SERANGOON GARDEN SECONDARY SCHOOL

Vision: Critical Thinkers, Thoughtful Leaders
Mission: Love to Learn, Learn to Lead

PRELIMINARY EXAMINATION 2017

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS — 4047/01
Paper 1 — 23 August 2017
Secondary 4 Express — 2 hours
1200 – 1400

Additional Materials: Writing Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Areas for Improvement		
Error	**Penalty**	**Qn. No.(s)**
Accuracy of non-exact answers	– 1	
Missing/ wrong units	– 1	
Presentation/ Not using ink	– 1	

Name/Signature of Parent/Guardian ______________ Date ______________

FOR MARKER'S USE
80

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Setter: Mr Ng HJ — Vetter: Ms Tay HY
SGS/A Maths/4Exp/2017/PRELIMS/4047/P1/QP

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc\cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

Answer all the questions.

1 Find the range of values of k for which the line $y = kx - 2$ meets the curve $y^2 = 4x - x^2$. [3]

Hence, describe the relationship between the line and the curve if $k = 1$. [1]

2 (i) Sketch, on the same diagram, the graphs of $y = |x| - 1$ and $y = |x^2 - 2x|$, including all the important features of the graphs and the intersections with the x- and y-axes. [4]

(ii) Hence, determine the value of a such that the equation

$$|x| - |x^2 - 2x| = a + 1$$

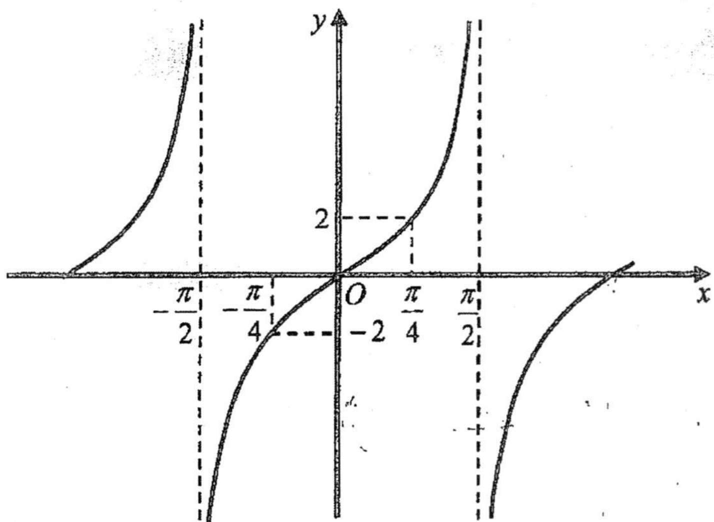
has exactly one solution. [1]

3 (a) State the values between which each of the following must lie:

(i) the principal value of $\sin^{-1} x$, [1]

(ii) the principal value of $\tan^{-1} x$. [1]

(b)



The diagram shows part of the graph of $y = a\tan(bx)$.

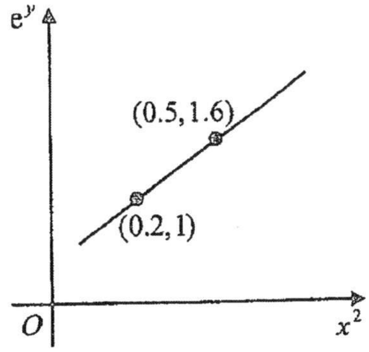
(i) Find the value of each of the constants a and b. [2]

(ii) Find the gradient of the curve at $x = \dfrac{\pi}{4}$. [2]

4 The quadratic equation $x^2 + mx + 2m = 0$, where m is a non-zero constant, has roots α anc tion with roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$. [4]

[Turn over

5 Variables x and y are such that, when e^y is plotted against x^2, a straight line passing through the points $(0.2, 1)$ and $(0.5, 1.6)$ is obtained.



(i) Find the value of e^y when $x = 0$. [2]

(ii) Express y in terms of x. [1]

6 (a) (i) For what values of x is $\log_x \sqrt{(x+1)(2-x)}$ defined? [2]

(ii) Differentiate $\ln \sqrt{(x+1)(2-x)}$ with respect to x. [2]

(b) Solve the equations $9^y + 5(3^y - 10) = 0$. [3]

(c) If $x^2 + y^2 = 11xy$, show that $\lg(x-y) = a\lg x + b\lg y + \lg c$, where a, b and c are constants to be determined. [5]

7 A circle has equation $x^2 + y^2 - 4x - 8y = 25$.

(i) Show that the radius of the circle is $3\sqrt{5}$ units and state the coordinates of the centre of the circle. [4]

(ii) Determine whether the point $(8, 8)$ lies inside or outside the circle. [2]

(iii) C and D are the points where the line $y + 2x = 8$ crosses the circle.

(a) Find the coordinates of C and D. [3]

(b) Show that CD is a diameter of the circle. [1]

8 (i) Prove the identity

$$\sin^2\theta\cos^2\theta = \frac{1}{8}(1-\cos 4\theta).$$ [3]

(ii) Hence,

(a) show that

$$\int_0^{\frac{\pi}{3}} \sin^2\theta\cos^2\theta \, d\theta = \frac{1}{8}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{8}\right),$$ [3]

(b) solve, for $0° \le \theta \le 180°$, the equation

$$\sin^2\theta\cos^2\theta = \frac{1}{10}.$$ [4]

9 A particle moves in a straight line, so that, t seconds after passing a fixed point A on the line, its velocity, v m/s, is given by

$$v = pt^2 + qt + 24,$$

where p and q are constants. When $t = 1$, the acceleration of the particle is -4 m/s^2. It comes to rest at a point B when $t = 4$.

(i) Find the value of p and of q. [4]

(ii) Find the distance AB. [3]

10 The equation of a curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{3x+1}{(x+2)(x-3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [2]

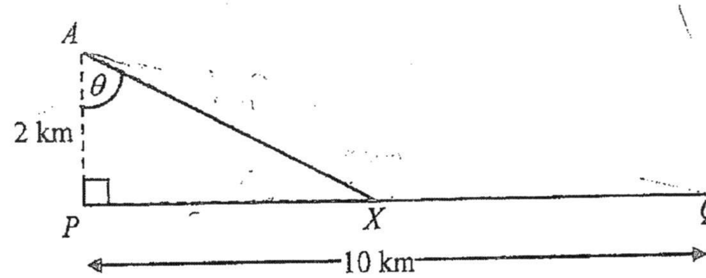
(ii) Hence find $\mathrm{f}'(x)$ and determine if $y = \mathrm{f}(x)$ is increasing or decreasing. [3]

(iii) Find $\displaystyle\int_4^6 \frac{3x+1}{(x+2)(x-3)}\,dx$. [3]

11 (i) By considering $\sec\theta$ as $(\cos\theta)^{-1}$, show that

$$\frac{d}{d\theta}(\sec\theta) = \frac{\sin\theta}{\cos^2\theta}.$$ [2]

(ii) The diagram shows a main straight road joining two towns, P and Q, 10 km part. An ambulance is at point A, where AP is perpendicular to PQ and AP is 2 km. The ambulance wishes to reach the hospital at Q as quickly as possible and travels in a straight line along a rocky road to meet the road at point X, where angle $\angle PAX = \theta$ radians.



The ambulance travels along AX at a speed of $10\,\text{kmh}^{-1}$ but on reaching the main road, it travels at a speed of $60\,\text{kmh}^{-1}$ along XQ.

(a) Given that the ambulance takes T hours to travel from A to Q, show that

$$T = \frac{\sec\theta}{5} + \frac{1}{6} - \frac{\tan\theta}{30}.$$ [4]

(b) Given that θ can vary, find the distance PX for which T has a stationary value. [5]

END OF PAPER

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PRELIMINARY EXAMINATION 2017

CANDIDATE NAME

CLASS REGISTER NUMBER

ADDITIONAL MATHEMATICS
Paper 2

4047/02
24 August 2017
2 hours 30 minutes
1000 - 1230

Secondary 4 Express

Materials needed: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and class register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal] in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Areas for Improvement		
Error	**Penalty**	**Qn. No.(s)**
Accuracy of non-exact answers	– 1	
Missing/ wrong units (for Paper 2 only)	– 1	
Presentation/ Not using ink	– 1	

Name/Signature of Parent/Guardian Date

FOR MARKER'S USE

100

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Setter: Ms Tay HY **Vetter: Mr Ng HJ**

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$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

Answer all the questions.

1 It is given that $f(x)=2x^3+ax^2+bx+6$ has a factor of $(x+2)$ and leaves a reminder of 15 when divided by $(2x-6)$.

(i) Find the value of a and of b. [4]

(ii) Solve $f(x)=0$, leaving your answers in exact value. [3]

(iii) Hence, solve the equation $8y^3-4y^2-9y+3=0$, leaving your answers in exact value. [2]

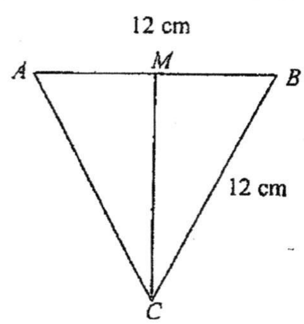
2 (i) In the binomial expansion of $\left(x+\frac{k}{x}\right)^7$, where k is a negative constant, the ratio of the coefficients of $\frac{1}{x}$ and x^3 is 15: 1.

(a) Show that $k=-3$. [3]

(b) Hence, find the coefficient of x in the expansion of $\left(1-\frac{1}{3}x^2\right)\left(x+\frac{k}{x}\right)^7$. [2]

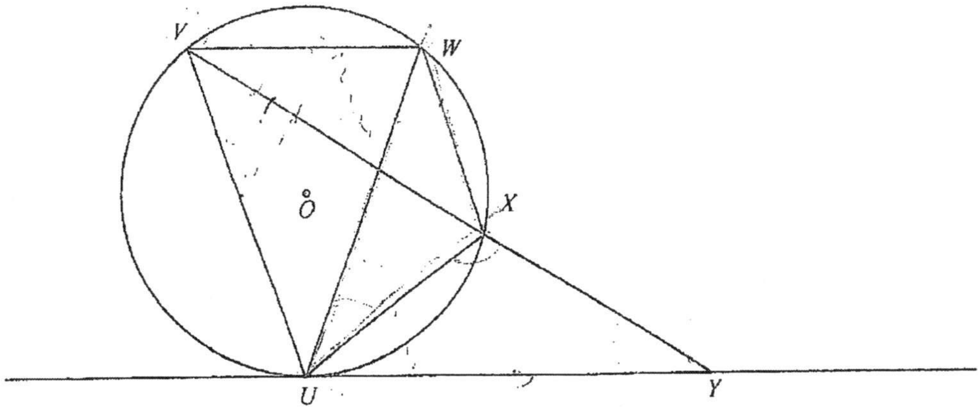
(ii) In the binomial expansion of $(1+bx)^n$, the first three terms are $1+\frac{9}{4}x+\frac{9}{4}x^2+\ldots$ Calculate the value of n and of b. [6]

3 (a) The diagram below shows a conical cup with slant height and diameter being 12 cm each. There is a tiny spider at C. Given that the spider climbs at a constant speed of $\frac{6-3\sqrt{3}}{4}$ cm/s, find the time, in seconds, taken by the spider to climb up along CM, giving your answer in the form $a\sqrt{3}+b$ where a and b are integers. You may assume that the spider is of negligible size. [4]



(b) Find the value of k, given that $125^k=\sqrt[3]{25\sqrt{5}}$ and k is a fraction. [3]

4 In the figure below, $UV = UW$ and the line UY is a tangent to the circle at the point U. VX is produced to meet the tangent at point Y.



Prove that

(i) VW is parallel to UY, [3]

(ii) ΔVUY is similar to ΔWXU, [2]

(iii) $VU^2 = WX \times VY$. [2]

5 (i) Find $\int \frac{1}{\sqrt{(4x-1)^3}} dx$. [2]

(ii) Show that $\frac{d}{dx}\left[\frac{8x+4}{\sqrt{4x-1}}\right] = \frac{16(x-1)}{\sqrt{(4x-1)^3}}$. [3]

(iii) Hence, evaluate $\int_1^2 \frac{x}{\sqrt{(4x-1)^3}} dx$, giving your answer correct to 4 significant figures. [5]

6 (i) It is given that a curve has an equation $y = (x+2)^3(x-k)$, where k is a positive constant. Find the x - coordinates of the stationary points of the curve, leaving your answers in terms of k where necessary. [4]

(ii) Determine the nature of each of the stationary points found in (i), showing your working clearly. [5]

7 The table below shows the experimental values of x and y which are known to be related by the equation $ya^x = b+1$, where a and b are constants. It is known that one value of y has been incorrectly recorded.

x	1	1.5	2	2.5	3	3.5	4
y	3.8	2.9	2.2	1.5	1.3	1	0.77

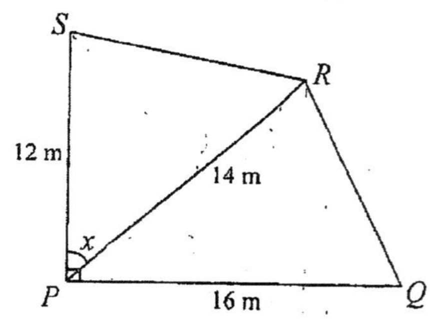
(i) On graph paper, plot $\lg y$ against x and draw a straight line graph. [3]

(ii) Use your graph to
(a) estimate the value of a and of b, [4]

(b) identify the abnormal value of y and estimate the correct value of it. [2]

(iii) On the same graph paper, draw the straight line representing the equation $y = 10^{0.4x-0.1}$ and hence find the value of x for which the two lines intersect. [3]

8 A playground $PQRS$ is formed by two triangles, ΔPQR and ΔPRS, where $PQ = 16$ m, $PR = 14$ m, $PS = 12$ m, $\angle RPS = x$ radians, $x < \pi$ and $\angle SPQ = \frac{\pi}{2}$.
The area of the playground is A m^2.



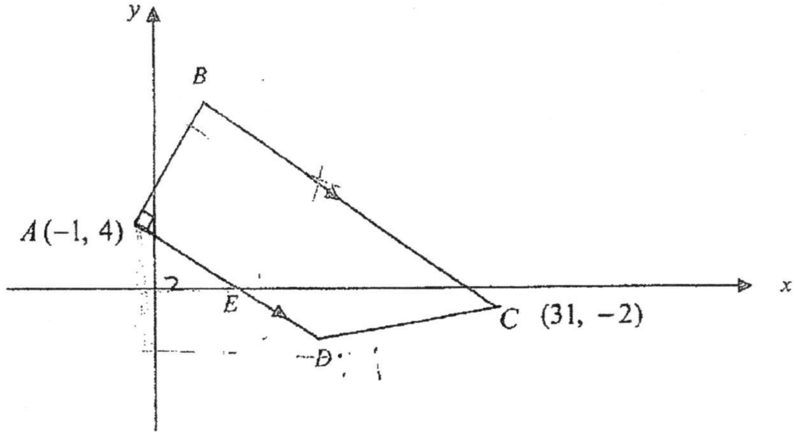
(i) Show clearly that $A = 112\cos x + 84\sin x$. [3]

(ii) Express A in the form $R\cos(x-\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4]

(iii) There are two contractors who worked on estimating the area of the playground. Contractor A concluded that the area of the playground was more than 160 m^2 but Contractor B disagreed. Explain whether you agree with Contractor B, stating your reason clearly. [2]

(iv) Find the ... rea of the playground was 130 m^2. [2]

9 The diagram below, not drawn to scale, shows a trapezium $ABCD$ in which AD is parallel to BC and AB is perpendicular to BC and AD. The coordinates of A and C are $(-1,\ 4)$ and $(31,\ -2)$ respectively. AD cuts the $x-$axis at E. The gradient of AB is 2.



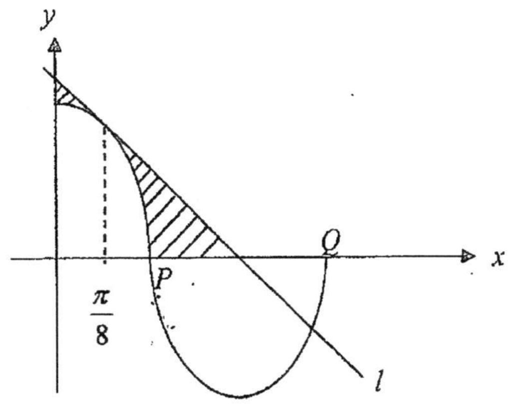
(i) Find the coordinates of B and E. [5]

(ii) Given that $AE:ED$ is $2:3$, find the coordinates of D. [2]

(iii) Find the area of trapezium $ABCD$. [2]

(iv) F is a point on the line BC such that $ABFE$ is a rhombus. Find the coordinates of F. [3]

10 The diagram below shows part of the curve $y=\sin\left(2x+\frac{\pi}{2}\right)$. The straight line, l, is a tangent to the curve at $x=\frac{\pi}{8}$. The points P and Q are on the x - axis.



$y=\sin\left(2x+\frac{\pi}{2}\right)$

Find the

(i) coordinates of P and Q, [3]

(ii) equation of the line l, [4]

(iii) sum of the areas of the shaded regions. [5]

END OF PAPER

Serangoon Garden Secondary School
Prelim Exam 2017
Sec 4E Add Maths (Paper 2)

Qn	Solution	Marks
1(i)	$f(x)=2x^3+ax^2+bx+6$	
	Since $(x+2)$ is a factor, $f(-2)=2(-2)^3+a(-2)^2+b(-2)+6=0$	
	$-16+4a-2b+6=0$	
	$4a-2b=10$--------------- (1)	M1
	$f(3)=2(3)^3+a(3)^2+b(3)+6=15$	
	$54+9a+3b+6=15$	
	$9a+3b=-45$	M1
	$a=-2$ and $b=-9$	A1, A1
1(ii)	$f(x)=2x^3-2x^2-9x+6=0$	
	$(x+2)(2x^2+kx+3)=0$	
	$kx^2+4x^2=-2x^2$	
	$k=-6$	
	$(x+2)(2x^2-6x+3)=0$	M1
	$x=-2$ or $x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$	
	$x=-2$ or $x=\frac{3\pm\sqrt{3}}{2}$	A2 for all three answers
1(iii)	$8y^3-4y^2-9y+3=0$	
	$16y^3-8y^2-18y+6=0$	
	$2(2y)^3-2(2y)^2-9(2y)+6=0$	
	Consider $2y=x$	M1
	$2y=-2$ or $2y=\frac{3\pm\sqrt{3}}{2}$	
	$y=-1$ or $y=\frac{3\pm\sqrt{3}}{4}$	A1 for all three answers
	Total for Q1	**9m**

2(i)(a)	$\left(x+\frac{k}{x}\right)^7$	
	$= x^7 + {}^7C_1(x)^6\left(\frac{k}{x}\right)^1 + {}^7C_2(x)^5\left(\frac{k}{x}\right)^2 + {}^7C_3(x)^4\left(\frac{k}{x}\right)^3 + {}^7C_4(x)^3\left(\frac{k}{x}\right)^4 + \ldots$	M1
	$= x^7 + \ldots + 21k^2x^3 + \ldots + 35k^4\frac{1}{x} + \ldots$	
	$\frac{35k^4}{21k^2} = 15$	M1
	$k = 3$ (reject) or $k = -3$	A1
2(i)(b)	$\left(1-\frac{1}{3}x^2\right)\left(x-\frac{3}{x}\right)^7$	
	$= \left(1-\frac{1}{3}x^2\right)(x^7 - 21x^5 + 189x^3 - 945x + 2835\frac{1}{x} + \ldots)$	M1
	$= -945x - 945x$	
	$= -1890x$	
	Therefore, coefficient of x is -1890.	A1
2(ii)	$(1+bx)^n = 1 + \binom{n}{1}(bx)^1 + \binom{n}{2}(bx)^2 + \ldots$	M1
	$= 1 + nbx + \frac{n(n-1)}{1\times 2}(bx)^2 + \ldots$	M1
	$= 1 + \frac{9}{4}x + \frac{9}{4}x^2 + \ldots$	
	Hence, $bn = \frac{9}{4}$ ------- (1)	
	$\frac{n^2-n}{2}b^2 = \frac{9}{4}$ -------- (2)	
	From (1), $b = \frac{9}{4n}$ --------- (3)	M1
	Sub (3) into (2) $\frac{n^2-n}{2}\left(\frac{9}{4n}\right)^2 = \frac{9}{4}$	M1
	$\frac{n^2-n}{2}\left(\frac{9}{4n^2}\right) = 1$	
	$9n^2 - 9n = 8n^2$	
	$n^2 - 9n = 0$	
	$n(n-9) = 0$	
	$n = 0$ **(reject)** **or** $n = 9$	A1
	Hen	A1
	Total for Q2	11m

3(a)	$CM=\sqrt{12^2-6^2}$	
	$CM=\sqrt{108}$	M1
	$CM=6\sqrt{3}$	
	Time taken by spider $=6\sqrt{3}\div\dfrac{6-3\sqrt{3}}{4}$	M1
	$=\dfrac{24\sqrt{3}}{6-3\sqrt{3}}\times\dfrac{6+3\sqrt{3}}{6+3\sqrt{3}}$	M1
	$=\dfrac{144\sqrt{3}+216}{(6)^2-(3\sqrt{3})^2}$	
	$=\dfrac{144\sqrt{3}+216}{9}$	
	$=16\sqrt{3}+24$ seconds	A1
3(b)	$125^k=\sqrt[3]{25\sqrt{5}}$	
	$5^{3k}=\sqrt[3]{5^2\sqrt{5}}$	M1
	$5^{3k}=\sqrt[3]{5^2(5)^{0.5}}$	
	$5^{3k}=\sqrt[3]{5^{2.5}}$	M1
	$5^{3k}=\left(5^{2.5}\right)^{\frac{1}{3}}$	
	$3k=\dfrac{5}{6}$	
	$k=\dfrac{5}{18}$	A1
	Total for Q3	7 m

4(i)	$\angle WUY = \angle UVW$ (Alt Segment Theorem)	M1
	$\angle UVW = \angle VWU$ ($UV = UW$)	
	$\angle WUY = \angle VWU$	M1
	$\therefore VW$ is parallel to UY (Alternate Angles)	A1
4(ii)	$\angle UVY = \angle XWU$ ($\angle$ in same segment)	M1
	$\angle VYU = \angle WVX$ (alternate $\angle$)	
	$\angle WVX = \angle WUX$ ($\angle$ in same segment)	
	$\angle VYU = \angle WUX$	M1
	$\therefore \Delta VUY$ is similar to ΔWXU by AA test.	
4(iii)	Since ΔVUY is similar to ΔWXU, $\frac{VU}{WX} = \frac{VY}{WU}$	M1
	$\frac{VU}{WX} = \frac{VY}{VU}$ as $UV = UW$	
	Hence, $VU^2 = WX \times VY$.	A1
	Total for Q4	7 m

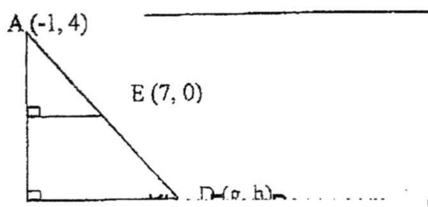
5(i)	$\int \frac{1}{\sqrt{(4x-1)^3}}dx = \int (4x-1)^{-1.5}dx$	
	$= \frac{(4x-1)^{-0.5}}{-0.5} \times \frac{1}{4} + c$	M1
	$= -\frac{1}{2\sqrt{(4x-1)}} + c$	A1
5(ii)	$\frac{d}{dx}\left[\frac{8x+4}{\sqrt{4x-1}}\right] = \frac{8(4x-1)^{0.5} - (8x+4)\times\frac{1}{2}(4x-1)^{-0.5}\times 4}{(4x-1)}$	M2
	$= \frac{(4x-1)^{-0.5}[8(4x-1) - 2(8x+4)]}{(4x-1)}$	
	$= \frac{16(x-1)}{\sqrt{(4x-1)^3}}$	A1
5(iii)	$\int_1^2 \frac{16(x-1)}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	M1
	$\int_1^2 \frac{16x}{\sqrt{(4x-1)^3}}dx - \int_1^2 \frac{16}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	M1
	$16\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx - 16\int_1^2 \frac{1}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	
	$16\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx - 16\left[-\frac{1}{2\sqrt{4x-1}}\right]_1^2 = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2$	M1
	$16\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx = \left[\frac{8x+4}{\sqrt{(4x-1)}}\right]_1^2 - \left[\frac{16}{2\sqrt{4x-1}}\right]_1^2$	
	$\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx = \frac{1}{16}\left\{\left[\frac{20}{\sqrt{7}} - \frac{12}{\sqrt{3}}\right] - \left[\frac{8}{\sqrt{7}} - \frac{8}{\sqrt{3}}\right]\right\}$	M1
	$\int_1^2 \frac{x}{\sqrt{(4x-1)^3}}dx = 0.1391$ (correct to 4 s.f)	A1
	Total for Q5	**10m**

6(i)	$y=(x+2)^3(x-k)$	
	$\frac{dy}{dx}=3(x+2)^2(x-k)+(x+2)^3$	M1
	$\frac{dy}{dx}=(x+2)^2[3(x-k)+(x+2)]$	
	$\frac{dy}{dx}=(x+2)^2(4x-3k+2)$	
	$\frac{dy}{dx}=(x+2)^2[4x-(3k-2)]$	
	To find stationary point, let $\frac{dy}{dx}=0$	
	$(x+2)^2[4x-(3k-2)]=0$	M1
	$x=-2$ or $x=\frac{3k-2}{4}$	A2
6(ii)	$\frac{d^2y}{dx^2}=2(x+2)(4x-3k+2)+(x+2)^2(4)$	M1
	$\frac{d^2y}{dx^2}=(x+2)[12x-6k+12]$	
	$\frac{d^2y}{dx^2}=6(x+2)[2x-k+2]$	
	Sub $x=\frac{3k-2}{4}$, $\frac{d^2y}{dx^2}=6(\frac{3k-2}{4}+2)\left[2\left(\frac{3k-2}{4}\right)-k+2\right]$	M1
	$\frac{d^2y}{dx^2}=6(\frac{3k-2+8}{4})\left[\frac{6k-4-4k+8}{4}\right]$	
	$\frac{d^2y}{dx^2}=6(\frac{3k+6}{4})\left[\frac{2k+4}{4}\right]>0$ since $k>0$	
	Hence, the stationary point at $x=\frac{3k-2}{4}$ is a minimum point.	A1
	Sub $x=-2$, $\frac{d^2y}{dx^2}=6(x+2)[2x-k+2]$	
	$\frac{d^2y}{dx^2}=0$	
	Hence, 2nd derivative test fails when $x=-2$.	
	Therefore, use 1st derivative test for $x=-2$.	
	Since $k>0$, $3k>0$, $3k-2>-2$, $-(3k-2)<2$	
	(table below)	M1
	Hen… $=-2$ is an inflexion point.	A1
	Total for Q6	9m

x	-2^- e.g. -2.01	-2	-2^+ e.g. -1.99
$\frac{dy}{dx}$	-ve since $(x+2)^2>0$ and $4x-(3k-2)<0$	0	-ve since $(x+2)^2>0$ and $4x-(3k-2)<0$
Slope	\	—	\

8(i)	$A =$ Area of ΔPRS + Area of ΔPQR	
	$A=\frac{1}{2}(12)(14)\sin x+\frac{1}{2}(14)(16)\sin\left(\frac{\pi}{2}-x\right)$	M1, M1
	$A=84\sin x+112\sin\left(\frac{\pi}{2}-x\right)$	
	$A=84\sin x+112\cos x$ since $\sin\left(\frac{\pi}{2}-x\right)=\cos x$	
	$A=112\cos x+84\sin x$ (shown)	A1
	Note: Students must state the result $\sin\left(\frac{\pi}{2}-x\right)=\cos x$ to get A1.	
8(ii)	$R=\sqrt{112^2+84^2}$	
	$R=140$	M1
	$\tan\alpha=\frac{84}{112}$	M1
	$\alpha=0.64350$	
	$\alpha=0.644$	M1
	$A=140\cos(x-0.644)$	A1
8(iii)	Since $-1\le\cos(x-0.644)\le 1$,	B1
	the maximum area of the playground is 140 m^2.	B1
	Hence, I agree with Contractor B that Contractor A is wrong.	
	[Note: If student does not state whether he agrees or disagrees with Contractor B, minus 1 mark]	
8(iv)	$140\cos(x-0.64350)=130$	
	$\cos(x-0.64350)=\frac{13}{14}$	
	$x-0.64350=0.38025$	M1
	$x=1.02375$	
	$x=1.02$ (Given that $x<\pi$)	A1
	Total for Q8	**11m**

9(i)	Equation of AB is $y = 2x + c$	
	Sub. (-1, 4) into equation	
	$4 = 2(-1) + c$	
	$c = 6$	
	Equation of AB is $y = 2x + 6$.	M1
	Gradient of $BC = -\frac{1}{2}$	
	Equation of BC is $y = -\frac{1}{2}x + d$	
	Sub. (31, -2) into equation	
	$-2 = -\frac{1}{2}(31) + d$	
	$d = \frac{27}{2}$	
	Equation of BC is $y = -\frac{1}{2}x + \frac{27}{2}$	M1
	B is the point of intersection between the lines AB and BC, so solve the 2 equations.	
	$2x + 6 = -\frac{1}{2}x + \frac{27}{2}$	
	$x = 3$	
	$y = 12$	
	Hence, B is (3, 12)	A1
	Equation of AE is $y = -\frac{1}{2}x + e$	
	Sub. (-1, 4) into equation	
	$4 = -\frac{1}{2}(-1) + e$	
	$e = \frac{7}{2}$	
	Equation of AE is $y = -\frac{1}{2}x + \frac{7}{2}$	M1
	Sub $E(k, 0)$ into equation of AE.	
	$0 = -\frac{1}{2}k + \frac{7}{2}$	
	$k = 7$	
	E is (7, 0).	A1
9(ii)		



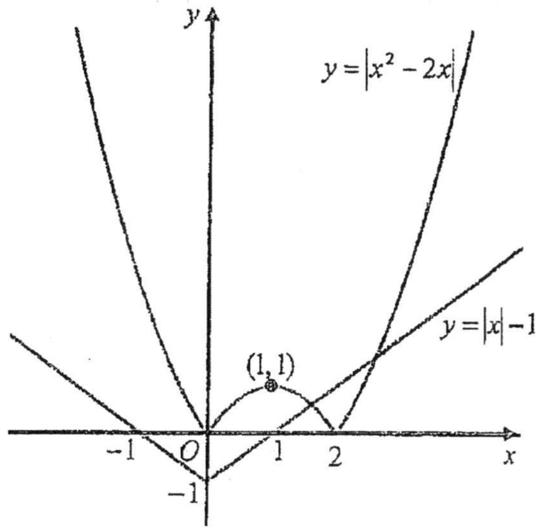
	2 units 2 units 3 units 2 units 3 units 5 units	
	Let D be (g, h)	
	$\frac{4-0}{0-h}=\frac{2}{3}$	
	$h=-6$	M1 for both parts
	$\frac{7-(-1)}{g-(-1)}=\frac{2}{5}$	
	$g=19$	
	Hence, D is (19, -6)	A1
9(iii)	Area of trapezium $ABCD=\frac{1}{2}\begin{vmatrix}-1 & 19 & 31 & 3 & -1\\ 4 & -6 & -2 & 12 & 4\end{vmatrix}$	M1
	$=\frac{1}{2}[(352)-(-128)]$	
	$=240$ units^2.	A1
9(iv)	Midpoint of $BE=\left(\frac{3+7}{2},\frac{12+0}{2}\right)$	
	$=(5, 6)$	M1
	Let F be (x, y)	
	Midpoint of $AF=(5, 6)$ $\left(\frac{-1+x}{2},\frac{4+y}{2}\right)=(5, 6)$	M1
	$x=11,\ y=8$	
	Hence, F is (11, 8)	A1
	Total for Q9	**12m**

10(i)	$y=\sin\left(2x+\frac{\pi}{2}\right)$	
	$\sin\left(2x+\frac{\pi}{2}\right)=0$	M1
	$2x+\frac{\pi}{2}=0,\pi,2\pi$	
	$x=-\frac{\pi}{4}$ (reject), $x=\frac{\pi}{4}$ or $x=\frac{3\pi}{4}$	
	P is $\left(\frac{\pi}{4},0\right)$ and Q is $\left(\frac{3\pi}{4},0\right)$	A1, A1
10(ii)	$\frac{dy}{dx}=2\cos\left(2x+\frac{\pi}{2}\right)$	M1
	At $x=\frac{\pi}{8}$, $\frac{dy}{dx}=2\cos\left(2\left(\frac{\pi}{8}\right)+\frac{\pi}{2}\right)$	
	$\frac{dy}{dx}=-\sqrt{2}$	
	At $x=\frac{\pi}{8}$, $y=\sin\left(2\left(\frac{\pi}{8}\right)+\frac{\pi}{2}\right)$	M1 for both parts.
	$y=\frac{\sqrt{2}}{2}$	
	The equation of l is $y=-\sqrt{2}x+c$	
	$\frac{\sqrt{2}}{2}=-\sqrt{2}\left(\frac{\pi}{8}\right)+c$	M1
	$c=\frac{\sqrt{2}(4+\pi)}{8}$	
	The equation of l is $y=-\sqrt{2}x+\frac{\sqrt{2}(4+\pi)}{8}$	A1
10(iii)	$0=-\sqrt{2}x+\frac{\sqrt{2}(4+\pi)}{8}$	
	$x=\frac{4+\pi}{8}$	
	Sub $x=0$, $y=\frac{\sqrt{2}(4+\pi)}{8}$	
	Area of triangle $=\frac{1}{2}\times\left(\frac{4+\pi}{8}\right)\times\left(\frac{\sqrt{2}(4+\pi)}{8}\right)$	M1

	Area under curve = $\int_0^{\frac{\pi}{4}}\left(\sin\left(2x+\frac{\pi}{2}\right)\right)dx$	M1
	$=\left[-\frac{1}{2}\cos(2x+\frac{\pi}{2})\right]_0^{\frac{\pi}{4}}$	M1
	$=\left[-\frac{1}{2}\cos(\frac{\pi}{2}+\frac{\pi}{2})\right]-\left[-\frac{1}{2}\cos(\frac{\pi}{2})\right]$	
	$=\frac{1}{2}-0$	
	$=\frac{1}{2}$ units^2	M1
	Sum of areas of shaded regions = $\frac{1}{2}\times\left(\frac{4+\pi}{8}\right)\times\left(\frac{\sqrt{2}(4+\pi)}{8}\right)-\frac{1}{2}$	
	= 0.063502 units^2	
	= 0.0635 units^2	A1
	Total for Q10	**12m**

--END OF PAPER--

Sec 4E Add Math Prelims P1 Suggested Mark Scheme

Qn	Solution	Mark Scheme
1	$y = kx - 2$ --- ①	
	$y^2 = 4x - x^2$ --- ②	
	Substitute ① into ②:	
	$(kx-2)^2 = 4x - x^2$	M1
	$k^2x^2 - 4kx + 4 - 4x + x^2 = 0$	
	$(k^2+1)x^2 + (-4k-4)x + 4 = 0$	
	For the line to meet the curve, $D \geq 0$	
	$(-4k-4)^2 - 4(k^2+1)(4) \geq 0$	M1
	$16k^2 + 32k + 16 - 16k^2 - 16 \geq 0$	
	$32k \geq 0$	
	$k \geq 0$	A1
	If $k = 1$, the line intersects the curve at two distinct points.	B1
		Total for Q1: 4
2(i)		G1: Shape of $y = \lvert x^2 - 2x \rvert$ G1: Shape of $y = \lvert x \rvert - 1$ G1: Maximum point G1: All intercepts
(ii)	$\lvert x \rvert - \lvert x^2 - 2x \rvert = a + 1$ $\lvert x \rvert - 1 = \lvert x^2 - 2x \rvert + a$ For exactly one solution, $a = 1$.	B1
		Total for Q2: 5

3(a)	(i) Principal value of $\sin^{-1}x$: $-\dfrac{\pi}{2} \le \sin^{-1}x \le \dfrac{\pi}{2}$ OR $-90^\circ \le \sin^{-1}x \le 90^\circ$ (ii) Principal value of $\tan^{-1}x$: $-\dfrac{\pi}{2} < \tan^{-1}x < \dfrac{\pi}{2}$ OR $-90^\circ < \tan^{-1}x < 90^\circ$	B1 B1
(b)	(i) $y = a\tan(bx)$ Period $= 2\pi \Rightarrow b = 1$ $\left(\dfrac{\pi}{4}, 2\right)$: $2 = a\tan\left(\dfrac{\pi}{4}\right) \Rightarrow a = 2$	B1 B1
	(ii) $y = 2\tan(x) \Rightarrow \dfrac{dy}{dx} = 2\sec^2 x$ At $x = \dfrac{\pi}{4}, \dfrac{dy}{dx} = \dfrac{2}{\cos^2(\frac{\pi}{4})} = \dfrac{2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 4$	M1 A1
		Total for Q3: 6
4	$x^2 + mx + 2m = 0$ $\alpha + \beta = -m$ $\alpha\beta = 2m$ $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{\alpha^2 + \beta^2}{\alpha\beta}$ $= \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$ $= \dfrac{m^2 - 4m}{2m} = \dfrac{m-4}{2}$ $\left(\dfrac{\alpha}{\beta}\right)\left(\dfrac{\beta}{\alpha}\right) = 1$ Required equation: $x^2 - \dfrac{m-4}{2}x + 1 = 0$ $2x^2 - (m-4)x + 2 = 0$	B1 for both M1 M1 A1
		Total for Q4: 4
5(i)	$e^y - 1 = \dfrac{1.6-1}{0.5-0.2}(x^2 - 0.2)$ $e^y - 1 = 2(x^2 - 0.2)$ $e^y = 2x^2 + 0.6$ When $x = 0, e^y = 0.6$	B1 B1
(ii)	$\ln e^y = \ln(2x^2 + 0.6)$ $y = \ln(2x^2 + 0.6)$	B1
		Total for Q5: 3

6(a) (i)	$\log_x \sqrt{(x+1)(2-x)}$ is defined when $x>0, x\neq 1$ and $(x+1)(2-x)>0 \Rightarrow -1<x<2$ Thus $0<x<2, x\neq 1$	M1 A1
(ii)	Let $y=\ln\sqrt{(x+1)(2-x)}=\frac{1}{2}[\ln(x+1)+\ln(2-x)]$ $\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x+1}-\frac{1}{2-x}\right)$	M1 A1
(b)	$9^y+5(3^y-10)=0$ $(3^y)^2+5(3^y-10)=0$ Let $u=3^y$ $u^2+5u-50=0$ $(u+10)(u-5)=0$ $u=-10$ or $u=5$ $3^y=-10$ (no soln) or $3^y=5$ $y\lg 3=\lg 5$ $y=\frac{\lg 5}{\lg 3}=1.46$	M1 M1 A1
(c)	$x^2+y^2=11xy$ $x^2+y^2-2xy=9xy$ $(x-y)^2=9xy$ $\lg(x-y)^2=\lg(9xy)$ $2\lg(x-y)=\lg 9+\lg x+\lg y$ $\lg(x-y)=\frac{1}{2}\lg x+\frac{1}{2}\lg y+\frac{1}{2}\lg 9$ $=\frac{1}{2}\lg x+\frac{1}{2}\lg y+\lg 3$ Thus $a=\frac{1}{2}, b=\frac{1}{2}, c=3$	M1 M1 M1 A1 for a and b, A1 for c
		Total for Q6: 12

7(i)	$x^2+y^2-4x-8y=25$ $(x-2)^2-4+(y-4)^2-16=25$ $(x-2)^2+(y-4)^2=45$ Centre: $(2,4)$, Radius $=\sqrt{45}=\sqrt{(9)(5)}=3\sqrt{5}$ units (shown)	 M1 M1 A1, A1
(ii)	Distance between the centre and the point $(8, 8)$ $=\sqrt{(8-2)^2+(8-4)^2}$ $=\sqrt{52}$ Since $\sqrt{52}>\sqrt{45}$, $(8, 8)$ lies outside the circle.	 M1 A1
(iii)	$y+2x=8$ $x^2+y^2-4x-8y=25$ $x^2+(8-2x)^2-4x-8(8-2x)=25$ $x^2+64-32x+4x^2-4x-64+16x-25=0$ $5x^2-20x-25=0$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=-1$ or $x=5$ $y=10$ $\quad y=-2$ Thus $C(-1,10)$ and $D(5,-2)$. Distance between C and D $=\sqrt{(5-(-1))^2+(-2-10)^2}$ $=\sqrt{6^2+(-12)^2}$ $=\sqrt{180}$ $=2\sqrt{45}$ Since $CD=2\sqrt{45}$, CD is the diameter of the circle (shown).	 M1 A1, A1 B1
		Total for Q7: 10

8(i)	$\sin^2\theta\cos^2\theta$	
	$=\left(\frac{1}{2}(2\sin\theta\cos\theta)\right)^2$	M1
	$=\frac{1}{4}(\sin 2\theta)^2$	
	$=\frac{1}{4}\left(\frac{1}{2}(1-\cos 4\theta)\right)$ since $\cos 4\theta = 1-2\sin^2 2\theta$	M1
	$=\frac{1}{8}(1-\cos 4\theta)$ (shown)	AG1
(ii)	(a) $\int_0^{\frac{\pi}{3}} \sin^2\theta\cos^2\theta \, d\theta = \int_0^{\frac{\pi}{3}} \frac{1}{8}(1-\cos 4\theta)\, d\theta$	
	$=\frac{1}{8}\int_0^{\frac{\pi}{3}} (1-\cos 4\theta)\, d\theta$	
	$=\frac{1}{8}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{3}}$	M1
	$=\frac{1}{8}\left[\frac{\pi}{3} - \frac{\sin\left(\frac{4\pi}{3}\right)}{4}\right] - 0$	M1
	$=\frac{1}{8}\left[\frac{\pi}{3} - \frac{1}{4}\left(-\frac{\sqrt{3}}{2}\right)\right]$	
	$=\frac{1}{8}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$ (shown)	AG1
	(b) $\sin^2\theta\cos^2\theta = \frac{1}{10}$	
	$\frac{1}{8}(1-\cos 4\theta) = \frac{1}{10}$	
	$1-\cos 4\theta = \frac{8}{10}$	
	$\cos 4\theta = \frac{1}{5}$	M1
	Basic angle $= 78.463^\circ$	M1
	$0^\circ \le \theta \le 180^\circ \Rightarrow 0^\circ \le 4\theta \le 720^\circ$	
	$4\theta = 78.463^\circ, 360^\circ - 78.463^\circ,$	
	$78.463^\circ + 360^\circ, 360^\circ - 78.463^\circ + 360^\circ,$	A2
	$\theta = 19.6^\circ, 70.4^\circ, 109.6^\circ, 160.4^\circ$ (1 d.p.)	
		Total for Q8: 10

9(i)	$v = pt^2 + qt + 24$ $a = \frac{dv}{dt} = 2pt + q$ $t = 1, a = -4: \quad 2p + q = -4 \text{ ---(1)}$ $t = 4, v = 0: \quad 16p + 4q = -24 \text{ ---(2)}$ $(2) \div 4: \quad 4p + q = -6 \text{ ---(3)}$ $(3) - (1): \quad 2p = -2 \Rightarrow p = -1, q = -2$	 M1 M1 A2
(ii)	$v = -t^2 - 2t + 24$ $s = \int v\,dt$ $= \int -t^2 - 2t + 24\,dt$ $= -\frac{t^3}{3} - t^2 + 24t + c$ When $t = 0, s = 0, c = 0$ $s = -\frac{t^3}{3} - t^2 + 24t$ Distance $AB = -\frac{(4)^3}{3} - (4)^2 + 24(4) = 58\frac{2}{3}$ m	 M1 M1 A1
		Total for Q9: 7

10(i)	$f(x)=\dfrac{3x+1}{(x+2)(x-3)}=\dfrac{A}{x+2}+\dfrac{B}{x-3}$ $3x+1=A(x-3)+B(x+2)$ Let $x=-2$: $-5=-5A\Rightarrow A=1$ Let $x=3$: $10=5B\Rightarrow B=2$ $\dfrac{3x+1}{(x+2)(x-3)}=\dfrac{1}{x+2}+\dfrac{2}{x-3}$	 **B1** **B1**
(ii)	$f'(x)=\dfrac{d}{dx}\left(\dfrac{1}{x+2}+\dfrac{2}{x-3}\right)$ $=-\dfrac{1}{(x+2)^2}-\dfrac{2}{(x-3)^2}$ Since $(x+2)^2>0$ and $(x-3)^2>0$, $-\dfrac{1}{(x+2)^2}<0$ and $-\dfrac{2}{(x-3)^2}<0$ Thus $f'(x)<0$ and f is a decreasing curve.	 **B1** **M1** **A1**
(iii)	$\int_4^6 \dfrac{3x+1}{(x+2)(x-3)}\,dx$ $=\int_4^6 \dfrac{1}{x+2}+\dfrac{2}{x-3}\,dx$ $=\left[\ln(x+2)+2\ln(x-3)\right]_4^6$ $=(\ln(6+2)+2\ln(6-3))-(\ln 6+2\ln 1)$ $=\ln 8+\ln 9-\ln 6$ $=\ln 12$	 **M1** **M1** **A1**
		Total for Q10: 8

11(i)		$\frac{d}{d\theta}(\sec\theta)$ $=\frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right)$ $=\frac{d}{d\theta}(\cos\theta)^{-1}$ $=-(\cos\theta)^{-2}(-\sin\theta)$ $=\frac{\sin\theta}{\cos^2\theta}$ (shown)	M1 AG1
(ii)	(a)	$\cos\theta=\frac{2}{AX}\Rightarrow AX=\frac{2}{\cos\theta}$ Time taken to travel along $AX=\frac{AX}{10}=\frac{2}{\cos\theta}\times\frac{1}{10}=\frac{\sec\theta}{5}$ $\tan\theta=\frac{PX}{2}\Rightarrow PX=2\tan\theta$ $XQ=10-2\tan\theta$ Time taken to travel along XQ $=\frac{XQ}{60}=\frac{10-2\tan\theta}{60}=\frac{1}{6}-\frac{\tan\theta}{30}$ Thus total time taken T $=\frac{\sec\theta}{5}+\frac{1}{6}-\frac{\tan\theta}{30}$	M1 M1 M1 AG1
	(b)	$T=\frac{\sec\theta}{5}+\frac{1}{6}-\frac{\tan\theta}{30}$ $\frac{dT}{d\theta}=\frac{1}{5}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{1}{30}\sec^2\theta$ For stationary T, $\frac{dT}{d\theta}=0$ $\frac{1}{5}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{1}{30}\sec^2\theta=0$ $6\sin\theta(\sec^2\theta)-\sec^2\theta=0$ $\sec^2\theta[6\sin\theta-1]=0$ $\sec^2\theta=0$ or $\sin\theta=\frac{1}{6}$ $\cos\theta=0$ or basic angle $\theta=9.5941°$ $\theta=\frac{\pi}{2}$ (rejected) or $\theta=9.6°$ OR $\frac{1}{5}\left(\frac{\sin\theta}{\cos^2\theta}\right)-\frac{1}{30}\sec^2\theta=0$ $\frac{6\sin\theta-1}{30\cos^2\theta}=0$ $\sin\theta=\frac{1}{6}$ basic angle $\theta=9.5941°$ $\theta=9.6°$ Thus $PX=2\tan\theta=2\tan 9.5941°=0.33806\text{km}$	M1 M1 M1 M1, A1
			Total for Q11: 11