum x = 970.02, \qquad \sum x^2 = 11\,326.$$

(i) Calculate unbiased estimates of the population mean and variance. [2]

The mean walking age of a toddler is claimed to be 11.2 months.

(ii) Test at the 5% level of significance, whether the sample indicates that this claim is incorrect. [4]

(b) A podiatric researcher measures the walking ages of a random sample of 20 toddlers from City B. She carries out a test, at the 5% significance level, of whether the mean walking age of a toddler in this city is less than 11.2 months. She assumes that the walking ages of toddlers are normally distributed with standard deviation 10.3 months.

(i) State appropriate hypotheses for the test. [1]

The sample mean walking age is k months.

(ii) Find the set of values of k for which the null hypothesis would not be rejected. [3]

13	(a)(i) Unbiased estimate of population mean, $\bar{x} = \frac{\sum x}{n} = \frac{970.02}{90} = 10.778$ Unbiased estimate of population variance, $s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right)$ $= \frac{1}{89}\left(11326 - \frac{(970.02)^2}{90}\right)$ $= 9.78792 = 9.79$ (3 s.f.)

(a)(ii)

Let X months be the r.v. denoting the walking age of a toddler and μ months be the population mean walking age.

To test $\quad H_0 : \mu = 11.2$

against $\quad H_1 : \mu \neq 11.2$ at 5% level of significance

Since n is large, under H_0, approximately by Central Limit Theorem,

Test statistic: $Z = \dfrac{\bar{X} - 11.2}{\sqrt{\dfrac{9.7879}{90}}} \sim N(0,1)$

Using G.C, p-value = 0.20067= 0.201 (3s.f)

Since p-value = 0.201 > 0.05, do not reject H_0 and conclude that there is insufficient evidence to conclude at 5% level of significance that the toddlers' walking ability is not equal to 11.2 months.

(b)(i)

To test $\quad H_0 : \mu = 11.2$

against $\quad H_1 : \mu < 11.2$

(b)(ii)

At 5% level of significance

Under H_0, $Z = \dfrac{\bar{X} - 12}{\dfrac{10.3}{\sqrt{20}}} \sim N(0,1)$

We do not reject H_0 if $z > -1.64485$

$$\frac{k - 11.2}{\dfrac{10.3}{\sqrt{20}}} > -1.64485$$

$$k - 11.2 > -1.64485\left(\frac{10.3}{\sqrt{20}}\right)$$

$$k > 7.4117$$

Set of values of k = $\{k \in \mathbb{R} : k > 7.41\}$ (3 s.f.)