TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2021
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

Thursday 19 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **21** printed pages and **3** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

1(i) Find the set of values of the constant k for which $y = kx - 2$ meets the curve $y^2 = 4x - x^2$. [4]

(ii) Given that $2x^2 + bx + c$ is always positive, write down and simplify the relationship between b and c. Explain why c is never negative. [3]

2 Given that θ is an acute angle and $\sin\,\theta = m$, express in terms of m,

(i) $\cos\,\theta$, [1]

(ii) $\cot\,\theta$. [1]

Another acute angle α is such that $\tan(\theta-\alpha)=-\frac{1}{2}$.

(iii) What can be deduced about the size of θ relative to α ? [1]

(iv) Given further that $\tan\alpha=\frac{3}{2}$, show $\tan\theta=\frac{4}{7}$.
Hence, find the value of m. [3]

3 Given that $y = A - B\cos 4x - \frac{1}{2}\sin 2x$ and $\frac{d^2y}{dx^2} + 4y = 3\cos 4x + 1$, find the value of each of the following constants A and B. [4]

4 Given that $y = \frac{4}{5}\left(\frac{x}{12} - 1\right)^6$ and that both x and y vary with time, find the value of y when the rate of change of y is $12\frac{4}{5}$ times the rate of change of x. [5]

5 Given that $y = \dfrac{6x^2}{2-5x}$, find the range of values of x for which y is a decreasing function of x.

[4]

6 (i) A virus was spreading at a chicken farm such that 2% of chickens were infected in one day. The infected chickens were culled daily. If N is the total number of chickens before the start of the virus infection, and assuming that the virus continues to spread at the same rate, explain why the number of chickens expected to be alive after n days is given by $(0.98)^n N$. [2]

(ii) It is known that the virus will infect x % of chicken in 7 days. Calculate to the nearest 2 significant figures, the value of x. [2]

(iii) Given that the number of chickens expected to be alive after n days can be expressed as Ne^{kn}, find the value of the constant k. [2]

7 **(i)** Write down the general term in the binomial expansion of $\left(x+\frac{5}{x}\right)^n$. [1]

(ii) Given that there is a constant term in the expansion of $\left(x+\frac{5}{x}\right)^n$, show that n must be an even number. [2]

(iii) Find the term in x^6 in the expansion of $\left(x+\frac{5}{x}\right)^{10}$. [2]

(iv) Hence or otherwise, find the value of a, given that the coefficient of x^6 is 43875 in the expansion $\left(ax^2-1\right)\left(x+\frac{5}{x}\right)^{10}$. [3]

8 A gardener plans to construct two flower beds, one a circle of radius r m, the other a square of side x m. To protect the young flowers, the two beds are to be surrounded by wire netting. The total length of wire netting to be used is 30 m.

(i) Express x in terms of r and π. [1]

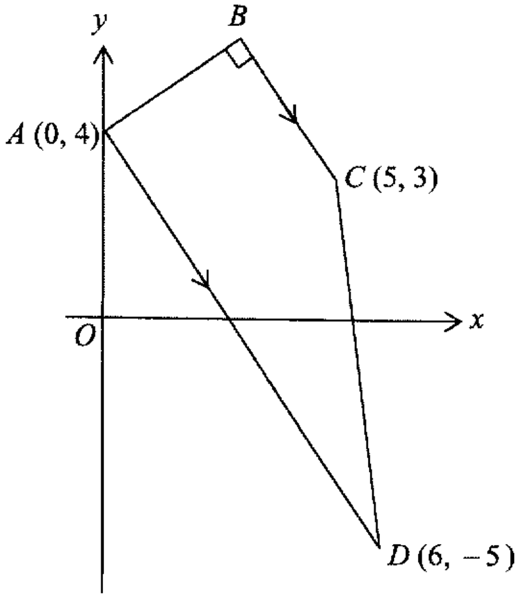
(ii) The combined area of the two flower beds is A m^2.
Show that $A=\left(\frac{1}{4}\pi^2+\pi\right)r^2-\frac{15}{2}\pi r+\frac{225}{4}$. [2]

(iii) Given that r can vary, find the value of r which gives a stationary value of A. [3]

(iv) Show that, when A is stationary, the side of the square is equal in length to the diameter of the circle. [2]

(v) Determine whether the stationary value of A is a maximum or minimum. [2]

9



The diagram shows a trapezium *ABCD* in which *AD* is parallel to *BC* and angle $ABC = 90°$.
The coordinates of *A*, *C* and *D* are (0, 4) , (5, 3) and (6, −5) respectively.

(i) Show that the coordinates of *B* are (3, 6). [4]

(ii) The point E lies on the line DA produced such that $\frac{\text{area of } \Delta BED}{\text{area of } \Delta BAD} = \frac{4}{3}$.

Find the coordinates of point E. [2]

(iii) Given that $ECDF$ form a kite in which FC is perpendicular to ED, find the coordinates of F. Hence, find the area of $ECDF$. [4]

10 A particle moves in a straight line, so that t seconds after passing through a fixed point O, its acceleration, a ms^{-2}, is given by $a = -12e^{2t}$. The initial velocity is 9 ms^{-1}.

(i) Show that the particle is instantaneously at rest at $t = \frac{1}{2}\ln\frac{15}{6}$. [5]

(ii) Find an expression for the displacement of the particle from O in terms of t.
Hence, find the total distance travelled by the particle when it returns to O. [4]

11 It is given that $f(x) = 2 - 3\sin 3x$ and $g(x) = 4\cos\left(\frac{x}{2}\right) - 1$.

(i) State the least and greatest values of f (x). [2]

(ii) State the least and greatest values of g (x). [2]

(iii) State the period of f (x). [1]

(iv) State the period of g (x). [1]

(v) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0° \leq x \leq 180°$. [4]

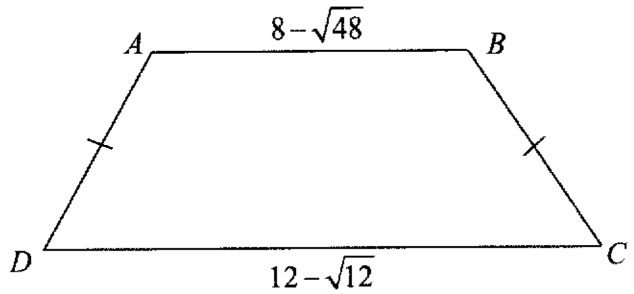
(vi) Find the number of solutions to the equation $2\cos\frac{x}{2} + \frac{3}{2}\sin 3x = \frac{3}{2}$ for $0° \leq x \leq 180°$. [2]

12 **(a)** Find the values of x and y which satisfy the equations

$$2^x \div 4^y = \frac{1}{8},$$

$$9^y(\sqrt{3})^{2x} = 3\sqrt{3}.$$ [5]

(b)



ABCD is a trapezium with an area of $12+11\sqrt{3}$ cm², $AB = 8-\sqrt{48}$ cm, $DC = 12-\sqrt{12}$ cm and $AD = BC$. Without using a calculator, obtain an expression for the perpendicular distance between *AB* and *DC* in the form $a+b\sqrt{3}$, where *a* and *b* are integers. [4]

End of Paper

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2021
Secondary 4

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Tuesday 24 August 2021

2 hours 15 minutes

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **20** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

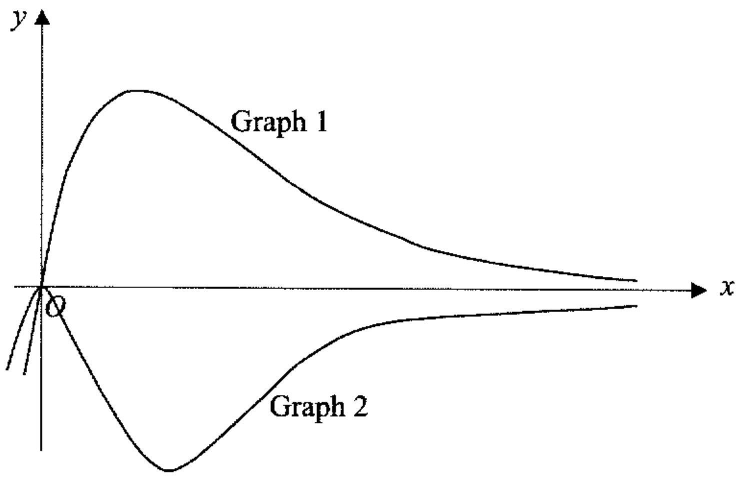
$$\Delta = \frac{1}{2}bc \sin A$$

Answer **all** questions.

1 **(i)** Differentiate $2xe^{1-2x}$ with respect to x. [2]

(ii) Hence find the exact value of $\int_0^1 2xe^{1-2x}\,dx$. [4]

(iii) Determine which one of two graphs below is that of $y = 2xe^{1-2x}$. Explain your answer. [2]



2 **(i)** Prove the identity $\dfrac{\tan^2 y - \sin^2 y}{\sec^2 y\left(\operatorname{cosec}^2 y - 1\right)} = \tan^2 y \sin^4 y$. [4]

(ii) Hence solve the equation $\dfrac{\tan^2 y - \sin^2 y}{\sec^2 y\left(\operatorname{cosec}^2 y - 1\right)} = \dfrac{1}{16}\tan^2 y$ for $0° \le y \le 180°$. [4]

3 **(i)** Show that $x-2$ is a factor of $x^3-12x+16$. [1]

(ii) Express $\dfrac{2+x^2}{x^3-12x+16}$ as the sum of three partial fractions. [7]

4 **(a)** Given that $\log_a 8 + 3\log_a N = 3$ and $N > \frac{1}{4}$, find the range of values of a. [4]

(b) Solve the equation $\log_3 x - \log_x 3x - 2 = 0$. [5]

5 It is given that $f'(x)=\sin 3x-\frac{1}{2x+1}$ and $f(0)=\frac{2}{3}$.

Find an expression for $6f(x)+f''(x)$. [6]

6 **(i)** Show that $\frac{d}{d\theta}\left(\cos^4\theta + \sin^4\theta\right) = p\sin q\theta$, where p and q are integers. [4]

(ii) Hence find the exact value of θ, for which the graph $y = \cos^4\theta + \sin^4\theta$ has a horizontal tangent for $0 < \theta < \frac{\pi}{2}$. [2]

7 The equation of a circle, centre C, is $x^2+y^2+px+\left(\frac{p}{2}+4\right)y+k=0$, where p and k are constants. It is given that C lies on the line $3x-2y-8=0$.

(i) Show that $p=-4$. [2]

(ii) Find the coordinates of C. [1]

(iii) Find the value of k, given that $x=-8$ is a tangent to the circle. [3]

The coordinates of point A is $(14, -8)$.

(iv) Show that A lies outside of the circle. [2]

(v) Point X lies on the circle such that it is furthest from A.

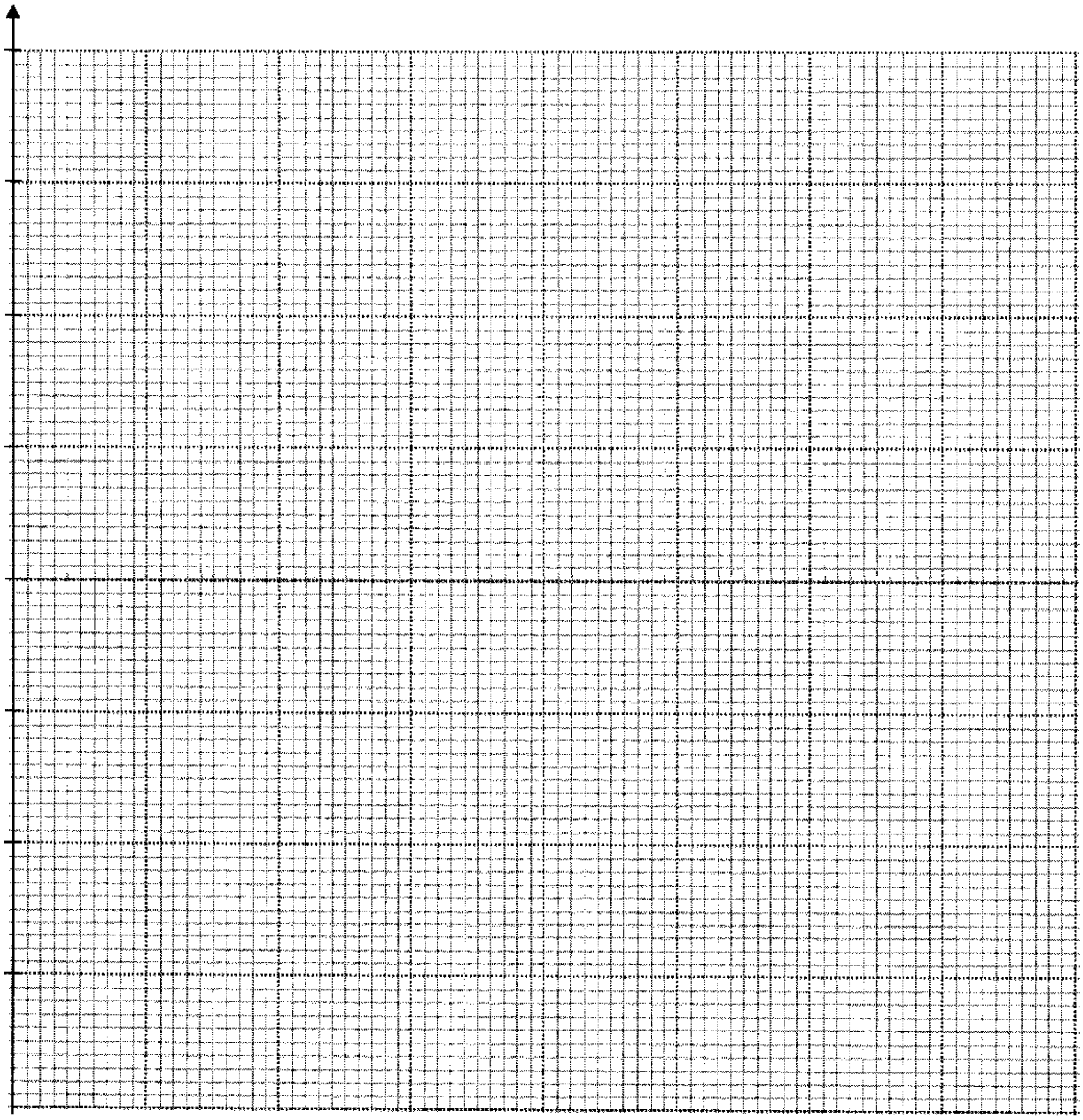
State the geometrical relationship between points A, C and X. [1]

8 Measured values of x and y are given in the following table.

x	1.2	1.6	2.0	2.6	3.5	4.5
y	11.1	6.29	3.98	2.36	1.55	0.79

It is known that x and y are to obey the formula $x^a y = c$, where a and c are constants to be found.

(i) Explain how a straight line graph may be drawn to represent the given formula. [2]

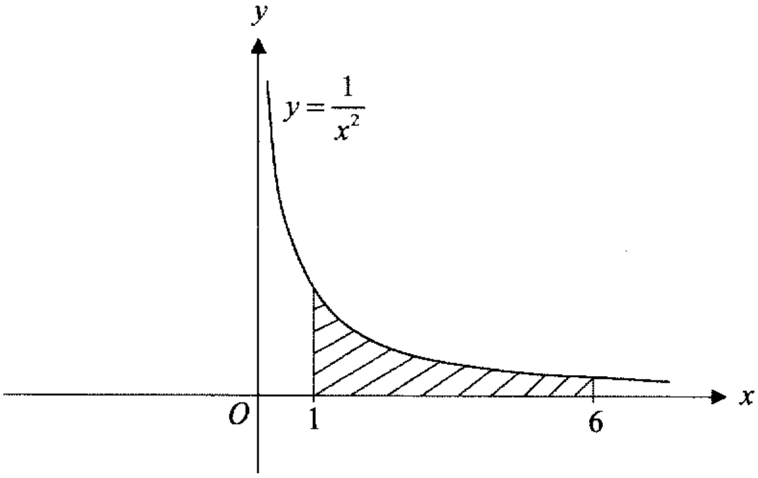


(ii) On the grid above, draw the straight line graph for the given data and **show** that 5 of the readings are a close fit to the formula. [4]

(iii) Assuming that x is correct, state which value of y appears to be wrong and estimate what it should be to fit the formula.; [2]

(iv) Find the value of a and of c. [3]

9 The diagram shows part of the curve $y = \frac{1}{x^2}$.



(i) Find the area bounded by the curve $y = \frac{1}{x^2}$, the lines $x = 1$, $x = 6$ and the x-axis. [3]

(ii) Show that $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} < \frac{5}{6}$. [2]

10 **(i)** Show that $4\sin x + 2\cos\left(x + \frac{\pi}{6}\right) = a\cos x + b\sin x$, where a and b are exact values. [3]

(ii) Express $4\sin x + 2\cos\left(x + \frac{\pi}{6}\right)$ in the form $k\cos(x - \alpha)$ where $k > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Find the value of k and of α. [3]

(iii) State the minimum value of $4\sin x + 2\cos\left(x+\frac{\pi}{6}\right)$ and the corresponding value of x. [2]

(iv) Solve the equation $4\sin x + 2\cos\left(x+\frac{\pi}{6}\right) = -2$ for $0 < x < 6$. [3]

11 **(i)** Find the gradient function of the curve $y = 2x^2 - \ln x + c$. [1]

(ii) The curve touches the positive x-axis at point T. Find the coordinates of T. [2]

(iii) Show that $c = -\ln 2\sqrt{e}$. [3]

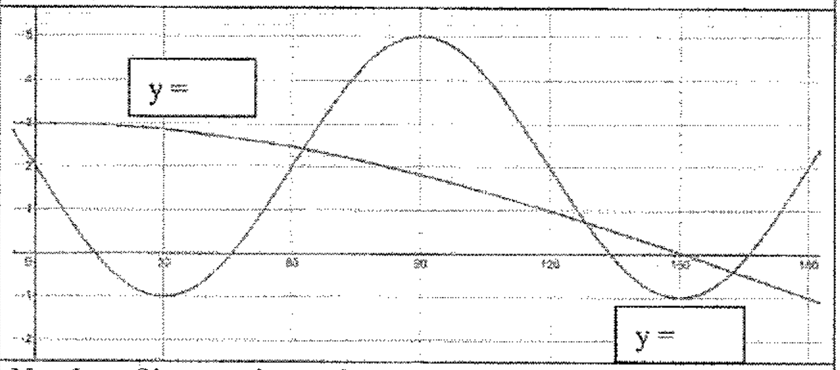
(iv) Find the value of $\frac{d^2y}{dx^2}$ at T and hence determine the nature of point T. [3]

End of Paper

ANSWERS

Qns	Solutions
1(i)	$k \geq 0$
(ii)	$2x^2 + bx + c > 0$ $b^2 - 4(2)(c) < 0$ $b^2 < 8c$ $8c > b^2$ $c > \frac{b^2}{8}$ $\sin ce\, b^2 \geq 0,$ $\frac{b^2}{8} \geq 0, c \geq 0$ Hence, c is never negative
2(i)	$\cos\theta = \sqrt{1-m^2}$
(ii)	$\cot\theta = \frac{\sqrt{1-m^2}}{m}$
(iii)	$\alpha > \theta$
(iv)	$m = \frac{4}{\sqrt{65}}$
3	$B = \frac{1}{4}, A = \frac{1}{4}$
4	$y = 51.2$
5	$x < 0 \quad or \quad x > \frac{4}{5}$
6(i)	Number of chicken alive after 1 day = (1-0.02)N = (0.98)N Number of chicken alive after 2 days = (0.98)(0.98)N =$(0.98)^2N$ Number of chicken alive after 3 days = $(0.98)(0.98)^2N$ =$(0.98)^3N$ Therefore the number of chickens expected to be alive after n days is $(0.98)^nN$.
(ii)	$x = 13$
(iii)	$k = -0.0202(3sf)$

Qns	Solutions
7(i)	$\binom{n}{r}5^r(x)^{n-2r}$
(ii)	For constant term, x^0 $n-2r=0$ $n=2r$ Hence, n must be an even number
(iii)	$1125x^6$
(iv)	$a=3$
8(i)	$x=\frac{15}{2}-\frac{\pi r}{2}$
(ii)	$x=\frac{15}{2}-\frac{\pi r}{2}$ $A=\pi r^2+x^2$ $=\pi r^2+\left(\frac{15}{2}-\frac{\pi r}{2}\right)^2$ $=\pi r^2+\left(\frac{225}{4}-\frac{15\pi r}{2}+\frac{\pi^2 r^2}{4}\right)$ $A=\left(\frac{1}{4}\pi^2+\pi\right)r^2-\frac{15}{2}\pi r+\frac{225}{4}$
(iii)	$r=2.1004$
(iv)	Diameter, 2r = 2(2.1004) = 4.2007 $x=\frac{15}{2}-\frac{\pi(2.1004)}{2}=4.2007$ Hence, x = *diameter*
(v)	Area is a minimum
9(ii)	$E=(-2,7)$
(iii)	Area $ECDF$ = 52 units^2
10(ii)	4.74 m
11(i)	least value of f (x) = -1 greatest value of f (x) = 5
(ii)	least value of g (x) = -5 greatest value of g (x) = 3
(iii)	period of f (x) = 120°
(iv)	period of g (x) = 720°

Qns	Solutions
(v)	
(vi)	Number of intersections = 3
12a	$x=\frac{-3}{4}, y=1\frac{1}{8}$
12b	$height=3+2\sqrt{3}$

Answers

1(i) $2e^{1-2x} - 4xe^{1-2x}$

1(ii) $\frac{1}{2}\left(e - \frac{3}{e}\right)$

1(iii) $\frac{1}{2}\left(e - \frac{3}{e}\right) > 0$, Graph 1 as graph is above the x-axis.

2(ii) $y = 0°, 30°, 150°, 180°$

3(ii) $\frac{2+x^2}{x^3-12x+16} = \frac{1}{2(x-2)} + \frac{1}{(x-2)^2} + \frac{1}{2(x+4)}$

4(a) $a > \frac{1}{2}, \quad a \neq 1$

4(b) $x = 37.7,\ 0.717$

5 $\cos 3x - 3\ln(2x+1) + 6 + \frac{2}{(2x+1)^2}$

6(i) $\frac{d}{dx}\left(\cos^4\theta + \sin^4\theta\right) = -\sin 4\theta$

6(ii) $\theta = \frac{\pi}{4}$

7(ii) $C = (2,\ -1)$

7(iii) $k = -95$

7(v) Points A, C and X are collinear.

8(i) $\lg y = -a\lg x + \lg c$

By plotting $\lg y$ against $\lg x$, a straight line will be obtained with $-a$ as the gradient and $\lg c$ as the vertical intercept.

8(iii) $y = 1.30$

8(iv) $a = 2$, $c = 16.2$

9(i) $\frac{5}{6}$

10(i) $4\sin x + 2\cos\left(x + \frac{\pi}{6}\right) = \sqrt{3}\cos x + 3\sin x$

10(ii) $k = 2\sqrt{3}$, $\alpha = \frac{\pi}{3}$, $4\sin x + 2\cos\left(x + \frac{\pi}{6}\right) = 2\sqrt{3}\cos\left(x - \frac{\pi}{3}\right)$

10(iii) Minimum value $= -2\sqrt{3}$, $x = \frac{4\pi}{3}$

10(iv) $x = 3.23,\ 5.14$

11(i) $\frac{dy}{dx} = 4x - \frac{1}{x}$

11(ii) $T = \left(\frac{1}{2},\ 0\right)$

11(iv) $\frac{d^2y}{dx^2} = 8$, minimum point

Additional Mathematics Paper 1

Qns	Solutions	Marks	Remarks
1(i)	$y^2 = 4x - x^2$ meets $y = kx - 2$		
	$(kx-2)^2 = 4x - x^2$	B1	Equate both eqns
	$k^2x^2 - 4kx + 4 - 4x + x^2 = 0$		
	$(k^2+1)x^2 - (4k+4)x + 4 = 0$	B1	Discriminant >=0
	$D \geq 0$	B1	Correct substitution of their a, b and c
	$(4k+4)^2 - 4(k^2+1)(4) \geq 0$		
	$16k^2 + 32k + 16 - 16k^2 - 16 \geq 0$		
	$32k \geq 0$	B1	
	$k \geq 0$		
(ii)	$2x^2 + bx + c > 0$		
	$b^2 - 4(2)(c) < 0$	B1	Attempt to use D
	$b^2 < 8c$	B1	relationship
	$8c > b^2$		
	$c > \frac{b^2}{8}$		
	$\sin ce\ b^2 \geq 0,$		
	$\frac{b^2}{8} \geq 0, c \geq 0$	B1	
	Hence, c is never negative		
2(i)	$\sin\theta = m$		
	$\cos\theta = \sqrt{1-m^2}$	B1	
(ii)	$\tan\theta = \frac{m}{\sqrt{1-m^2}}$		
	$\cot\theta = \frac{\sqrt{1-m^2}}{m}$	B1	
(iii)	$\tan(\theta - \alpha) = -\frac{1}{2}$	B1	
	$\alpha > \theta$		
(iv)	$\frac{\tan\theta - \tan\alpha}{1 + \tan\theta\tan\alpha} = -\frac{1}{2}$		
	$\frac{\tan\theta - \frac{3}{2}}{1 + \frac{3}{2}\tan\theta} = -\frac{1}{2}$	B1	Substitute ratio using compound angle formula
		AG	
	$\tan\theta = \frac{4}{7}$		

Qns	Solutions	Marks	Remarks
	$\frac{4}{7}=\frac{m}{\sqrt{1-m^2}}$	M1	Equate from reciprocal of their (ii)
	$4\sqrt{1-m^2}=7m$		
	$16(1-m^2)=49m^2$		
	$m^2=\frac{16}{65}$		
	$m=\frac{4}{\sqrt{65}}$ $\quad (reject\ \frac{-4}{\sqrt{65}})$	A1	
3	$y=A-B\cos 4x-\frac{1}{2}\sin 2x$		
	$\frac{dy}{dx}=4B\sin 4x-\cos 2x$	B1	Either 4Bsin4x or – cos2x see
	$\frac{d^2y}{dx^2}=16B\cos 4x+2\sin 2x$	B1	Second derivative
	$\frac{d^2y}{dx^2}+4y$		
	$=16B\cos 4x+2\sin 2x+4\left[A-B\cos 4x-\frac{1}{2}\sin 2x\right]$	M1	Their second derivative + 4y
	$=12B\cos 4x+4A$		
	$\therefore 12B\cos 4x+4A=3\cos 4x+1$		
	$B=\frac{1}{4}, A=\frac{1}{4}$	A1	Both correct

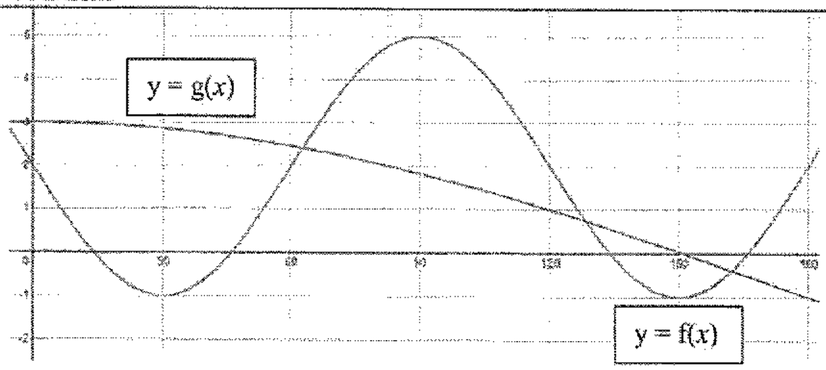
Qns	Solutions	Marks	Remarks
4	$y=\frac{4}{5}\left(\frac{x}{12}-1\right)^6$ $\frac{dy}{dx}=\frac{24}{5}\left(\frac{x}{12}-1\right)^5\left(\frac{1}{12}\right)=\frac{2}{5}\left(\frac{x}{12}-1\right)^5$	B1	1^{st} derivative using chain rule
	$given \frac{dy}{dt}=\frac{64}{5}\times\frac{dx}{dt}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\frac{64}{5}\times\frac{dx}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $\therefore \frac{dy}{dx}=\frac{64}{5}$	B1	Form connected rates of change Or dy/dx = 64/5
	$\frac{2}{5}\left(\frac{x}{12}-1\right)^5=\frac{64}{5}$ $\left(\frac{x}{12}-1\right)^5=32$	M1	Their 1^{st} derivative to 64/5
	$x=36$	A1	
	$y=51.2$	A1	
5	$y=\frac{6x^2}{2-5x}$ $\frac{dy}{dx}=\frac{12x(2-5x)-6x^2(-5)}{(2-5x)^2}$ $=\frac{24x-60x^2+30x^2}{(2-5x)^2}$ $=\frac{24x-30x^2}{(2-5x)^2}$	M1	Quotient rule with (2-5x)( ) – $6x^2$( ) seen
	For decreasing function, $\frac{dy}{dx}<0$ $\frac{24x-30x^2}{(2-5x)^2}<0$	B1	Their dy/dx < 0
	$(2-5x)^2>0$ $24x-30x^2<0$ $6x(4-5x)<0$	M1	Their nume < 0 provided deno is squared.
	$x<0 \quad or \quad x>\frac{4}{5}$	A1	

Qns	Solutions	Marks	Remarks
6(i)	Number of chicken alive after 1 day $= (1-0.02)N$ $= (0.98)N$ Number of chicken alive after 2 days $= (0.98)(0.98)N$ $=(0.98)^2N$ Number of chicken alive after 3 days $= (0.98)(0.98)^2N$ $=(0.98)^3N$ Therefore the number of chickens expected to be alive after n days is $(0.98)^nN$.	B1 B1	0.98 N seen Show the exponential relationship between no of days and 0.98
(ii)	$\frac{100-x}{100}\times N=(0.98)^7 N$ $\frac{100-x}{100}=(0.98)^7$ $100-x=100(0.98)^7$ $x=13$	M1 A1	o.e.
(iii)	$Ne^{kn}=(0.98)^n N$ $e^{kn}=(0.98)^n$ $e^k=(0.98)$ $k=\ln 0.98$ $=-0.0202(3sf)$	M1 A1	Equate (interpret info from qn)
7(i)	$\binom{n}{r}(x)^{n-r}(5x^{-1})^r$ $\binom{n}{r}5^r(x)^{n-2r}$	B1	
(ii)	For constant term, x^0 n-2r =0 n = 2r Hence, n must be an even number	M1 A1	Equate power to 0 With conclusion
(iii)	$6=10-2r$ $r=2$ $\binom{10}{2}5^2(x)^{10-2(2)}$ $=1125x^6$	B1 B1	Find r = 2 or 8 soi
(iv)	Term in x^4: $4=10-2r$ $r=3$ $\binom{10}{3}5^3(x)^{10-2(3)}$ $=15000x^4$	M1	Find the term in x^4

Qns	Solutions	Marks	Remarks
	$\left(ax^2-1\right)\left(x+\frac{5}{x}\right)^{10}$ $=\left(ax^2-1\right)(.........+1125x^6+15000x^4+.......)$ 15000a − 1125 = 43875 a = 3	 M1 A1	compare terms to form x^6 term
8(i)	$2\pi r+4x=30$ $x=\frac{15}{2}-\frac{\pi r}{2}$	B1	
(ii)	$x=\frac{15}{2}-\frac{\pi r}{2}$ $A=\pi r^2+x^2$ $=\pi r^2+\left(\frac{15}{2}-\frac{\pi r}{2}\right)^2$ $=\pi r^2+\left(\frac{225}{4}-\frac{15\pi r}{2}+\frac{\pi^2 r^2}{4}\right)$ $A=\left(\frac{1}{4}\pi^2+\pi\right)r^2-\frac{15}{2}\pi r+\frac{225}{4}$	B1 B1	Substitute their (i) Expand quadratic AG
(iii)	$\frac{dA}{dr}=2\left(\frac{1}{4}\pi^2+\pi\right)r-\frac{15}{2}\pi$ $\frac{dA}{dr}=0$ $2\left(\frac{1}{4}\pi^2+\pi\right)r=\frac{15}{2}\pi$ $r=2.1004$	B1 M1 A1	1st derivative Their dA/dr = 0
(iv)	Diameter, 2r = 2(2.1004) = 4.2007 $x=\frac{15}{2}-\frac{\pi(2.1004)}{2}=4.2007$ Hence, $x=diameter$	M1 A1	Use their r to find either diameter or length of square side Correct values of x and r seen
(v)	$\frac{d^2A}{dr^2}=2\left(\frac{1}{4}\pi^2+\pi\right)$ $\frac{d^2A}{dr^2}>0$ Area is a minimum	B1 √ B1	o.e. first derivative test with table conclusion

Qns	Solutions	Marks	Remarks
9(i)	$gradient\ AD = \frac{4-(-5)}{0-6} = \frac{-3}{2}$ $gradient\ BC = \frac{-3}{2}$ $Eqn\ BC:\quad y-3 = \frac{-3}{2}(x-5)$ $y = \frac{-3}{2}x + 10\frac{1}{2}$ ----------(1) $Eqn\ AB:\quad y-4 = \frac{2}{3}x$ $y = \frac{2}{3}x + 4$ -----------(2) $(1)-(2):\quad 0 = -2\frac{1}{6} + 6\frac{1}{2}$ $x = 3$ $y = \frac{2}{3}(3) + 4$ $= 6$ $\therefore B = (3,6)$	M1 M1 M1 A1	Attempt to find eqn BC or AB Use reciprocal of gradient to find another eqn Solve their eqns Both x and y correct with working shown, leading to answer
(ii)	$\frac{ED}{AD} = \frac{4}{3}$ $E = (-2, 7)$	M1 A1	Any method including drawing on diagram with ration seen.
(iii)	By counting, $F = (-1, -1)$ Area ECDF $= 2 \times \frac{1}{2}\begin{vmatrix} -2 & -1 & 6 & -2 \\ 7 & -1 & -5 & 7 \end{vmatrix}$ $= [2+5+42]-[10-6-7]$ $= 49+3$ $= 52 units^2$	B1 M1 M1 A1	Coor of F Determinant method Or Find ED Evaluate determinant or find area of triangles o.e.

Qns	Solutions	Marks	Remarks
10(i)	$a=-12e^{2t}$ $v=\int -12e^{2t}\,dt$ $=-6e^{2t}+c$ $t=0, V=9,$ $9=-6e^{2(0)}+c$ $c=15$ $\therefore v=-6e^{2t}+15$ $v=0,$ $-6e^{2t}+15=0$ $e^{2t}=\frac{15}{6}$ $2t=\ln\frac{15}{6}$ $t=\frac{1}{2}\ln\frac{15}{6}$	B1 M1 A1 M1 A1	$-6e^{2t}$ seen Attempt to find c Their v = 0 Taking ln, leading to CAG
(ii)	$v=-6e^{2t}+15$ $s=\int -6e^{2t}+15\,dt$ $=-3e^{2t}+15t+c$ $t=0, s=0,$ $0=-3e^{2(0)}+15(0)+c$ $c=3$ $\therefore s=-3e^{2t}+15t+3$ Distance when particle stops to change direction: Sub $t=\frac{1}{2}\ln\frac{15}{6}$, s = 2.372m When particle returns to *O*, total distance travelled = 2(2.372) = 4.74m	B1 √ M1 A1 B1	Integrate their v to find s Attempt to find c
11(i)	least value of f (x) = -1 greatest value of f (x) = 5	B1 B1	
(ii)	least value of g (x) = -5 greatest value of g (x) = 3	B1 B1	
(iii)	period of f (x) = 120	B1	Accept $\frac{2\pi}{3}$
(iv)	period of g (x) = 720	B1	Accept 4π

Qns	Solutions	Marks	Remarks
(v)		G1 G1 G1 G1	For cosine graph fully correct For cosine graph 1.5 cycles seen Starts and ends at 2 Graph is fully correct -1mark for not labelling
(vi)	$2\cos\frac{x}{2}+\frac{3}{2}\sin 3x=\frac{3}{2}$ $4\cos\frac{x}{2}+3\sin 3x=3$ $4\cos\frac{x}{2}+3\sin 3x=2+1$ $4\cos\frac{x}{2}-1=2-3\sin 3x$ Number of intersections = 3	M1 A1 √	Manipulation seen Their number of intersections provided that manipulation is seen
12a	$2^x \div 4^y=\frac{1}{8}$ $2^x \div 2^{2y}=2^{-3}$ $x-2y=-3 \quad ----(1)$ $9^y(\sqrt{3})^{2x}=3\sqrt{3}$ $3^{2y}\times 3^{\frac{1}{2}(2x)}=3^{\frac{3}{2}}$ $2y+x=\frac{3}{2} \quad ----(2)$ $(1)+(2):\ 2x=-1.5$ $x=\frac{-3}{4}$ $y=1\frac{1}{8}$	M1 M1 M1 A1 A1	Change to same base for either eqn Index rule seen for either eqn Comparing index

Qns	Solutions	Marks	Remarks
12b	$\frac{1}{2}\left(8-\sqrt{48}+12-\sqrt{12}\right) height = 12+11\sqrt{3}$	M1	Forming equation
	$\frac{1}{2}\left(8-2\sqrt{12}+12-\sqrt{12}\right) height = 12+11\sqrt{3}$ $\left(20-3\sqrt{12}\right) height = 24+22\sqrt{3}$ $\left(20-6\sqrt{3}\right) height = 24+22\sqrt{3}$	M1	Attempt to reduce / simplify surds (take out largest perfect square o.e.)
	$height = \frac{12+11\sqrt{3}}{\left(10-3\sqrt{3}\right)} \times \frac{\left(10+3\sqrt{3}\right)}{\left(10+3\sqrt{3}\right)}$ $= \frac{120+36\sqrt{3}+110\sqrt{3}+99}{100-27}$	M1	Simplify with their conjugate seen
	$= \frac{219+146\sqrt{3}}{73}$ $= 3+2\sqrt{3}$	A1	

Qn	Working	
1(i)	$\frac{d}{dx}2xe^{1-2x}$ $=2e^{1-2x}+2x\left(-2e^{1-2x}\right)$ $=2e^{1-2x}-4xe^{1-2x}$ $=2e^{1-2x}(1-2x)$	B1 for $2e^{1-2x}$ B1 for $2x\left(-2e^{1-2x}\right)$ isw
1(ii)	$\int_0^1 2e^{1-2x}-4xe^{1-2x}dx=\left[2xe^{1-2x}\right]_0^1$ $-2\int_0^1 2xe^{1-2x}dx=\left(2e^{-1}-0\right)-\int_0^1 2e^{1-2x}dx$ $-2\int_0^1 2xe^{1-2x}dx=\frac{2}{e}-\left[\frac{2}{-2}e^{1-2x}\right]_0^1$ $\int_0^1 2xe^{1-2x}dx=-\frac{1}{e}+\frac{1}{2}\left[-e^{-1}+e^{1}\right]$ $\int_0^1 2xe^{1-2x}dx=-\frac{1}{e}-\frac{1}{2e}+\frac{e}{2}$ $\int_0^1 2xe^{1-2x}dx=-\frac{3}{2e}+\frac{e}{2}=\frac{1}{2}\left(e-\frac{3}{e}\right)=\frac{e^2-3}{2e}$	B1 Using diff for integral M1 Splitting the integrals, attempt to manipulate B1 Integrate $\int_0^1 2e^{1-2x}dx=-\frac{2}{2}e^{1-2x}$ A1 exact o.e.
1(iii)	When $x>0$, $2x>0$ and $e^{1-2x}>0$ $\therefore\ 2xe^{1-2x}>0$ $y>0$ $\therefore$ Graph 1 is $y=2xe^{1-2x}$. **Or** Since area under the graph is positive because, $\int_0^1 2xe^{1-2x}dx=-\frac{3}{2e}+\frac{e}{2}=\frac{1}{2}\left(e-\frac{3}{e}\right)>0$ thus, graph is above the x-axis. $\therefore$ Graph 1 is $y=2xe^{1-2x}$.	B1 Any logical reasoning B1 Graph 1

2(i)	Taking LHS, $\dfrac{\tan^2 y-\sin^2 y}{\sec^2 y\left(\operatorname{cosec}^2 y-1\right)}$	
	$=\dfrac{\tan^2 y-\sin^2 y}{\sec^2 y\left(\cot^2 y\right)}$	B1 Reciprocal of sec/cosec B1 Use of any sq identities
	$=\dfrac{\tan^2 y-\sin^2 y}{\dfrac{1}{\cos^2 y}\left(\dfrac{\cos^2 y}{\sin^2 y}\right)}$	B1 $\tan y=\dfrac{\sin y}{\cos y}$
	$=\sin^2 y\left(\tan^2 y-\sin^2 y\right)$	B1 Manipulate leading to correct answer
	$=\sin^2 y\left(\dfrac{\sin^2 y}{\cos^2 y}\right)-\sin^4 y$	
	$=\sin^4 y\left(\dfrac{1}{\cos^2 y}-1\right)$	
	$=\sin^4 y\left(\sec^2 y-1\right)$	
	$=\tan^2 y\sin^4 y$ (shown)	AG
2(ii)	$\dfrac{\tan^2 y-\sin^2 y}{\sec^2 y\left(\operatorname{cosec}^2 y-1\right)}=\dfrac{1}{16}\tan^2 y$	
	From (i),	
	$\tan^2 y\sin^4 y=\dfrac{1}{16}\tan^2 y$	B1 Using part (i)
	$16\tan^2 y\sin^4 y-\tan^2 y=0$	
	$\tan^2 y\left(16\sin^4 y-1\right)=0$	B1 Factorisation
	$\tan^2 y=0$ or $16\sin^4 y-1=0$	
	$\tan y=0$; $\sin y=\pm\dfrac{1}{2}$	
	$y=0°,180°$; $y=30°,150°$	B2 2 sets of answers
	$\therefore\ y=0°,30°,150°,180°$	

3(i)	Let $f(x)=x^3-12x+16$ Sub. $x=2$, $f(2)=(2)^3-12(2)+16$ $=0$ By Factor Theorem since $f(2)=0$, $(x-2)$ is a factor. (shown)	B1 Substitution and Factor Theorem/ Remainder Theorem must be quoted

3(ii)	$x^3-12x+16=(x-2)(x^2+px-8)$
	
	By comparing coefficient of x^2,
	$0=p-2$
	$p=2$
	
	$\therefore\ f(x)=(x-2)(x^2+2x-8)$
	$=(x-2)(x-2)(x+4)$
	$=(x-2)^2(x+4)$
	
	$\dfrac{2+x^2}{x^3-12x+16}=\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}+\dfrac{C}{x+4}$
	$2+x^2=A(x-2)(x+4)+B(x+4)+C(x-2)^2$
	Sub $x=2$, $B=1$
	Sub $x=-4$, $C=\dfrac{1}{2}$
	Sub $x=0$, $A=\dfrac{1}{2}$
	$\dfrac{2+x^2}{x^3-12x+16}=\dfrac{1}{2(x-2)}+\dfrac{1}{(x-2)^2}+\dfrac{1}{2(x+4)}$
		M1 Inspection/Long Div
		
		B1 Quad factor soi
		
		B1 3 Factors soi
		Lose 3m if use calculator
		
		B1 Correct cases √ (must be cubic)
		
		M1 Subst/Comparing coef
		
		A2 *A, B, C*
		
		[− 1m of not stated]

4(a)	$\log_a 8+3\log_a N=3$ $\log_a 8N^3=3$ $a^3=8N^3$ $a=2N$ $N=\frac{a}{2}$ $\frac{a}{2}>\frac{1}{4}$ since $N>\frac{1}{4}$ $a>\frac{1}{2},\ a\neq 1$	M1 $\log_a 8N^3$ (combine) M1 $a^3=8N^3$ (remove log) M1 (Their N) $\frac{a}{2}>\frac{1}{4}$ A1 Answer
4(b)	$\log_3 x-\log_x 3x-2=0$ $\log_3 x-\frac{\log_3 3x}{\log_3 x}-2=0$ $(\log_3 x)^2-\log_3 3x-2\log_3 x=0$ $(\log_3 x)^2-(\log_3 3+\log_3 x)-2\log_3 x=0$ $(\log_3 x)^2-1-3\log_3 x=0$ Let $u=\log_3 x$, $u^2-3u-1=0$ $u=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-1)}}{2(1)}$ $u=3.30278$ or $u=-0.30278$ $\log_3 x=3.30278$ or $\log_3 x=-0.30278$ $x=3^{3.30278}$ $x=3^{-0.30278}$ $x=37.7$ (3 sf) $x=0.717$ (3 sf)	B1 Change base soi B1 Addition log law soi M1 Forming equation (Subst) M1 Solving Quad A1 both answers

5	$f''(x)=3\cos 3x+\frac{2}{(2x+1)^2}$ $f(x)=-\frac{1}{3}\cos 3x-\frac{1}{2}\ln(2x+1)+c$ Sub $f(0)=\frac{2}{3}$, $\frac{2}{3}=-\frac{1}{3}\cos 3(0)-\frac{1}{2}\ln(1)+c$ $c=1$ $f(x)=-\frac{1}{3}\cos 3x-\frac{1}{2}\ln(2x+1)+1$ $6f(x)+f''(x)$ $=-2\cos 3x-3\ln(2x+1)+6+3\cos 3x+\frac{2}{(2x+1)^2}$ $=\cos 3x-3\ln(2x+1)+6+\frac{2}{(2x+1)^2}$	B1 $3\cos 3x$ seen B1 $\frac{2}{(2x+1)^2}$ seen B1 $-\frac{1}{3}\cos 3x$ seen B1 $-\frac{1}{2}\ln(2x+1)$ seen M1 Finding c A1 Substitution √ o.e. isw
6(i)	$\frac{d}{d\theta}(\cos^4\theta+\sin^4\theta)$ $=4\cos^3\theta(-\sin\theta)+4\sin^3\theta(\cos\theta)$ $=-4\cos^3\theta\sin\theta+4\sin^3\theta\cos\theta$ $=-4\sin\theta\cos\theta(\cos^2\theta-\sin^2\theta)$ $=-2\sin 2\theta\cos 2\theta$ $=-\sin 4\theta$	M1 Chain rule A1 cao M1 Use of $\cos 2\theta$ / $\sin 2\theta$ formula A1
6(ii)	$\frac{d}{d\theta}(\cos^4\theta+\sin^4\theta)=0$ $-\sin 4\theta=0$ $4\theta=\pi$ $\theta=\frac{\pi}{4}$	B1 their $\frac{dy}{dx}=0$ B1 exact cao

7(i)	$x^2+y^2+px+\left(\frac{p}{2}+4\right)y+k=0$ Centre, $C=\left(-\frac{p}{2},\ -\frac{p}{4}-2\right)$ Sub into eqn of line; $3\left(-\frac{p}{2}\right)-2\left(-\frac{p}{4}-2\right)-8=0$ $3p-p-8+16=0$ $p=-4$ (shown)	B1 Centre of circle B1 Substitution √ (wrt centre) AG
7(ii)	$C=(2,\ -1)$	B1 √
7(iii)	Radius $=10$ $10=\sqrt{(2)^2+(-1)^2-k}$ $100=4+1-k$ $k=-95$	B1 M1 Subst of formula (ignore r) A1
7(iv)	Length $CA=\sqrt{(2-14)^2+(-1-(-8))^2}$ $=13.89>10$ $\therefore$ A lies outside of the circle. (shown)	M1 Finding length of point from centre of circle A1 cao
7(v)	ACX will be a straight line. OR Points A, C and X are collinear.	B1 Statement

8(i)

$x^a y = c$

$\lg x^a + \lg y = \lg c$

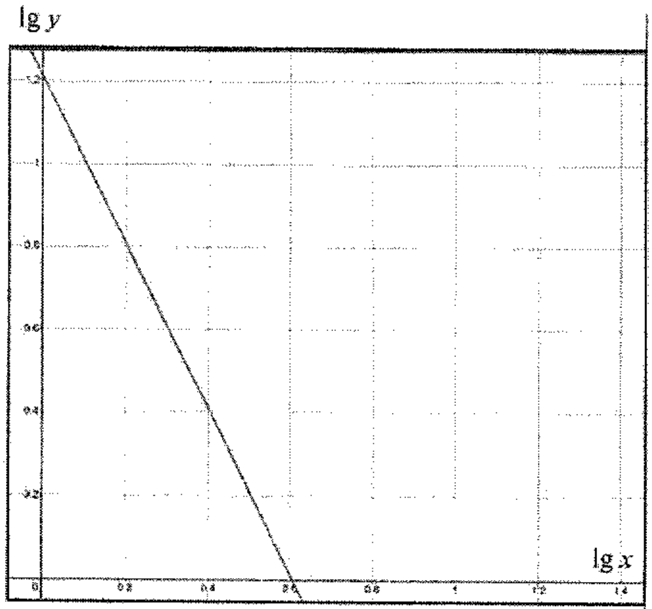
$\lg y = -a \lg x + \lg c$

By plotting $\lg y$ against $\lg x$, a straight line will be obtained with $-a$ as the gradient and $\lg c$ as the vertical intercept.

B1 $Y = mX + c$
B1 Statement

8(ii)

$\lg x$	0.08	0.20	0.30	0.41	0.54	0.65
$\lg y$	1.05	0.80	0.60	0.37	0.19	−0.10



T1 New table of values
Accept only 2, 3, 4 dp accuracy
P1 Pts correctly plotted
L1 Line of best fit
L1 Vertical intercept seen with one outlier point

Or

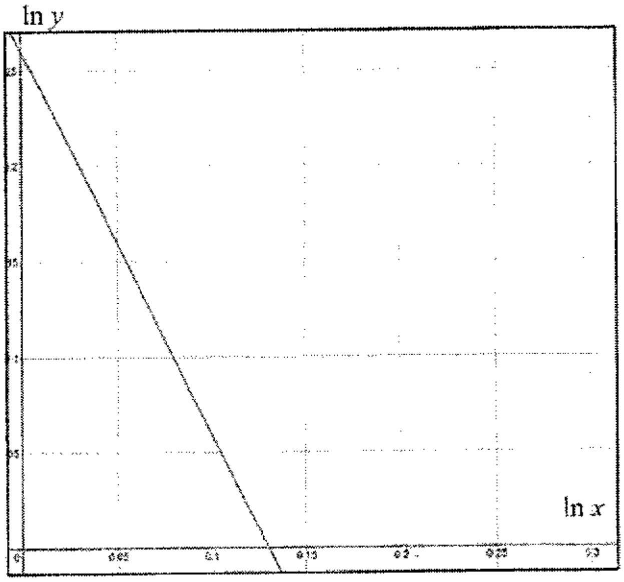
$x^a y = c$

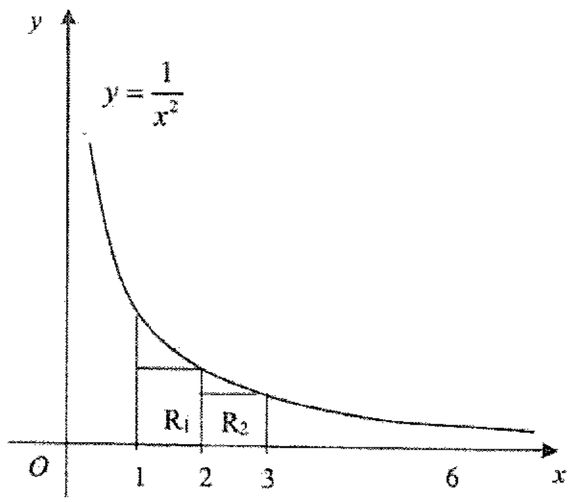
$\ln x^a + \ln y = \ln c$

$\ln y = -a \ln x + \ln c$

By plotting $\ln y$ against $\ln x$, a straight line will be obtained with $-a$ as the gradient and $\ln c$ as the vertical intercept.

$\ln x$	0.18	0.47	0.69	0.96	1.25	1.50
$\ln y$	2.41	1.84	1.38	0.86	0.44	−0.24

	 As shown from the plotted graph, **one point is an outlier point** from the straight line $Y = mX + C$ which is at $\lg x = 0.54$ / $\ln x = 1.25$.	Not required. Teacher to reiterate when papers are returned – explain qns
8(iii)	$y = 1.55$ For graph of $\lg y$ against $\lg x$, $\lg y = 0.11$ $y = 10^{0.11}$ $y = 1.29$ **Or** For graph of $\ln y$ against $\ln x$, $\ln y = 0.26$ $y = e^{0.26}$ $y = 1.30$	B1 Incorrect value of y B1 [1.26 to 1.32]
8(iv)	For graph of $\lg y$ against $\lg x$, $\lg c = 1.21$ $c = 10^{1.21}$ $c = 16.2$ (3sf) $-a = \dfrac{0.9-0.2}{0.15-0.5}$ $a = 2$ **Or** For graph of $\ln y$ against $\ln x$, $\lg c = 2.76$ $c = e^{2.76}$ $c = 15.8$ (3sf) $-a = \dfrac{2.36-0.4}{0.2-1.18}$ $a = 2$	M1 Vertical intercept used [1.20 to 1.22] A1 [15.5 to 16.5] A1 [−1.94 to −2.06] [2.74 to 2.78]

9(i)	Area $= \int_1^6 \frac{1}{x^2} dx$ $= \left[-\frac{1}{x}\right]_1^6$ $= -\frac{1}{6} + \frac{1}{1}$ $= \frac{5}{6}$	B1 $\int_1^6 \frac{1}{x^2} dx$ seen B1 $-\frac{1}{x}$ soi B1
9(ii)	 Area of R_1 = Length × Breadth $= 1 \times \frac{1}{2^2}$ $= \frac{1}{2^2}$ Area of R_2 = Length × Breadth $= 1 \times \frac{1}{3^2}$ $= \frac{1}{3^2}$ Area of $\left(R_1 + R_2 + R_3 + R_4 + R_5\right) <$ Shaded area $\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} < \frac{5}{6}$ (shown)	B1 Identify the rectangles B1 Conclusion AG

10(i)	$4\sin x + 2\cos\left(x+\frac{\pi}{6}\right)$ $= 4\sin x + 2\left(\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6}\right)$ $= 4\sin x + 2\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right)$ $= 4\sin x + \sqrt{3}\cos x - \sin x$ $= \sqrt{3}\cos x + 3\sin x$	M1 soi $\cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6}$ B1 $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ soi A1
10(ii)	Let $\sqrt{3}\cos x + 3\sin x = k\cos(x-\alpha)$ $k = \sqrt{\left(\sqrt{3}\right)^2 + 3^2}$ $k = \sqrt{12}$ $k = 2\sqrt{3}$ $\alpha = \tan^{-1}\frac{3}{\sqrt{3}}$ $\alpha = \frac{\pi}{3}$ $4\sin x + 2\cos\left(x+\frac{\pi}{6}\right) = 2\sqrt{3}\cos\left(x-\frac{\pi}{3}\right)$	B1 Find k o.e. B1 Finding α o.e. B1 Answer √ o.e.
10(iii)	Minimum value $= -\sqrt{12} = -2\sqrt{3}$ Occurs when $x = \frac{4\pi}{3}$	B1 √ o.e. B1
10(iv)	$\sqrt{12}\cos\left(x-\frac{\pi}{3}\right) = -2$ $\cos\left(x-\frac{\pi}{3}\right) = \frac{-2}{\sqrt{12}}$ (2^{nd} / 3^{rd} quads) Basic angle of $x-\frac{\pi}{3} = 0.95532$ $x-\frac{\pi}{3} = 2.1863,\ 4.0969$ $x = 3.23,\ 5.14$ (3sf) Domain: $0 < x < 6$ $-1.0472 < x-\frac{\pi}{3} < 4.9528$	B1 Statement √ o.e. B2 Answers Accept 5.15

11(i)	$y = 2x^2 - \ln x + c$ $\frac{dy}{dx} = 4x - \frac{1}{x}$	B1 $4x - \frac{1}{x}$ soi
11(ii)	$4x - \frac{1}{x} = 0$ $4x^2 - 1 = 0$ $x = \frac{1}{2}$ $(\because x > 0)$ $T = \left(\frac{1}{2}, 0\right)$	B1 $=0$ √ B1 Coordinates
11(iii)	Sub $\left(\frac{1}{2}, 0\right)$, $0 = 2\left(\frac{1}{2}\right)^2 - \ln\frac{1}{2} + c$ $c = \ln\frac{1}{2} - 2\left(\frac{1}{4}\right)$ $c = \ln 2^{-1} - \frac{1}{2}$ $c = \ln 2^{-1} - \frac{1}{2}\ln e$ $c = \ln 2^{-1} - \ln e^{\frac{1}{2}}$ $c = \ln\frac{2^{-1}}{\sqrt{e}}$ $c = \ln\frac{1}{2\sqrt{e}}$ $c = \ln\left(2\sqrt{e}\right)^{-1}$ $c = -\ln 2\sqrt{e}$	M1 Substitution M1 Combine 2 logs A1 $\ln x^y = y\ln x$ o.e. AG
11(iv)	$\frac{d^2y}{dx^2} = 4 + \frac{1}{x^2}$ Sub $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = 4 + \frac{1}{(\frac{1}{2})^2}$ $\frac{d^2y}{dx^2} = 8 \quad > 0$ Since $\frac{d^2y}{dx^2} > 0$, therefore point T is a minimum point.	M1 Finding $\frac{d^2y}{dx^2}$ √ M1 Value of $\frac{d^2y}{dx^2}$ √ A1 Minimum point seen