EX

JURONG SECONDARY SCHOOL
2018 MID-YEAR EXAMINATION
SECONDARY 3 EXPRESS

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4047

3 May 2018
2 hours

Additional Materials : Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks given in the brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

This document consists of 5 printed pages including this page.
[Turn over]

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

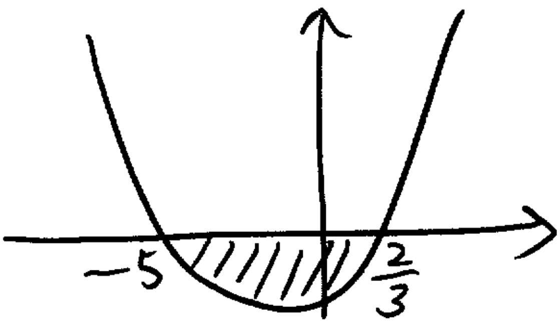
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

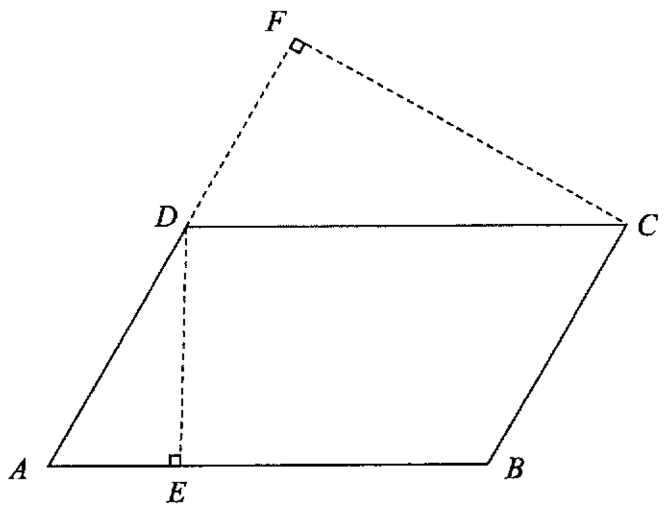
1	Solve the simultaneous equations.		
	$2y+x=3$ $\frac{1}{y}-\frac{4}{x}=3$		[4]
	$x = 3 - 2y$ Substitute: $\frac{1}{y} - \frac{4}{3-2y} = 3$ $6y^2 - 15y + 3 = 0$ $2y^2 - 5y + 1 = 0$ $y = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$ $y = 2.28078$ or $y = 0.21922$ $x = -1.56156 \quad x = 2.56156$ $\therefore x = -1.56, y = 2.28$ or $x = 2.56, y = 0.219$		M1 – substitute M1 – correct quadratic equation M1 – formula for solving A1 (accept exact value)
2	Given that $(1, k)$ is a point of intersection between the curve $4x^2-4xy+y^2=1$ and the line		
	$y=7-4x$, find the		
	(a)	value of k,	[1]
		$k = 3$	B1
	(b)	coordinates of the other point of intersection.	[3]
		$4x^2 - 4x(7-4x) + (7-4x)^2 = 1$ $3x^2 - 7x + 4 = 0$ $(3x-4)(x-1) = 0$ $x = \frac{4}{3}$ or $x = 1$(reject.) $y = \frac{5}{3}$ $\therefore (\frac{4}{3}, \frac{5}{3})$	M1 – simultaneous equation M1 – factorized quadratic/ formula A1
3	(a)	Given that $2x^3+5x^2-x-2=(Ax+3)(x+B)(x-1)+C$, where A, B and C are	
		constants, find the values of A, B and C.	[3]
		When $x = 1, 2 + 5 - 1 - 2 = C$ $C = 4$ When $x = 0, -2 = 3(B)(-1) + 4$ $-6 = -3B$ $B = 2$ When $x = -1, -2 + 5 + 1 - 2 = (-A + 3)(-1 + 2)(-2) + 4$ $A = 5$	M1M1 – substitution /expansion + comparing coefficients A1 – 3 values

	(b)	Given that x^2+mx+n and x^2+ax+b have the same remainder when divided by	
		$x+p$, express p in terms of a, b, m and n.	[3]
		Substitute $x=-p$, $(-p)^2+m(-p)+n=(-p)^2+a(-p)+b$ $ap-pm=b-n$ $p=\frac{b-n}{a-m}$	M1 – remainder theorem (R=0 reject) M1 – equal remainder A1
4	The function $f(x)=54x^4-9x^3-6a^2x^2+7x+2$ has a factor of $3x+a$.		
	(i)	Show that $a^3-7a+6=0$.	[2]
		When $x=-\frac{a}{3}$, $54\left(-\frac{a}{3}\right)^4-9\left(-\frac{a}{3}\right)^3-6a^2\left(-\frac{a}{3}\right)^2+7\left(-\frac{a}{3}\right)+2=0$ $\frac{1}{3}a^3-\frac{7}{3}a+2=0$ $a^3-7a+6=0$ (shown)	M1 – factor theorem A1
	(ii)	Hence, find the possible values of a.	[4]
		Let $a=1$ $1-7+6=0$ $(a^3-7a+6)\div(a+1)=a^2+a-6=(a+3)(a-2)$ $\therefore a=1,2,-3$	M1 – factor theorem M1 – long division M1 – factorise A1
5	Express $\frac{2x^3+5x^2-x+1}{(x-2)(x^2+1)}$ in partial fractions.		[5]
	$\frac{2x^3+5x^2-x+1}{(x-2)(x^2+1)}=2+\frac{9x^2-3x+5}{(x-2)(x^2+1)}$ $\frac{9x^2-3x+5}{(x-2)(x^2+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+1}$ $x^2(A+B)+x(-2B+C)+(A-2C)=9x^2-3x+5$ $A+B=9$ $-2B+C=-3$ $A-2C=5$ $\therefore A=7, B=2, C=1$ $\therefore 2+\frac{7}{x-2}+\frac{2x+1}{x^2+1}$		M1 – long division M1 – partial fractions M1 – compare coefficient/ substitution M1 – values of A, B, C A1

6	The curve $y=(k+2)x^2-(2k+1)x+k$ has a minimum point and lies completely above the		
	x-axis. Find the range of values of k.		[4]
	$k+2>0, k>-2$ $(-2k-1)^2-4(k+2)(k)<0$ $-4k+1<0$ $k>\frac{1}{4}$ $\therefore\ k>\frac{1}{4}$		M1 – coefficient of x^2 M1 – discriminant <0 A1 A1 - conclusion
7	(i)	Find the values of m for which the curve $y=x^2-5x+m+8$ touches the line $y=mx$	
		only once.	[4]
		$x^2-5x+m+8=mx$ $x^2+x(-5-m++m+8=0$ $(-5-m)^2-4(1)(m+8)=0$ $m^2+6m-7=0$ $(m+7)(m-1)=0$ $m=-7, m=1$	M1 – simultaneous equation M1 – discriminant $=0$ M1 – factorise A1
	(ii)	Hence, state the range of values of m for which the curve $y=x^2-5x+m+8$ cuts	
		the line $y=mx$ twice.	[1]
		$m<-7, m>1$	B1
	(iii)	Using answers in **(i)** and **(ii)**, state what can be deduced about the curve	
		$y=x^2-5x+11$ and the straight line $y=3x$, giving a reason for your answer.	[2]
		$m=3$, which is >1. Since the value of m is more than 1, the curve will cut the line twice.	B1 – value of m + comparison B1 – conclusion (ecf)
8	The roots of the equation $3x^2=2kx-k-4$ are α and β. If $\alpha^2+\beta^2=\frac{16}{9}$, find possible		
	values of k.		[5]
		$3x^2-2kx+k+4=0$ $\alpha+\beta=\frac{2k}{3}$ $\alpha\beta=\frac{k+4}{3}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{16}{9}$ $(\frac{2k}{3})^2-2\left(\frac{k+4}{3}\right)=\frac{16}{9}$ $2k^2-3k-20=0$ $(2k+5)(k-4)=0$	M1 – sum and product of roots M1 – formula M1 – substitution of values M1 – factorise

		$\therefore k = 4, k = -\frac{5}{2}$	A1
9	The equation $2x^2 + 4x + 5 = 0$ has roots α and β.		
	(a)	Find $\alpha + \beta$ and $\alpha\beta$.	[2]
		$\alpha + \beta = -2$ $\alpha\beta = \frac{5}{2}$	B1 B1
	(b)	Show that $\alpha^3 + \beta^3 = 7$.	[3]
		$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ $= (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$ $= (-2)\left[(-2)^2 - 3\left(\frac{5}{2}\right)\right]$ $= 7$	M1 – formula M1 – simplifying A1
	(c)	Find the quadratic equation, with integer coefficients, whose roots are $\frac{\alpha}{\beta^2}$	
		and $\frac{\beta}{\alpha^2}$.	[3]
		$\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^2} = \frac{7}{\left(\frac{5}{2}\right)^2} = \frac{28}{25}$ $\left(\frac{\alpha}{\beta^2}\right)\left(\frac{\beta}{\alpha^2}\right) = \frac{1}{\alpha\beta} = \frac{2}{5}$ $\therefore 25x^2 - 28x + 10 = 0$	M1 – sum substitution M1 – product substitution A1
			-1 (a & b)
10	Find the range of values for which $\frac{-5}{3x^2 + 13x - 10} > 0$.		[3]
	$3x^2 + 13x - 10 < 0$ $(3x - 2)(x + 5) < 0$  $\therefore -5 < x < \frac{2}{3}$		M1 – denominator inequality M1 – diagram with shaded area (ecf) A1

11	Without using a calculator, find the value of 6^x, given that $3^{2x+2}=4^{-3-x}$.		[4]
	$(3^{2x})(3^2)=2^{-6-2x}$ $9(3^{2x})=2^{-6}(2^{-2x})$ $(3^{2x})(2^{2x})=(2^{-6})(3^{-2})$ $6^{2x}=\frac{1}{2^6 3^2}=\frac{1}{576}$ $6^x=\frac{1}{24}$ (reject negative)		M1 – base 2 M1 – combine power x M1 A1
12	(a)	Solve the equation $4^{x+1}=2-7(2^x)$.	[3]
		$2^{2x+2}=2-7(2^x)$ Let $2^x=y$ $4y^2+7y-2=0$ $(4y-1)(y+2)=0$ $y=\frac{1}{4}$ or $y=-2$ $2^x=\frac{1}{4}$ $2^x=-2$ (rej.) $\therefore x=-2$	M1 – quadratic M1 – solve for y (no marks if reject at y) A1 – with reject
	(b)	Solve the simultaneous equations	
		$4^{x-2}=\frac{64}{2^y}$ $\log_x(y+2)-1=\log_x 4$	[5]
		$4^{x-2}=\frac{64}{2^y}$ $2^{2x-4}=2^{6-y}$ $2x-4=6-y$ $x=5-\frac{y}{2}$ $log_x(y+2)-log_x x=log_x 4$ $\frac{y+2}{x}=4$ $y+2=4x$ $y+2=4\left(5-\frac{y}{2}\right)$ $3y=18$ $y=6$ $x=2$	M1 – base of 2 M1 – compare power M1 – compare logarithm M1 – substitution A1

13	Solve the equation $\sqrt{x-5} = \sqrt{x} + 2$.	[3]
	$x - 5 = (\sqrt{x} + 2)^2$ $x - 5 = x + 4\sqrt{x} + 4$ $4\sqrt{x} = -9$ $\sqrt{x} = -\frac{9}{4}$ $x = \frac{81}{16}$	M1 – correctly squaring both sides M1 – square root x A1
14	In the diagram, *ABCD* is a parallelogram, with heights *DE* and *CF*. It is given that	
	$CD = (7 + 4\sqrt{2})$ cm, $BC = (11 - 2\sqrt{2})$ cm and $DE = (3 + 3\sqrt{2})$ cm.	
		
	Find	
	(a) the perimeter of *ABCD*,	[2]
	$2(11 - 2\sqrt{2} + 7 - 4\sqrt{2}) = 36 + 4\sqrt{2}$ cm	M1 – formula A1 – with units
	(b) the area of *ABCD*,	[2]
	$(3 + 3\sqrt{2})(7 + 4\sqrt{2}) = 21 + 12\sqrt{2} + 21\sqrt{2} + 12(2)$ $= 45 + 33\sqrt{2}$ cm	M1 – formula A1
	(c) the length of *CF*.	[3]
	$(CF)(11 - 2\sqrt{2}) = 45 + 33\sqrt{2}$ $CF = \frac{45 + 33\sqrt{2}}{11 - 2\sqrt{2}} \times \frac{11 + 2\sqrt{2}}{11 + 2\sqrt{2}}$ $= \frac{627+453\sqrt{2}}{113}$ cm	M1 M1 – rationalizing A1

15	**(a)**	Solve $\log_3(\frac{1}{x})=2$.	[2]
		$log_3\frac{1}{x}=2$ $3^2=\frac{1}{x}$ $\frac{1}{x}=9$ $\therefore x=\frac{1}{9}$	M1 – exponential A1
	(b)	Given that $\lg x=a$ and $\lg y=b$, express $\lg(\frac{1000x^2}{y})$ in terms of a and b.	[2]
		$\lg\left(\frac{1000x^2}{y}\right)=lg1000+2lgx-lgy$ $=3+2a-b$	M1 – product and quotient law A1
	(c)	Solve $\ln(x-1)=3$, leaving your answer in exact terms.	[2]
		$\ln(x-1)=3$ $e^3=x-1$ $x=1+e^3$	M1 – exponential equation A1

End of Paper