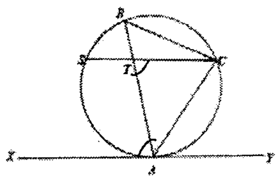
2022 Prelim A Maths Paper 1 Marking Scheme

Qn	Solution	Marks	Remarks
1	$y=x^2+3x-8$ —— (1)		
	$y=\frac{3}{2}x-\frac{7}{2}$ —— (2)		
	(1) = (2)		
	$x^2+3x-8=\frac{3}{2}x-\frac{7}{2}$	[M1]	
	$2x^2+3x-9=0$		
	$(2x-3)(x+3)=0$	[M1]	
	$x=\frac{3}{2}$ or $x=-3$	[M1]	
	$y=-\frac{5}{4}$ or $y=-8$		
	Coord of A and B are $\left(\frac{3}{2}, -\frac{5}{4}\right)$ & $(-3,-8)$.		
	Distance btw A and B		
	$=\sqrt{(1.5+3)^2+(-1.25+8)^2}$		
	$=8.11$ units	[A1]	Accept exact surd $\frac{\sqrt{1053}}{4}$
2	$\frac{7x+8}{(2x+1)(x-1)^2}=\frac{A}{2x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$	[M1]	
	$7x-8=A(x-1)^2+B(2x+1)(x-1)+C(2x+1)$	[M1]	
	Let $x=1$, $15=3C$, $C=5$		
	Let $x=-\frac{1}{2}$, $\frac{9}{2}=A\left(-\frac{3}{2}\right)^2$, $A=2$	[A3]	
	Let $x=0$, $8=2(-1)^2+B(1)(-1)+5(1)$, $B=-1$		
	$\frac{7x+8}{(2x+1)(x-1)^2}=\frac{2}{2x+1}-\frac{1}{x-1}+\frac{5}{(x-1)^2}$		

3	$a+b\sqrt{3}=\dfrac{\left(4-\sqrt{3}\right)^2}{2+\sqrt{3}}$ $=\dfrac{19-8\sqrt{3}}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $=\dfrac{38-19\sqrt{3}-16\sqrt{3}+24}{2^2-3}$ $=62-35\sqrt{3}$ $a=62,\quad b=-35$	[M1] [M1] [A1]	Accept alternative method $\left(a+b\sqrt{3}\right)\left(2+\sqrt{3}\right)=\left(4-\sqrt{3}\right)^2$
4	$y=a\left(x+2\right)^2+k$ At $(-1,4)$, $4=a(-1+2)^2+k$ $a+k=4$ Since $a>0$ & $k>0$ as curve lies above x-axis, let $a=1$, then $k=3$ A possible equation for the curve is $y=\left(x+2\right)^2+3$	[M1] [M1] [M1] [A1]	(Any other suitable equation satisfying the relevant conditions accepted)
5(a)	$r=\dfrac{\max+\min}{2}=\dfrac{1+(-3)}{2}=-1$ (shown)	[B1]	
(b)	Period of curve $=\dfrac{2\pi}{\frac{1}{q}}=8\pi,\quad q=4$ Amplitude $=\dfrac{\max-\min}{2}=\dfrac{1-(-3)}{2}=2,\ p=-2$	[B1] [B1]	
(c)	Equation of the curve is $y=-2\sin\dfrac{x}{4}-1$	[B1]	
6(a)	$f'(x)=\int\left(6x+2\right)dx$ $=3x^2+2x+c$ At $x=-1$, $f'(x)=11$ $11=3(-1)^2+2(-1)+c$ $c=10$ $f'(x)=3x^2+2x+10$	[M1] [M1] [A1]	

(b)	$f(x)=\int(3x^2+2x+10)\,dx$ $=x^3+x^2+10x+d$ At $(-1,10)$, $10=(-1)^3+(-1)^2+10(-1)+d$ $d=20$ $f(x)=x^3+x^2+10x+20$	[M1] [A1]	
(c)	For $y=f(x)$ to have stationary points, set $f'(x)=0$. $3x^2+2x+10=0$ $b^2-4ac=2^2-4(3)(10)=-116<0$ $f'(x)=0$ has no real solution, ie no stationary points.	 [M1] [M1] [A1]	Solve equation to show no real roots accepted No marks if no real roots is not mentioned
7(a)	$x=4.5,\quad V=\frac{1}{3}\pi(4.5)^2(18-4.5)$ $=\frac{729}{8}\pi$ $\frac{dV}{dt}=\frac{\frac{729}{8}\pi}{9}=\frac{81}{8}\pi$ cm/s (or $10\frac{1}{8}\pi$ cm/s)	[M1] [M1.A1]	Accept 10.125π *or* 31.8 cm/s
(b)	$V=\frac{\pi}{3}(18x^2-x^3)$ $\frac{dV}{dx}=\frac{\pi}{3}(36x-3x^2)$ *At* $x=$ 4.5, $\frac{dV}{dx}=\frac{\pi}{3}(36(4.5)-3(4.5)^2)$ $=\frac{135}{4}\pi$ Using $\frac{dV}{dt}=\frac{dV}{dx}\ \frac{dx}{dt}$ $\frac{dx}{dt}=\frac{\frac{81}{8}\pi}{\frac{135}{4}\pi}=0.3$ The water level is rising at a rate of 0.3 cm/s	[M1] [M1] [M1. A1]	

8(a)	$\sin^3 x + \cos^3 x$ $= (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)$	[B1]	
(b)	$LHS = \dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$ $= \dfrac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\sin x + \cos x}$ $= \sin^2 x - \frac{1}{2}(2\sin x \cos x) + \cos^2 x$ $= 1 - \frac{1}{2}\sin 2x = RHS$	[M1] [A1, A1]	No mark awarded if student do not show $\sin x \cos x = \frac{1}{2}(2\sin x \cos x)$
(c)	$1 - \frac{1}{2}\sin 2x = \frac{5}{4}$ $\frac{1}{2}\sin 2x = -\frac{1}{4}$ $\sin 2x = -\frac{1}{2}$ Basic angle, $\alpha = 30^o$ $2x = 210^o, 330^o, 570^o, 690^o$ $x = 105^o, 165^o, 285^o, 345^o$	[M1] [M1] [M1] [A1]	
9(a)	$\angle ACB = \angle ATC$ (given) $\angle BAC = \angle CAT$ (common) $\therefore \Delta ABC \& \Delta ACT$ are similar (AA Test)	[M1] [A1]	Either statement
(b)	Since $\Delta ABC \& \Delta ACT$ are similar, $\dfrac{AB}{AC} = \dfrac{AC}{AT}$ $AC^2 = (AB)(AT)$ $= (AT + TB)(AT)$ $= AT^2 + AT \times TB$ $AC^2 - AT^2 = AT \times TB$ (shown)	[M1] [M1] [A1]	
(c)	$\angle BAX = \angle ACB$ (alt. segment thm) $\& \angle ACB = \angle ATC$ (given) $\therefore \angle BAX = \angle ATC$ By the alternate angle property, SC and XY are parallel.	[M1] [M1] [A1]	

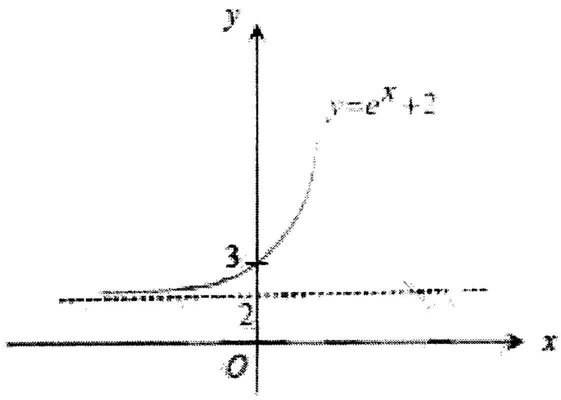
10(a)	$m_{AB}=\frac{p-10}{3}$ $m_{CB}=\frac{6-p}{1}$ Since $\angle ABO=\angle CBO$, $m_{AB}=-m_{CB}$ $\frac{p-10}{3}=-(6-p)$ $p-10=-18+3p$ $-2p=-8$ $p=4$ (shown)	[M1] [M1] [A1]	Accept method involving tan θ
(b)	$m_{AB}=-2$ $m_{AD}=\frac{1}{2}$ Equation of line AD is $y-10=\frac{1}{2}(x+3)$ $y=\frac{1}{2}x+\frac{23}{2}$ (shown) $CD\,//\,AB$, $m_{CD}=-2$ Equation of line CD is $y-6=-2(x-1)$ $y=-2x+8$ At D, $\frac{1}{2}x+\frac{23}{2}=-2x+8$ $\frac{5}{2}x=-\frac{7}{2}$ $x=-\frac{7}{5}, y=-2\left(-\frac{7}{5}\right)+8=\frac{54}{5}$ Coord of D is $\left(-\frac{7}{5},\frac{54}{5}\right)$	[M1] [A1] [M1] [M1] [A1]	Accept (-1.4, 10.8)

(c)	Area of $ABCD$ $=\frac{1}{2}\begin{vmatrix}0 & 1 & -1.4 & -3 & 0\\ 4 & 6 & 10.8 & 10 & 4\end{vmatrix}$ $==\frac{1}{2}\left[(0+10.8-14-12)-(4-8.4-32.4)\right]$ $=10.8$ units^2	[M1] [A1]	
11(a)	$x^2+4x+3=(x+3)(x+1)$ By Factor Theorem, $P(-3)=0$ & $P(-1)=0$ $5(-3)^3+a(-3)^2+3+b=0$ $9a+b=132$ ----(1) $5(-1)^3+a(-1)^2+1+b=0$ $a+b=4$ ----(2) (1)−(2): $8a=128$ $a=16,\quad b=-12$	[M1] [M1] [M1] [A1]	*overall minus 1 mark if any expression is not set to 0
(b)	$5x^3+16x^2-x-12=(x+3)(x+1)(px+q)$ By comparison, $p=5$ & $q=-4$ $P(x)=0$ $(x+3)(x+1)(5x-4)=0$ $x=-3$ or -1 or $\frac{4}{5}$	[M1] [M1] [A1]	
(c)	$5(x^2)^3+a(x^2)^2-x^2+b=0$ ---- (1) Let $u=x^2$, (1) becomes $5u^3+au^2-u+b=0$ From (b), $u=-3(rej)$ *or* $-1(rej)$ *or* $\frac{4}{5}$ $\therefore x^2=\frac{4}{5}$ $x=\frac{2}{\sqrt{5}}$ or $-\frac{2}{\sqrt{5}}$	[M1] [A1]	Accept $\pm\frac{2\sqrt{5}}{5}$

12(a)	$y=\frac{7}{6x+1}=7(6x+1)^{-1}$ $\frac{dy}{dx}=-7(6x+1)^{-2}(6)$ $=-\frac{42}{(6x+1)^2}$ At K, $x=1$, $\frac{dy}{dx}=-\frac{42}{7^2}=-\frac{6}{7}$ Equation of tangent at K is $y-1=-\frac{6}{7}(x-1)$ $y=-\frac{6}{7}x+\frac{13}{7}$ Coord of B is $\left(0,\frac{13}{7}\right)$	[M1] [M1] [M1] [M1] [A1]	
(b)	Area of the shaded region = Area under curve – Area of trapezium $=\int_0^1 \frac{7}{6x+1}dx-\frac{1}{2}(1)\left(1+\frac{13}{7}\right)$ $=7\left[\frac{1}{6}\ln(6x+1)\right]_0^1-\frac{10}{7}$ $=\frac{7}{6}(\ln 7-\ln 1)-\frac{10}{7}$ $=0.842$ units^2	[M1,,M1] [M1] [M1] [A1]	
13(a)	$(\ln x)^2+\frac{2}{\frac{\ln e}{\ln x}}=3$ $(\ln x)^2+2\ln x-3=0$ Let $u=\ln x$ $u^2+2u-3=0$ $(u+3)(u-1)=0$ $u=-3$ or $u=1$ $\ln x=-3$ or $\ln x=1$ $x=e^{-3}=\frac{1}{e^3}$ or $x=e$	[M1] [M1] [M1] [A1]	Accept $x=0.0498$ or $x=2.72$

(b)	$\lg\left(\frac{p}{2q}\right)=\lg(p+2q)$ $\frac{p}{2q} = p+2q$ (i) $p = 2pq+4q^2$ $p(1-2q)=4q^2$ $p=\frac{4q^2}{1-2q}$	[M1] [A1]	
	(ii) Range of p is $p>0$ $\therefore \frac{4q^2}{1-2q}>0$ Since $q>0$ for lg $2q$ to be defined, $4q^2>0$ & $1-2q>0$ $q<\frac{1}{2}$ $\therefore 0<q<\frac{1}{2}$ (shown)	[B1] [B1]	
(c)	By observing the shape of the curve, a logarithmic function, ie equation (B) $y=a\ln x+b$ is a suitable model since the rate of growth of the head circumference gets much slower when the baby gets older over the months.	[B1] [B1]	-for correct model -for correct reasoning

Marking Scheme for A Maths Paper 2

Qn	Working	Marks	Remarks
1(a)		B1 B1	Shape of the curve y-intercept and horizontal asymptote
(b)	$3-e^{-x}=2e^{x}$ $3-\frac{1}{e^{x}}=2e^{x}$ $3e^{x}-1=2e^{2x}$ $2e^{2x}-3e^{x}+1=0$	M1	
	Let $y=e^{x}$ $2y^{2}-3y+1=0$ $(2y-1)(y-1)=0$	M1	
	$y=\frac{1}{2}$ or $y=1$ $e^{x}=\frac{1}{2}$ or $e^{x}=1$	M1	
	$x=\ln\frac{1}{2}$ or $x=\ln 1$ $x=-0.693$ or $x=0$	A1	

2(i)	Since roots of $f(x) = 0$ are 1, k and k^2 $f(x) = -(x-1)(x-k)(x-k^2)$ $f(2) = -7$ $-(2-1)(2-k)(2-k^2) = -7$ $(2-k)(2-k^2) = 7$ $4-2k^2-2k+k^3 = 7$ $k^3+2k^2+2k-3=0$ (shown)	M1 M1 A1	
(ii)	Let $g(k) = k^3-2k^2-2k-3$ $g(3) = 3^3-2(3)^2-2(3)-3$ $= 0$ Since $g(3) = 0$, $k-3$ is a factor of $g(k)$. $k^3-2k^2-2k-3=0$ $(k-3)(k^2+k+1)=0$ $k^2+k+1=0$ $k=3$ or $b^2-4ac = 1^2-4(1)(1)$ $= -3 < 0$ (no real roots) Therefore $k^3-2k^2-2k-3=0$ has only 1 real root.	M1 M1 M1, M1 M1 B1	

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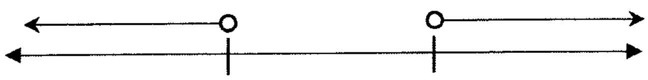
3(i)	$y=(x+2)\sqrt{x-1}$ $\frac{dy}{dx}=(x+2)[\frac{1}{2}(x-1)^{-\frac{1}{2}}]+\sqrt{x-1}(1)$ $=(x-1)^{-\frac{1}{2}}[\frac{1}{2}x+1+x-1]$ $=(x-1)^{-\frac{1}{2}}(\frac{3}{2}x)$ $=\frac{3x}{2\sqrt{x-1}}$	M1 M1 M1 A1	
(ii)	By Chain Rule, $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $2=\frac{3x}{2\sqrt{x-1}}\times\frac{dx}{dt}$ $\frac{dx}{dt}=2\div\frac{3(2)}{2\sqrt{2-1}}$ $=\frac{2}{3}$ units/s	M1 A1	
(iii)	$\int_2^5\frac{x}{\sqrt{x-1}}\,dx=\frac{2}{3}\int_2^5\frac{3x}{2\sqrt{x-1}}\,dx$ $=\frac{2}{3}[(x+2)\sqrt{x-1}]_2^5$ $=\frac{2}{3}[7\sqrt{4}-4\sqrt{1}]$ $=\frac{2}{3}(10)$ $=6\frac{2}{3}$	M1 M1 A1	

4(i)	Let E be the point on AB such that BE is perpendicular to CE. F is the point on CE such that CF is perpendicular to FD. $\cos\theta = \frac{BE}{4}$ $BE = 4\cos\theta$ $\sin\theta = \frac{FD}{1}$ $FD = \sin\theta$ $AB = BE + FD$ $= 4\cos\theta + \sin\theta$ (shown)	M1 for either one correct A1	
(ii)	$AB = \sqrt{4^2+1^2}\cos\left(\theta - \tan^{-1}\frac{1}{4}\right)$ $= \sqrt{17}\cos(\theta - 14.036°)$ $= \sqrt{17}\cos(\theta - 14.0°)$	M1 for finding R, M1 for finding α A1	
(iii)	Max $AB = \sqrt{17}$ m For AB to be maximum, the value of $\cos(\theta - 14.036°)$ must be 1. $\cos(\theta - 14.036°) = 1$ $\theta - 14.036° = 0°$ $\theta = 14.036°$ $= 14.0°$ (1 d.p.)	B1 B1	
(iv)	$3 = \sqrt{17}\cos(\theta - 14.036°)$ $\cos(\theta - 14.036°) = \frac{3}{\sqrt{17}}$ $\theta - 14.036° = 43.313°$ $\theta = 43.313° + 14.036°$ $= 57.349°$ $= 57.3°$ (1 d.p.)		

[Turn over

5(a)	$y = 2x^2 - 6x + c$ --------- (1) $y + 2x = 8$ --------------- (2) Equating (1) & (2): $2x^2 - 6x + c = 8 - 2x$ $2x^2 - 4x + c - 8 = 0$ The line is a tangent to the curve, $b^2 - 4ac = 0$ $(-4)^2 - 4(2)(c-8) = 0$ $16 - 8c + 64 = 0$ $c = 10$	M1 M1 A1	
(b)	$3(x^2 - 5) > x - 1$ $3x^2 - 15 - x + 1 > 0$ $3x^2 - x - 14 > 0$ $(3x-7)(x+2) > 0$ $x < -2$ or $x > 2\frac{1}{3}$ 	M1 A1 A1	
(c)	$-2x^2 + x - p$ $a = -2,\ b = 1,\ c = -p$ For real roots, $b^2 - 4ac \geq 0$ $1 - 4(-2)(-p) \geq 0$ $1 - 8p \geq 0$ $p \leq \frac{1}{8}$ Greatest value of integer $p = 0$	M1 M1 A1	

6(i)	$\left(\frac{1}{2}-2x\right)^5$ $=\left(\frac{1}{2}\right)^5+\binom{5}{1}\left(\frac{1}{2}\right)^4(-2x)^1+\binom{5}{2}\left(\frac{1}{2}\right)^3(-2x)^2+\binom{5}{3}\left(\frac{1}{2}\right)^2(-2x)^3+$ $=\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+$	M1 A1	
(ii)	$\left(1+ax+3x^2\right)\left(\frac{1}{2}-2x\right)^5$ $=\left(1+ax+3x^2\right)\left(\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+...\right)$ Considering x^2, $5x^2-\frac{5}{8}ax^2+\frac{3}{32}x^2=\frac{13}{2}x^2$ $5-\frac{5}{8}a+\frac{3}{32}=\frac{13}{2}$ $a=-2\frac{1}{4}$	M1 M1 M1 A1	
(iii)	$(0.47)^5=\left(\frac{1}{2}-2x\right)^5$ $\frac{1}{2}-2x=0.47$ $x=0.015$ $(0.47)^5=\frac{1}{32}-\frac{5(0.015)}{8}+5(0.015)^2-20(0.015)^3+...$ ≈ 0.02293	M1 M1 A1	

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<table>
<tr>
<td>7(i)</td>
<td>$y = 10 + Ab^x$
$y - 10 = Ab^x$
Take log to the base 10 on both sides,
$\lg(y-10) = \lg b(x) + \lg A$
Plot $\lg(y-10)$ against x to obtain a straight line.
<table>
<tr><td>x</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td></tr>
<tr><td>y</td><td>15.9</td><td>19.1</td><td>23.4</td><td>30.2</td></tr>
<tr><td>$y - 10$</td><td>5.9</td><td>9.1</td><td>13.4</td><td>20.2</td></tr>
<tr><td>$\lg(y-10)$</td><td>0.77</td><td>0.96</td><td>1.13</td><td>1.31</td></tr>
</table>
</td>
<td>M1, G1</td>
<td>1 mark for drawing the straight line</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\lg A =$ vertical intercept of the graph
$= 0.60$ (Accept ± 0.02)
$A = 10^{0.60}$
≈ 3.98

$\lg b =$ Gradient of the graph
$= \dfrac{1.31 - 0.60}{2 - 0}$
$= 0.355$ (Accept ± 0.02)
$b = 10^{0.355}$
≈ 2.26</td>
<td>M1

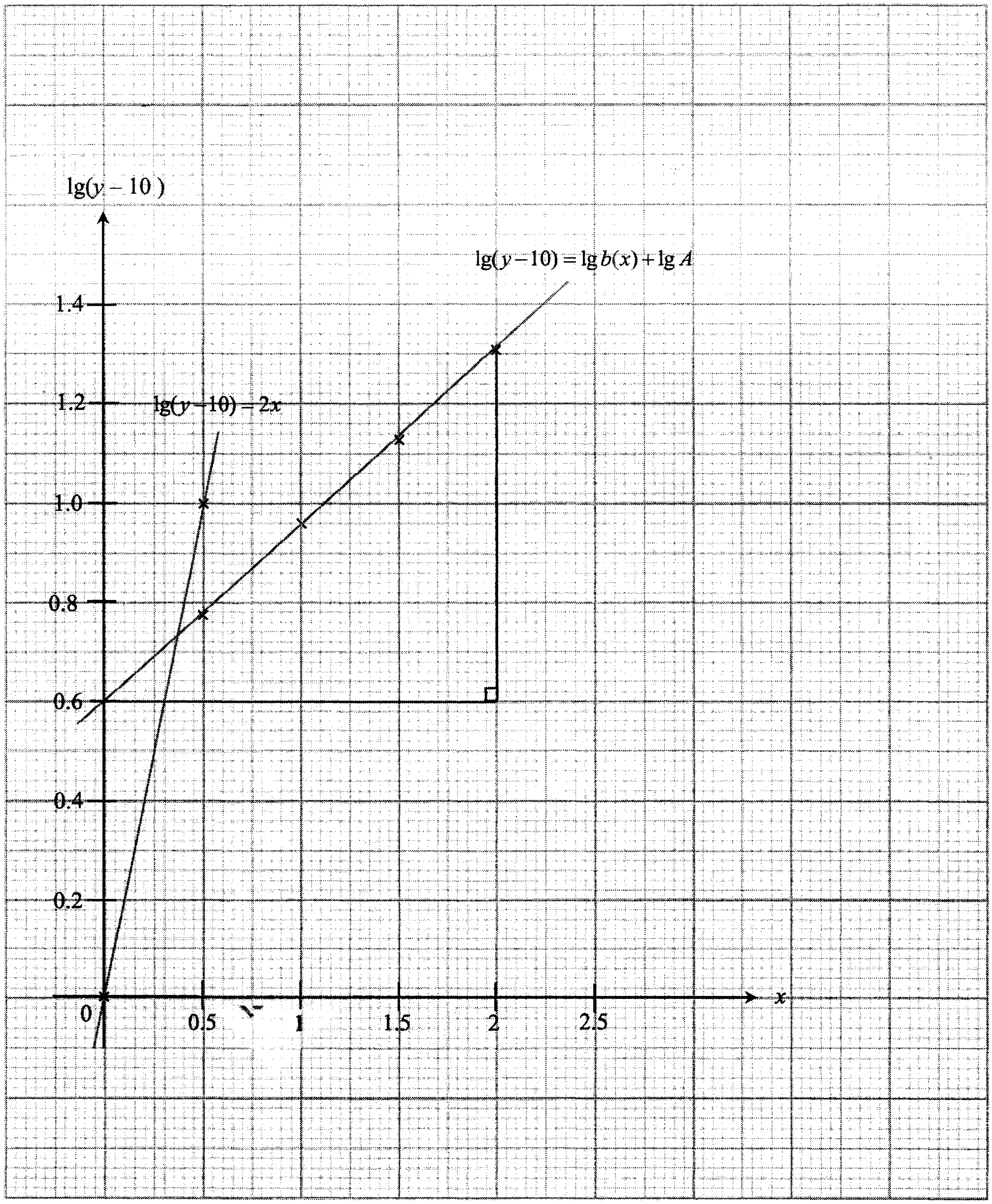
A1

M1

A1</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>$Ab^x = 10^{2x}$
$\lg A + x \lg b = 2x$
$\therefore$ draw $\lg(y-10) = 2x$

From the graph, $x \approx 0.375$ (Accept ± 0.01)</td>
<td>M1
M1

A1</td>
<td></td>
</tr>
</table>



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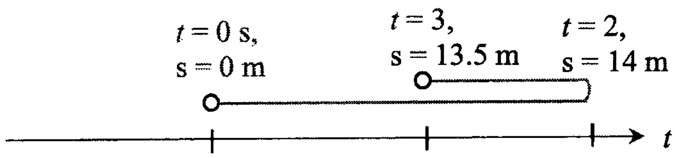
8(i)	$v=3t^2+kt+18$ $a=\dfrac{dv}{dt}$ $=6t+k$ When $t=1$, $6(1)+k=-9$ $k=-15$ (shown)	M1 A1	
(ii)	$3t^2-15t+18=0$ $t^2-5t+6=0$ $(t-2)(t-3)=0$ $t=2$ or $t=3$	M1 A1 for both answers	
(iii)	$s=\int v\ dt$ $=\int\left(3t^2-15t+18\right)\ dt$ $=t^3-\dfrac{15}{2}t^2+18t+C$ When $t=0, s=0, C=0$ $\therefore s=t^3-\dfrac{15}{2}t^2+18t$ when $t=2$ s, $s=14$ m when $t=3$ s, $s=13.5$ m  Distance travelled in first 3 s = 14 + (14 – 13.5) = 14.5 m	M1 M1 M1 A1	

9(i)	when $y=0$, $4-e^{-2x}=0$ $e^{-2x}=4$ $-2x=\ln 4$ $x=-\ln 2$ $\therefore A(-\ln 2,0)$ when $x=0$, $y=4-e^{0}$ $=3$ $\therefore B(0,3)$	B1 B1	
(ii)	$y=4-e^{-2x}$ $\frac{dy}{dx}=-(-2)e^{-2x}$ $=2e^{-2x}$ when $x=0$, $\frac{dy}{dx}=2e^{0}$ $=2$ Gradient of $BC=-\frac{1}{2}$ Let the coordinates of C be (x, y). $\frac{3-0}{0-c}=-\frac{1}{2}$ $c=6$ $\therefore C(6,0)$	M1 M1 M1 A1	
(iii)	Area of shaded region $=\int_{-\ln 2}^{0}(4-e^{-2x})\ dx+\frac{1}{2}(6)(3)$ $=\left[4x+\frac{1}{2}e^{-2x}\right]_{-\ln 2}^{0}+9$ $=\left(0+\frac{1}{2}\right)-\left(-4\ln 2+\frac{1}{2}e^{2\ln 2}\right)+9$ $=7\frac{1}{2}+4\ln 2$ ≈ 10.3 units^2	M1, M1 M1 M1 A1	

[Turn over

10(i)	$(x-3)^2+(y+2)^2=12+9+4$ $(x-3)^2+(y+2)^2=25$ $C(3,-2)$ Radius $=\sqrt{25}$ $=5$ units	B1 B1	
(ii)	Normal of a circle passes through centre of a circle. $3(-2)=m-4(3)$ $m=6$	M1 A1	
(iii)	$3y=6-4x$ $y=-\frac{4}{3}x+2$ ---------------- (1) $(x-3)^2+(y+2)^2=25$ ---------------- (2) $(x-3)^2+(-\frac{4}{3}x+2+2)^2=25$ $(x-3)^2+(4-\frac{4}{3}x)^2=25$ $x^2-6x+9+(\frac{16}{9}x^2-\frac{32}{3}x+16)=25$ $\frac{25}{9}x^2-\frac{50}{3}x=0$ $25x^2-150x=0$ $25x(x-6)=0$ $x=6$ (rejected) or $x=0$ when $x=0$, $y=-\frac{4}{3}(0)+2$ $y=2$ $\therefore A(0,2)$ (shown)	M1 M1 A1 A1	
(iv)	$(3,-2)$ is midpoint of AB and the required coordinates B be $B(e,f)$ $(\frac{0+e}{2},\frac{2+f}{2})=(3,-2)$ $B(6,-6)$ Equation of tangent at B: $y+6=\frac{3}{4}(x-6)$ $y=\frac{3}{4}x-\frac{21}{2}$	M1 M1 A1	