Name:__________________________ **Class:**_____________

JURONG JUNIOR COLLEGE

JC2 Preliminary Examinations 2018

MATHEMATICS
Higher 2

9758/01

28 August 2018

Paper 1

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

1 To watch the daily matches of a badminton tournament in a stadium, each spectator is to buy either a daily normal ticket for an adult spectator or a daily concession ticket for a student spectator. Tickets are purchased from either ticket booth A, B or C. The number of tickets sold and the total amount of money collected by each ticket booth are shown in the following table.

Ticket booth	Number of daily normal tickets sold	Number of daily concession tickets sold	Total amount collected
A	$5n$	n	\$5976
B	$7n$	$2n$	\$8712
C	357	51	\$5763

Find the price of each daily normal ticket and each daily concession ticket and determine the value of n. [4]

2 The curve C has equation $y=\dfrac{ax^2+bx+c}{x+d}$, where $x\in\mathbb{R}$, $x\neq -d$ and a, b, c and d are constants.

It is given that C has stationary points at $x=0$ and $x=-2$. The lines $x=-1$ and $y=x$ are asymptotes to C.

(i) Write down the value of d, and determine the values of a, b and c. [6]

With the values of a, b, c and d found in **(i)**,

(ii) find the range of values that y can take using an algebraic method, [4]

(iii) sketch the graph of $y=\dfrac{x+d}{ax^2+bx+c}$, indicating clearly the coordinates of the points where the graph crosses the axes, the turning points and the equations of any asymptotes. [3]

3 **(i)** Find $\displaystyle\int \frac{x}{\left(4+3x^2\right)^2}\,\mathrm{d}x$. [2]

(ii) Hence find the exact value of $\displaystyle\int_0^{\frac{2}{\sqrt{3}}} \frac{2x^2}{\left(4+3x^2\right)^2}\,\mathrm{d}x$. [4]

4 A curve C has parametric equations

$$x = \theta - \sin\theta, \quad y = 1 - \cos\theta, \quad \text{for } 0 < \theta < 2\pi.$$

(i) Find the equation of the tangent that is parallel to the x-axis. [3]

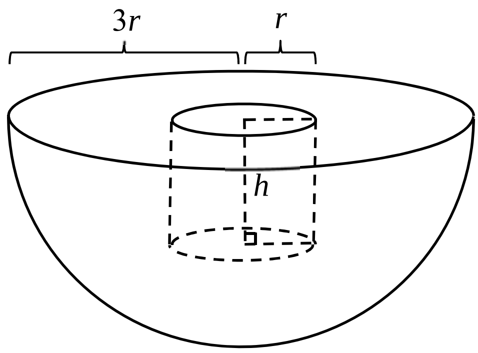
(ii) The normal to the curve at the point with parameter $\frac{2\pi}{3}$ meets the x- and y-axes at P and Q respectively. Show that the equation of the normal is $y = -\sqrt{3}x + \frac{2\sqrt{3}\pi}{3}$.

Hence find the exact area of the triangle OPQ. [5]

(iii) Given that θ is increasing at a rate of 2 radians per second, find the rate of change of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$. [3]

5 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$.]

A toy is constructed from a hemisphere with radius $3r$ cm by removing a circular cylinder of radius r cm and height h cm where $h < 3r$ as shown in the diagram below. As r and h vary, the total cost of coating the surface with a protective film on the entire toy is a constant \$$C$. The cost of coating on the flat surfaces is \$$k$ per cm^2 and that on the curved surfaces is \$$2k$ per cm^2, where k is a positive constant.



Show that the volume, V cm^3, of the toy is

$$V = \frac{117}{4}\pi r^3 - \frac{Cr}{4k}. \quad [3]$$

(i) Find the value of r in terms of C and k which gives a stationary value of V. [2]

(ii) Find also the ratio of the height to the radius, $\frac{h}{r}$, in this case, simplifying your answer. [2]

(iii) Explain why it is not possible for this toy to have a stationary value of V. [1]

6 The position vectors of points A, B and C of a triangle are **a**, **b** and **c** respectively, relative to an origin O.

(i) By considering the area of triangle ABC, show that the shortest distance from B to AC is

$$\frac{|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}|}{|\mathbf{c}-\mathbf{a}|}.$$ [4]

R is a point on AB such that $\overrightarrow{AR}=\frac{1}{3}\overrightarrow{AB}$. S is a point on AC such that $\overrightarrow{AS}=\frac{2}{3}\overrightarrow{AC}$. $OACB$ is a kite with $OA=OB$, $CA=CB$ and OC is perpendicular to AB.

(ii) Show that $\angle SRA=90°$. [6]

7 **(a)** A function f is said to be self-inverse if $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$ for all x in the domain of f.

The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x+k}{x-b},\ x\in\mathbb{R},\ x\neq b,$$ where k and b are constants.

(i) Find the value of b and the set of values of k such that f is self-inverse. [3]

Using the value of b found in (i), another function g is defined such that

$$\mathrm{fg}: x \mapsto 2x-1,\ x\neq 2.$$

(ii) Find in terms of k, an expression for $\mathrm{g}(x)$. [2]

(b) The function h is defined as follows:

$$\mathrm{h}(x)=\begin{cases}-4x+8, & \text{for } 1\le x<2,\\ -x^2+8x-12, & \text{for } 2\le x<4,\end{cases}$$

and that $\mathrm{h}(x+3)=\mathrm{h}(x)$ for all real values of x.

(i) Sketch the graph of $y=\mathrm{h}(x)$ for $-4\le x\le 6$, indicating the axial intercepts and endpoints clearly. [3]

(ii) Find $\int_{-\frac{3}{2}}^{3}\mathrm{h}(x)\,\mathrm{d}x$. [3]

8 When a plague of locusts attacks a wheat crop, the proportion of the crop destroyed after t hours is denoted by x. In a model, it is assumed that the rate at which the crop is destroyed is proportional to $x(1-x)$. A plague of locusts is discovered in a wheat crop when one-third of the crop has been destroyed and the rate of destruction at this instant is $\frac{1}{6}$.

(i) Show that $\frac{\mathrm{d}x}{\mathrm{d}t} = kx(1-x)$, where k is a constant to be determined. [3]

(ii) Find the percentage of the crop destroyed two hours after the plague of locusts is first discovered. [9]

9 **Do not use a calculator in answering this question.**

(a) Find the roots of the equation $z^2 + (\mathrm{i}-4)z + (6-2\mathrm{i}) = 0$, giving your answers in cartesian form $a + \mathrm{i}b$. [2]

(b) The complex number w has modulus r and argument θ, where $0 < \theta < \frac{\pi}{2}$, and w^* denotes the conjugate of w. State the modulus and argument of p, where $p = \frac{w}{w^*}$. [2]

Given that p^6 is real and positive, find the possible values of θ. [3]

(c) The polynomial $\mathrm{P}(z)$ of degree 4 has real coefficients. Two of the roots of the equation $\mathrm{P}(z) = 0$ are $z = 1 + \mathrm{i}$ and $z = 2$.

(i) State the number of complex roots of $\mathrm{P}(z) = 0$, justifying your answer. [1]

(ii) By expressing $\mathrm{P}(z)$ as a product of linear factors, find the remaining roots of the equation $\mathrm{P}(z) = 0$ given that $\mathrm{P}(\mathrm{i}) = 10 + 10\mathrm{i}$. [5]

10 Building contractors are constructing a rock climbing wall at the corner wall of a gymnasium. Points (x, y, z) are defined relative to a ground anchor point at $(0,0,0)$, where units are metres.

Support beams are laid in straight lines and the thickness of the support beams and rock climbing wall can be neglected.

The three support beams of the rock climbing wall, S_1, S_2 and S_3 start at the ground anchor point and go in the direction $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\1\\0\end{pmatrix}$, and $\begin{pmatrix}0\\0\\1\end{pmatrix}$ respectively. The support beams S_1 and S_2 are on the ground level. The vertices A, B and C of the rock climbing wall lie on the support beams S_1, S_2 and S_3 respectively. The rock climbing wall lies on the plane π with vector equation $\mathbf{r} = \begin{pmatrix}1\\5\\-1\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\-12\end{pmatrix} + \mu\begin{pmatrix}1\\2\\-7\end{pmatrix}$, where λ, $\mu \in \mathbb{R}$.

(i) Find the cartesian equation of the plane π and hence show that the coordinates of A are $(4,0,0)$. [4]

One of the building safety standards stipulates that the rock climbing wall should be inclined to the horizontal ground at an acute angle not exceeding $80°$.

(ii) Determine if this building safety standard is met. [3]

For additional stability, a fourth support beam from the ground anchor point to a point N on the rock climbing wall is laid. This support beam is the shortest in length.

(iii) Find the coordinates of N and the exact length of this support beam. [5]

Name:__________________________ **Class:**_____________

JURONG JUNIOR COLLEGE

JC2 Preliminary Examinations 2018

MATHEMATICS
Higher 2

9758/02

12 September 2018

Paper 2

3 hours

Additional materials: Answer Paper
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Section A: Pure Mathematics [40 marks]

1 Given that $f(x) = e^{\sin x}$, use the standard series to find the series expansion for $f(x)$ in the form $a + bx + cx^2 + dx^3$, where a, b, c and d are constants to be determined.

Hence show that the first three non-zero terms for the expansion of $\frac{1}{\left(e^{\sin x}\right)^2}$ in ascending powers of x is $1 - 2x + 2x^2$. [4]

The function $y = g(x)$ satisfies $4\frac{dy}{dx} = (y+1)^2$ and $y = 1$ at $x = 0$.

(i) By further differentiation, find the series expansion for $g(x)$, up to and including the term in x^3.

Hence show that when x is small,

$$g(x) - f(x) \approx \frac{1}{4}x^3.$$ [5]

(ii) By using the result in **(i)**, justify whether $f(x)$ is a good approximation to $g(x)$ for values of x close to zero. [1]

2 In 2004, 10 000 cases of obesity among Singaporeans aged 18-30 years old were reported. Each year after that, the number of cases reported increased by 7%. If this pattern were to continue, how many obesity cases would be reported in 2018? Leave your answer to the nearest whole number. [3]

John, who was obese, started on a weight-loss programme. The number of calories he burned in the first week of his exercise regime was a. As the intensity of the exercise regime increased, the number of calories John burned each week was increased by d. On the other hand, the number of calories John consumed each week is a geometric sequence such that the numbers of calories he consumed in the first, second and third week equal the numbers of calories he burned through exercising in the seventh, third and first week respectively.

(i) Show that $d = \frac{a}{2}$. [2]

(ii) John burned 3000 calories in the seventh week through exercising. Find the least number of weeks required for the total number of calories John burned to exceed the total number of calories he consumed by at least 200 000. [5]

3 **(i)** Using the method of differences, find $\sum_{r=1}^{n} \frac{1}{r(r+1)}$. [3]

Hence find $\sum_{r=1}^{n} \left[3^{-r} - \frac{1}{r(r+1)} \right]$. [3]

(ii) Use your result in part **(i)** to show

$$\sum_{r=3}^{2N} \left[3^{1-r} - \frac{1}{r(r-1)} \right] = -\frac{1}{3} + \frac{1}{2}\left(\frac{1}{N} - \frac{1}{3^{2N-1}} \right).$$ [3]

Hence find $\sum_{r=3}^{\infty} \left[3^{1-r} - \frac{1}{r(r-1)} \right]$. [1]

4 **(i)** By using the substitution $x = \frac{1}{3}\sin^2\theta$, where $0 \le \theta < \frac{\pi}{2}$, find the exact value of

$$\int_0^{\frac{1}{4}} \sqrt{\frac{x}{1-3x}}\, dx.$$ [5]

The region R is bounded by the curve $y = \sqrt{\frac{x}{1-3x}}$, the line $y = 1$ and the y-axis.

(ii) Using your answer in **(i)**, find the exact value of the area of R. [2]

(iii) Find the volume of revolution when R is rotated completely about the y-axis. Give your answer correct to 4 decimal places. [3]

Section B: Probability and Statistics [60 marks]

5 There are ten boys and twelve girls in a school table tennis club. A team of seven boys and seven girls will be selected randomly to represent the school in a table tennis friendly match.

(i) In how different ways can the team be formed? [2]

(ii) Jason is the youngest boy and Joyce is the youngest girl in the club. What is the probability that the team includes both Jason and Joyce? [2]

(iii) Joel is the oldest boy in the club. Given that Joel is selected for the team, what is the probability that the team includes Jason or Joyce, but not both? [4]

6 Alice and Betty each throw a fair cubical die simultaneously.

The random variable X is the larger number shown on the two dice or the common number of the dice if the numbers are equal.

(i) Show that $P(X \le x) = \left(\frac{x}{6}\right)^2$, for $x = 1, 2, \ldots, 6$. [2]

(ii) Deduce that $P(X = x) = \frac{2x-1}{36}$, for $x = 1, 2, \ldots, 6$. [1]

(iii) Show that $E(X) = \frac{161}{36}$ and $\text{Var}(X) = \frac{2555}{1296}$. [3]

(iv) Forty independent observations of X are taken. Using a suitable approximation, estimate the probability that the mean of these observations is at least 4.5. [3]

7 The number of employees, y, who stay back and continue to work in the office t minutes after 5 pm on a particular day in a company is recorded. The results are shown in the table.

t	15	30	45	60	75	90	105
y	30	19	15	13	12	11	10

(i) Draw a scatter diagram for these values, labeling the axes clearly. [1]

(ii) Find, correct to 4 decimal places, the product moment correlation coefficient between

(a) t and y,

(b) $\sqrt{t}$ and y,

(c) $\frac{1}{t}$ and y.

Hence, state with a valid reason, which of the above models is the most appropriate model of the relationship between t and y. [4]

(iii) Using the model you chose in part **(ii)**, find the equation for the relationship between t and y. [2]

(iv) Predict, to the nearest whole number, the number of employees who stay back and continue to work in the office at 7 pm on that particular day. Comment on the reliability of your prediction. [2]

8 A jar contains 10 blue and 8 red marbles. Five marbles are randomly drawn from the box, one by one and without replacement.

(i) Explain why it is inappropriate to model the number of blue marbles by a binomial distribution. [1]

(ii) Find the probability that exactly three marbles are blue. [3]

Another jar contains 20 blue and 12 red marbles. n marbles are randomly drawn from the box, one by one and with replacement. The number of red marbles drawn is denoted by R.

(iii) Given that the mean of R is 4.5, find n and $\mathrm{P}\left(R > 4\right)$. [3]

(iv) Given instead that $\mathrm{P}(R = 0 \text{ or } 1) < 0.01$, write down an inequality for n and find the least value of n. [3]

9 Durians and melons are sold by weight. The masses, in kg, of durians and melons are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean Mass	Standard Deviation
Durians	2.1	0.25
Melons	0.6	0.16

Durians are sold at \$15 per kg and melons at \$6 per kg.

(i) Find the probability that the mass of a randomly chosen durian is less than four times the mass of a randomly chosen melon. [3]

(ii) Two durians and eight melons are randomly selected. Find the probability that the average mass of these ten fruits exceeds 1 kg. [4]

(iii) Find the probability that the total selling price of a randomly chosen durian and a randomly chosen melon is less than \$40. [4]

(iv) Without any further calculation, explain why the probability of the event that both a randomly chosen durian has a selling price less than \$35 and a randomly chosen melon has a selling price less than \$5 is less than the answer to part **(iii)**. [1]

10 There was a complaint that the average waiting time for a patient to see a doctor in a local polyclinic is longer than 60 minutes. A public relation officer in the polyclinic investigated the waiting times, x minutes, for 70 randomly chosen patients. The data are summarised by

$$\sum(x-50)=1071, \quad \sum(x-50)^2=73158.$$

(i) Explain whether the public relation officer should use a 1-tail or a 2-tail test. [1]

(ii) Explain why the public relation officer is able to carry out a hypothesis test without knowing anything about the population distribution of the waiting times for the patients to see a doctor. [1]

(iii) Find unbiased estimates of the population mean and variance. [2]

(iv) Test, at the 5% significance level, whether the complaint is valid. [4]

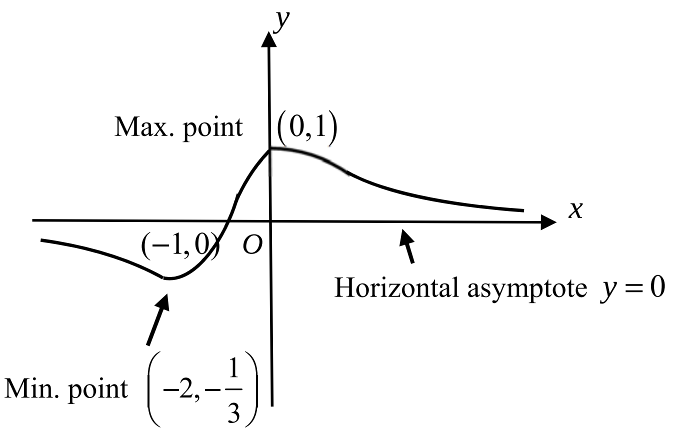
In another test, using the same set of data and also at the 5% significance level, the hypotheses are as follows:

H_0 : the population mean waiting time is equal to k minutes.
H_1 : the population mean waiting time is not equal to k minutes.

(v) Given that the null hypothesis is rejected in favour of the alternative hypothesis, find the set of possible values of k. [4]

Jurong Junior College
2018 JC2 H2 Mathematics Prelim Paper 1 Solution

Qn	Solution
1	Let \$$x$ and \$$y$ be the price of each daily normal ticket and each daily concession ticket respectively. Booth A : $5nx+ny=5976$ -----(1) Booth B : $7nx+2ny=8712$ -----(2) Booth C : $357x+51y=5763$ -----(3)
	$5x+y-5976\left(\frac{1}{n}\right)=0$ L L (1) $7x+2y-8712\left(\frac{1}{n}\right)=0$ L L (2) $357x+51y+0\left(\frac{1}{n}\right)=5763$ L L (3)
	Or $\frac{5x+y}{7x+2y}=\frac{5976}{8712} \Rightarrow 1728x-3240y=0$
	From GC : $x=15$, $y=8$, $\frac{1}{n}=\frac{1}{72}$ Each daily normal ticket costs \$15, and each daily concession ticket costs \$8 and $n=72$.

Qn	Solution	Marks	Remarks
2(i)	Since $x=-1$ is an asymptote, $\Rightarrow \quad d=1$	[B1]	
	Since $y=x$ is an asymptote, $y=x+\dfrac{A}{x+1}=\dfrac{x^2+x+A}{x+1}=\dfrac{ax^2+bx+c}{x+1}$	[M1]	Use long division $y=\dfrac{ax^2+bx+c}{x+1}$ $=ax+b-a+\dfrac{a+c-b}{x+1}$
	$\Rightarrow \quad a=1 \quad$ and $\quad b=1$	[A2]	A1 for $a=1$ A1 for $b=1$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=1-\dfrac{A}{(x+1)^2}$ when $x=0$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$ $\Rightarrow \quad A=1$	[M1]	Or : when $x=-2$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$
	$\Rightarrow \quad c=1$	[A1]	
(ii)	$y=\dfrac{x^2+x+1}{x+1}$ $y(x+1)=x^2+x+1$ $x^2+(1-y)x+(1-y)=0$	[M1]	Form a quadratic equation in *x*.
	For real *x*, $(1-y)^2-4(1)(1-y)\geq 0$	[M1]	Use Discriminant
	$(1-y)^2-4(1)(1-y)\geq 0$ $1-2y+y^2-4+4y\geq 0$ $y^2+2y-3\geq 0$ $(y+3)(y-1)\geq 0$	[M1]	Solve *y*
	$\Rightarrow \quad y\leq -3 \quad$ or $\quad y\geq 1$ (ans)	[A1]	
(iii)		[G3]	G1 for shape G1 for $y=0$ & $(-1,0)$ G1 for $(0,1)$ & $\left(-2,-\dfrac{1}{3}\right)$

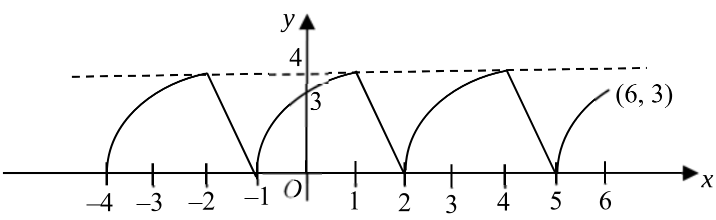
Qn	Solution	Marks	Remarks
3(i)	$\displaystyle\int \frac{x}{\left(4+3x^2\right)^2}\ \mathrm{d}x = \frac{1}{6}\int (6x)\left(4+3x^2\right)^{-2}\ \mathrm{d}x$	[M1]	
	$\displaystyle= -\frac{1}{6\left(4+3x^2\right)}+c$	[A1]	
(ii)	$u = 2x \qquad \dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{x}{\left(4+3x^2\right)^2}$ $\dfrac{\mathrm{d}u}{\mathrm{d}x}=2 \qquad v=-\dfrac{1}{6\left(4+3x^2\right)}$ $\displaystyle\int_0^{\frac{2}{\sqrt{3}}} \frac{2x^2}{\left(4+3x^2\right)^2}\ \mathrm{d}x = \left[-\frac{2x}{6\left(4+3x^2\right)}\right]_0^{\frac{2}{\sqrt{3}}} + \frac{1}{3}\int_0^{\frac{2}{\sqrt{3}}} \frac{1}{4+3x^2}\ \mathrm{d}x$	[M1]	Integration by parts with the correct u and $\dfrac{\mathrm{d}v}{\mathrm{d}x}$.
	$\displaystyle= -\frac{1}{12\sqrt{3}}+\frac{1}{3\sqrt{3}}\int_0^{\frac{2}{\sqrt{3}}} \frac{\sqrt{3}}{2^2+\left(\sqrt{3}x\right)^2}\ \mathrm{d}x$ $\displaystyle= -\frac{1}{12\sqrt{3}}+\left[\frac{1}{6\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)\right]_0^{\frac{2}{\sqrt{3}}}$	[M1]	Award mark for $\tan^{-1}\left(\dfrac{\sqrt{3}x}{2}\right)$
	$\displaystyle= \frac{\pi}{24\sqrt{3}}-\frac{1}{12\sqrt{3}}$	[M1]	Substitution of limits to the two anti-derivatives
	$\displaystyle= \frac{\pi-2}{24\sqrt{3}}$	[A1]	

Qn	Solution	Marks	Remarks
4(i)	$\frac{dy}{dx}=\frac{\sin\theta}{1-\cos\theta}$	[M1]	
	$\frac{dy}{dx}=0\Rightarrow\sin\theta=0\Rightarrow\theta=\pi$	[M1]	Award mark for $\frac{dy}{dx}=0$ and attempt to solve for θ
	$y=1-\cos\pi=2$	[A1]	
(ii)	At $\theta=\frac{2\pi}{3}$, $x=\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$, $y=\frac{3}{2}$, $\frac{dy}{dx}=\frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}}=\frac{\sqrt{3}}{3}$.	[M1]	
	Gradient of normal $=-\sqrt{3}$	[A1]	
	Eqn of normal is $y-\frac{3}{2}=-\sqrt{3}\left[x-\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)\right]$ $y=-\sqrt{3}x+\frac{2\sqrt{3}\pi}{3}$ (Shown)	[M1]	Show working. AG
	$x=0,\ y=\frac{2\sqrt{3}\pi}{3}\Rightarrow Q\left(0,\ \frac{2\sqrt{3}\pi}{3}\right)$ $y=0,\ x=\frac{2\pi}{3}\quad\Rightarrow P\left(\frac{2\pi}{3},\ 0\right)$	[M1]	
	Area of triangle = $\frac{1}{2}\times\frac{2\pi}{3}\times\frac{2\sqrt{3}\pi}{3}=\frac{2\sqrt{3}\pi^2}{9}$ units^2	[A1]	
(iii)	$\frac{dy}{dx}=\frac{\sin\theta}{1-\cos\theta}$ $\frac{d\left(\frac{dy}{dx}\right)}{d\theta}=\frac{\cos\theta(1-\cos\theta)-\sin^2\theta}{(1-\cos\theta)^2}=\frac{1}{\cos\theta-1}$	[M1]	Differentiate wrt θ Accept $\frac{d^2y}{d\theta^2}=\frac{1}{\cos\theta-1}$
	$\frac{d\left(\frac{dy}{dx}\right)}{dt}=\frac{d\left(\frac{dy}{dx}\right)}{d\theta}\cdot\frac{d\theta}{dt}=\frac{1}{\cos\frac{\pi}{3}-1}(2)$	[M1]	Use chain rule $\frac{d\left(\frac{dy}{dx}\right)}{d\theta}\cdot\frac{d\theta}{dt}$
	$=-4$ units/s	[A1]	

Qn	Solution	Marks	Remarks
5	$C=\pi(3r)^2 k+2\pi(3r)^2(2k)+2\pi rh(2k)$ $=45\pi r^2 k+4\pi rhk$	[M1]	
	$\Rightarrow h=\dfrac{C-45\pi r^2 k}{4\pi rk}$	[M1]	
	$V=\dfrac{2}{3}\pi(3r)^3-\pi r^2 h$ $=18\pi r^3-\pi r^2\left(\dfrac{C}{4\pi rk}-\dfrac{45}{4}r\right)$ $=\dfrac{117}{4}\pi r^3-\dfrac{Cr}{4k}$	[M1]	Show working. AG
(i)	$\dfrac{\mathrm{d}V}{\mathrm{d}r}=\dfrac{117}{4}\pi(3r^2)-\dfrac{C}{4k}$	[M1]	
	At stationary value of V, $\dfrac{\mathrm{d}V}{\mathrm{d}r}=0$ $\dfrac{117}{4}\pi(3r^2)-\dfrac{C}{4k}=0$ $\Rightarrow r^2=\dfrac{C}{351\pi k}\Rightarrow r=\sqrt{\dfrac{C}{351\pi k}}$	[A1]	
(ii)	$\dfrac{h}{r}=\dfrac{C-45\pi r^2 k}{4\pi r^2 k}=\dfrac{C}{4\pi r^2 k}-\dfrac{45}{4}$ $=\dfrac{C}{4\pi k\left(\dfrac{C}{351\pi k}\right)}-\dfrac{45}{4}$	[M1]	
	$=\dfrac{153}{2}$	[A1]	
(iii)	Since $h=76.5r$ does not satisfy $h<3r$, $\therefore$ it is not possible for this toy to have a stationary value of V.	[B1]	

Qn	Solution	Marks	Remarks
6(i)	Area of triangle $ABC = \frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|$	[M1]	
	$= \frac{1}{2}\left\|(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})\right\|$ $= \frac{1}{2}\left\|\mathbf{b}\times\mathbf{c}-\mathbf{b}\times\mathbf{a}-\mathbf{a}\times\mathbf{c}+\mathbf{a}\times\mathbf{a}\right\|$	[M1]	
	$= \frac{1}{2}\left\|\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a}\right\|$	[A1]	
	$\Rightarrow \frac{1}{2}\left\|\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a}\right\| = \frac{1}{2}\left\|\mathbf{c}-\mathbf{a}\right\|\times\text{height}$ shortest distance from B to AC $= \frac{\left\|\mathbf{b}\times\mathbf{c}+\mathbf{a}\times\mathbf{b}+\mathbf{c}\times\mathbf{a}\right\|}{\left\|\mathbf{c}-\mathbf{a}\right\|}$ (Shown)	[M1]	Show working AG
6(ii)	$\overrightarrow{RA} = \frac{1}{3}\overrightarrow{BA} = \frac{1}{3}(\mathbf{a}-\mathbf{b})$	[B1]	
	$\overrightarrow{RS} = \overrightarrow{OS}-\overrightarrow{OR}$ $= \left(\frac{2\mathbf{c}+\mathbf{a}}{3}\right)-\left(\frac{2\mathbf{a}+\boldsymbol{b}}{3}\right)$ $= -\frac{1}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}+\frac{2}{3}\mathbf{c}$ or $\overrightarrow{RS} = \overrightarrow{RA}+\overrightarrow{AS}$ $= \frac{1}{3}(\mathbf{a}-\mathbf{b})+\frac{2}{3}(\mathbf{c}-\mathbf{a})$ $= -\frac{1}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}+\frac{2}{3}\mathbf{c}$	[M1] [A1]	
	$\overrightarrow{RA}\cdot\overrightarrow{RS} = \frac{1}{3}(\mathbf{a}-\mathbf{b})\cdot\left(-\frac{1}{3}\mathbf{a}-\frac{1}{3}\mathbf{b}+\frac{2}{3}\mathbf{c}\right)$ $= \frac{1}{9}\left[(\mathbf{a}-\mathbf{b})\cdot(-\mathbf{a}-\mathbf{b}+2\mathbf{c})\right]$ $= \frac{1}{9}\left[-\mathbf{a}\cdot\mathbf{a}-\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{a}+\mathbf{b}\cdot\mathbf{b}+2(\mathbf{a}-\mathbf{b})\cdot\mathbf{c}\right]$	[M1]	
	$= \frac{1}{9}\left[-\left\|\mathbf{a}\right\|^2+\left\|\mathbf{b}\right\|^2+2(\mathbf{a}-\mathbf{b})\cdot\mathbf{c}\right]$	[M1]	
	$OACB$ is a kite with $OA=OB$, $CA=CB$ and $\overrightarrow{BA}\perp\overrightarrow{OC}$ $\Rightarrow \left\|\mathbf{a}\right\|=\left\|\mathbf{b}\right\|$ and $(\mathbf{a}-\mathbf{b})\cdot\mathbf{c}=0$ $\therefore \overrightarrow{RA}\cdot\overrightarrow{RS}=0$ (Shown)	[M1]	

Qn	Solution	Marks	Remarks
7(a) **(i)**	$y=\dfrac{3x+k}{x-b}$ $xy-by=3x+k$ $x(y-3)=by+k$ $x=\dfrac{by+k}{y-3}$ $\mathrm{f}^{-1}(x)=\dfrac{bx+k}{x-3}$	[M1]	
	For $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$, $\dfrac{3x+k}{x-b}=\dfrac{bx+k}{x-3}$ $\therefore b=3$	[A1]	
	Also, $3x+k\neq m(x-3)$ since f is a one to one function. $\therefore\ k\neq -9$	[A1]	
(ii)	$\mathrm{fg}(x)=2x-1$ $\mathrm{g}(x)=\mathrm{f}^{-1}(2x-1)$ $=\mathrm{f}(2x-1)$ $=\dfrac{3(2x-1)+k}{(2x-1)-3}$	[M1]	
	$=\dfrac{6x-3+k}{2x-4},\ x\neq 2$	[A1]	

Qn	Solution	Marks	Remarks
7(b) **(i)**		[G3]	M1: Correct shape, maximum and minimum points on the interval $1\le x\le 4$. A1: y-intercept at (0, 3) and interpreting $\mathrm{h}(x+3)=\mathrm{h}(x)$ A1: Endpoints at (–4,0) and (6, 3)
(ii)	$\int_{-1}^{2}\mathrm{h}(x)\ \mathrm{d}x=\int_{2}^{4}(-x^2+8x-12)\ \mathrm{d}x+\frac{1}{2}(1)(4)$ $=\dfrac{22}{3}$	[M1]	Finding $\int_{-1}^{2}\mathrm{h}(x)\ \mathrm{d}x$ or equivalent
	$\int_{-\frac{3}{2}}^{3}\mathrm{h}(x)\ \mathrm{d}x=\frac{1}{2}\left(\frac{1}{2}\right)(2)+\left(\frac{22}{3}\right)+\int_{2}^{3}(-x^2+8x-12)\ \mathrm{d}x$	[M1]	
	$=\dfrac{19}{2}=9.5$	[A1]	

Qn	Solution	Marks	Remarks
8(i)	$\dfrac{dx}{dt}=kx(1-x)$	[B1]	
	When $x=\dfrac{1}{3}$, $\dfrac{dx}{dt}=\dfrac{1}{6}=k\left(\dfrac{1}{3}\right)\left(\dfrac{2}{3}\right)$	[M1]	
	$\therefore\ k=\dfrac{3}{4}$ i.e. $\dfrac{dx}{dt}=\dfrac{3}{4}x(1-x)$	[A1]	
(ii)	$\int \dfrac{1}{x(1-x)}\,dx=\int \dfrac{3}{4}\,dt$	[M1]	
	$\int \left(\dfrac{1}{x}+\dfrac{1}{1-x}\right)dx=\int \dfrac{3}{4}\,dt$	[M1]	Use of partial fractions (or equivalent)
	$\ln\lvert x\rvert-\ln\lvert 1-x\rvert=\dfrac{3}{4}t+c$	[M1]	
	$\ln\left\lvert\dfrac{x}{1-x}\right\rvert=\dfrac{3}{4}t+c$ $\dfrac{x}{1-x}=e^{\frac{3}{4}t+c}$ $\dfrac{x}{1-x}=Ae^{\frac{3}{4}t}$, where $A=e^{c}$	[M1]	
	when $t=0$, $x=\dfrac{1}{3}$, $\dfrac{1}{2}=Ae^{0}$	[M1]	
	$A=\dfrac{1}{2}$ i.e. $\dfrac{x}{1-x}=\dfrac{1}{2}e^{\frac{3}{4}t}$	[A1]	
	when $t=2$, $\dfrac{x}{1-x}=\dfrac{1}{2}e^{\frac{3}{2}}$	[M1]	
	$2x=e^{\frac{3}{2}}-xe^{\frac{3}{2}}$ $x\left(2+e^{\frac{3}{2}}\right)=e^{\frac{3}{2}}$ $x=\dfrac{e^{\frac{3}{2}}}{2+e^{\frac{3}{2}}}$	[M1]	
	% of crop destroyed = $\dfrac{e^{\frac{3}{2}}}{2+e^{\frac{3}{2}}}\times 100=69.1\%$	[A1]	

Qn	Solution	Marks	Remarks
9(a)	$z^2+(\mathrm{i}-4)z+(6-2\mathrm{i})=0$ $z=\dfrac{-(\mathrm{i}-4)\pm\sqrt{(\mathrm{i}-4)^2-4(6-2\mathrm{i})}}{2}$	[M1]	
	$z=\dfrac{-\mathrm{i}+4\pm\sqrt{\mathrm{i}^2-8\mathrm{i}+16-24+8\mathrm{i}}}{2}$ $z=2+\mathrm{i},\ 2-2\mathrm{i}$	[A1]	
9(b)	$w=r\mathrm{e}^{\mathrm{i}\theta}$ and $w^*=r\mathrm{e}^{-\mathrm{i}\theta}$ $p=\dfrac{r\mathrm{e}^{\mathrm{i}\theta}}{r\mathrm{e}^{-\mathrm{i}\theta}}=\mathrm{e}^{\mathrm{i}(2\theta)}$	[M1]	
	$\lvert p\rvert=1$ and $\arg(p)=2\theta$	[A1]	
	$p^6=\mathrm{e}^{\mathrm{i}(12\theta)}=\cos(12\theta)+\mathrm{i}\sin(12\theta)$	[M1]	
	$0<\theta<\dfrac{\pi}{2}\Rightarrow\ 0<12\theta<6\pi$ For p^6 to be real, $\sin(12\theta)=0$, i.e. $12\theta=\pi,\ 2\pi,\ 3\pi,\ 4\pi,\ 5\pi$	[M1]	
	For p^6 to be positive $\Rightarrow\cos(12\theta)>0$ $\Rightarrow 12\theta=2\pi,\ 4\pi$ $\theta=\dfrac{\pi}{6},\ \dfrac{\pi}{3}$	[A1]	
9(c) (i)	2 complex roots. Since $\mathrm{P}(z)$ has real coefficients and $z=1+\mathrm{i}$ is a complex root, its conjugate is another root. There cannot be a third complex root since $z=2$ is a real root.	[B1]	
(ii)	$\mathrm{P}(z)=(z-(1+\mathrm{i}))(z-(1-\mathrm{i}))(z-2)(az+c)$	[M1]	Accept $(z+c)$ Or $\mathrm{P}(z)=k(z-(1+\mathrm{i}))$ $(z-(1-\mathrm{i}))(z-2)(z+c)$
	$\mathrm{P}(\mathrm{i})=(\mathrm{i}-(1+\mathrm{i}))(\mathrm{i}-(1-\mathrm{i}))(\mathrm{i}-2)(a\mathrm{i}+c)=10+10\mathrm{i}$	[M1]	
	$(-1)(-1+2\mathrm{i})(\mathrm{i}-2)(a\mathrm{i}+c)=10+10i$ $(5\mathrm{i})(a\mathrm{i}+c)=10+10\mathrm{i}$ $-5a+5c\mathrm{i}=10+10\mathrm{i}$	[M1]	
	Comparing real and imaginary parts, $a=-2$ and $c=2$	[A1]	
	$\therefore\ -2z+2=0\ \Rightarrow z=1$ Hence the other 2 roots are $z=1-\mathrm{i}$ and $z=1$.	[A1]	

Qn	Solution	Marks	Remarks
10 (i)	$\begin{pmatrix}2\\3\\-12\end{pmatrix}\times\begin{pmatrix}1\\2\\-7\end{pmatrix}=\begin{pmatrix}3\\2\\1\end{pmatrix}$	[M1]	
	$\begin{pmatrix}1\\5\\-1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}=12$	[M1]	
	Cartesian equation of plane $\pi: 3x+2y+z=12$	[A1]	
	At support beam S_1: $y=z=0 \Rightarrow$ vertex $A=(4,0,0)$	[M1]	AG
10 (ii)	Acute angle of inclination of wall $=\cos^{-1}\dfrac{\left\|\begin{pmatrix}0\\0\\1\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}\right\|}{\sqrt{9+4+1}}$	[M2]	M1 for using formula to find angle between 2 vectors M1 for $\begin{pmatrix}0\\0\\1\end{pmatrix}$
	$=74.498^{\circ} \approx 74.5^{\circ}$ Since $74.5^{\circ}<80^{\circ}$, the safety standard is met.	[A1]	
10 (iii)	$\overrightarrow{ON}=\lambda\begin{pmatrix}3\\2\\1\end{pmatrix}$ for some λ	[B1]	
	$\Rightarrow \begin{pmatrix}3\lambda\\2\lambda\\\lambda\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}=12$ $\lambda=\dfrac{6}{7}$	[M1]	
	Point $N:\left(\dfrac{18}{7},\dfrac{12}{7},\dfrac{6}{7}\right)$	[A1]	
	Length of 4th support beam $=\dfrac{6}{7}\sqrt{9+4+1}$	[M1]	
	$=\dfrac{6\sqrt{14}}{7}$	[A1]	

Jurong Junior College

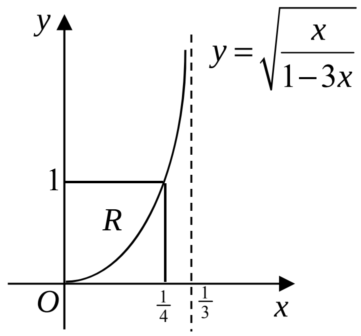
2018 JC2 H2 Preliminary Examination Paper 2 Solutions

Qn	Solution
1	$\text{f}(x) = \text{e}^{\sin x}$ $= \text{e}^{x-\frac{x^3}{3!}+...}$ $=1+\left(x-\frac{x^3}{3!}\right)+\frac{\left(x-\frac{x^3}{3!}\right)^2}{2!}+\frac{\left(x-\frac{x^3}{3!}\right)^3}{3!}+.....$ $=1+x-\frac{x^3}{6}+\frac{x^2}{2}+\frac{x^3}{6}+...$ $=1+x+\frac{x^2}{2}+....$ $\therefore\ a=1,\ b=1,\ c=\frac{1}{2}$ and $d=0.$
	$\frac{1}{\left(\text{e}^{\sin x}\right)^2}\approx\left(1+x+\frac{x^2}{2}\right)^{-2}$
	$=1+(-2)\left(x+\frac{x^2}{2}\right)+\frac{(-2)(-3)}{2!}\left(x+\frac{x^2}{2}\right)^2+.....$ $\approx 1-2x+2x^2$
(i)	$4\frac{\text{d}y}{\text{d}x}=(y+1)^2$ Differentiating with respect to *x*, $4\frac{\text{d}^2y}{\text{d}x^2}=2(y+1)\frac{\text{d}y}{\text{d}x}$ $4\frac{\text{d}^3y}{\text{d}x^3}=2(y+1)\left(\frac{\text{d}^2y}{\text{d}x^2}\right)+2\left(\frac{\text{d}y}{\text{d}x}\right)^2$
	Sub $x=0$, $y=1$, $\frac{\text{d}y}{\text{d}x}=1$, $\frac{\text{d}^2y}{\text{d}x^2}=1$, $\frac{\text{d}^3y}{\text{d}x^3}=\frac{3}{2}$
	Using Maclaurin's formula, $\text{g}(x)=1+x+\frac{x^2}{2!}+\left(\frac{3}{2}\right)\frac{x^3}{3!}+.....$
	$\text{g}(x)=1+x+\frac{x^2}{2}+\frac{x^3}{4}+.....$
	$\text{g}(x)-\text{f}(x)=\left(1+x+\frac{x^2}{2}+\frac{x^3}{4}+.....\right)-\left(1+x+\frac{1}{2}x^2+.....\right)$ $\approx\frac{x^3}{4}$

(ii)	As $x \to 0$, $\mathrm{g}(x) - \mathrm{f}(x) \approx \frac{1}{4}x^3 \to 0$. Therefore, $\mathrm{f}(x)$ is a good approximation to $\mathrm{g}(x)$ for values of x close to zero.

2	GP: $a = 10000,\ r = 1.07$	
	$U_{15} = 10000(1.07)^{14}$	
	$U_{15} = 25785.34 \approx 25785$	
(i)	First three terms of G.P.: $a+6d$, $a+2d$, a $\Rightarrow \dfrac{a+2d}{a+6d} = \dfrac{a}{a+2d}$	
	$a^2 + 4ad + 4d^2 = a^2 + 6ad$ $4d^2 = 2ad$ $d \neq 0 \Rightarrow d = \dfrac{a}{2}$	
(ii)	Given $a + 6d = 3000$, where $d = \dfrac{a}{2}$ from (i) $a + 3a = 3000 \Rightarrow a = \dfrac{3000}{4} = 750$	
	Total calories loss $S_n = \dfrac{n}{2}\left[2(750) + (n-1)(375)\right]$ $= \dfrac{375n}{2}(3+n)$	
	G.P. $U_1 = 3000,\ r = \dfrac{a}{a+2\left(\frac{a}{2}\right)} = \dfrac{1}{2}$ Total calories gain $S_n = \dfrac{3000\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = 6000\left(1-\left(\frac{1}{2}\right)^n\right)$	
	$\dfrac{375n}{2}(3+n) - 6000\left(1-\left(\frac{1}{2}\right)^n\right) \geq 200000$ $\dfrac{3n}{16}(3+n) - 6\left(1-\left(\frac{1}{2}\right)^n\right) \geq 200$	
	From GC, $n \geq 31.68$ (or 32) Least number of weeks $= 32$.	

3(i)	$\sum_{r=1}^{n}\frac{1}{r(r+1)}=\sum_{r=1}^{n}\left[\frac{1}{r}-\frac{1}{(r+1)}\right]$ $=\left\{\begin{array}{l}\frac{1}{1}-\frac{1}{2}\\+\frac{1}{2}-\frac{1}{3}\\+\ldots\\+\frac{1}{n-1}-\frac{1}{n}\\+\frac{1}{n}-\frac{1}{n+1}\end{array}\right\}$ $=1-\frac{1}{n+1}$	
	$\sum_{r=1}^{n}\left[3^{-r}-\frac{1}{r(r+1)}\right]=\sum_{r=1}^{n}3^{-r}-\sum_{r=1}^{n}\frac{1}{r(r+1)}$ $=\frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}}-\left(1-\frac{1}{n+1}\right)$ $=\frac{1}{2}\left(1-\frac{1}{3^n}\right)-1+\frac{1}{n+1}$ $=-\frac{1}{2}\left(\frac{1}{3^n}\right)+\frac{1}{n+1}-\frac{1}{2}$	
(ii)	$\sum_{r=3}^{2N}\left[3^{1-r}-\frac{1}{r(r-1)}\right]=\sum_{r=2}^{2N-1}\left[3^{-r}-\frac{1}{r(r+1)}\right]$ $=\sum_{r=1}^{2N-1}\left[3^{-r}-\frac{1}{r(r+1)}\right]-\left(\frac{1}{3}-\frac{1}{2}\right)$ $=-\frac{1}{2}\left(\frac{1}{3^{2N-1}}\right)+\frac{1}{2N}-\frac{1}{2}+\frac{1}{6}$ $=-\frac{1}{3}+\frac{1}{2}\left(\frac{1}{N}-\frac{1}{3^{2N-1}}\right)$ [Shown]	
	$\sum_{r=3}^{\infty}\left[3^{1-r}-\frac{1}{r(r-1)}\right]=\lim_{N\to\infty}\left[-\frac{1}{3}+\frac{1}{2}\left(\frac{1}{N}-\frac{1}{3^{2N-1}}\right)\right]=-\frac{1}{3}$	

4(i)	$x=\frac{1}{3}\sin^2\theta \Rightarrow \frac{dx}{d\theta}=\frac{2}{3}\sin\theta\cos\theta$ When $x = 0$, $\theta = 0$; when $x=\frac{1}{4}$, $\theta=\frac{\pi}{3}$. $\int_0^{\frac{1}{4}}\sqrt{\frac{x}{1-3x}}\,dx = \int_0^{\frac{\pi}{3}}\sqrt{\frac{\frac{1}{3}\sin^2\theta}{\cos^2\theta}}\left(\frac{2}{3}\sin\theta\cos\theta\ d\theta\right)$	
	$=\frac{2}{3\sqrt{3}}\int_0^{\frac{\pi}{3}}\sin^2\theta\ d\theta$ $=\frac{2}{3\sqrt{3}}\int_0^{\frac{\pi}{3}}\frac{1-\cos 2\theta}{2}\ d\theta$	
	$=\frac{1}{3\sqrt{3}}\left[\theta-\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{3}}$	
	$=\frac{1}{3\sqrt{3}}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]$	
(ii)		
	Area of $R=\frac{1}{4}-\frac{1}{3\sqrt{3}}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right]$ $=\frac{1}{3}-\frac{\pi}{9\sqrt{3}}$	
(iii)	$y=\sqrt{\frac{x}{1-3x}} \Rightarrow y^2=\frac{x}{1-3x}$ $y^2-3xy^2=x$ $x(1+3y^2)=y^2$ $x=\frac{y^2}{1+3y^2}$	
	Volume $=\pi\int_0^1\left(\frac{y^2}{1+3y^2}\right)^2\ dy$ $= 0.0761$ (4 dp)	

5(i)	No. of different ways = ${}^{10}C_7 \times {}^{12}C_7$	
	$= 95040$	
(ii)	No. of teams including Jason and Joyce $= {}^{9}C_6 \times {}^{11}C_6$	
	Required probability $= \dfrac{{}^{9}C_6 \times {}^{11}C_6}{95040}$ $= \dfrac{38808}{95040}$ $= \dfrac{49}{120}$ or 0.408	
(iii)	No. of teams including Joel $= {}^{9}C_6 \times {}^{12}C_7$ = 66528	
	No. of teams including Joel and Jason but not Joyce $= {}^{8}C_5 \times {}^{11}C_7$ = 18480 No. of teams including Joel and Joyce but not Jason $= {}^{8}C_6 \times {}^{11}C_6$ = 12936	
	Required probability $= \dfrac{18480 + 12936}{66528}$	
	$= \dfrac{17}{36}$ or 0.472	

6(i)	Table of outcomes:	

X_2 \ X_1	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

P(one number $\leq x$) = $\frac{x}{6}$

$\text{P}(X \leq x) = \text{P}(\text{both numbers} \leq x), \ x = 1, 2, \ldots, 6.$

$= \text{P}(X_1 = 1, 2, \ldots x \ \& \ X_2 = 1, 2, \ldots x)$

$= \left(\frac{x}{6}\right)\left(\frac{x}{6}\right)$

$= \left(\frac{x}{6}\right)^2$

(ii)

$\text{P}(X = x) = \text{P}(X \leq x) - \text{P}(X \leq x-1)$

$= \left(\frac{x}{6}\right)^2 - \left(\frac{x-1}{6}\right)^2$

$= \frac{2x-1}{36}$

(iii)

x	1	2	3	4	5	6
$\text{P}(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$\text{E}(X) = \sum_{\text{all } r} x\text{P}(X = x)$

$= 1\times\frac{1}{36} + 2\times\frac{3}{36} + 3\times\frac{5}{36} + 4\times\frac{7}{36} + 5\times\frac{9}{36} + 6\times\frac{11}{36} = \frac{161}{36}$

[Shown]

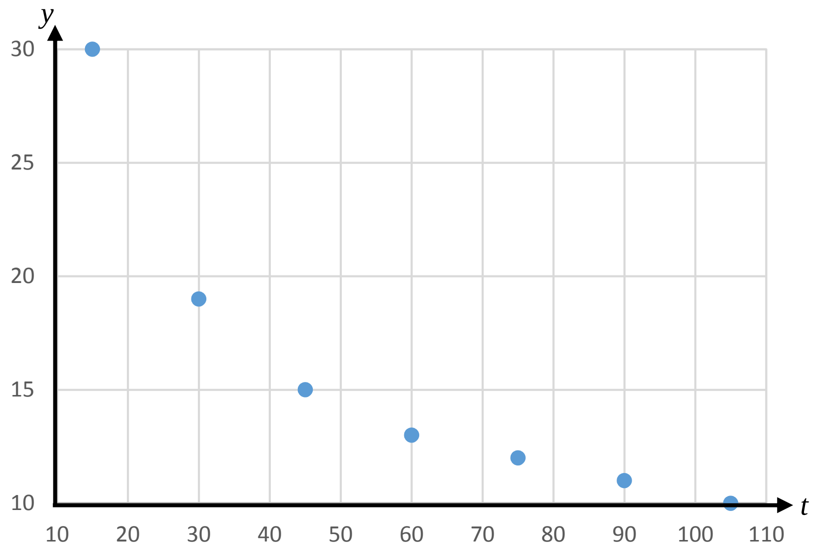
$\text{E}(X^2) = \sum_{\text{all } r} x^2\text{P}(X = x)$

$= 1^2\times\frac{1}{36} + 2^2\times\frac{3}{36} + 3^2\times\frac{5}{36} + 4^2\times\frac{7}{36} + 5^2\times\frac{9}{36} + 6^2\times\frac{11}{36}$

$= \frac{791}{36}$

$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$

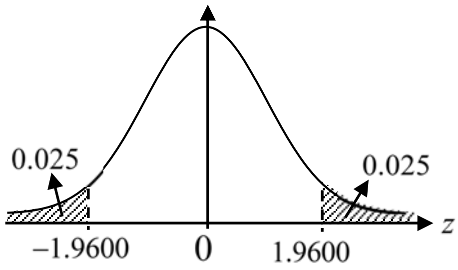
	$= \frac{791}{36} - \left(\frac{161}{36}\right)^2$ $= \frac{2555}{1296}$ [Shown]	
(iv)	Since $n = 40$ is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(\frac{161}{36}, \frac{2555}{1296 \times 40}\right)$ approximately.	
	$\mathrm{P}(\overline{X} \geq 4.5) = 0.45021 \approx 0.450$ (3 sig figs)	

7(i)		
(ii)	From G.C., (a) $r = -0.8745$ (ans) (b) $r = -0.9288$ (ans) (c) $r = 0.9993$ (ans)	
	Model (c) is the most appropriate model for the relationship between t and y since its value of $\lvert r \rvert$ is closest to 1.	

(iii)	From G.C., $y = 7.2048 + 344.60\left(\frac{1}{t}\right)$ $\underline{\underline{y = 7.20 + 345\left(\frac{1}{t}\right) \quad \text{(3s.f.)(ans)}}}$	
(iv)	When $t = 120$, $y = 7.2048 + 344.60\left(\frac{1}{120}\right) = 10.076$ $\underline{\underline{y ;\ 10 \quad \text{(ans)}}}$	
	Since $t = 120$ is outside the given range of the values of *t*, this is an extrapolation and thus the prediction may not be reliable.	
8(i)	It is inappropriate to model the number of blue marbles by a binomial distribution because the marbles are drawn without replacement, the colour of the marbles depends on that of the previous draw.	
(ii)	Required probability $= \frac{10}{18} \times \frac{9}{17} \times \frac{8}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{5!}{3! \times 2!}$ or $\frac{{}^{10}C_3 \times {}^{8}C_2}{{}^{18}C_5}$	
	$= \frac{20}{51}$ or 0.392	
(iii)	$R \sim \text{B}\left(n, \frac{12}{32}\right) = \text{B}\left(n, \frac{3}{8}\right)$ $\text{E}(R) = 4.5$ $\Rightarrow \frac{3}{8}n = 4.5$ $n = 12$	
	$\text{P}(R > 4) = 1 - \text{P}(R \le 4)$ $= 0.48972 \approx 0.490$ (3 sig figs)	
(iv)	$R \sim \text{B}\left(n, \frac{3}{8}\right)$ $\text{P}(R = 0 \text{ or } 1) < 0.01$ $\Rightarrow \text{P}(R = 0) + \text{P}(R = 1) < 0.01$	
	$\Rightarrow \left(\frac{5}{8}\right)^n + n\left(\frac{3}{8}\right)\left(\frac{5}{8}\right)^{n-1} < 0.01$	
	From GC, least $n = 15$	

9(i)	Let D = Mass of a durian and R = Mass of a melon. Then $D \sim \text{N}(2.1,\ 0.25^2)$ and $R \sim \text{N}(0.6,\ 0.16^2)$. $D-4R \sim \text{N}\left(2.1-4(0.6),\ 0.25^2+4^2\times0.16^2\right)$	
	$D-4R \sim \text{N}\left(-0.3, 0.4721\right)$	
	$\text{P}\left(D-4R<0\right) \approx 0.66881 = 0.669$ (to 3 sig figs)	
(ii)	Let $M = \dfrac{D_1+D_2+R_1+R_2+\ldots+R_8}{10}$ $\text{E}(M) = \dfrac{1}{10}[2(2.1)+8(0.6)] = 0.9$ $\text{Var}(M) = \dfrac{1}{10^2}[2(0.25)^2+8(0.16)^2] = 0.003298$	
	$M \sim \text{N}\left(0.9, 0.003298\right)$	
	$\text{P}(M>1) = 0.040815 = 0.0408$ (to 3 sig figs)	
(iii)	Let $S = 15D + 6R$ $15D \sim \text{N}\left(15(2.1),\ 15^2\times0.25^2\right)$ and $6R \sim \text{N}\left(6(0.6),\ 6^2\times0.16^2\right)$	
	$S \sim \text{N}\left(31.5+3.6,\ 14.0625+0.9216\right) = \text{N}\left(35.1,\ 14.9841\right)$	
	$\text{P}(S<40) = 0.89722 = 0.897$	
(iv)	Event in **part (iii)** includes the event in **part (iv)** plus some other cases. [For example, the case where $15D < 33$ and $6R < 7$ is included in **(iii)** but not in **(iv)**.]	

10(i)	A 1-tail test should be used because he is investigating for an average time longer than 60 minutes	
(ii)	Since the sample size is large, the public relation officer can apply the Central Limit Theorem to approximate the distribution of the sample mean ($\overline{X}$) by a normal distribution to conduct a hypothesis test.	
(iii)	unbiased estimate of population mean $=\overline{x}$ $=50+\dfrac{1071}{70}$ $=\dfrac{653}{10}$	
	Unbiased estimate of population variance $=s^2$ $=\dfrac{1}{69}\left(73158-\dfrac{1071^2}{70}\right)$ $=\dfrac{189239}{230}$	
(iv)	Let μ be the population mean of *X*. $\text{H}_0 : \mu = 60$ $\text{H}_1 : \mu > 60$	
	Under H_0, since *n* = 70 is large, by the Central Limit Theorem, $\overline{X} \sim \text{N}\left(60, \dfrac{189239}{230(70)}\right)$ approximately. Test Statistic : $Z = \dfrac{\overline{X}-60}{\sqrt{\dfrac{189239}{230(70)}}} \sim \text{N}(0,1)$ approximately	
	Level of significance: 5% By using G.C., $p-\text{value}=0.0611$ (3 s.f)	
	Since *p*-value = 0.0611 > 0.05, we do not reject H_0 at the 5% level of significance and conclude that there is insufficient evidence that the population mean waiting time is longer than 60 minutes. i.e. The complaint is not valid.	

(v)	$H_0 : \mu = k$ $H_1 : \mu \neq k$ Level of significance: 5%  For H_0 to be rejected, $z \le -1.9600$ or $z \ge 1.9600$	
	$\dfrac{\overline{x}-k}{s/\sqrt{70}} \le -1.9600$ or $\dfrac{\overline{x}-k}{s/\sqrt{70}} \ge 1.9600$ $\dfrac{65.3-k}{\sqrt{822.778/70}} \le -1.9600$ or $\dfrac{65.3-k}{\sqrt{822.778/70}} \ge 1.9600$ $65.3-k \le -6.7197$ or $65.3-k \ge 6.7197$	
	$\Rightarrow$ $k \le 58.6$ or $k \ge 72.0$ (ans)	