Name :	Class	Index no.

Preliminary Examination 2016
Secondary Four Express/Five Normal Academic

ADDITIONAL MATHEMATICS

4047/01

Paper 1

14 September 2016
0900 – 1100h
2 hours

Additional Materials: Writing Paper (8 sheets)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write your calculator model on the top right-hand corner of the first page of your answer script.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate writing paper or graph paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange all your answer scripts in order of the questions answered and fasten them together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

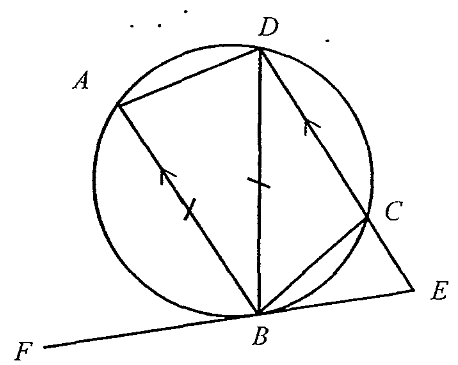
1 Given that $y=\dfrac{x}{1+x^2}$, show that y is decreasing for $x>1$ and $x<-1$. [3]

2 The graphs of $y=x^n$ and $y^2=kx$, where n and k are integers, intersect at the point $\left(\dfrac{1}{2},\ 2\right)$.

(i) Find the value of n and of k. [2]

(ii) On the same diagram, sketch the graphs of $y=x^n$ and $y^2=kx$ and indicate the point $\left(\dfrac{1}{2},\ 2\right)$ on the graphs. [2]

3 The diagram shows points A, B, C and D lying on a circle. The point E is such that DCE is a straight line and EBF is a tangent to the circle at point B. AB is parallel to DC and $AB = DB$.



(i) Prove angle CBE = angle ABD. [2]

(ii) Prove that triangle CBE is similar to triangle ABD. [3]

4 **(a)** Given that $\int_0^a (\sin\theta+\cos\theta)\ \mathrm{d}\theta=1$ and $0\le\theta\le\dfrac{\pi}{2}$, find the value of a. [3]

(b) A curve passes through the point $(1,\ 2)$ and has gradient $\dfrac{6}{(3x-4)^2}$. Find the equation of the curve. [3]

5 The roots of a quadratic equation are α and β, where $\alpha+\beta=5$ and $\alpha\beta=7$.

(i) Write down the quadratic equation in the form of $x^2+px+q=0$. [2]

(ii) Find, in the same form, the quadratic equation with roots α^3 and β^3. [4]

6 **(i)** Differentiate $3x^2 \ln x$ with respect to x. [3]

(ii) Hence evaluate $\int_1^2 x \ln x \, dx$. [3]

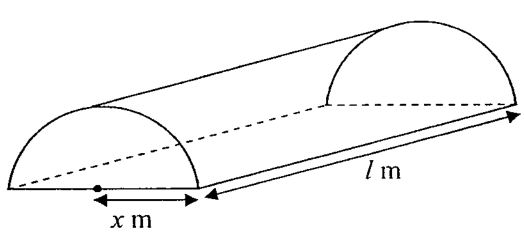
7 **(a)** Water is leaking from a plastic bag. After t seconds, the volume of water in the plastic bag is given by $V = \left(260e^{-0.1t} + 20\right)$ cm^3.

(i) Find the time taken when $\frac{3}{4}$ of the water in the plastic bag has leaked out. [3]

(ii) Find, in terms of t, the rate of decrease of volume of water in the plastic bag. [3]

(b) The variables x and y are connected by the equation $y = 2\cos^2\left(x - \frac{\pi}{6}\right)$. Given that y is decreasing at 0.5 radian per second, find the corresponding rate of change of x when $x = \frac{\pi}{3}$. [3]

8 A farmer wants to build a greenhouse that stands on a horizontal rectangular base. The vertical semicircle ends and the curved roof are made from polyethylene film for insulation. The radius of each semicircle is x m and the length of the greenhouse is l m.



(i) Given that 120 m^2 of polyethylene film is used for the greenhouse, show that the volume, V m^3, of the greenhouse is given by $V = 60x - \frac{\pi x^3}{2}$. [3]

(ii) Given x may vary, find the value of x for which the volume of the greenhouse is stationary. [4]

(iii) Explain why this value of x gives the farmer the largest volume possible. Hence find the largest possible volume of the greenhouse. [2]

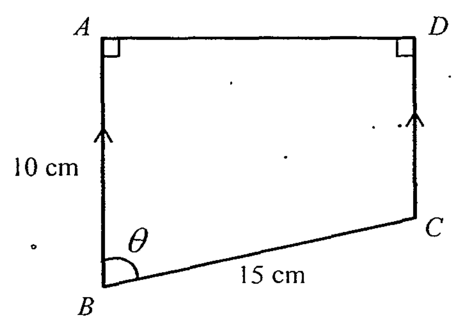
9 **(a)** Given that $p = \lg 5$, express the following in terms of p,

(i) $\lg 50$, [2]

(ii) $\lg 16$. [3]

(b) Solve $\log_2(x-3)+\log_2(x-2)=1+\log_4(x^2-4x+4)$. [5]

10 The diagram shows a trapezium *ABCD* where *AB* is parallel to *DC*. Angle *BAD* and angle *CDA* are right angles. Angle $ABC = \theta$ radians, where $0 < \theta < \frac{\pi}{2}$ and the lengths of *AB* and *BC* are 10 cm and 15 cm respectively.



(i) Show that L cm, the perimeter of the trapezium, can be expressed in the form $p + q\sin\theta + r\cos\theta$ where p, q and r are constants. [3]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$ where $R > 0$ and α is an acute angle. [4]

(iii) Given that the perimeter, L is 45 cm, find the value of θ. [3]

11 A circle, C_1, has equation $x^2 + y^2 + 4x - 6y - 36 = 0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

A second circle C_2, has a diameter *PQ*. The point *P* has coordinates (–5, 5) and the equation of the tangent to C_2 at *Q* is $y = 2x - 5$.

(ii) Find the equation of the diameter *PQ* and hence the coordinates of *Q*. [4]

(iii) Find the radius and the coordinates of the centre of C_2. [3]

(iv) Show, with working, that the point $(-6, 2)$ lies within only one of the circles, C_1 and C_2. [2]

~ End of paper ~

Name :	Class	Index no.

Preliminary Examination 2016
Secondary Four Express/Five Normal Academic

ADDITIONAL MATHEMATICS

4047/02

Paper 2

16 September 2016
0800 – 1030h
2 hours 30 minutes

Additional Materials: Writing Paper (8 sheets)
Graph Paper (1 sheet)

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Mathematical Formulae

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Quadratic Equation

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$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

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Identities

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$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 **(a)** Find the range of values of x for which $(2x+1)(4-x)>4$. [3]

(b) Find the range of values of m for which $2x-x^2<6-m$ for all values of x. [3]

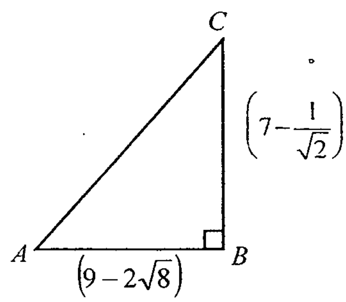
2 **(i)** The binomial expansion of $(1+px)^n$, where $n>0$, in ascending powers of x is

$$1-12x+28p^2x^2+...$$

Find the value of n and of p. [4]

(ii) Hence find the coefficient of x in the expansion of $\left(2x+\frac{5}{x}\right)(1+px)^n$. [2]

3 The diagram shows a right-angled triangle ABC where the height BC is $\left(7-\frac{1}{\sqrt{2}}\right)$ cm and the base AB is $\left(9-2\sqrt{8}\right)$ cm.



Express in the form of $a+b\sqrt{2}$, where a and b are rational numbers,

(i) the area of the triangle, [3]

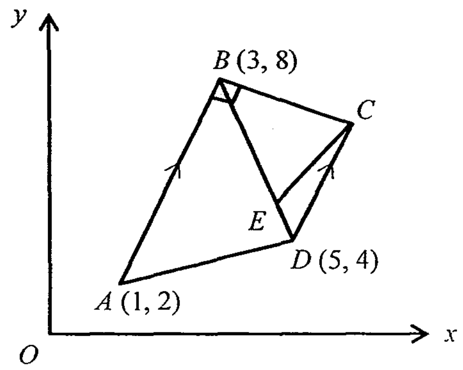
(ii) AC^2. [3]

4 A curve has the equation $y=x(x-6)^2$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Hence, find coordinates of each of the stationary points and determine the nature of each point. [4]

5 The diagram shows the trapezium $ABCD$ where coordinates of A, B and D are (1, 2), (3, 8) and (5, 4) respectively. Given that angle $ABC = 90^\circ$ and AB is parallel to DC.



(i) Find the coordinates of C. [4]

The point E lies on BD such that the area of triangle CDE is $\frac{1}{4}$ of the area of triangle CDB. Find

(ii) the coordinates of E, [2]

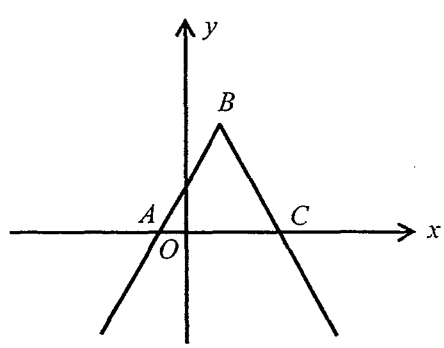
(iii) the area of triangle CDB. [2]

6 A particle, moving in a straight line, passes through a fixed point O with a speed 28 m/s. The acceleration, a m/s^2, of the particle, t s after through O, is given by $a = -24e^{-\frac{1}{3}t}$.

(i) Find the value of t when the particle is at instantaneous rest. [4]

(ii) Find the distance travelled in the first 3 seconds. [5]

7 (a) The diagram shows part of the graph of $y = 2 - |2x - 1|$.



(i) Find the coordinates of the points A, B and C. [3]

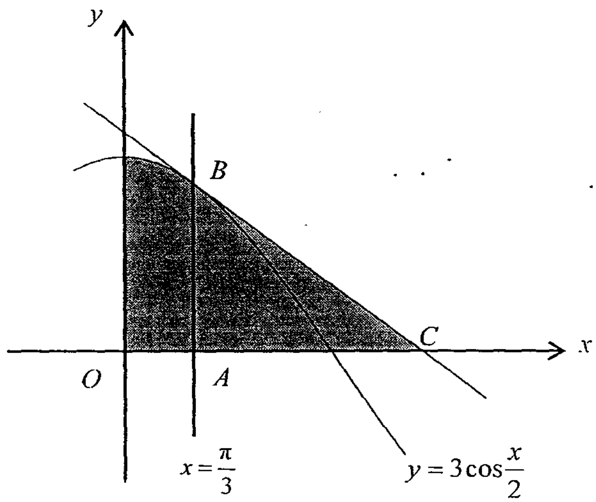
(ii) Solve the equation $2 - |2x - 1| = 3x$. [3]

(b) A curve has the equation $y=(x-1)^2-5$.

(i) Explain why $(1, -5)$ is the lowest point on the curve. [1]

(ii) Sketch the graph of $y=\left|(x-1)^2-5\right|$. [3]

8 The diagram shows part of the curve $y=3\cos\frac{x}{2}$. The line $x=\frac{\pi}{3}$ meets the curve at B and the x-axis at A. The tangent to the curve at B meets the x-axis at C.



Find

(i) the coordinates of B and C, [6]

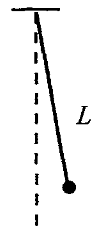
(ii) the area of the shaded region. [5]

9 **(a)** Find the value of k for which $x^2+(k-1)x+k^2-16$ is exactly divisible by $x-3$ but not by $x+4$. [5]

(b)(i) Express $\dfrac{5x^2-6x-13}{(x+3)(x-2)^2}$ in partial fractions. [4]

(b)(ii) Hence find $\int_3^5 \dfrac{5x^2-6x-13}{(x+3)(x-2)^2}\,dx$, giving your answer to 4 significant figures. [3]

10 An experiment in Physics was conducted to determine the gravitational acceleration, g, using a pendulum. The period of oscillation of the pendulum, T, is related to the length of the string, L, where the bob is attached.



The table below gives the value of the period recorded for the different length of the string of pendulum.

L/m	0.2	0.4	0.6	0.8	1.0
T/s	0.897	1.27	1.55	1.87	2.01

The variables, T and L are related by the following formula, $T = 2\pi\sqrt{\frac{L}{g}}$, where π and g are constants. It is known that one of the readings for T is incorrect and the air resistance is negligible in the experiment.

(i) Plot T^2 against L and draw a straight line graph for $0 \le L \le 1$. [3]

(ii) Using the graph,

(a) determine which value of T, in the table above, is the incorrect recording, [1]

(b) estimate a value of T to replace the incorrect recording of T found in part **(a)**, [1]

(c) estimate the value of g. [2]

The same experiment is repeated on the planet of Mars, where the gravitational acceleration is 3.71 m/s^2.

(iii) For this experiment, a straight line is to be drawn on the same graph. Determine the gradient of this line. [1]

(iv) Draw this straight line on the same graph. [1]

(v) Using the graph, determine the period for a pendulum of length 0.4 m on Mars. [2]

(vi) Compare and comment, using the graphs, on the period of a pendulum on the Earth and Mars. [1]

11 **(a)(i)** Prove the identity $\sin^4\theta - \cos^4\theta + \cos 2\theta = 0$. [3]

(a)(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta = 1$ for $0 \le \theta \le \pi$. [2]

(b)(i) On the same axes, sketch, for $0° \le x \le 120°$, the graphs of

$y = \frac{3}{2}\cos 6x + 1$ and $y = 2 - \frac{1}{2}\sin 3x$. [6]

(b)(ii) Explain how the graphs can be used to solve the equation $3\cos 6x + \sin 3x - 2 = 0$. [2]

~ End of paper ~

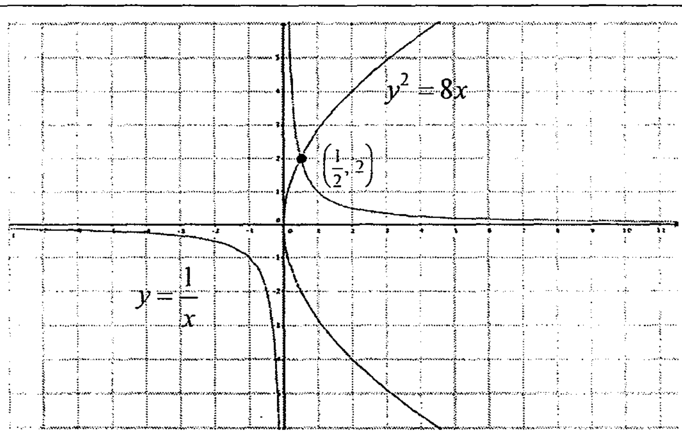
1 Given that $y=\frac{x}{1+x^2}$, show that y is decreasing for $x>1$ and $x<-1$. [3]

$y=\frac{x}{1+x^2}$	
$\frac{dy}{dx}=\frac{(1+x^2)(1)-x(2x)}{(1+x^2)^2}$	Find dy/dx **[M1]**
$=\frac{-x^2+1}{(1+x^2)^2}$	
For $x<-1$ and $x>1$, $(1+x^2)^2>0,\ -x^2+1<0$.	Explanation **[M1]**
$\therefore \frac{dy}{dx}<0$. Hence y is decreasing for $x>1$ and $x<-1$.	Conclude dy/dx <0 hence decreasing fn **[A1]**

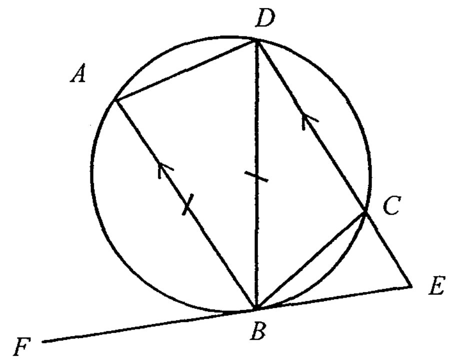
2 The graphs of $y=x^n$ and $y^2=kx$, where n and k are integers, intersect at the point $\left(\frac{1}{2}, 2\right)$.

(i) Find the value of n and of k. [2]

(ii) On the same diagram, sketch the graphs of $y=x^n$ and $y^2=kx$ and indicate the point $\left(\frac{1}{2}, 2\right)$ on the graphs. [2]

(i)	sub $\left(\frac{1}{2}, 2\right)$ into $y=x^n$ $2=\left(\frac{1}{2}\right)^n$ $n=-1$	sub $\left(\frac{1}{2}, 2\right)$ into $y^2=kx$ $2^2=k\left(\frac{1}{2}\right)$ $k=8$	Answer for n **[B1]** Answer for k **[B1]**
(ii)			Graph of y^2=8x **[A1]** Graph of y=1/x **[A1]**

3 The diagram shows points A, B, C and D lying on a circle. The point E is such that DCE is a straight line and EBF is a tangent to the circle at point B. AB is parallel to DC and $AB = DB$.



(i) Prove angle CBE = angle ABD. [2]

(ii) Prove that triangle CBE is similar to triangle ABD. [3]

(i)	$\angle CBE = \angle BDC$ (alternate segment theorem) $\angle CBE = \angle ABD$ (alt $\angle s$, $AD // DE$) $\therefore \angle CBE = \angle ABD$	CBE = CBD **[B1]** CBD = ABD **[B1]**
(ii)	Let $\angle BCE = x$ $\angle DCB = 180° - x$ ($\angle$s on a str line) $\angle DAB = 180° - (180° - x)$ ($\angle$s in opp seg) $= x$ $\therefore \angle DAB = \angle BCE$ From (i), $\angle CBE = \angle ABD$ $\angle CEB = \angle ADB$ ($\angle$ sum of Δ) $\therefore$ Triangle CBE is similar to triangle ABD (AAA similarity)	Show DAB = BCE **[M1]** DAB = BCE **[B1]** Deduced correctly **[B1]**

4 **(a)** Given that $\int_0^a (\sin\theta + \cos\theta)\ d\theta = 1$ and $0 \le \theta \le \frac{\pi}{2}$, find the value of a. [3]

(b) A curve passes through the point $(1, 2)$ and has gradient $\frac{6}{(3x-4)^2}$. Find the equation of the curve. [3]

(a)	$\int_0^a (\sin\theta + \cos\theta)\ d\theta = 1$	
	$[-\cos\theta + \sin\theta]_0^a = 1$	Integrate correctly **[A1]**
	$-\cos a + \sin a + 1 = 1$	
	$\sin a = \cos a$	Solve: Use tan function **[M1]**
	$\tan a = 1$	
	$a = \frac{\pi}{4}$	Answer **[A1]**
(b)	$\frac{dy}{dx} = \frac{6}{(3x-4)^2}$	
	$y = \int \frac{6}{(3x-4)^2}\ dx$	Integrate **[M1]**
	$= \int 6(3x-4)^{-2}\ dx$	
	$= 6\left[\frac{(3x-4)^{-1}}{-3}\right] + c$	
	$= -\frac{2}{(3x-4)} + c$	
	sub $(1, 2)$ into y,	
	$2 = -\frac{2}{3(1)-4} + c$	Sub point to find c **[M1]**
	$c = 0$	
	$\therefore y = -\frac{2}{(3x-4)}$	Answer **[A1]**

5 The roots of a quadratic equation are α and β, where $\alpha+\beta=5$ and $\alpha\beta=7$.

(i) Write down the quadratic equation in the form of $x^2+px+q=0$. [2]

(ii) Find, in the same form, the quadratic equation with roots α^3 and β^3. [4]

(i)	Quadratic equation: $x^2-5x+7=0$	Answer -5x **[B1]** Answer +7 **[B1]**
(ii)	new sum of roots: $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=(\alpha+\beta)\left[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta\right]$ $=5\left[5^2-(3)7\right]$ $=20$ new product of roots: $\alpha^3(\beta^3)=(\alpha\beta)^3$ $=7^3$ $=343$ The new quadratic equation is $x^2-20x+343=0$	Find new sum of roots **[M1]** Use algebraic result **[M1]** Find new product of roots **[M1]** Answer **[A1]**

6 **(i)** Differentiate $3x^2\ln x$ with respect to x. [3]

(ii) Hence evaluate $\int_1^2 x\ln x\,dx$. [3]

(i)	$\frac{d}{dx}3x^2\ln x$ $=(3x^2)\left(\frac{1}{x}\right)+(\ln x)(6x)$ $=3x+6x\ln x$	Use product rule differentiate $(3x^2)\left(\frac{1}{x}\right)$ correctly **[M1]** Use product rule differentiate $(\ln x)(6x)$ correctly **[M1]** Answer **[A1]**
(ii)	$\int_1^2 3x+6x\ln x\,dx=\left[3x^2\ln x\right]_1^2$ $\int_1^2 3x\,dx+\int_1^2 6x\ln x\,dx=\left[3x^2\ln x\right]_1^2$ $\int_1^2 6x\ln x\,dx=\left[3x^2\ln x\right]_1^2-\int_1^2 3x\,dx$ $6\int_1^2 x\ln x\,dx=\left[3x^2\ln x\right]_1^2-\left[\frac{3}{2}x^2\right]_1^2$ $=12\ln 2-\frac{9}{2}$ $\int_1^2 x\ln x\,dx=\frac{1}{6}\left(12\ln 2-\frac{9}{2}\right)$ $=0.636\ (3sf)$	Use (i) **[M1]** simplify **[M1]** Integrate 3x **[M1]** Answer **[A1]**

7 **(a)** Water is leaking from a plastic bag. After t seconds, the volume of water in the plastic bag is given by $V = \left(260e^{-0.1t} + 20\right)$ cm^3.

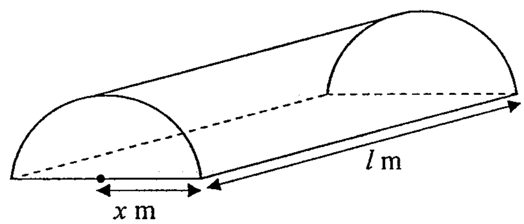
(i) Find the time taken when $\frac{3}{4}$ of the water in the plastic bag has leaked out. [3]

(ii) Find, in terms of t, the rate of decrease of volume of water in the plastic bag. [3]

(b) The variables x and y are connected by the equation $y = 2\cos^2\left(x - \frac{\pi}{6}\right)$. Given that y is decreasing at 0.5 radian per second, find the corresponding rate of change of x when $x = \frac{\pi}{3}$. [3]

(ai)	Volume of remaining water = 70cm^3 $\left(260e^{-0.1t} + 20\right) = 70$ $260e^{-0.1t} = 50$ $e^{-0.1t} = \frac{50}{260}$ $-0.1t = \ln\frac{50}{260}$ $t = 16.5\text{s (3sf)}$	Simplify: make e the subject **[M1]** Take ln **[M1]** Answer **[A1]**
(aii)	$\frac{dV}{dt} = -0.1(260)\left(e^{-0.1t}\right)$ $= -26e^{-0.1t}$ cm^3/s $\therefore$ rate of decrease $= 26e^{-0.1t}$ cm^3/s	Differentiate **[M1]** Answer **[A1]** Answer **[A1]**
(b)	$y = 2\cos^2\left(x - \frac{\pi}{6}\right)$ $\frac{dy}{dx} = 4\cos\left(x - \frac{\pi}{6}\right)\left[-\sin\left(x - \frac{\pi}{6}\right)\right](1)$ $= -4\cos\left(x - \frac{\pi}{6}\right)\sin\left(x - \frac{\pi}{6}\right)$ at $x = \frac{\pi}{3}$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $-0.5 = -4\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.289$ rad/s (3sf) Rate of change of x = 0.289 rad/s	Differentiate **[M1]** Use chain rule **[M1]** Answer **[A1]**

8 A farmer wants to build a greenhouse that stands on a horizontal rectangular base. The vertical semicircle ends and the curved roof are made from polyethylene film for insulation. The radius of each semicircle is x m and the length of the greenhouse is l m.



(i) Given that 120 m^2 of polyethylene film is used for the greenhouse, show that the volume, V m^3, of the greenhouse is given by $V = 60x - \frac{\pi x^3}{2}$. [3]

(ii) Given x may vary, find the value of x for which the volume of the greenhouse is stationary. [4]

(iii) Explain why this value of x gives the farmer the largest volume possible. Hence find the largest possible volume of the greenhouse. [2]

(i)	$surface\ area = \pi x^2 + \pi x l$	
	$120 = \pi x^2 + \pi x l$	Make l the subject **[M1]**
	$l = \frac{120 - \pi x^2}{\pi x}$	
	$V = \frac{1}{2}\pi x^2 l$	Sub into V **[M1]**
	$= \frac{1}{2}\pi x^2 \left(\frac{120 - \pi x^2}{\pi x}\right)$	
	$= \frac{x\left(120 - \pi x^2\right)}{2}$	
	$= 60x - \frac{\pi x^3}{2}$	Answer **[A1]**
(ii)	$\frac{dV}{dx} = 60 - \frac{3\pi x^2}{2}$	Differentiate for dv/dx **[M1]**
	For stationary value,	
	$\frac{dV}{dx} = 0$	dv/dx = 0 **[M1]**
	$60 - \frac{3\pi x^2}{2} = 0$	
	$\frac{3\pi x^2}{2} = 60$	
	$x^2 = \frac{40}{\pi}$	Simplify and solve dv/dx = 0 **[M1]**
	$x = 3.5682$ or -3.5682 (N.A.)	Answer **[A1]**
	$\therefore x = 3.57$ (3sf)	

(iii)	$\frac{d^2V}{dx^2} = -3\pi < 0$ Since $\frac{d^2V}{dx^2} < 0$, the volume is a maximum. Sub $x = 3.5682$ into V, $V = 143$ cm^3 (3sf)	Show $d^2v/dx^2 < 0$ and conclude maximum **[A1]** Answer for V **[A1]**

9 (a) Given that $p = \lg 5$, express the following in terms of p,

(i) $\lg 50$, [2]

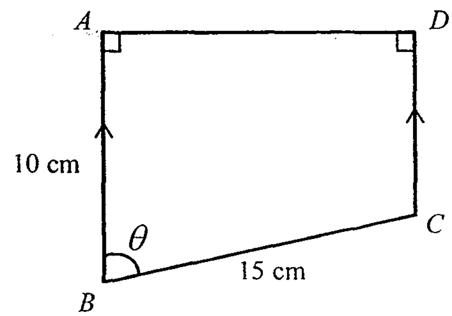
(ii) $\lg 16$. [3]

(b) Solve $\log_2(x-3) + \log_2(x-2) = 1 + \log_4(x^2 - 4x + 4)$. [5]

(ai)	$\lg 50 = \lg(5 \times 10)$ $= \lg 5 + \lg 10$ $= p + 1$	Product law **[M1]** Answer **[A1]**
(aii)	$\lg 16 = \lg 2^4$ $= 4\lg\left(\frac{10}{5}\right)$ $= 4(\lg 10 - \lg 5)$ $= 4(1-p)$	Power law **[M1]** Quotient law **[M1]** Answer **[A1]**
(b)	$\log_2(x-3) + \log_2(x-2) = 1 + \log_4(x^2 - 4x + 4)$ $\log_2[(x-3)(x-2)] = \log_2 2 + \frac{\log_2(x^2 - 4x + 4)}{\log_2 4}$ $= \log_2 2 + \frac{\log_2(x-2)^2}{2}$ $= \log_2 2 + \frac{2\log_2(x-2)}{2}$ $= \log_2 2 + \log_2(x-2)$ $= \log_2 2(x-2)$ $(x-3)(x-2) = 2(x-2)$ $(x-3)(x-2) - 2(x-2) = 0$ $(x-2)(x-3-2) = 0$ $(x-2)(x-5) = 0$ $x = 5$ or 2 (N.A.)	Product law for LHS **[M1]** Change of base for RHS **[M1]** Remove log on both sides **[M1]** Solve quad eqn **[M1]** Answer **[A1]**

[Turn over

10 The diagram shows a trapezium $ABCD$ where AB is parallel to DC. Angle BAD and angle CDA are right angles. Angle $ABC = \theta$ radians, where $0 \le \theta \le \frac{\pi}{2}$ and the lengths of AB and BC are 10 cm and 15 cm respectively.



(i) Show that L cm, the perimeter of the trapezium, can be expressed in the form $p + q\sin\theta + r\cos\theta$ where p, q and r are constants. [3]

(ii) Express L in the form $p + R\sin(\theta - \alpha)$ where $R > 0$ and α is an acute angle. [4]

(iii) Given that the perimeter, L is 45 cm, find the value of θ. [3]

(i)	Let X be the foot of the perpendicular line from C to AB. $\sin\theta = \frac{TC}{15}$ $TC = 15\sin\theta$ $AD = TC = 15\sin\theta$ $\cos\theta = \frac{TB}{15}$ $TB = 15\cos\theta$ $DC = 10 - 15\cos\theta$ $L = 15\sin\theta + (10 - 15\cos\theta) + 10 + 15$ $= 35 + 15\sin\theta - 15\cos\theta$	Express TC in terms sin θ **[M1]** Express DC in terms of cos θ **[M1]** Answer for L **[A1]**
(ii)	Let $15\sin\theta - 15\cos\theta = R\sin(\theta - \alpha)$ $R = \sqrt{15^2 + 15^2}$ $\quad$ $\tan\alpha = 1$ $= \sqrt{450}$ $\quad$ $\alpha = \frac{\pi}{4}$ $= 15\sqrt{2}$ $\therefore L = 35 + 15\sqrt{2}\sin\left(\theta - \frac{\pi}{4}\right)$	Find R **[M1]** Find α **[M1]** Answer **[A2]** **[must include 35, 1m for R, 1m for α]**

(iii)	$35+15\sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right)=45$	Form equation **[M1]**
	$15\sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right)=10$	
	$\sin\left(\theta-\frac{\pi}{4}\right)=\frac{2}{3\sqrt{2}}$	Simplify to make $\sin\left(\theta-\frac{\pi}{4}\right)$ the subject **[M1]**
	$\beta=0.490882$	
	$\theta=1.27628$	Answer **[A1]**
	$\theta=1.28$ rad (3sf)	

11 A circle, C_1, has equation $x^2+y^2+4x-6y-36=0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

A second circle C_2, has a diameter PQ. The point P has coordinates $(-5, 5)$ and the equation of the tangent to C_2 at Q is $y=2x-5$.

(ii) Find the equation of the diameter PQ and hence the coordinates of Q. [4]

(iii) Find the radius and the coordinates of the centre of C_2. [3]

(iv) Show, with working, that the point $(-6, 2)$ lies within only one of the circles, C_1 and C_2. [2]

(i)	$x^2+y^2+4x-6y-36=0$	
	$(x+2)^2+(y-3)^2-4-9-36=0$	Complete sq or use $x^2+y^2+2gx+2fy+c=0$ **[M1]**
	$(x+2)^2+(y-3)^2=49$	
	$(x+2)^2+(y-3)^2=7^2$	Answer for centre **[A1]** Answer for radius **[A1]**
	centre: $(-2,3)$, radius = 7 units	
(ii)	$m_{PQ}=-\frac{1}{2}$	Use m of tangent to find m_{pq} **[M1]**
	sub $m_{PQ}=-\frac{1}{2}$ and $(-5, 5)$ into $y=mx+c$	
	$5=-\frac{1}{2}(-5)+c$	
	$c=\frac{5}{2}$	
	Equation of PQ: $y=-\frac{1}{2}x+\frac{5}{2}$ or $2y=-x+5$ ---(1)	Answer for eqn of PQ **[A1]** Solve simultaneous eqn **[M1]**
	$y=2x-5$ ---(2)	

	$(1)=(2),\ 2x-5=-\frac{1}{2}x+\frac{5}{2}$ $x=3$ $\therefore y=1$ $Q(3,\ 1)$	Answer for Q **[A1]**
(iii)	centre, $C_2=\left(\frac{-5+3}{2},\frac{5+1}{2}\right)$ $=(-1,\ 3)$ radius, $C_2=\sqrt{(-1+5)^2+(3-5)^2}$ $=\sqrt{20}$ or 4.47 units	Midpoint of PQ **[M1]** Answer for centre of C_2 **[A1]** Answer for radius of C_2 **[B1]**
(iv)	Dist of $(-6,\ 2)$ from C_1 $=\sqrt{(-6+2)^2+(2-3)^2}$ $=4.12$ units Dist of $(-6,\ 2)$ from C_2 $=\sqrt{(-6+1)^2+(2-3)^2}$ $=5.10$ units	Find dist of (-6, 2) from the centres **[M1]** Conclude and show lies only within C_1 **[A1]**

~ End of paper ~

1 **(a)** Find the range of values of x for which $(2x+1)(4-x)>4$. [3]

(b) Find the range of values of m for which $2x-x^2<6-m$ for all values of x. [3]

(a)	$(2x+1)(4-x)>4$	
	$8x-2x^2+4-x>4$	Expand **[M1]**
	$-2x^2+7x>0$	Factorise **[M1]**
	$x(2x-7)<0$	Answer **[A1]**
	$0<x<3\frac{1}{2}$	
(b)	$2x-x^2<6-m$	
	$x^2-2x+6-m>0$	$b^2-4ac<0$ **[M1]**
	Curve always positive => $b^2-4ac<0$	Solve **[M1]**
	$(-2)^2-4(1)(6-m)<0$	
	$4-24+4m<0$	Answer **[A1]**
	$m<5$	

2 **(i)** The binomial expansion of $(1+px)^n$, where $n>0$, in ascending powers of x is

$$1-12x+28p^2x^2+...$$

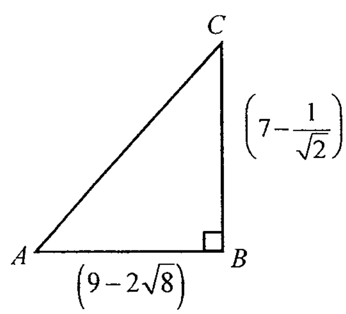
Find the value of n and of p. [4]

(ii) Hence find the coefficient of x in the expansion of $\left(2x+\frac{5}{x}\right)(1+px)^n$. [2]

(i)	$(1+px)^n$	
	$=1+\binom{n}{1}px+\binom{n}{2}(px)^2+...$	Expand using binomial theorem **[M1]**
	$=1+npx+\frac{n(n-1)}{2!}p^2x^2+...$	
	$=1+npx+\frac{1}{2}n(n-1)p^2x^2+...$	
	comparing coefficients,	Compare coefficients **[M1]**
	$x^2:\frac{1}{2}n(n-1)p^2=28p^2$	
	$\frac{1}{2}n(n-1)=28$	
	$n^2-n-56=0$	
	$(n+7)(n-8)=0$	
	$n=8$ *or* $n=-7$ (N.A.)	Answer **[A1]**
	$x:np=-12$	
	$8p=-12$	
	$p=-\frac{3}{2}$	Answer **[A1]**

(ii)	$\left(2x+\frac{5}{x}\right)(1+px)^n$ $=\left(2x+\frac{5}{x}\right)\left(1-\frac{3}{2}x\right)^8$ $=\left(2x+\frac{5}{x}\right)\left[1-12x+28\left(-\frac{3}{2}\right)^2x^2+...\right]$ $=\left(2x+\frac{5}{x}\right)(1-12x+63x^2+...)$ $=2x+315x$ $\doteq 317x$ The coefficient of $x=317$	Expand for x term **[M1]** Answer **[A1]**

3 The diagram shows a right-angled triangle ABC where the height BC is $\left(7-\frac{1}{\sqrt{2}}\right)$ cm and the base AB is $\left(9-2\sqrt{8}\right)$ cm.



Express in the form of $a+b\sqrt{2}$, where a and b are rational numbers,

(i) the area of the triangle, [3]

(ii) AC^2. [3]

(i)	$\text{Area} = \frac{1}{2}\left(9-2\sqrt{8}\right)\left(7-\frac{1}{\sqrt{2}}\right)$ $= \frac{1}{2}\left(63-\frac{9}{\sqrt{2}}-14\sqrt{8}+\frac{2\sqrt{8}}{\sqrt{2}}\right)$ $= \frac{1}{2}\left(63-\frac{9}{\sqrt{2}}-28\sqrt{2}+4\right)$ $= \frac{1}{2}\left(67-\frac{9\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}-28\sqrt{2}\right)$ $= \frac{1}{2}\left(67-\frac{9}{2}\sqrt{2}-28\sqrt{2}\right)$ $= \frac{1}{2}\left(67-\frac{65}{2}\sqrt{2}\right)$ $= \frac{67}{2}-\frac{65}{4}\sqrt{2}$ cm^2 **OR** Area $= \frac{1}{2}\left(9-2\sqrt{8}\right)\left(7-\frac{1}{\sqrt{2}}\right)$ $= \frac{1}{2}\left(9-2\sqrt{8}\right)\left(7-\frac{\sqrt{2}}{2}\right)$ $= \frac{1}{2}\left(63-\frac{9}{2}\sqrt{2}-28\sqrt{2}+4\right)$ $= \frac{1}{2}\left(67-\frac{65\sqrt{2}}{2}\right)$ $= \frac{67}{2}-\frac{65}{4}\sqrt{2}$ cm^2	Expansion **[M1]** Rationalize denominator **[M1]** Answer **[A1]** Rationalize denominator **[M1]** Expansion **[M1]** Answer **[A1]**
(ii)	AC^2 $= \left(9-2\sqrt{8}\right)^2+\left(7-\frac{1}{\sqrt{2}}\right)^2$ $= 81-36\sqrt{8}+32+49-\frac{14}{\sqrt{2}}+\frac{1}{2}$ $= 81-72\sqrt{2}+32+49-\frac{14\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}+\frac{1}{2}$ $= 81-72\sqrt{2}+32+49-7\sqrt{2}+\frac{1}{2}$ $= \frac{325}{2}-79\sqrt{2}$ **OR** AC^2	Expansion **[M1]** Rationalize denominator **[M1]** Answer **[A1]**

	$= \left(9-4\sqrt{2}\right)^2 + \left(7-\frac{\sqrt{2}}{2}\right)^2$	Rationalize denominator **[M1]**
	$=81-72\sqrt{2}+32+49-7\sqrt{2}+\frac{1}{2}$	Expansion **[M1]**
	$=\frac{325}{2}-79\sqrt{2}$	Answer **[A1]**

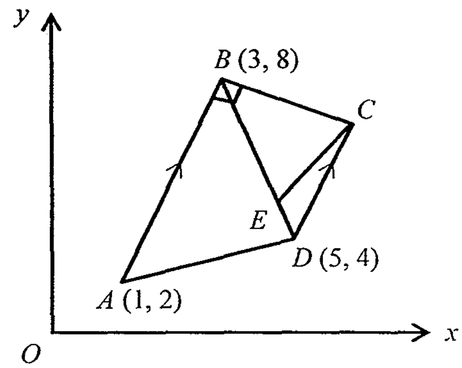
4 A curve has the equation $y = x(x-6)^2$.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Hence, find coordinates of each of the stationary points and determine the nature of each point. [4]

(i)	$\frac{dy}{dx} = x\left[2(x-6)(1)\right]+(x-6)^2(1)$ $= 2x(x-6)+(x-6)^2$ $= (x-6)(3x-6)$ or $3x^2-24x+36$ $\frac{d^2y}{dx^2} = 6x-24$	Differentiate using product rule **[M1]** Answer dy/dx **[A1]** Answer d^2/y^2 **[A1]**
(ii)	For stationary points, $\frac{dy}{dx} = 0$ $(x-6)(3x-6) = 0$ $x = 6$ or $x = 2$ sub $x = 6$ into y, $y = 0$ sub $x = 2$ into y, $y = 32$ $\therefore$ (6, 0) and (2, 32) at $x = 2$, $\frac{d^2y}{dx^2} = -12 < 0$ at $x = 6$, $\frac{d^2y}{dx^2} = 12 > 0$ (6, 0) is a minimum point (2, 32) is a maximum point	Answer (6,0) **[B1]** Answer (2,32) **[B1]** Answer (6,0) min **[B1]** Answer (2,32) max **[B1]**

5 The diagram shows the trapezium $ABCD$ where coordinates of A, B and D are (1, 2), (3, 8) and (5, 4) respectively. Given that angle $ABC = 90^\circ$ and AB is parallel to DC.



(i) Find the coordinates of C. [4]

The point E lies on BD such that the area of triangle CDE is $\frac{1}{4}$ of the area of triangle CDB.

(ii) Find the coordinates of E. [2]

(iii) Find the area of triangle CDB. [2]

(i)	Grad of $AB = \frac{8-2}{3-1} = 3$ Grad of $BC = -\frac{1}{3}$ sub $B(3, 8)$ *and* $m = -\frac{1}{3}$ into $y = mx + c$ $8 = -\frac{1}{3}(3) + c$ $c = 9$ Eqn of $BC : y = -\frac{1}{3}x + 9$	Find equation of BC **[M1]** Find equation of DC **[M1]** Solve simultaneous equations **[M1]** Answer **[A1]**

	Grad of $DC = 3$ sub $D(5, 4)$ *and* $m = 3$ into $y = mx + c$ $4 = 3(5) + c$ $c = -11$ Eqn of $DC: y = 3x - 11$ ---(2) $3x - 11 = -\frac{1}{3}x + 9$ $9x - 33 = -x + 27$ $10x = 60$ $x = 6$ sub into (2) $y = 7$ $\therefore C(6, 7)$	
(ii)	Both triangles have the same height. $\therefore DE = \frac{1}{4}DB$ Let M be the mid-point of DB. Hence E is the mid-point of DM. $M = \left(\frac{3+5}{2}, \frac{8+4}{2}\right)$ $= (4, 6)$ $E = \left(\frac{4+5}{2}, \frac{6+4}{2}\right)$ $= \left(\frac{9}{2}, 5\right)$	Find M mid-point of DB **[M1]** Answer **[A1]**
(iii)	Area of triangle CDB $= \frac{1}{2}\begin{vmatrix} 6 & 3 & 5 & 6 \\ 7 & 8 & 4 & 7 \end{vmatrix}$ $= \frac{1}{2}(48 + 12 + 35 - 21 - 40 - 24)$ $= \frac{1}{2}(10)$ $= 5 \text{ units}^2$	Find area using formula **[M1]** Answer **[A1]**

6 A particle, moving in a straight line, passes through a fixed point O with a speed 28 m/s. The acceleration, a m/s^2, of the particle, t s after through O, is given by $a=-24e^{-\frac{1}{3}t}$.

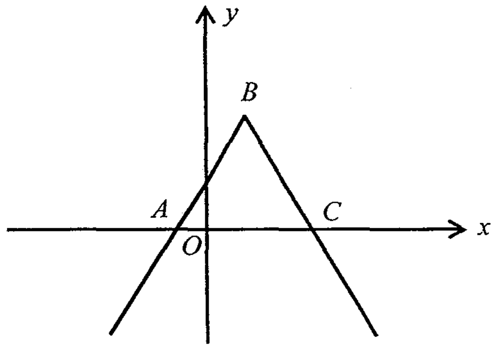
(i) Find the value of t when the particle is at instantaneous rest. [4]

(ii) Find the distance travelled in the first 3 seconds. [5]

(i)	$a=-24e^{-\frac{1}{3}t}$	
	$v=\int -24e^{-\frac{1}{3}t}\,dt$	Integrate a to find v **[M1]**
	$v=-24(-3)e^{-\frac{1}{3}t}+c$ when $t=0,\ v=28$ $28=72e^{0}+c$ $c=-44$	
	$v=72e^{-\frac{1}{3}t}-44$	Answer for v **[M1]**
	at instantaneous rest, $v=0$	V = 0 to find t **[M1]**
	$72e^{-\frac{1}{3}t}-44=0$ $e^{-\frac{1}{3}t}=\frac{44}{72}$ $-\frac{1}{3}t=\ln\frac{44}{72}$	
	$t=1.4774$	Answer **[A1]**
	$t=1.48s\ (3sf)$	
(ii)	$s=\int 72e^{-\frac{1}{3}t}-44\ dt$	Integrate v to find s **[M1]**
	$s=-216e^{-\frac{1}{3}t}-44t+d$ $when\ t=0,\ s=0$ $d=216$	
	$s=-216e^{-\frac{1}{3}t}-44t+216$	Find s **[M1]**
	$when\ t=1.4774$	Find s when t=1.4774 **[M1]**
	$s=18.993$ $when\ t=3$ $s=4.53804$ $dist\ travelled\ in\ 3s$	
	$=18.993+(18.993-4.53804)$	Find the distance **[M1]**
	$=33.44796$	Answer **[A1]**
	$=33.4\ m\ (3sf)$	

[Turn over

7 **(a)** The diagram shows part of the graph of $y=2-|2x-1|$.



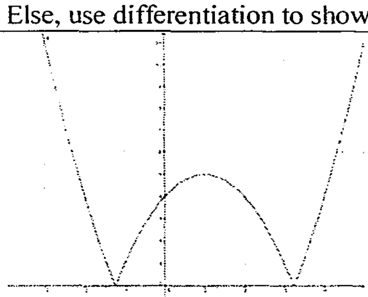
(i) Find the coordinates of the points A, B and C. [3]

(ii) Solve the equation $2-|2x-1|=3x$. [3]

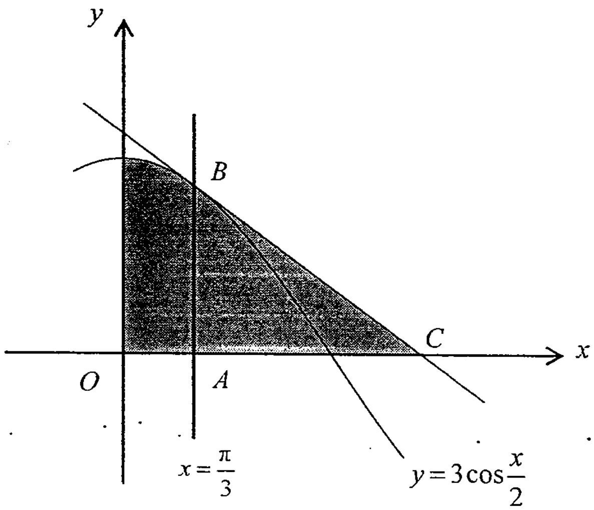
(b) A curve has the equation $y=(x-1)^2-5$.

(i) Explain why $(1,-5)$ is the lowest point on the curve. [1]

(ii) Sketch the graph of $y=|(x-1)^2-5|$. [3]

(ai)	$2-\|2x-1\|=0$ $x=\frac{3}{2}$ or $x=-\frac{1}{2}$ let $2x-1=0$ $x=\frac{1}{2}$ $A\left(-\frac{1}{2},0\right)$, $B\left(\frac{1}{2},2\right)$ and $C\left(\frac{3}{2},0\right)$	Answer **[B3]** **1m for each coordinates**
(aii)	$2-\|2x-1\|=3x$ $-\|2x-1\|=3x-2$ $\|2x-1\|=2-3x$ $2x-1=2-3x$ or $2x-1=3x-2$ $x=\frac{3}{4}$ or $x=1$ (N.A.) $\therefore x=\frac{3}{5}$ **OR** $(2x-1)^2=(2-3x)^2$ [M1] $x=\frac{3}{5}$ *or* $x=1$ (N.A.) [A1] [A1]	Simplify **[M1]** Solve modulus equations **[M1]** Answer x=3/5 **[A1]** Need to N.A. x = 1
(bi)	Since $(x-1)^2\geq 0$, for the smallest y, $(x-1)^2=0, y=5$ when $x=1$. Else, use differentiation to show.	Answer with explanation **[A1]**
(bii)		Shape **[A1]** Correct turning point **[A1]** Correct y-intercept **[A1]**

8 The diagram shows part of the curve $y=3\cos\frac{x}{2}$. The line $x=\frac{\pi}{3}$ meets the curve at B and the x-axis at A. The tangent to the curve at B meets the x-axis at C.



(i) Find the coordinates of B and C. [6]

(ii) Find the area of the shaded region. [5]

(i)	$y=3\cos\frac{x}{2}$	
	$\frac{dy}{dx}=-3\sin\frac{1}{2}x\left(\frac{1}{2}\right)$	differentiate **[M1]**
	$=-\frac{3}{2}\sin\frac{1}{2}x$	
	at $x=\frac{\pi}{3}$,	Sub x= π /3 into *y* **[M1]**
	$y=3\cos\frac{1}{2}\left(\frac{\pi}{3}\right)$	
	$=\frac{3\sqrt{3}}{2}$	
	$\therefore B\left(\frac{\pi}{3},\frac{3\sqrt{3}}{2}\right)$	Answer for B **[A1]**
	at $x=\frac{\pi}{3}$,	Sub x= π /3 into d*y/dx* **[M1]**
	$\frac{dy}{dx}=-\frac{3}{2}\sin\frac{1}{2}\left(\frac{\pi}{3}\right)$	
	$=-\frac{3}{4}$	

	Eqn of BC:	
	$y-\frac{3\sqrt{3}}{2}=-\frac{3}{4}\left(x-\frac{\pi}{3}\right)$	Sub x= π /3 into d*y*/*dx* **[M1]**
	$y=-\frac{3}{4}x+\frac{3\sqrt{3}}{2}+\frac{\pi}{4}$	Find eqn of BC **[M1]**
	Let $y=0$	
	$-\frac{3}{4}x+\frac{3\sqrt{3}}{2}+\frac{\pi}{4}=0$	
	$-3x+6\sqrt{3}+\pi=0$	
	$x=\frac{1}{3}\pi+2\sqrt{3}$	
	$\therefore C\left(\frac{1}{3}\pi+2\sqrt{3},\ 0\right)$	Answer for C **[A1]**
(ii)	shaded area	
	$=\int_0^{\frac{\pi}{3}}3\cos\frac{1}{2}x\ dx+\frac{1}{2}\left(2\sqrt{3}\right)\left(\frac{3\sqrt{3}}{2}\right)$	Area under curve 0 to $\pi/3$ **[M1]**
		Add to area of triangle **[M1]**
	$=\left[3(2)\sin\frac{1}{2}x\right]_0^{\frac{\pi}{3}}+\frac{9}{2}$	Integrate to obtain area under curve **[M1]**
	$=6\sin\frac{1}{2}\left(\frac{\pi}{3}\right)-0+\frac{9}{2}$	Correct integration 6sin1/2x **[A1]**
	$=6\left(\frac{1}{2}\right)+\frac{9}{2}$	
	$=7\frac{1}{2}$ sq units	Answer **[A1]**

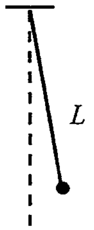
9 **(a)** Find the value of k for which $x^2+(k-1)x+k^2-16$ is exactly divisible by $x-3$ but not divisible by $x+4$. [5]

(bi) Express $\frac{5x^2-6x-13}{(x+3)(x-2)^2}$ in partial fractions. [4]

(bii) Hence find $\int_3^5 \frac{5x^2-6x-13}{(x+3)(x-2)^2}\ dx$, giving your answer to 4 significant figures. [3]

(a)	Let $f(x)=x^2+(k-1)x+k^2-16$ By factor theorem, $f(3)=0$ $(3)^2+3(k-1)+k^2-16=0$ $9+3k-3+k^2-16=0$ $k^2+3k-10=0$ $(k+5)(k-2)=0$ $k=-5$ or $k=2$ When $k=-5$, When $k=2$, $f(-4)=49$ $f(-4)=0$ $\therefore k=-5$	f(3) = 0 **[M1]** Factorise and solve **[M1]** Both answers **[A1]** Sub to check or justify **[M1]** Answer **[A1]**
(bi)	Let $\dfrac{5x^2-6x-13}{(x+3)(x-2)^2}=\dfrac{A}{(x+3)}+\dfrac{B}{(x-2)}+\dfrac{C}{(x-2)^2}$ $5x^2-6x-13=A(x-2)^2+B(x+3)(x-2)+C(x+3)$ sub $x=2$, $-5=5C$ $C=-1$ sub $x=-3$, $50=25A$ $A=2$ sub $x=0$, $-13=5-6B$ $B=3$ $\therefore \dfrac{5x^2-6x-13}{(x+3)(x-2)^2}=\dfrac{2}{(x+3)}+\dfrac{3}{(x-2)}-\dfrac{1}{(x-2)^2}$	Express $\dfrac{5x^2-6x-13}{(x+3)(x-2)^2}$ in partial fraction form **[M1]** Solve for unknowns. Substitution or comparing coefficients **[M1]** Answer **[A2]** Minus 1m for each wrong fraction
(bii)	$\int_3^5 \dfrac{5x^2-6x-13}{(x+3)(x-2)^2}\,dx$ $=\int_3^5 \dfrac{2}{(x+3)}+\dfrac{3}{(x-2)}-\dfrac{1}{(x-2)^2}\,dx$ $=\left[2\ln(x+3)+3\ln(x-2)+\dfrac{1}{(x-2)}\right]_3^5$ $=\left[2\ln8+3\ln3+\dfrac{1}{3}\right]-\left[2\ln6+3\ln1+1\right]$ $=2\ln8+3\ln3-2\ln6-\dfrac{2}{3}$ $=3.205\ (4sf)$	Use bii **[M1]** Integrate **[M1]** Answer to 4 sf **[A1]**

10 An experiment in Physics was conducted to determine the gravitational acceleration, g, using a pendulum. The period of oscillation of the pendulum, T, is related to the length of the string, L, where the bob is attached.



The table below gives the value of the period recorded for the different length of the string of pendulum.

L/m	0.2	0.4	0.6	0.8	1.0
T/s	0.897	1.27	1.55	1.87	2.01

The variables, T and L are related by the following formula, $T = 2\pi\sqrt{\frac{L}{g}}$, where π and g are constants. It is known that one of the readings for T is incorrect and the air resistance is negligible in the experiment.

(i) Plot T^2 against L and draw a straight line graph for $0 \le L \le 1$. [3]

(ii) Using the graph,

(a) determine which value of T, in the table above, is the incorrect recording, [1]

(b) estimate a value of T to replace the incorrect recording of T found in part **(a)**, [1]

(c) estimate the value of g. [2]

The same experiment is repeated on the planet of Mars, where the gravitational acceleration is 3.71 m/s^2.

(iii) For this experiment, a straight line is to be drawn on the same graph. Determine the gradient of this line. [1]

(iv) Draw this straight line on the same graph. [1]

(v) Using the graph, determine the period for a pendulum of length 0.4m on Mars. [2]

(vi) Compare and comment, using the graphs, on the period of a pendulum on the Earth and Mars. [1]

11 **(ai)** Prove the identity $\sin^4\theta-\cos^4\theta+\cos 2\theta=0$. [3]

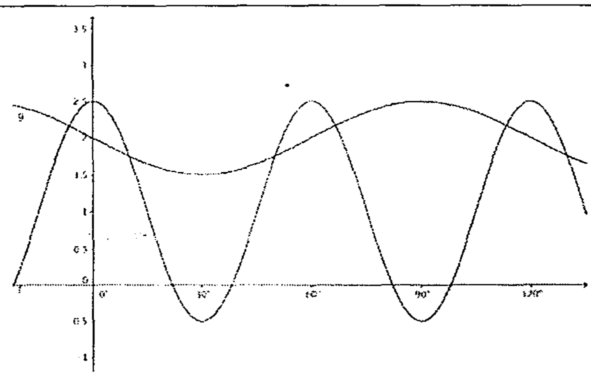
(aii) Hence solve the equation $\sin^4\theta-\cos^4\theta=1$ for $0\le\theta\le\pi$. [2]

(ai)	LHS $=\sin^4\theta-\cos^4\theta+\cos 2\theta$ $=(\sin^2\theta+\cos^2\theta)(\sin^2\theta-\cos^2\theta)+\cos 2\theta$ $=(1)(-\cos 2\theta)+\cos 2\theta$ $=0$	Factorise **[M1]** Double angle formula for $\cos 2\theta$ **[M1]** Answer **[A1]**
(aii)	$\sin^4\theta-\cos^4\theta=1$ From (ai), $-\cos 2\theta=1$ $\cos 2\theta=-1$ $2\theta=\pi$ $\theta=\frac{\pi}{2}$	Use (i) **[M1]** Answer **[A1]**

(bi) On the same axes, sketch, for $0°\le x\le 120°$, the graphs of

$$y=\frac{3}{2}\cos 6x+1 \quad \text{and} \quad y=2-\frac{1}{2}\sin 3x.$$ [6]

(bii) Explain how the graphs can be used to solve the equation $3\cos 6x+\sin 3x-2=0$. [2]

(bi)		**Each graph 3m x 2 = 6m** Correct shape/no.of cycles **[A1]** Correct y-intercept **[A1]** Correct range **[A1]**
(bii)	$\frac{3}{2}\cos 6x+1=2-\frac{1}{2}\sin 3x$ $3\cos 6x+2=4-\sin 3x$ $3\cos 6x+\sin 3x-2=0$ The x-coordinates of the points of intersection of the graphs are the solutions of the equation $3\cos 6x+\sin 3x-2=0$.	Equate the equations and simplify to required equation for solving **[M1]** Explanation using x-coord of points of intersection **[A1]**

~ End of paper ~

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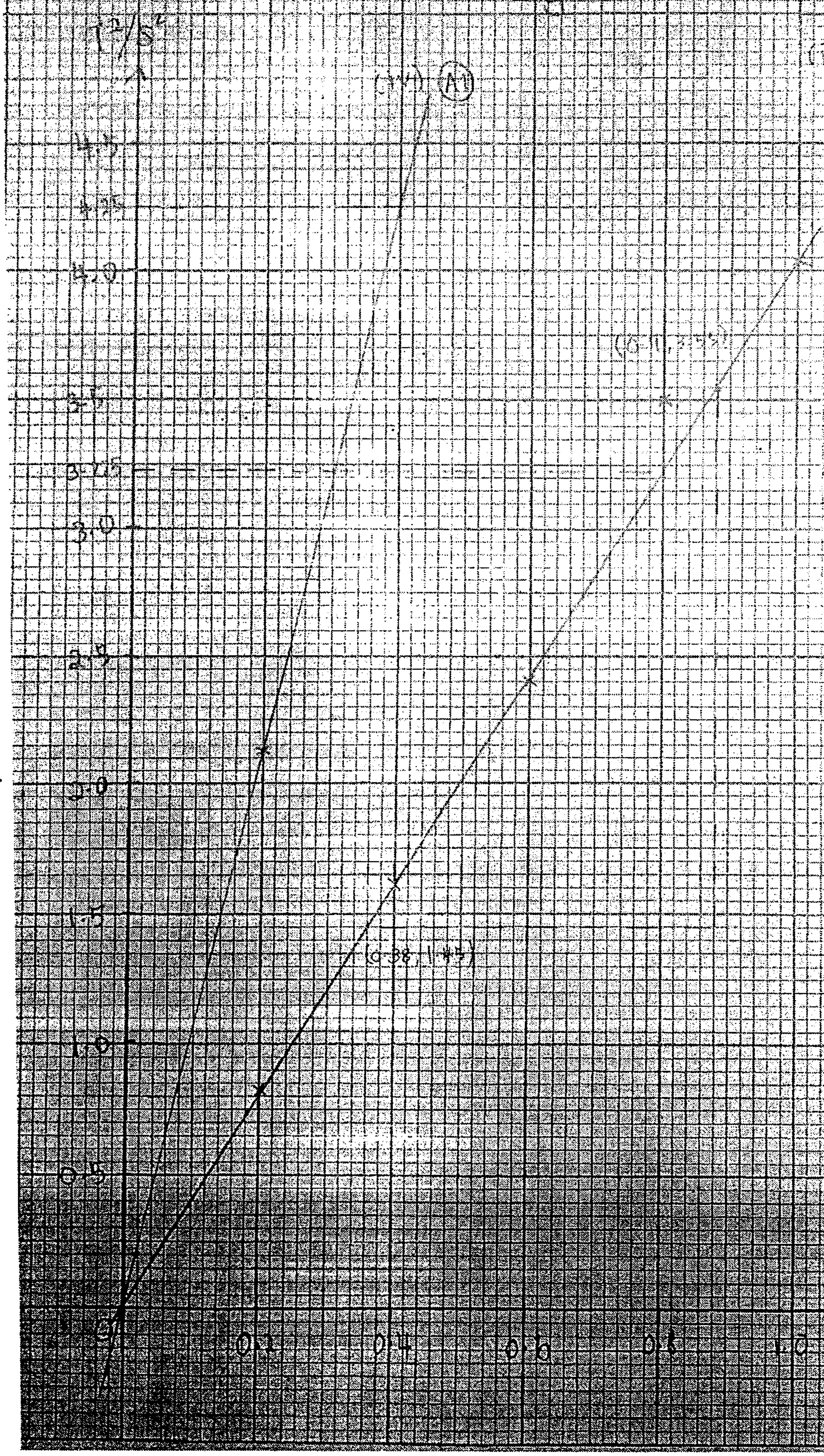
$T = 2\pi\sqrt{\frac{L}{g}}$

$T^2 = \frac{4\pi^2}{g} L$

L	0.2	0.4	0.6	0.8	1.0
T^2	0.805	1.61	2.40	3.50	4.04



(i) Points — ①
y-intercept at 0 — ①
Best fit line — ①

(iia) incorrect $T = 1.87$ (A1)

(iib) $T^2 = 3.225$

$T = 1.7958$

$= 1.80$ (A1)

(iic) grad

$= \frac{3.52 - 1.45}{0.9 - 0.38}$ (M1)

$= \frac{2.1}{0.52}$

$= 4.0346$

$\frac{4\pi^2}{g} = 4.0346$

$g = 9.7754$ 9.79

$= 9.78$ (3sf) (M1)

(iv) grad $= \frac{4\pi^2}{3.71}$

$= 10.64$

$= 10.6$ (3sf) (A1)

(v) $L = 0.4$ m (M1)

$T^2 = 4.25$

$T = 2.06$ s (3sf)

2.06 (A1)

(vi) The period of a pendulum on Mars is slower/longer than on Earth.