	Class	Register Number
Name ______________________		

4049/01 **22/S4PR/AM/1**

ADDITIONAL MATHEMATICS **PAPER 1**

Monday **29 August 2022** **2 hours 15 minutes**

NIL SINE LABORE

VICTORIA SCHOOL

PRELIMINARY EXAMINATION
SECONDARY FOUR

Candidates answer on the Question Paper.

No Additional Materials are required.

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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setters: Mdm Ernie Bte Abdullah and Ms Emmeline Lau

This paper consists of **25** printed pages, including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 A cuboid has a base area of $\left(7+6\sqrt{3}\right)$ cm^2 and a volume of $\left(107+58\sqrt{3}\right)$ cm^3.

Find, without using a calculator, the height of the cuboid, in cm, in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [3]

2 The line $3x-2y-12=0$ intersects the curve $xy=18$ at the points P and Q.
Find the x-coordinate of P and of Q. [3]

3 **(a)** Express $11-9x-2x^2$ in the form $a(x+b)^2+c$. [2]

Hence

(b) state the coordinates of the turning point of the curve $11-9x-2x^2$, [1]

(c) write down a possible value of k such that the number of real roots to the equation $11-9x-2x^2=k$ is 0. [1]

4 Integrate $3\sqrt{4+5x}+\dfrac{2}{x^3}+\dfrac{6}{7x-1}$ with respect to x. [4]

5 Express $\dfrac{8x^2+4x+1}{(x+1)(x^2+4)}$ in partial fractions. [6]

6 A polynomial, P, is $2x^3 + x^2 + hx - 12$, where h is an integer.

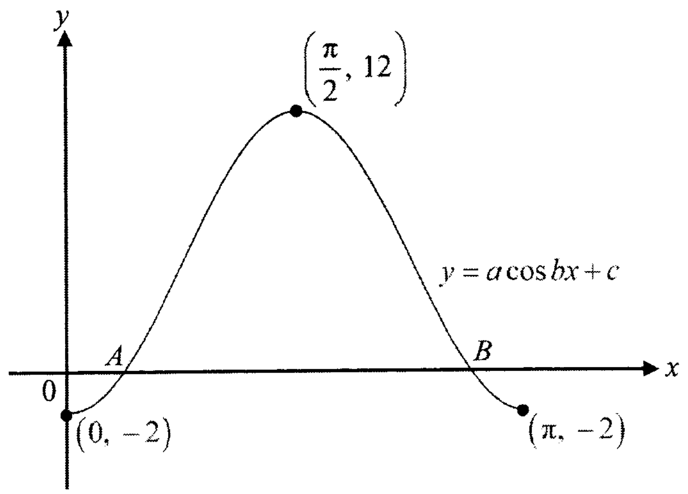
(a) Find the value of h given that P leaves a remainder of -16 when divided by $2x+1$. [2]

(b) In the case where $h = -19$, the quadratic expression $2x^2 + gx - 4$ is a factor of P.

(i) Find the value of the constant g. [3]

(ii) Hence explain why $P = 0$ has 3 real roots. [2]

7



The diagram shows the curve $y = a\cos bx + c$ for $0 \le x \le \pi$ radians. The curve has a maximum point at $\left(\dfrac{\pi}{2},\ 12\right)$ and two minimum points at $(0,\ -2)$ and $(\pi,\ -2)$.

(a) Explain why $c = 5$. [1]

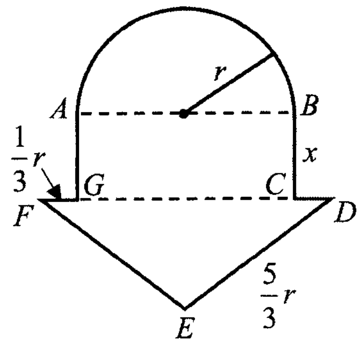
(b) Explain why $b = 2$. [1]

(c) Hence find the equation of the curve. [2]

The curve intersects the x-axis at A and at B.

(d) Find, in radians, the values of x at A and at B for which $y = 0$. [2]

8



A baker uses 131 cm of wire to enclose a cake mould of the shape shown in the diagram. The shape consists of a semicircle with diameter AB, a rectangle $ABCG$ and an isosceles triangle FED such that $FE = ED$.

It is given that $AB = 2r$ cm, $BC = x$ cm, $ED = \frac{5}{3}r$ cm and $FG = CD = \frac{1}{3}r$ cm.

(a) Express x in terms of r and π. [2]

(b) Show that the area of the mould, P cm^2, is given by

$$P = 131r - \frac{\pi}{2}r^2 - \frac{8}{3}r^2.$$ [3]

(c) Given that r can vary, find the value of r which gives a stationary value of P. [3]

(d) The baker's son claimed that his father will be disappointed with the nature of this stationary value. Explain why you would agree or disagree with the baker's son. [2]

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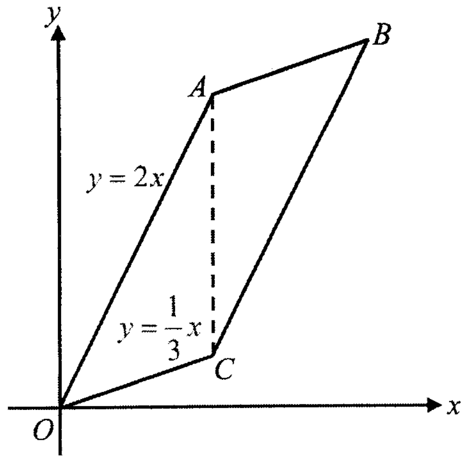


The diagram shows a parallelogram $OABC$, where O is the origin. The side OA has equation $y = 2x$ and the side OC has equation $y = \frac{1}{3}x$. The diagonal AC is parallel to the y-axis and the x-coordinate of C is k.

(a) Show that $AC = \frac{5}{3}k$. [1]

(b) Find the coordinates of B in terms of k. [2]

It is now given that $k = 6$.

(c) Find the area of the parallelogram $OABC$. [2]

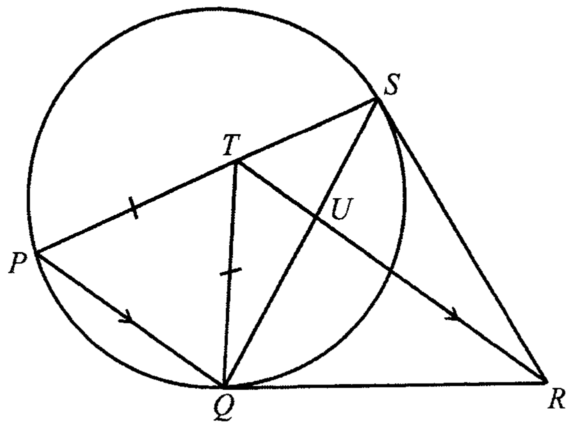
D is a point such that $ABDC$ is a kite.

(d) Hence state the area of $ABDC$. [1]

10 **(a)** Prove the identity $\dfrac{1-\sin x}{\cos x} - \dfrac{\cos x}{\sin x - 1} = 2\sec x$. [4]

(b) Hence solve the equation $\frac{1-\sin 2x}{\cos 2x} - \frac{\cos 2x}{\sin 2x - 1} = -3$ for $-\pi \le x \le \pi$. [4]

11



In the diagram, P, Q and S lie on a circle. The tangents to the circle at Q and S meet at R and PQ is parallel to TR. SQ and TR intersect at U and $PT = QT$.

(a) Prove that ΔTQU and ΔSRU are similar. [4]

(b) **(i)** Hence show that a circle can be drawn passing through *Q*, *R*, *S* and *T*. [1]

(ii) Explain the conclusion that can be made for angle *QTS* and angle *QRS*. [1]

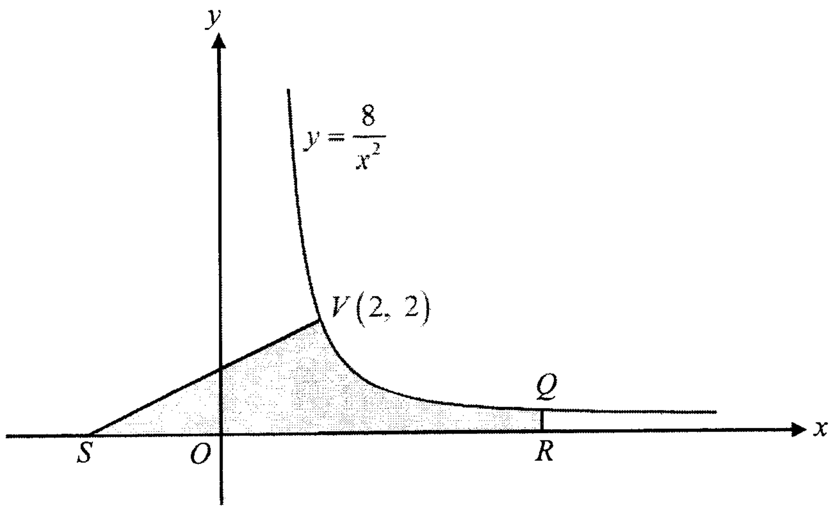
12 **(a)** Solve the equation $6^x + 8 - 6^{2-x} = 17$. [5]

(b) Express the equation $\log_p\left(\frac{1-4x}{x}\right)=\log_{\sqrt{p}}(2-x)$, where $p>0$ and $p\neq 1$, as a cubic equation in x. [4]

13 **(a)** $PQRS$ is a rectangle with $PQ = x$ cm and $PS = (17 - x)$ cm. The sides of the rectangle vary with time such that x increases at a rate of 0.4 cm per second. Find the rate of decrease of the length of the diagonal when $x = 5$ cm. [4]

(b) Air is pumped into a spherical balloon at a rate of 250 cm^3 per second.
At a particular instant, the radius of the balloon is increasing at a rate of $\frac{5}{18\pi}$ cm per second. Find the rate of change of the surface area of the balloon at that instant. [4]

14



The diagram shows part of the curve $y = \dfrac{8}{x^2}$. The point $V(2,\ 2)$ lies on the curve and the normal to the curve at V meets the x-axis at S. The x-coordinate of the points Q and R is 5.

(a) Find the coordinates of S. [5]

(b) Find the area of the shaded region bounded by the curve, the x-axis, the normal VS and the line QR. [5]

End of Paper

Class | Register Number

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4049/02

22/S4PR/AM/2

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PAPER 2

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1 Show that $x-1$ is a factor of $2x^3-x^2-3x+2$ and hence solve the equation $2x^3-x^2-3x+2=0$ completely. [5]

2 **(a)** Show that the equation $2e^x - 1 = 3e^{-x}$ has only one solution and find its exact value. [4]

(b) Explain how the solution of $2e^{\ln 2x} - 1 = 3e^{\ln\frac{1}{2x}}$ can be deduced from your answer in part **(a)** and find the solution. [2]

3 **(a)** Given that $y = x\sqrt{4x-3}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x-3}{\sqrt{4x-3}}$. [3]

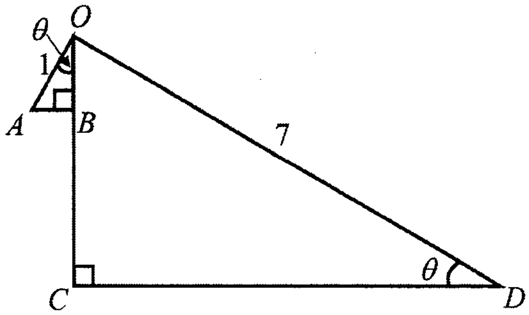
(b) Hence find the value of $\int_1^7 \frac{6x}{\sqrt{4x-3}}\,\mathrm{d}x$. [5]

4 An ant moves in a straight line such that, t seconds after leaving a fixed point O, its velocity is modelled by $v = 8 + 2t - t^2$.

(a) Find the velocity of the ant when its acceleration is 1 cm/s^2. [3]

(b) Find the distance travelled by the ant in the first 5 seconds. [5]

5



The diagram above shows the plan of a yard.
It is given that angle $ODC =$ angle $AOB = \theta$, $OD = 7$ m and $OA = 1$ m.
AB and CD are each perpendicular to OC. A fence is to be built along AB, BC and CD.

(i) Show that $AB + BC + CD = (8\sin\theta + 6\cos\theta)$ m. [3]

(ii) Express $AB + BC + CD$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [2]

(iii) Explain why the length of the fence needed can never be 11 m. [1]

(iv) Find the values of θ for which the length of the fence is 8.5 m. [3]

6 **(a)** Show that the second term in the expansion, in ascending powers of x, of $\left(2+\frac{x}{8}\right)^n$, is $n2^{n-4}x$, where n is a positive integer greater than 2 and find the third term in a similar form. [4]

(b) The first two terms in the expansion, in ascending powers of x, of $(1-x)\left(2+\frac{x}{8}\right)^n$ are $p+qx^2$, where p and q are constants.

(i) Show that the value of n is 16. [3]

(ii) Hence find the value of p and of q. [2]

7 **(a)** The population of cheetahs, P, in n years, can be modelled by $P = ab^n$, where a and b are constants. Explain how a straight line graph can be drawn to represent the formula, and state how the values of a and b could be obtained from the line. [3]

(b) Drone A moves along a horizontal straight line. Its displacement, s m, from a fixed point O, t seconds after it passes through O is recorded in the table below.

s	10	32	66	112
t	2	4	6	8

A physicist believed that these figures can be modelled by $s = ut + \frac{1}{2}at^2$, where u is the initial velocity of Drone A and a is its constant acceleration.

(i) Draw a straight line graph to show that the model is reasonable. [4]

(ii) Use your graph to estimate the displacement of Drone A when $t = 10$. [1]

(iii) Drone B moves along the same horizontal straight line as Drone A from O four seconds after Drone A. Its displacement, s m, from O, t seconds after Drone A passes through O can be modelled by $s = 3t^2 - 12t$. By using your graph in part **(i)**, explain how you can estimate when the drones will meet. [2]

8 **(a)** Show that the equation

$$(p+1)x^2+(p+3)x-(p+2)=0$$

has two real roots for all real values of p. [4]

(b) The equation of a curve is $y=3x^2-5ax+2a^2$, where a is a positive constant.

(i) Find, in terms of a, the set of values of x for which the curve lies below the line $y=10a^2$ and represent this set on a number line. [4]

(ii) Find the value of a for which the curve touches the line $y=1-3ax$. [3]

9 The equation of a circle C, with centre O, is $x^2 + y^2 - 4x - 6y - 5 = 0$.

(i) Find the coordinates of O and the exact radius of C. [3]

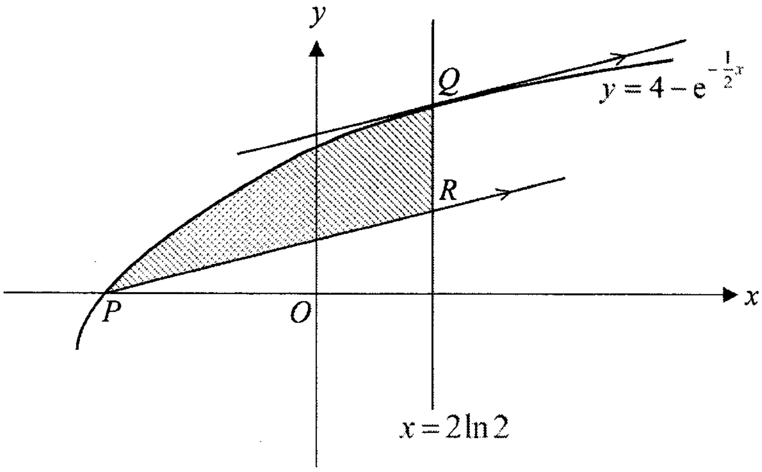
The line l is a tangent to the circle at the point $P(5, 6)$.

(ii) Find the equation of l. [3]

(iii) Points A and B are on C such that AB is a diameter of C and is also parallel to l. Find the equation of AB. [2]

(iv) Hence find the coordinates of A and of B. [4]

10



The diagram shows part of a curve with equation $y = 4 - e^{-\frac{1}{2}x}$ meeting the x-axis at the point P. A line $x = 2\ln 2$ intersects the curve at the point Q. R is a point on the line $x = 2\ln 2$ such that PR is parallel to the tangent to the curve at Q. Show that the area of the shaded region is $a(\ln 2)^2 + b\ln 2 - c$, where a, b and c are constants to be determined. [12]

Continuation of working space for Question 10.

End of Paper