CANBERRA SECONDARY SCHOOL

O

2021 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/01

26 Aug 2021
2 hours 15 minutes
1000h – 1215h

Name: ______________________________ (   ) **Class:** ________

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

This question paper consists of 17 printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0, a \neq 0,$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Given that $y = p\sin 2x + q$, where p and q are positive integers. The maximum and minimum values of y are 8 and -2 respectively.

(a) Find the value of p and of q. [2]

(b) Hence, sketch the graph of y for $0^\circ \leq x \leq 360^\circ$. [3]

2 **(a)** Express each of $4x^2 - 6x - 1$ and $-x^2 - 10x + 3$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [4]

(b) Using your answers in **(a)**, show that the curves $y = 4x^2 - 6x - 1$ and $y = -x^2 - 10x + 3$ will intersect. [3]

3 The equation of a curve is given by $y = 5xe^{2x+1}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Hence, find $\int xe^{2x+1}\,dx$. [3]

(c) Determine the range of values of x for which y is a decreasing function. [2]

4 **(a)** A curve has equation $y = x^2 + x + 2k - 4$ and a line has equation $y - 2x = k$, where k is a constant. Find the set of values of k for which the line and curve do not intersect and represent this set on a number line. [5]

(b) Given that the curve with equation $y = ax^2 + 8x + c$, where a and c are constants, lies completely below the x-axis.
Write down the conditions which must apply to a and c. [3]

5 Given that a point P(1 , 2) lies on the curve $y = \frac{5}{2-x} - 3$, find

(a) **(i)** the value of $\frac{dy}{dx}$ at point P, [2]

(ii) the equation of the normal to the curve at point P. [3]

(b) Explain whether a stationary point will exist on the curve. [1]

(c) If x is increasing at a rate of 0.2 units per second, find the rate of change of y at the instant when $y = -4$. [3]

6 **(a)** Given that $\dfrac{\cos\dfrac{\pi}{6}-\tan\dfrac{\pi}{4}}{\sin\dfrac{\pi}{3}+\cos\dfrac{\pi}{3}}=\dfrac{\sqrt{3}+a}{\sqrt{3}+b}$, where a and b are integers.

Find the values of a and b. [3]

(b) Given that $\sin A = -\frac{5}{13}$ and $\cos B = \frac{4}{5}$, where A and B are in the same quadrant. Find the exact value of the following:

(i) $\sin(A+B)$ [3]

(ii) $\sec 2B$ [3]

7 A dot on a computer screen moves in a straight line and passes through a fixed point A. The distance, d metres, that it runs in t s after it passes through A is given by

$$d = t^3 - 6t^2 + 9t \text{ for } t \geq 0.$$

(i) Find the dot's speed and acceleration 10 seconds after it passes A. [6]

(ii) Find the values of t at which the dot is instantaneously at rest. [3]

(iii) Find the distance travelled by the dot in the first 4 seconds. [3]

8 The area of a quadrilateral is $\left(13-\sqrt{48}\right)\text{cm}^2$.

(i) In the case where the quadrilateral is a rectangle with a width of $\left(2-\sqrt{3}\right)$ cm, find, without the use of a calculator, the length of the rectangle in the form of $\left(a+b\sqrt{3}\right)$ cm.

[4]

(ii) In the case where the quadrilateral is a square with side $\left(c+2\sqrt{3}\right)$cm, find, without the use of a calculator, the value of the constant c. [3]

9 The price of a new car on 1 January 2021 is \$120 888.00. Given that the value of the car depreciates in such a way that, n months after the purchase, the sale price, \$$P$, is determined by the formula

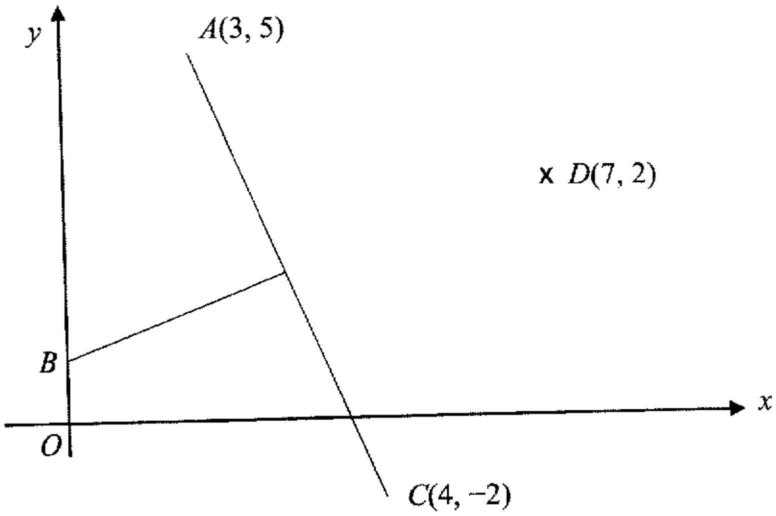
$$P = 120888e^{-0.015n},$$

estimate

(i) the sale price of the car after 1 year, giving your answer correct to the nearest \$1000, [2]

(ii) the month and year when the sale price of the car is less than \$50 000. [4]

10



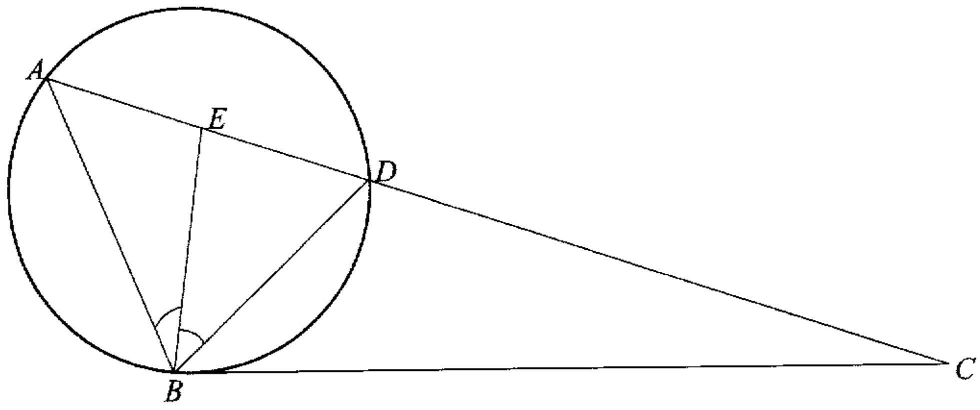
The figure shows three points A, C and D whose coordinates are (3, 5), (4, −2) and (7, 2) respectively.

(i) Find the equation of the perpendicular bisector of AC. [2]

(ii) Find the coordinate of B, the point where the perpendicular bisector cuts the y-axis. [1]

(iii) Show that *ABCD* is a square. [3]

11 In the diagram, AC is a straight line intersecting a circle at A and D. The point B lies on the circle and BC is a tangent to the circle. The point E lies on AC such that the line BE bisects angle ABD.



(i) State a reason why angle DBC and angle BAD are equal. [1]

(ii) Show that angle EBC = angle BEC. [3]

(iii) Hence name an isosceles triangle and explain why it is so. [2]

12 Find, in ascending powers of x, the first three terms in the expansion of

(i) $(1+4x)^7$, [3]

(ii) $(2-x)^7$. [3]

Hence, find the coefficient of x^2 in the expansion of $(2+7x-4x^2)^7$. [2]

- End of paper -

CANBERRA SECONDARY SCHOOL

2021 Preliminary Examination

Secondary Four Express

ADDITIONAL MATHEMATICS
4049/02

27th August 2021
2 hours 15 minutes
0800h – 1015h

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Setter: Mr Lathif and Mrs Wee

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Answer **all** the questions

1 Given that $f(x)=2x^3+px^2+7x+q$.

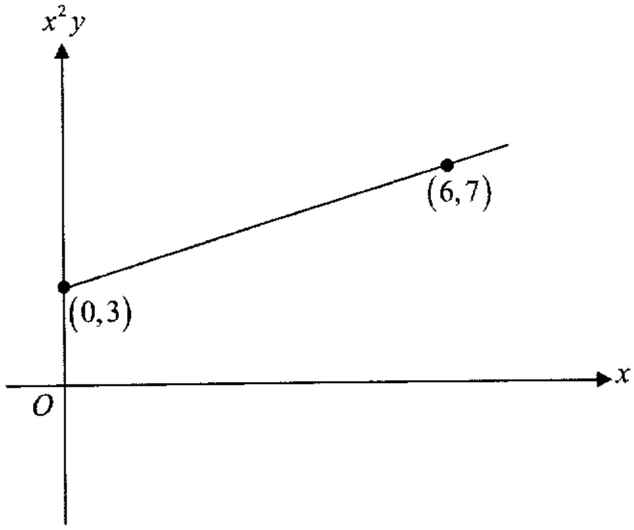
Find the value of p and of q for which $(x-3)$ and $(x-2)$ are factors of $f(x)$.

Hence solve $f(x)=0$. [8]

2 **(i)** Prove the identity $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=2\sec x$. [4]

(ii) Hence, solve the equation $\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=4$ for $0<x<2\pi$. [4]

3 The diagram shows part of a straight line obtained by plotting x^2y against x.

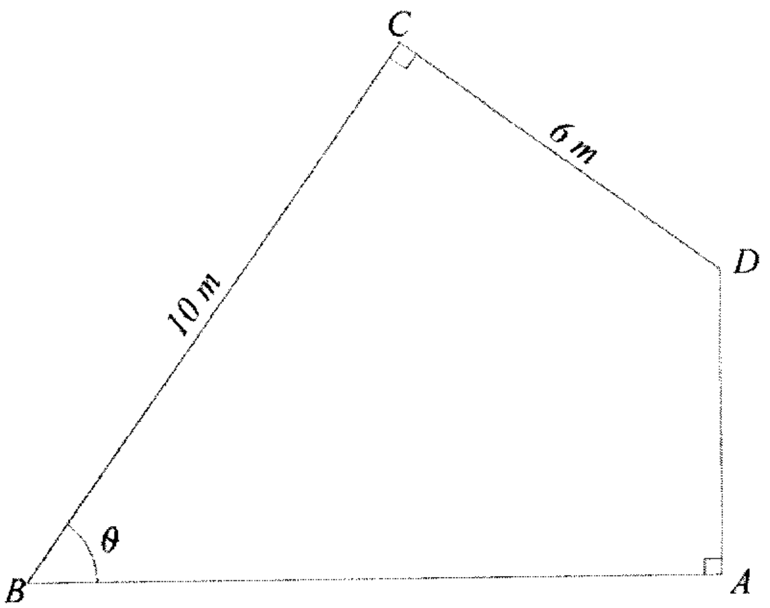


(i) Find an expression for y in terms of x. [4]

(ii) Find x when $y=\frac{5}{x^2}$. [2]

(iii) For the above equation found in part **(i)**, suggest another expression for the vertical and horizontal axes such that a straight line can also be plotted.
What is the value of the gradient and y – intercept of this new equation? [3]

4 The diagram below shows the side view of a warehouse $ABCD$ such that $BC = 10$ m, $CD = 6$ m, $\angle BCD = \angle BAD = 90°$ and $\angle CBA = \theta$.



(i) Show that $AB = 6\sin\theta + 10\cos\theta$. [2]

(ii) Express AB in the form $R\sin(\theta+\alpha)$, where R is positive and α is an acute angle.
Hence, find the maximum value of AB and the value of θ for which this occurs. [6]

(iii) Explain why this value of θ is the smallest possible. [2]

5 Points $A(3,8)$ and $B(6,5)$ are on a circle.

(i) Find the equation of perpendicular bisector of AB. [3]

(ii) Given that $y+x=8$ passes through the center of the circle, find the coordinates of the center and the radius of the circle. [3]

(iii) Hence, find the equation of the circle. [1]

(iv) Explain if point $C(6,3)$ is inside or outside the circle. [2]

(v) Which axis is tangent to the circle? Explain your answer. [2]

6 **(a)** Given that $y = \sec x$, show that $\dfrac{dy}{dx} = \sec x \tan x$.

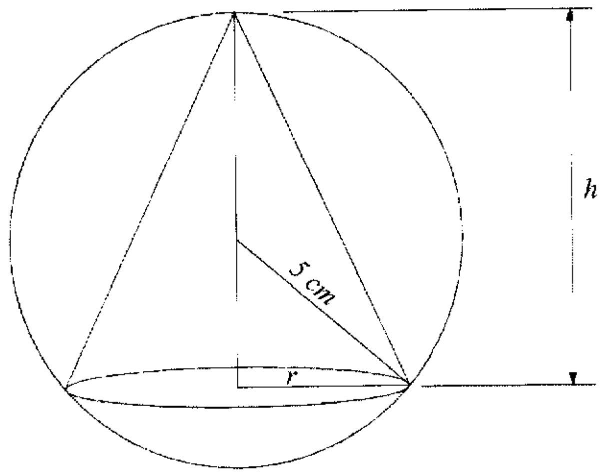
Hence show that $y^4 - y^2 = \left(\dfrac{dy}{dx}\right)^2$. [5]

(b) Given that $\frac{6x+1}{2x+1}$ can be written in the form $A+\frac{B}{2x+1}$, find A and B.

Hence, or otherwise $\int \frac{6x+1}{2x+1}\,dx$. [5]

7 A cone of height, h and radius, r fits exactly inside a sphere of radius 5 cm as shown below.

$\left(\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right)$



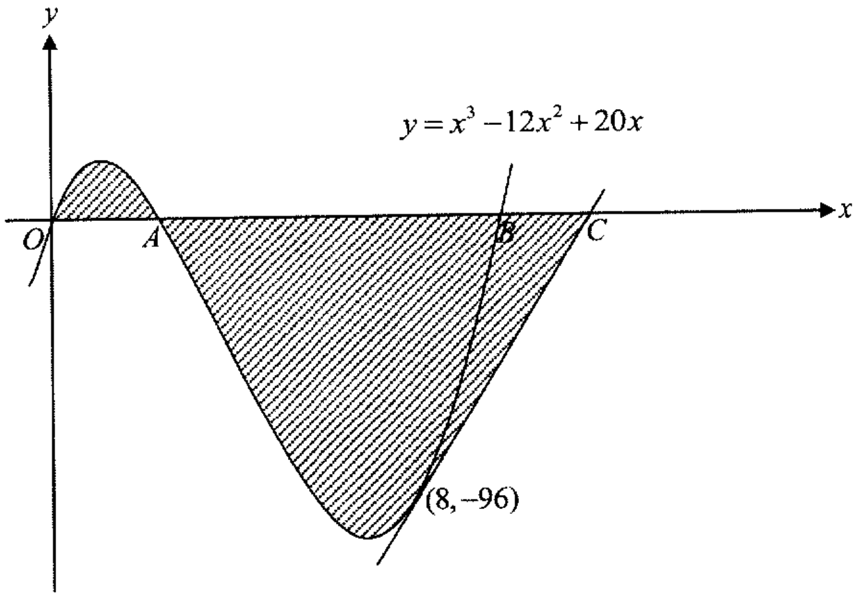
(i) Express r in terms of h. [2]

(ii) Show that the volume of cone is given by $V = \frac{\pi h^2(10-h)}{3}$. [2]

(iii) Find the value of h for which V is stationary. [3]

(iv) Find the volume of the cone for this value of h and determine if this volume is a maximum or minimum.
Justify your answer. [4]

8 The diagram below shows the curve $y = x^3 - 12x^2 + 20x$, which crosses the x – axis at the origin O and the points A and B.



(i) Find the coordinates of A and B. [3]

The tangent to the curve at the point $(8, -96)$ crosses the x – axis at the point C.

(ii) Find the equation of the tangent and hence, the coordinates of C. [4]

(iii) Find the area of the shaded region. [4]

9 **(a)** Solve $9^{x+1} - 3^{x+2} = 2\left(3^{x+1} - 2\right)$. [4]

(b) Given that $\log_3(2x-1) = \log_9(2x-1) + 1$, find the value of x. [3]

(c) Given that $\log_a xy = m$ and $\log_a \frac{x^3}{y^2} = n$, find $5\log_a\left(\frac{x}{y}\right)$ in terms of m and n. [5]

1	$2x^3-9x^2+7x+6=0$ $(2x+1)(x-3)(x-2)=0$ $x=-\frac{1}{2}$, $x=3$ or $x=2$
2ii	$x=\frac{\pi}{3},\frac{5\pi}{3}$
3i	$y=\frac{2}{3x}+\frac{3}{x^2}$
3ii	$x=3$
3iii	Plot *xy* against $\frac{1}{x}$ where the gradient is 3 and the *y*-intercept is $\frac{2}{3}$, o.e
4ii	$6\sin\theta+10\cos\theta=\sqrt{136}\sin(\theta+59.0°)$ Maximum *AB* = $\sqrt{136}$ or $2\sqrt{34}$ or 11.7 m when $\theta=31.0°$
4iii	For $AB=\sqrt{136}=\sqrt{10^2+6^2}$, *AD* must be 0, so that triangle *BCD* becomes a right-angled triangle. This will result that $\angle CBA=\theta$ to be smallest possible.
5i	$y=x+2$
5ii	Center $(3,5)$ Radius $=3$ units
5iii	$(x-3)^2+(y-5)^2=9$
5iv	*C* is outside of the circle.
5v	Since the *x*–coordinate of the center of the circle is equal to the radius, the *y*–axis is tangent to the circle.
6b	$3x-\ln(2x+1)+c$
7i	$r=\sqrt{10h-h^2}$
7iii	$h=6\frac{2}{3}$ cm
7iv	V=155 cm^3 (3 s.f.) Area is maximum.
8i	$A(2,0)$ and $B(10,0)$
8ii	$C(12.8,0)$
8iii	638.4 units^2
9a	$x=-1$ or $x\approx 0.262$
9b	$x=5$
9c	$5\log_a\left(\frac{x}{y}\right)=2n-m$

Answers

1(a)	$P=5, q=3$
2(a)	$-(x+5)^2+28$
3(a)	$5e^{2x+1}(2x+1)$
(b)	$\frac{1}{2}xe^{2x+1}-\frac{1}{4}e^{2x+1}+c$
(c)	$x<-\frac{1}{2}$
4(a)	$k>4\frac{1}{4}$
(b)	$a<0$ and $ac>16$
5(a)(i)	When $x=1$, $\frac{dy}{dx}=5$
(a)(ii)	$5y+x-11=0$
(b)	*Since* $(2-x)^2\geq 0$, $\frac{dy}{dx}=\frac{5}{(2-x)^2}\geq 0$
(c)	$\frac{1}{25}$
6(a)	$a=-2,\ b=1$
(b)(i)	$-\frac{56}{65}$
(b)(ii)	$\frac{25}{7}$
7i	48 m/s^2
7ii	$t=1$ or $t=3$
7iii	12 metres
8i	$\left(14+5\sqrt{3}\right)$ cm
8ii	$c=-1$
9i	$ 101 000 (to the nearest $1000)
9ii	The month and year is November 2025
10i	Equation of perpendicular bisector is $y=\frac{1}{7}x+1$
10ii	B has the coordinates (0, 1)
12i	$1+28x+336x^2+\ldots$
12ii	$128-448x+672x^2+\ldots$
12iii	31136