CONVENT OF THE HOLY INFANT JESUS SECONDARY
Semestral Assessment 2 in preparation for
the General Certificate of Education Ordinary Level 2017

ADDITIONAL MATHEMATICS

4047/01

Paper 1

23 Aug 2017

2 hours

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 (i) Write down and simplify the fourth term in the binomial expansion of $\left(x-\frac{p}{x}\right)^n$, where $n>0$. [2]

(ii) Given that the fourth term is equal to 160, find the value of n. [2]

(iii) With this value of n found in part **(ii)**, calculate the value of p. [2]

2 (i) Express $\frac{3}{3x^2-4x+1}$ in partial fractions. [3]

(ii) Find the second derivative of $\frac{3}{3x^2-4x+1}$ with respect to x and express your answer in the form $\frac{a}{(3x-1)^3}+\frac{b}{(x-1)^3}$, where a and b are integers. [4]

3 (a) Find, correct to 2 decimal places, the value of x which satisfies the equation $3^{4x}=7^{3x-1}$. [3]

(b) **Without using a calculator**, express $\log_{25} h$ in terms of x, given that $h^x=125$. [4]

4 It is given that $f(x)=(2-3p)x^2+(4-p)x+2$, where p is an integer.

(i) Find the range of values of p for which $f(x)=0$ has no real roots. [3]

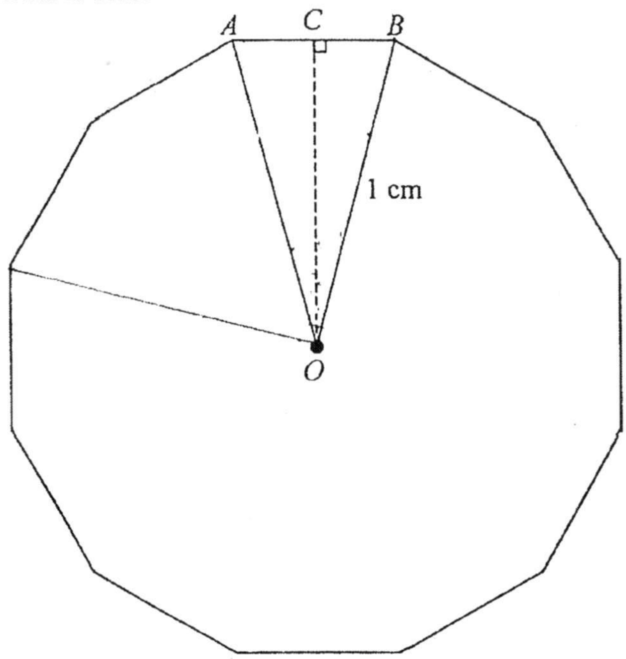
(ii) By considering the result of part **(i)**, explain whether the coefficient of x^2 is positive or negative. [1]

5 The equation of a curve is $y=\frac{x}{e^{2x-1}}$, where $x>\frac{1}{2}$.

(i) Using the **Quotient Rule**, find the gradient of the curve where $x=2$. [4]

(ii) Given that x is increasing at a rate of e^3 units per second, find the rate of decrease of y when $x=2$. [2]

6 **Diagram is not drawn to scale**



The diagram shows a regular 12-sided polygon with centre O. AB is one side of the polygon, C is the midpoint of AB and $OB = 1$ cm.

(i) Show that $AB = 2\sin 15°$. [2]

(ii) Express $\cos 30°$ in terms of $\sin 15°$ and show that $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$. [3]

7 (i) Prove that $\tan x + \cot x = \dfrac{2}{\sin 2x}$. [3]

(ii) Find all the angles between 0 and 4 which satisfy the equation

$$\frac{\tan x + \cot x}{4} = \frac{1}{\sqrt{3}}.$$ [4]

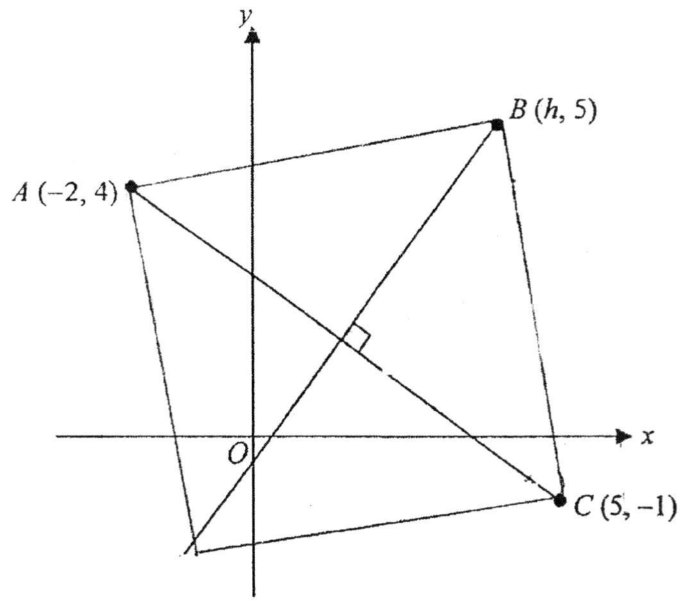
8 It is given that $y = \mathrm{e}^{2x}(A\sin 3x + B\cos 3x)$, where A and B are constant.

(i) Obtain an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the constants A and B such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 13\mathrm{e}^{2x}\sin 3x$. [4]

(iii) Hence fi [2]

9



In the diagram, the point A is $(-2, 4)$, the point B is $(h, 5)$ and the point C is $(5, -1)$. The point B lies on the perpendicular bisector of AC. Find

(i) the equation of the perpendicular bisector of AC, [3]

(ii) the value of h. [1]

The point D is such that $ABCD$ is a rhombus.

(iii) Find the coordinates of D. [2]

10 In an experiment, the mass, x grams, of a substance is measured at various times, t seconds. The two variables, x and t are related by a law of the form $2x - 10 = ab^{(t-3)}$, where a and b are constants. The table below shows some measured values of x and t and it is believed that one value of x does not conform to this law.

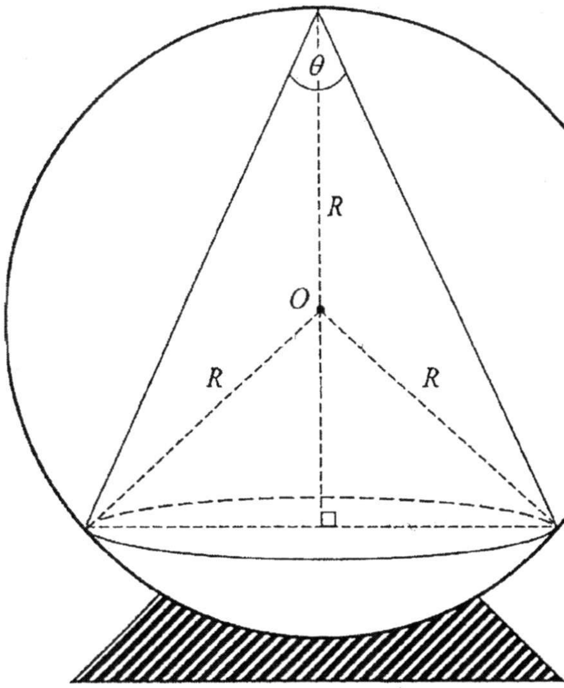
t (seconds)	7	11	15	19
x (grams)	7.4	8.5	10.2	15.0

(i) On graph paper, plot $\lg(2x-10)$ against $(t-3)$ and draw a straight line graph. The vertical $\lg(2x-10)$- axis should start at 0.4 and have a scale of 2 cm to 0.1. [2]

(ii) Use the graph to estimate the value of a and of b. [3]

(iii) Explain whether it is sensible to use these measured values to predict the mass when $t = 23$. [1]

11



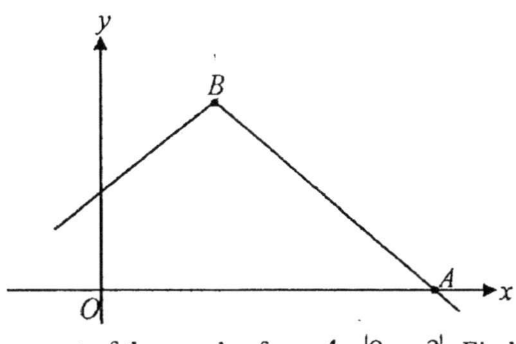
A snow globe is being designed to include a right circular cone inside a hollow sphere of fixed radius R cm and centre O, as shown in the diagram above. The vertical angle of the cone is θ.

(i) Given that the volume of the cone is V cm^3, show that

$$V = \frac{\pi}{3} R^3 (1 + \cos\theta) \sin^2 \theta.$$ [3]

(ii) Given that θ can vary, show that V has a stationary value when $\theta = \cos^{-1} \frac{1}{3}$. [5]

12 (a)



The diagram shows part of the graph of $y = 4 - |2x - 3|$. Find the coordinates of A and of B. [2]

(b) (i) Sketch the graph of $y = |2x^2 - 5x - 3|$ for $-1.5 \le x \le 4$, indicating on your graph the coordinates of the stationary point and of the points where the graph meets the vertical and horizontal axes. [4]

(ii) Calculate the values of x for which $|2x^2 - 5x - 3| = 3$, giving your answers to 2 de riate. [4]

--- End of Paper 1 ---

CONVENT OF THE HOLY INFANT JESUS SECONDARY
Semestral Assessment 2 in preparation for
the General Certificate of Education Ordinary Level 2017

ADDITIONAL MATHEMATICS

4047/02

Paper 2

12 September 2017

2 hours and 30 minutes

Additional Materials: Answer Paper
Graph Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

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Formulae for ΔABC

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1 Peter estimates that his score s marks in a class test is given by the formula

$$s = 95 - 58(1.8)^{-0.6t},$$

where t is the number of hours used to prepare for the test.

(i) How many marks can Peter expect to score if he did not prepare for the test? [1]

(ii) If Peter spends 2 hours 30 minutes to prepare for the test, what will his expected marks be? Give your answer correct to the nearest whole number. [1]

(iii) Peter's mother expects him to score at least 78 marks, how many hours must he put in in order not to disappoint his mother? Give your answer correct to the nearest half an hour. [4]

2 It is given that $2\log_5(2x-3)+\log_5(x+1)=3$.

(i) Show that $4x^3-8x^2-3x-116=0$. [3]

(ii) Factorise $4x^3-8x^2-3x-116$ completely. [3]

(iii) Hence, solve the equation $2\log_5(2x-3)+\log_5(x+1)=3$. Explain why there is only one real solution to the equation. [2]

3 The area of ΔABC is $11+7\sqrt{3}$ cm^2. $AC=5+3\sqrt{3}$ cm and $\angle ACB=\frac{\pi}{3}$ radian. Find the length of BC in the form $a+b\sqrt{3}$ where a and b are rational numbers.

[4]

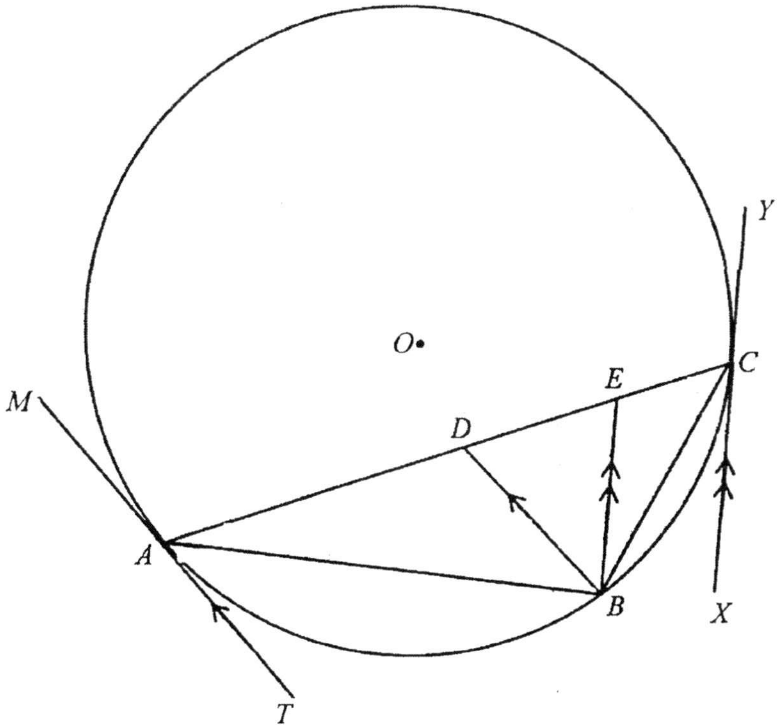
4 Given that α and β are the roots of the equation $3x^2-5x+4=0$, find

(i) the numerical value of $\alpha^3+\beta^3$, [3]

(ii) a quadratic equation whose roots are $\dfrac{\alpha}{\beta+2}$ and $\dfrac{\beta}{\alpha+2}$. [5]

5 Calculate the maximum value of $3uv^2$ given that u and v are two variables such that $u+v=15$. [5]

6



In the diagram, O is the centre of the circle. TAM is the tangent to the circle at A and XCY is the tangent to the circle at C. D and E are points on the chord AC where BD is parallel to TAM and BE is parallel to XCY. Prove that

(i) ΔABD and ΔBCE are similar, [4]

(ii) ΔBDE is isosceles, [2]

(iii) $AD \times CE = BE^2$. [2]

7 A particle moves in a straight line so that, at time t seconds after leaving a fixed point O, its displacement, s m, is given by $s = 8 - 8e^{-2t} - \frac{1}{8}t$. Calculate the

(i) initial velocity of the particle, [2]

(ii) time t when the particle is instantaneously at rest, [3]

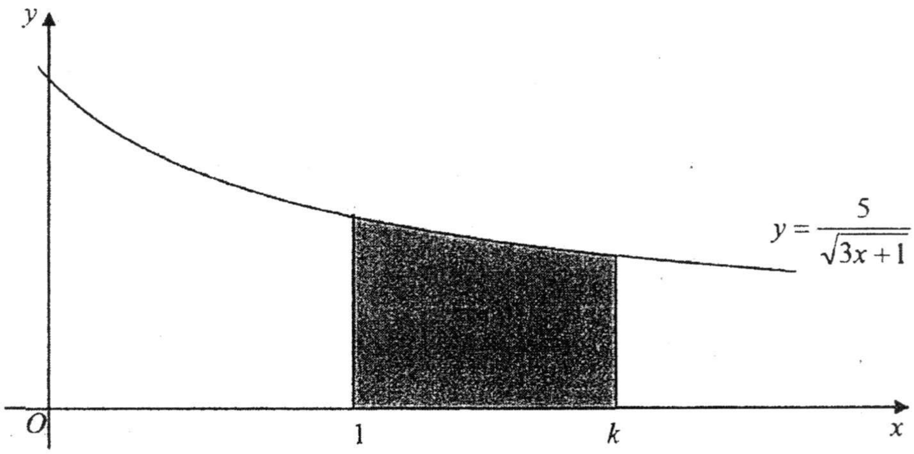
(iii) acceleration of the particle at the time $t = 2$ seconds, [2]

(iv) total distance travelled by the particle in the first 3 seconds. [4]

8 (a) Find $\int e^{\frac{x}{3}}\,dx$. [2]

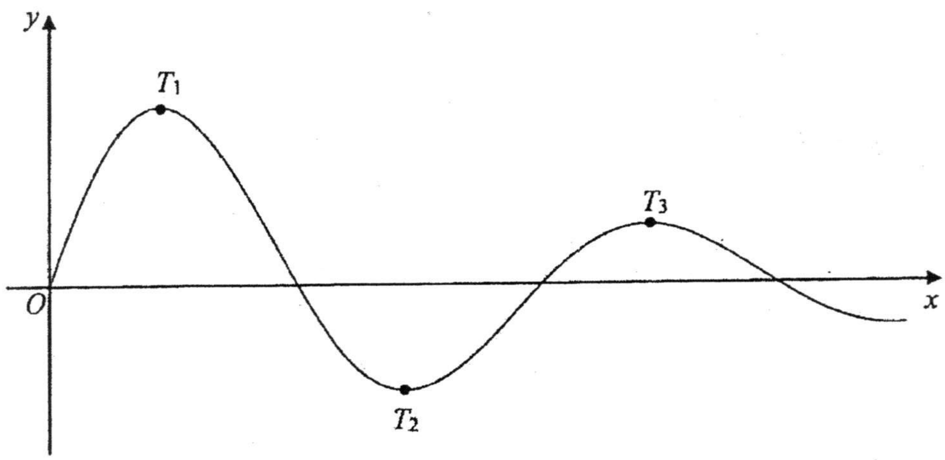
(b) Evaluate $\int_1^8 \left(\frac{3}{2x-1}+\frac{9e}{x^2}\right) dx$. [4]

(c)



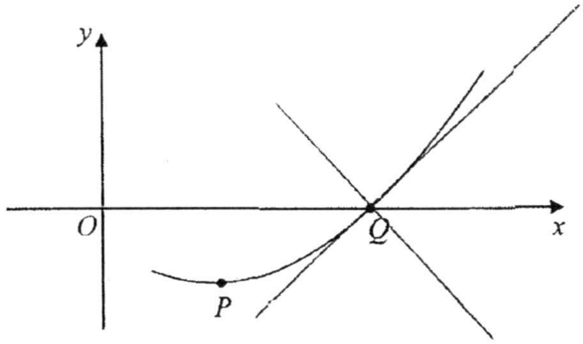
The shaded area bounded by the curve $y=\frac{5}{\sqrt{3x+1}}$, the x-axis and the lines $x=1$ and $x=k$ is 5 units, find the value of k. [6]

9



The diagram shows the graph of $y=6e^{-\left(\frac{x}{2}\right)}\sin 3x$ for $x \geq 0$. The first three stationary points are labelled T_1, T_2 and T_3. Find the x-coordinate of T_1 and of T_2, giving your answers correct to 3 decimal places. [6]

10



The diagram shows part of the curve $y = x^2 \ln x$ which intersects the x-axis at Q and has a minimum point at P.

(i) Find the equation of the normal to the curve at Q. [4]

(ii) Show that the x-coordinate of P is $\frac{1}{\sqrt{k}}$, where k is a constant and hence state the exact value of k. [4]

11 (i) Express $3\cos x + 4\sin x$ in the form $R\cos(x-\theta)$ where R is positive and $0° < \theta < 90°$. [4]

(ii) State the minimum value of $(3\cos x + 4\sin x)^2$ and the angles of x between $0°$ and $360°$ which give the minimum value. [2]

(iii) Find the maximum value of $3\cos x + 4\sin x - 2$ and the value of x between $0°$ and $90°$ which gives the maximum value. [3]

(iv) Find the principal value of $6\cos x + 8\sin x = 3$ for $0° < x < 360°$. [3]

12 A circle, C_1, has equation $x^2 + y^2 - 4x - 21 = 0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) Find the equation of the tangent to the circle C_1 at the point $(-1,4)$. [3]

The equation of another circle C_2 is $(x+5)^2 + (y-2)^2 = 49$.

(iii) The tangent to the circle C_1 at the point $(-1,4)$ cuts the circle C_2 at points P and Q. Find the x-coordinate of P and of Q, leaving your answers correct to 2 decimal places. [4]

(iv) Determine whether the point $(0,-4)$ is inside, outside or on the circle C_2. [2]

--- End of Paper 2 ---

CHIJ Sec Toa Payoh 2017 SA2 Paper 2 Numerical solutions

1 (i) 57 (ii) 71 (iii) 3.5 hours or 3h 30 min

2 (ii) $(4x^2+8x+29)(x-4)$

(iii) $x=4$, as $4x^2+8x+29=0$ has no real solutions as $b^2-4ac<0$

3 $-4+\frac{16}{3}\sqrt{3}$

4 (i) $-2\frac{1}{27}$ (ii) $x^2-\frac{31}{78}x+\frac{2}{13}=0$ or $78x^2-31x+12=0$

5 1500

6. (i) Let $T\hat{A}B=\theta=A\hat{C}B$ ($\angle$ in alt. segment)

$T\hat{A}B=A\hat{B}D=\theta$ (alt $\angle$, $TAM \parallel BD$)

Let $X\hat{C}B=\alpha=B\hat{A}C$ ($\angle$ in alt. segment)

$X\hat{C}B=\alpha=C\hat{B}E$ (alt $\angle$, $XCY \parallel BE$)

Since two angles of the triangles are equal, ΔABD and ΔBCE are similar (AA similarity test) A1

(ii) From (i) $B\hat{E}C=A\hat{D}B=\beta$

$\therefore B\hat{D}E=180°-\beta=B\hat{E}D$

$\therefore \Delta BDE$ is isosceles

(iii) From (i) $\frac{AB}{BC}=\frac{BD}{CE}=\frac{AD}{BE}$

$AD\times CE=BE\times BD$

$\therefore AD\times CE=BE^2$ ($\because BD=BE$ from (ii))

7 (i) $15\frac{7}{8}$ ms^{-1} (ii) 2.43 s (iii) -0.586 ms^{-2} (iv) 7.66 m

8 (a) $3e^{\frac{x}{3}}+c$ (b) 25.5 (c) k=3.75

9 0.469 or 1.516

10 (i) $y=-x+1$ (ii) k=e

11 (i) $5\cos(x-53.13°)$ (ii) 0, $x=143.1°$ or $323.1°$ (iii) 3, $x=53.1°$ (iv) PV of $y=125.7°$

12 (i) r=5, (2,0) (ii) $4y=3x+19$ (iii) 1.04 or -10.08 (iv) outside

CHIJ Sec Toa Payoh 2017 SA2 Paper 1 Numerical solutions

1 [illegible] $n=6$ (iii) $p=-2$

2 [illegible]

3 (a) $x = 1.348 = 1.35$ (to 2 dp) (b) $\therefore \log_{25} h = \frac{3}{2x}$

4 (i) $-16 < p < 0$ (ii) The coefficient of x^2 must be positive, because $(2-3p) > 0$ when $-16 < p < 0$

5 (i) $-\frac{3}{e^3}$ or $-0.14936 = -0.149$ (ii) y is decreasing at a rate of 3 units per second

6 (i) $\angle BOC = \frac{360°}{24} = 15°$, $\sin \angle BOC = \frac{BC}{1}$, $\sin 15° = BC$, $AB = 2 \times \sin 15°$ (Shown)

(ii) $\cos 30° = 1 - 2\sin^2 15°$, $2\sin^2 15° = 1 - \frac{\sqrt{3}}{2}$ $\sin^2 15° = \frac{2-\sqrt{3}}{4}$ $\sin 15° = \frac{1}{2}\sqrt{2-\sqrt{3}}$

7 (i) $\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\frac{1}{2}\sin 2x} = \frac{2}{\sin 2x}$ (ii) $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}$

8 (i) $\frac{dy}{dx} = 3Ae^{2x}\cos 3x - 3Be^{2x}\sin 3x + 2Ae^{2x}\sin 3x + 2Be^{2x}\cos 3x$

(ii) A=2, B=−3 (iii) $\int (e^{2x}\sin 3x)dx = \frac{1}{13}e^{2x}(2\sin 3x - 3\cos 3x) + c$ $y = e^{2x}(A\sin 3x + B\cos 3x)$

9 (i) $y = \frac{7}{5}x - \frac{3}{5}$ or $5y = 7x - 3$ (ii) $h = 4$ (iii) $\therefore D = (-1, -2)$

10 (ii) a= 3.24 (3 sf) b= 1.10 (3 sf) (iii) From the graph, the relationship between t and x applies for $7 \le t \le 15$. It clearly does not apply to $x = 19$, so it is not sensible to use the graph to predict the mass when $t = 23$.

12 (a) $A\left(\frac{7}{2}, 0\right)$, [redacted] $x = 3.39$ (2 dp) or $x = -0.89$ (2 dp)