PRELIMINARY EXAMINATION 2016
SECONDARY FOUR EXPRESS
ADDITIONAL MATHEMATICS 4047
PAPER 1

2 HOURS

Additional Materials: 7 sheets of writing paper

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your class and candidate number on the cover sheet.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either the calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

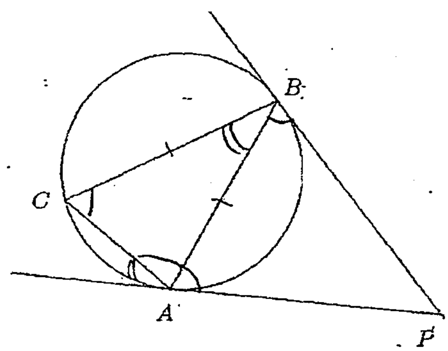
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}ab\sin C$$

1 (i) Show that the roots of the equation $ax^2+(2a+b)x+2b=0$ are real for all values of a and b. [2]

(ii) Hence write down a relation between a and b if the roots are real and equal. [2]

2



The diagram shows points A, B and C lying on the circumference of a circle. The point P is such that the lines PA and PB are tangents to the circle. Given that $BA = BC$, show that

(i) triangle BCA is similar to triangle PBA, [2]

(ii) $AC \times AP = (AB)^2$. [2]

3 Solve, for x and y, the simultaneous equations

$$\log_8(2x+3y)=\frac{1}{3},$$
$$\sqrt{54^x}=3^x6^y.$$ [5]

4 The roots of the quadratic equation $3x^2 = 7x - 1$ are α and β. Find

(i) Show that the value of $\alpha^3 + \beta^3$ is $10\frac{10}{27}$. [3]

(ii) Find the value of $\frac{\beta}{2\alpha} + \frac{\alpha}{2\beta}$. Hence form an equation whose roots are $\frac{\beta}{2\alpha}$ and $\frac{\alpha}{2\beta}$. [3]

5 At the beginning of 2010, a certain type of bacteria was found at the bottom of a seabed. It is known to grow with time, such that its population P, after t years is given by

$$P = 50\ 000e^{kt}, \text{ where } k \text{ is a constant.}$$

(i) Given that the population doubles in two years, show that $k = \frac{1}{2}\ln 2$. [2]

Hence find

(ii) the year in which the population reaches 450 000, [3]

(iii) the size of the population of bacteria in 2015, giving your answer correct to the nearest 10 000. [2]

6 (i) Show that $y = \ln\left(\frac{8+3x}{3x-4}\right)$ has no turning point for all values of x. [4]

(ii) Determine the range of values of x for which the graph of $y = \ln\left(\frac{8+3x}{3x-4}\right)$ is decreasing. [3]

7 (a) Given that $\sqrt{4\frac{1}{2}} - \frac{\sqrt{6}}{3} + \frac{15}{\sqrt{150}} = \frac{A\sqrt{2}+B\sqrt{6}}{6}$, find the constants A and B.

Hence, given that $\left(\sqrt{4\frac{1}{2}} - \frac{\sqrt{6}}{3} + \frac{15}{\sqrt{150}}\right) \times \frac{6}{\sqrt{3}+1} = m\sqrt{2} + n\sqrt{6},$ find the values of m and of n. [4]

(b) Solve the equation $\log_a 27 = \log_3 a - 2$. [4]

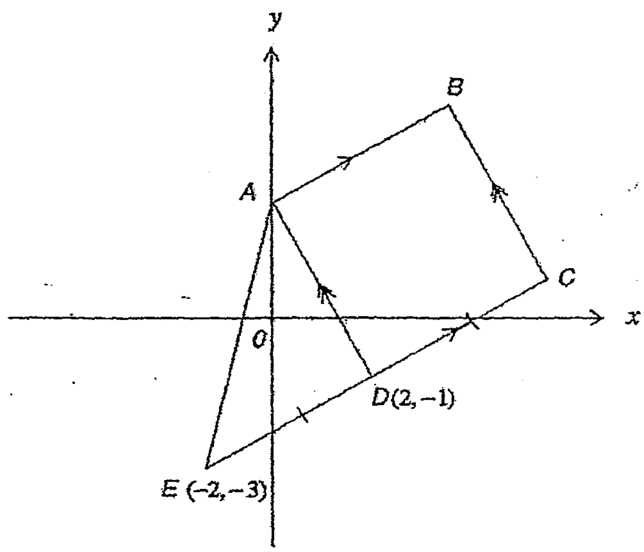
8 (a) Evaluate $\int_0^{\frac{\pi}{12}} 2\sin^2 3x \, dx$. [3]

(b) (i) Differentiate $\frac{2x}{\sqrt{x+1}}$ with respect to x. [2]

(ii) Hence evaluate $\int_3^8 \frac{3x+6}{\sqrt{(x+1)^3}} \, dx$. [3]

9 **Solutions to this question by accurate drawing will not be accepted.**

$ABCE$ is a trapezium in which AB is parallel to EC. D is a point on EC. AD is the perpendicular bisector of EC and is parallel to BC.□A lies on the y-axis and the coordinates of D and E are $(2,-1)$ and $(-2,-3)$ respectively.



(i) Find the coordinates of A, B and C. [6]

(ii) Prove that $ABCD$ is a square. [2]

(iii) Calculate the area of the trapezium $ABCE$. [2]

10 (a) ABC is a triangle and $\tan\frac{A}{2}=\frac{1}{4}$. **Without using a calculator,**

(i) show that $\tan A=\frac{8}{15}$, [2]

(ii) find the exact value of $\cos(B+C)$. [2]

(b) (i) Prove that $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=2\sec\theta$. [2]

(ii) Hence find all the angles for $0^\circ\le\theta\le360^\circ$ which satisfy the equation $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}=\tan^2\theta-2$. [4]

11 (a) A bank offers an annual interest rate of r%, compounded half-yearly, for a deposit of \$25 000. After t years, the total amount left in the bank will amount to $\$25000\left(1+\frac{r}{200}\right)^{2t}$.

(i) Given that $r=5$, use the expansion of $(1+x)^6$ in ascending powers of x as far as the term x^2, to **estimate** the amount of money received after 3 years. [5]

(ii) The percentage difference for the estimated amount in part **(i)** is given by $\frac{A-E}{A}\times100\%$, where A is the actual amount and E is the estimated amount. Calculate the percentage difference, correct to two decimal places. [3]

(b) Find the coefficient of the term in x^4 in the expansion of $\left(x-\frac{1}{2x^3}\right)^{16}$. [3]

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2 HOURS 30 MINUTES

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1 The variables x and y change in such a way that, when $x = 2$, the rate of increase of x with respect to time is $\frac{12}{13}$ times the rate of decrease of y with respect to time. Given that $y = k\sqrt{\frac{22}{x} - x}$ where k is a constant, find the value of k. [5]

2 (i) Express $\frac{8x+13}{(1+2x)(2+x)^2}$ in the form $\frac{A}{1+2x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$. [3]

(ii) Using the answer in part (i), find $\int \frac{8x+13}{(1+2x)(2+x)^2}\,dx$. [3]

3 The equation of a curve is $y = 3\sin 2x + 2$ for $0 \le x \le \pi$.

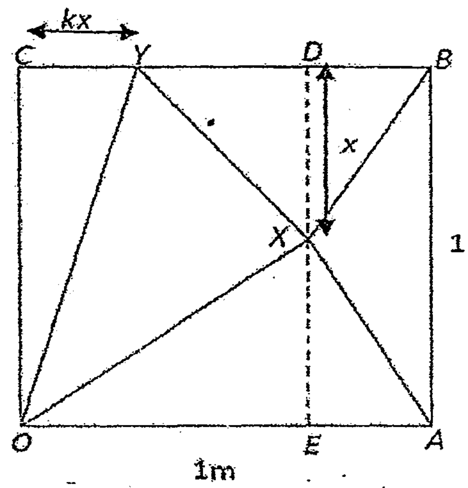
(i) Find the coordinates of the minimum point. [1]

(ii) Sketch the graph of $y = 3\sin 2x + 2$ for $0 \le x \le \pi$. [2]

(iii) On the same diagram, sketch the graph of $y = |2\cos x|$ for $0 \le x \le \pi$. [2]

(iv) State the number of solutions of $|\cos x| - 1 = \frac{3\sin 2x}{2}$ for $0 \le x \le \pi$. [1]

4



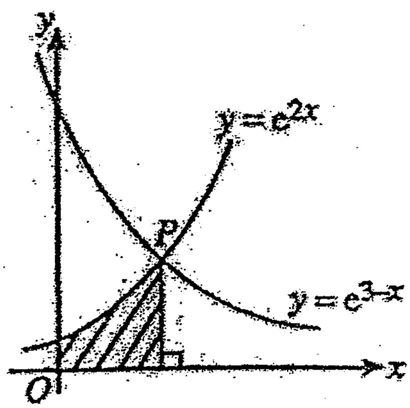
In the diagram, $OABC$ is a square of side 1m. D and E are points on BC and AO respectively. DE is parallel to the sides of CO and BA. X is a point on DE and Y is a point on CB such that DX is x m and CY is kx m, where k is a constant and $0 < k < 1$.

(i) Show that the sum, S m^2, of areas of triangles OXY and ABX, is given by

$S = \frac{1}{2}\left(1 - kx + kx^2\right)$. [3]

(ii) Find the value of x such that S a minimum. [3]

(iii) Given that x can vary, find the minimum value of S in terms of k. [1]



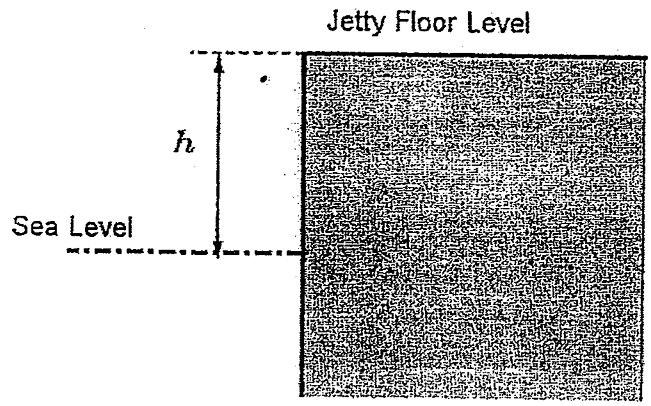
In the diagram, P is the point of intersection of the curves $y = e^{2x}$ and $y = e^{3-x}$.

(a) Find the x-coordinate of P. [2]

Hence, evaluate, to two decimal places

(b) **(i)** the area of the shaded region, [2]

(ii) the area of the region bounded by the two curves and the y-axis. [3]



The height difference, h m, between the jetty floor level and the sea level changes with time due to the tidal effects. It is given that h can be modelled mathematically as

$$h = 1.8 - 1.1 \sin kt$$

where k is a constant and t is the time in hours from midnight.

The time between two consecutive high tides is 12 hours.

(i) Show that the value of k is $\frac{\pi}{6}$. [2]

(ii) State the range of values of h. [2]

(iii) For boats to dock at the jetty, the difference in height must not be more than 1.5m. Find the total length of time in hours in a day when the boat landings are possible. [4]

7 The tangent to the curve $y = 3x^3 + 2x^2 - 5x + 1$ at the point where $x = -1$ meets the y-axis at the point P.

(i) Find the coordinates of the point P. [3]

The curve meets the y-axis at the point Q. The normal to the curve at the point Q meets the x-axis at the point R. The tangent to the curve at the point where $x = -1$ and the normal to the curve at Q meet at the point S.

(ii) Find the area of the triangle PRS. [6]

8 A particle P starts moving in a straight line with a displacement of 1 m from a fixed point O. After t seconds, its velocity v m/s is given by $v = 2\sin t + 3\cos t$. Find

(i) the time at which the particle first comes to an instantaneous rest, [3]

(ii) the acceleration at $t = \frac{\pi}{2}$ seconds, [2]

(iii) an expression for the displacement of the particle from O in terms of t, [2]

(iv) the distance travelled in the first 3 seconds. [3]

9 The equation of a circle C_1 is $2x^2 + 2y^2 - 16x + 4py = 22$, where $p > 0$. The radius of C_1 is 6 units.

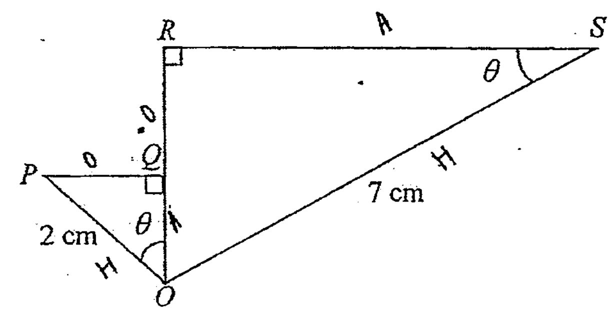
(i) Find the value of p and the coordinates of the centre of C_1. [4]

(ii) Determine whether the line $3x - y = 6$ passes through the centre of C_1. [2]

(iii) The circle C_1 is reflected in the line $x = 3$. Find the equation that represents the reflected circle C_2. [2]

(iv) Find the equation of another circle C_3 which passes through the origin and has the same centre as C_1. [2]

10



In the diagram, $\angle OSR = \angle POQ = \theta°$. $OS = 7$ cm and $OP = 2$ cm. PQ and SR are each perpendicular to OR.

(i) Show that the perimeter of the figure is $P = \left(9\sin\theta° + 5\cos\theta° + 9\right)$ cm. [2]

(ii) Express P in the form of $R\sin\left(\theta° + \alpha\right) + 9$, when $R > 0$ and α is acute. [3]

(iii) State the maximum value of the perimeter P and the corresponding value of θ. [3]

(iv) Find the value of θ for which $P = 17$ cm. [2]

11 **Answer the whole of this question on a sheet of graph paper.**

The variables x and y are related by the equation $yx^{-b}-2=a$, where a and b are constants. The table below shows values of x and y.

x	5	10	15	20	25
y	150	600	1350	2400	3750

(i) Use the data above to draw the straight line graph of $\lg y$ against $\lg x$, using a scale of 4 cm to represent 0.5 unit on the $\lg x$ axis and 4 cm to represent 1 unit on the $\lg y$ axis. [3]

(ii) Using your graph, estimate the values of a and of b. [4]

(iii) On the same graph, draw the straight line graph representing the equation $y=x^4$, and hence find the value of x for which $x^{4-b}-2=a$. [3]

12 Given that $f(x)=2x^3+ax^2+10x+b$ has a factor of $(2x+1)$ and that it leaves a remainder of 30 when divided by $(x-2)$.

(i) Find the values of a and of b. [4]

(ii) Solve the equation $f(x)=21x$. [5]

(iii) Hence obtain the solution for $2x^6-3x^4=11x^2-6$. [3]

Paper 1

Qns	Solutions
1i	$ax^2+(2a+b)x+2b=0$ b^2-4ac $=(2a+b)^2-4(a)(2b)$ $=4a^2+4ab+b^2-8ab$ $=4a^2-4ab+b^2$ $=(2a-b)^2\geq 0$ Since $b^2-4ac\geq 0$, the eq $ax^2+(2a+b)x+2b=0$ has real roots for all real values of a and b. (shown)
ii	If the roots are real and equal, $b^2-4ac=0$ $(2a-b)^2=0$ $b=2a$
2i	$PA=PB$ (externalangents to circle) $\angle PAB=\angle PBA$ (since triangle PAB is an isoseles triangle) But also $\angle PAB=\angle ACB$ (alternate segment thorem) Since also given $BA=BC$ then $\angle ACB=\angle CAB$ (since triangle PAB is an isoseles triangle) Hence by AA similarity triangle BCA is similar to triangle PBA
ii	Since triangle BCA is similar to triangle PBA => $\frac{AC}{AB}=\frac{AB}{AP}$ => $AC\times AP=AB^2$
3	=> $2x+3y=8^{\frac{1}{3}}$ => $2x+3y=2$ -------(1) => $\sqrt{(6\times 9)^x}=3^x6^y$ => $6^{\frac{x}{2}}3^{\frac{2x}{2}}=3^x6^y$ => $6^{\frac{x}{2}}=6^y$ => $x=2y$ ———(2) Substituting into (1) $2(2y)+3y=2$ $\therefore y=\frac{2}{7}$ and $x=\frac{4}{7}$

4i	$3x^2=7x-1$ $3x^2-7x+1=0$ $\alpha+\beta=-\frac{(-7)}{3}=\frac{7}{3}$ $\alpha\beta=\frac{1}{3}$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\left(\frac{7}{3}\right)^2-2\left(\frac{1}{3}\right)=4\frac{7}{9}$ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=\frac{7}{3}\left(4\frac{7}{9}-\frac{1}{3}\right)=10\frac{10}{27}$
ii	$\frac{\beta}{2\alpha}+\frac{\alpha}{2\beta}=\frac{\alpha^2+\beta^2}{2\alpha\beta}$ $=7\frac{1}{6}$ sum of new roots = $7\frac{1}{6}=\frac{43}{6}$ prod of new roots $=\frac{1}{4}$ $x^2-\frac{43}{6}x+\frac{1}{4}=0$ $12x^2-86x+3=0$

5i

When $t = 0$, $P = 50000$
When $t = 2$, $100000 = 50000e^{2k}$
$2 = e^{2k}$
$\Rightarrow \quad \ln 2 = 2k$
$\Rightarrow \quad k = \frac{1}{2}\ln 2$

ii

$450000 = 50000e^{\left(\frac{1}{2}t\ln 2\right)}$
$9 = e^{\frac{1}{2}t\ln 2}$
$\ln 9 = \frac{1}{2}t\ln 2$
$\Rightarrow \quad t \approx 6.34$ years

When the year in which the population will reach 450000 is 2016.

iii

When $t = 5$, $\quad P = 50000^{5\left(\frac{1}{2}\ln 2\right)}$
$P = 282842 \approx 280\ 000$

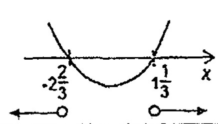
6i

$y = \ln\left(\frac{8+3x}{3x-4}\right)$
$= \ln(8+3x) - \ln(3x-4)$
$\frac{dy}{dx} = \frac{3}{8+3x} - \frac{3}{3x-4}$
$= \frac{3(3x-4)-3(8+3x)}{(8+3x)(3x-4)}$
$= \frac{-36}{(8+3x)(3x-4)}$
Since $-36 \neq 0, \frac{dy}{dx} \neq 0$.
$\therefore$ y has no turning point (shown)

ii

y is a decreasing function $\frac{dy}{dx} = \frac{-36}{(8+3x)(3x-4)} < 0$
$(8+3x)(3x-4) > 0$

$\therefore$ range of x are
$x < -2\frac{2}{3}$ or $x > 1\frac{1}{3}$



7a	$\frac{3}{\sqrt{2}}-\frac{\sqrt{6}}{3}+\frac{15}{5\sqrt{6}}$ $=\frac{3\sqrt{2}}{2}-\frac{\sqrt{6}}{3}+\frac{3\sqrt{6}}{6}$ $=\frac{9\sqrt{2}-2\sqrt{6}+3\sqrt{6}}{6}$ $=\frac{9\sqrt{2}+\sqrt{6}}{6}$ $A=9$ $B=1$ $\frac{9\sqrt{2}+\sqrt{6}}{6}\times\frac{6\left(\sqrt{3}-1\right)}{2}$ $=\frac{9\sqrt{6}-9\sqrt{2}+\sqrt{18}-\sqrt{6}}{2}$ $=4\sqrt{6}-3\sqrt{2}$ $m=-3,$ $n=4$
7b	$\frac{\log_3 27}{\log_3 a}=\log_3 a-2$ let $\log_3 a=x$ $\frac{3}{x}=x-2$ $x^2-2x-3=0$ $x=3$ or -1 $\therefore \log_3 a=3$ or $\log_3 a=-1$ $a=27$ $\quad a=\frac{1}{3}$

8a	$\int_0^{\frac{\pi}{12}} 2\sin^2 3x\,dx$ $=\int_0^{\frac{\pi}{12}} 1-\cos 6x\,dx$ $=\left[x-\frac{\sin 6x}{6}\right]_0^{\frac{\pi}{12}}$ $=\frac{\pi}{12}-\frac{1}{6}$ or 0.0951
bi	$\frac{d}{dx}\left(\frac{2x}{\sqrt{x+1}}\right)$ $=\frac{(x+1)^{\frac{1}{2}}(2)-2x\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}}{(x+1)}$ $=\frac{(x+1)^{-\frac{1}{2}}\left[2(x+1)-x\right]}{(x+1)}$ $=\frac{x+2}{(x+1)^{\frac{3}{2}}}$
ii	$\int_3^8 \frac{3(x+2)}{(x+1)^{\frac{3}{2}}}dx$ $=3\left[\frac{2x}{\sqrt{x+1}}\right]_3^8$ $=3\left[\frac{2(8)}{\sqrt{8+1}}-\frac{2(3)}{\sqrt{3+1}}\right]$ $=7$

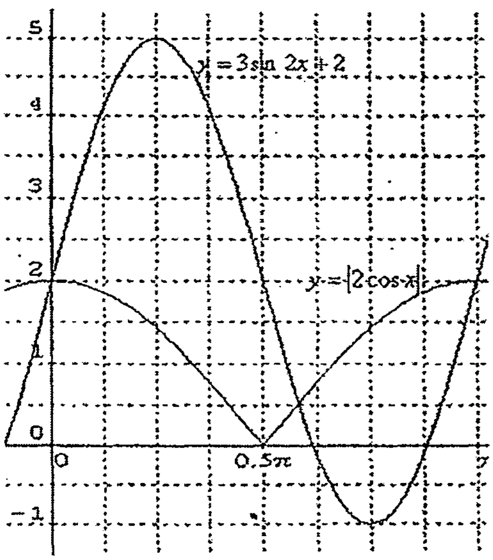
9i	let A be (0 , a) $\text{grad AD} = -\frac{1}{\text{grad BD}}$ $\frac{a+1}{0-2} = -\left(\frac{-2\times-2}{-3+1}\right)$ $q=3$ $\therefore$ coord of A is (0 , 3) Let C be (x , y) $(2, -1) = (\frac{-2+x}{2}, \frac{-3+y}{2})$ $\therefore \frac{-2+x}{2} = 2$ or $\frac{-3+y}{2} = -1$ $x=6$ $y=1$ $\therefore$ Coord of C is (6 , 1) Let B be (x_B, y_B) mid-pt of AC = (3 , 2) $(3, 2) = (\frac{2+x_B}{2}, \frac{-1+y_B}{2})$ $6 = 2+x_B$ and $4 = -1+y_B$ $x_B = 4$ $y_B = 5$ $\therefore$ Coord of B is (4 , 5)
ii	length of AD = $\sqrt{(2-0)^2+(-1-3)^2} = \sqrt{20}$ length of BC = $\sqrt{(6-4)^2+(1-5)^2} = \sqrt{20}$ since AD = BC, AB is parallel to CD and AD $\perp$ DC, $\therefore$ ABCD is a square.
iii	Area of trapezium = $\frac{1}{2}\begin{vmatrix} 0 & -2 & 6 & 4 & 0 \\ 3 & -3 & 1 & 5 & 3 \end{vmatrix}$ $= \frac{1}{2}\left|(0-2+30+12)-(-6-18+4+0)\right|$ $= 30$ units^2

10ai	$\tan A = \dfrac{2\tan\frac{A}{2}}{1-\left(\tan\frac{A}{2}\right)^2}$ $= \dfrac{2\left(\frac{1}{4}\right)}{1-\left(\frac{1}{4}\right)^2}$ $= \left(\dfrac{8}{15}\right)$
ii	$\cos(B+C)$ $=\cos(180-A)$ $=-\cos A$ $=-\dfrac{15}{17}$
bi	$\dfrac{\cos^2\theta+1+2\sin\theta+\sin^2\theta}{\cos\theta(1+\sin\theta)}$ $=\dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$ $=\dfrac{2}{\cos\theta}$ $=2\sec\theta(proven)$
ii	$2\sec\theta=\tan^2\theta-2$ $2\sec\theta=\sec^2\theta-1-2$ $\sec^2\theta-2\sec\theta-3=0$ $(\sec\theta-3)(\sec\theta+1)=0$ $\sec\theta=3$ or $\sec\theta=-1$ $\cos\theta=\dfrac{1}{3}$ $\cos\theta=-1$ $\theta=70.5^\circ,$ $\theta=180^\circ$ 289.5° $\therefore\theta=70.5^\circ,180^\circ,289.5^\circ$

11ai	$(1+x)^6=(1)^6+\binom{6}{1}(1)^5x+\binom{6}{2}(1)^4x^2+\ldots$ $=1+6x+15x^2+\ldots$ $25000\left(1+\frac{5}{200}\right)^{2(3)}$ $=25000(1+0.025)^6$ $=25000\left[1+6(0.025)+15(0.025)^2+\ldots\right]$ $=28984.38$
ii	Actual amt received after 3 years $=25000\left(1+\frac{5}{200}\right)^6=\$28992.34$ Percentage error $=\frac{28992.34-28984.38}{28992.34}\times 100\%$ $=0.027\%$
b	$T_{r+1}=\binom{16}{r}x^{16-r}\left(-\frac{1}{2x^3}\right)^r$ For x^4, $16-r-3r=4$ $r=3$ Coefficient of $x^4=\binom{16}{3}\left(-\frac{1}{2}\right)^3=-70$

Paper 2

Qns	Solutions
1	$\frac{dy}{dx}=\frac{1}{2}k\left(\frac{22}{x}-x\right)^{-\frac{1}{2}}\left(-\frac{22}{x^2}-1\right)$ when $x=2$, $\frac{dy}{dt}=\frac{1}{2}k\left(\frac{22}{2}-2\right)^{-\frac{1}{2}}\left(-\frac{22}{2^2}-1\right)\times\frac{dx}{dt}$ $\frac{dy}{dt}=\frac{1}{2}k\left(\frac{1}{3}\right)\left(-\frac{13}{2}\right)\times\frac{dx}{dt}$ given: $\frac{dx}{dt}=\frac{12}{13}\times\left(-\frac{dy}{dt}\right)$ $\frac{dy}{dt}=-\frac{13}{12}\times\frac{dx}{dt}$ $\therefore k=1$
2i	$\frac{8x+13}{(1+2x)(2+x)^2}=\frac{A}{1+2x}+\frac{B}{2+x}+\frac{C}{(2+x)^2}$ $8x+13=A(2+x)^2+B(1+2x)(2+x)+C(1+2x)$ Let $x=-2$, $C=1$. Let $x=-\frac{1}{2}$, $A=4$. Let $x=0$, $B=-2$. $\therefore\frac{8x+13}{(1+2x)(2+x)^2}=\frac{4}{1+2x}-\frac{2}{2+x}+\frac{1}{(2+x)^2}$
ii	$\int\frac{4}{1+2x}-\frac{2}{2+x}+\frac{1}{(2+x)^2}\,dx=\frac{4\ln(1+2x)}{2}-2\ln(2+x)+\frac{(2+x)^{-1}}{-1}+c$ $=2\ln(1+2x)-2\ln(2+x)-\frac{1}{2+x}+c$

3i	Minimum Point $= \left(\frac{3\pi}{4}, -1\right)$
ii and iii	
iv	$\|\cos x\| - 1 = \frac{3\sin 2x}{2}$ $\|2\cos x\| - 2 = 3\sin 2x$ $\|2\cos x\| = 3\sin 2x + 2$ 3 solutions

4i	Area of ΔOXY and ΔABX $= 1 - (\text{Area of } \Delta OCY + \text{Area of } \Delta BXY + \text{Area of } \Delta AXB)$ $= 1 - \left(\frac{1}{2} \times 1 \times kx + \frac{1}{2} \times x \times (1 - kx) + \frac{1}{2}(1 - x)(1)\right)$ $= \frac{1}{2} - \frac{1}{2}kx + \frac{1}{2}kx^2$ $= \frac{1}{2}(1 - kx + kx^2)$
ii	$\frac{dS}{dx} = 0$ $-\frac{1}{2}k + kx = 0$ $x = \frac{1}{2}$ $\frac{d^2S}{dx^2} = k > 0$ (minimum)
iii	$S = \frac{1}{2}\left(1 - \frac{1}{4}k\right)$
5a	$e^{2x} = e^{3-x}$ $e^{2x} = \frac{e^3}{e^x}$ $e^{3x} = e^3$ $\therefore x = 1$
bi	$\int_0^1 e^{2x} dx$ $= \left[\frac{e^{2x}}{2}\right]_0^1$ $= \frac{e^2}{2} - \frac{1}{2}$ $= 3.1945 \approx 3.19(2dp)$
ii	$\int_0^1 e^{3-x} dx - 3.1945$ $= \left[-e^{3-x}\right]_0^1 - 3.1945$ $= \left[-e^{3-1} + e^3\right] - 3.1945$ $= 9.50195 \approx 9.50(2dp)$

6i	Period = 12 h $\frac{2\pi}{k}=12$ $k=\frac{2\pi}{12}=\frac{\pi}{6}$
ii	Maximum value of h $=1.8-1.1(-1)=2.9$ m Minimum value of $h=1.8-1.1(1)=0.7$ m Range of h: $0.7 \text{ m} \le h \le 2.9 \text{ m}$
iii	$1.8-1.1\sin\frac{\pi}{6}t=1.5$ $\sin\frac{\pi}{6}t=0.2727$ $\alpha=0.27622$ $\frac{\pi}{6}t=0.27622,\ 2.8654,\ 6.5594,\ 9.14856$ $t=0.5275,\ 5.4724,\ 12.5275,\ 17.4724$ Length of time when boat landings are possible $=(5.4724-0.5275)+(17.4724-12.5275)$ $=9.8898$ h $=9.89$ h

7i	$y=3x^3+2x^2-5x+1$ $\frac{dy}{dx}=9x^2+4x-5$ When $x=-1$, $\frac{dy}{dx}=9(-1)^2+4(-1)-5=0$ Equation of tangent: $y=5$……………………(1) $\therefore$ Coord of P : $(0,5)$
ii	Sub $x=0$ in $y=3x^3+2x^2-5x+1$ $\therefore y=3(0)^3+2(0)^2-5(0)+1=1$ $\therefore$ Coord of Q: $(0,1)$ $\frac{dy}{dx}=9(0)^2+4(0)-5=-5$ Grad of normal is $\frac{1}{5}$ Equation of normal at $Q(0,1)$: $y-1=\frac{1}{5}(x-0)$ $y=\frac{1}{5}x+1$…………(2) When the normal cuts the x-axis, $y=0$ $0=\frac{1}{5}x+1$ $x=20$ $\therefore$ Coord of R: $(-5,0)$ Equations (1) and (2): $\frac{1}{5}x+1=5$ $x=20$ $\therefore$ Coord of S: $(20,5)$ Area PRS $=\frac{1}{2}\times 20\times 5=50\text{units}^2$

8i	$v=2\sin t+3\cos t=0$ $\tan t=-\frac{3}{2}$ $\alpha=0.98279$ $t=\pi-0.98279, \quad 2\pi-0.98279$ $=2.16, \ 5.30$ It first comes to instantaneous rest at $t=2.16$s.
ii	$a=2\cos t-3\sin t$ When $t=\frac{\pi}{2}$, $a=2\cos\frac{\pi}{2}-3\sin\frac{\pi}{2}$ $=-3 \text{ m/s}^2$
iii	$s=2\cos t+3\sin t+c$ When $t=0, s=1$ $\therefore c=3$ $\therefore s=2\cos t+3\sin t+3$
iv	When $t=2.16, s=6.6055$ m When $t=3, s=5.4033$ m $\therefore$ Distance travelled $=(6.6055-1)+(6.6055-5.4033)$ $=6.81$ m

9i	$2x^2+2y^2-16x+4py=22$ $x^2+y^2-8x+2py-11=0$ $2g=-8,\ 2f=2p,\ c=-11$ $g=-4, f=p, r=6$ $\sqrt{(-4)^2+p^2+11}=6$ $p^2=9$ $p=3$ Centre$=(4,-3)$
ii	At $(4,-3)$; LHS$=3(4)-(-3)$ $=15$ RHS$=6$ $15\neq 6$ $\therefore$ The line does not pass through C_1
iii	Centre of reflected circle $=(2,-3)$ $r=6$ Equation: $(x-2)^2+(y+3)^2=36$
iv	New $r=\sqrt{(4-0)^2+(-3)^2}$ $=5$ Equation: $(x-4)^2+(y-3)^2=25$

10i	Perimeter $= OP + PQ + QR + RS + SO$ $= 2 + 2\sin\theta + (7\sin\theta \square 2\cos\theta) + 7\cos\theta + 7$ $= (9\sin\theta + 5\cos\theta + 9)$cm
ii	$P = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha + 9$ $\tan\alpha = \frac{5}{9} \Rightarrow \alpha = 29.1°$ $R = \sqrt{81+25} = \sqrt{106}$ $P = \sqrt{106}\sin(\theta + 29.1°) + 9$ cm
iii	Maximum value of $P = \sqrt{106} + 9 = 19.3$cm $\sin(\theta + 29.1°) = \sin 90°$ $\theta = 60.9°$
iv	$\sqrt{106}\sin(\theta + 29.1°) + 9$ cm $= 17$ $\sin(\theta + 29.1°) = 0.777$ $\theta = 21.9°$
11ii	$3.01 < a < 4.31$ $b = 2.03\ (\pm 0.1)$
iii	$x = 2.37\ (\pm 0.1)$
12i	$f\left(-\frac{1}{2}\right) = 0$ $2\left(-\frac{1}{2}\right)^3 + a\left(-\frac{1}{2}\right)^2 + 10\left(-\frac{1}{2}\right) + b = 0$ $-\frac{1}{4} + \frac{1}{4}a - 5 + b = 0$ $a = 21 - 4b$......................Eq (1) $f(2) = 30$ $2(2)^3 + a(2)^2 + 10(2) + b = 30$ $4a + b = -6$......................Eq (2) Sub Eq 1 in Eq 2: $4(21 - 4b) + b = -6$ $b = 6$ $a = 21 - 4(6) = -3$

ii	$2x^3-3x^2+10x+6=21x$ $2x^3-3x^2-11x+6=0$ let $g(x)=2x^3-3x^2-11x+6$ By trial and error, $g(-2)=2(-2)^3-3(-2)^2-11(-2)+6=0$ Hence, (x+2) is a factor using long / synthetic division: quadratic factor: $2x^2-7x+3$ $(2x^2-7x+3)(x+2)=0$ $x=-2$ $x=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(3)}}{2(2)}$ $=3$ or 0.5
iii	$2x^6-3x^4-11x^2+6=0$ let $h(x)=2(x^2)^3-3(x^2)-11x^2+6$ $x^2=-2(NA)$ or $x=\pm\sqrt{3}$ or $x=\pm\sqrt{0.5}$