NAME:			CLASS:

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2017
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

DATE :	14 AUGUST 2017	DAY :	MONDAY
DURATION:	2 h	MARKS:	80

ADDITIONAL MATERIALS

Writing Paper x 6
Mathematics Cover Sheet x 1

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of **5** printed pages and 1 blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{4}{(x^2+4)(x-2)}$ in partial fractions. [5]

2 Solve the equations

(a) $4\log_4 x = 4 + \log_2(x+5)$, [5]

(b) $2(3^{2y+1}) = 6^{y-1}$. [3]

3 The equation of a curve is $y = px^2 - 4x + p$, where p is a constant.

Find the set of values of p for which the curve lies completely above the line $y = 3$. [4]

4 A rectangular block has a height of 3 unit with a square base of area $\left(\dfrac{x}{\sqrt{3}} + \dfrac{\sqrt{12}}{3}\right)$ square units. Given that the volume of the rectangular block is $x\sqrt{45}$ cubic units, without using a calculator, express the value of x in the form $\dfrac{a+\sqrt{b}}{7}$, where a and b are integers. [6]

5 Given that $4x^3 + 16x^2 + 13x + 3 = (x-k)\mathrm{f}(x)$ where f(x) is a polynomial and k is an integer.

(i) Find the value of k. [2]

(ii) Find f(x) and show that the x-axis is a tangent to the graph $y = \mathrm{f}(x)$. [4]

(iii) Deduce that there is no real solution for the equation $4x^6 + 16x^4 + 13x^2 + 3 = 0$. [2]

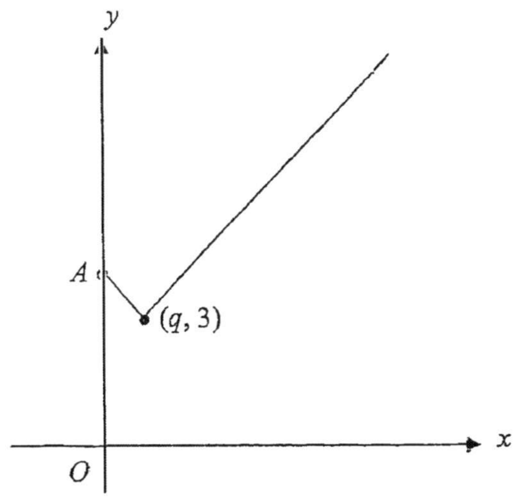
6 (i) Prove that $(\operatorname{cosec}\theta - \cot\theta)(1+\sec\theta) = \tan\theta$. [4]

(ii) Find, in radians, the acute angle θ for which

$$(\operatorname{cosec}\theta - \cot\theta)(1+\sec\theta) = \frac{1}{2}\sec^2\theta.$$ [3]

7 Two obtuse angles A and B are such that tan $(2A + B) = 4$ and $\sin B = \dfrac{1}{\sqrt{5}}$. Without using a calculator, explain why $135° < A < 180°$. [5]

8 The diagram shows the graph of $y = |1-2x| + p$ for $0 \le x \le 4$.



(i) Find the values of p and of q. [3]

(ii) Find the coordinates of A. [1]

(iii) Find the coordinates of the point(s) of intersection of the line $y = 3x$ and the graph of $y = |1-2x| + p$ for $0 \le x \le 4$. [4]

(iv) Determine the set of values of c for which the line $y = c$ intersects the graph of $y = |1-2x| + p$ at exactly one point for $0 \le x \le 4$. [2]

9 In the binomial expansion of $\left(x - \frac{k}{x^2}\right)^7$ in descending powers of x, k is a positive constant.

(i) Write down in terms of k, the first three terms of the expansion. [2]

(ii) Explain why there is no term which is independent of x in this expansion. [3]

(iii) The coefficient of x^4 in the expansion of $(2 + x^3)\left(x - \frac{k}{x^2}\right)^7$ is 7, find the value of k. [3]

10 The equation of a curve is $y = 2\sqrt{e^x} + \frac{4}{\sqrt{e^x}} + x$.

(i) Find the coordinates of the stationary point of the curve. [5]

(ii) Determine the nature of this stationary point. [3]

11 A triangle ABC is such that point A is (6, 6) and the point C is above point A and lies on the y-axis. Angle $ABC = 90°$ and $AB = BC = \sqrt{20}$ units. The equation of AB is $y + 2x = 18$.

(i) Find the coordinates of C and hence find the equation of BC. [5]

(ii) State the coordinates of M, the midpoint of AC. [1]

(iii) Show that the coordinates of B is (4, 10). [2]

(iv) Calculate the area of quadrilateral $OMBC$. [3]

End of Paper

Paper 1 Yishun Town 2017 SA2

2a) $4\log_4 x = 4 + \log_2(x+5)$

$4\left[\frac{\log_2 x}{2}\right] = 4\log_2 2 + \log_2(x+5)$

$\log_2 x^2 = \log_2 2^4(x+5)$

$x^2 = 16(x+5)$

$x^2 - 16x - 80 = 0$

$(x+4)(x-20) = 0$

$x = -4$ or $x = 20$
(NA)

2b) $2(3^{2y+1}) = 6^{y-1}$

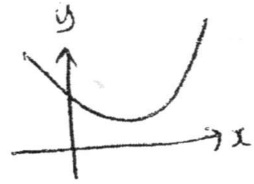
$2(3^{2y})(3) = (6^y)\left(\frac{1}{6}\right)$

$\frac{9^y}{6^y} = \frac{1}{36}$

$y\lg\left(\frac{9}{6}\right) = \lg\left(\frac{1}{36}\right)$

$y \approx -8.84$

3) $y = px^2 - 4x + p$
$y = 3$



$px^2 - 4x + p > 3$

$px^2 - 4x + p - 3 > 0$

$b^2 - 4ac < 0$

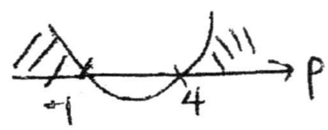
$(-4)^2 - 4(p)(p-3) < 0$

$16 - 4p^2 + 12p < 0$

$-4p^2 + 12p + 16 < 0$

$p^2 - 3p - 4 > 0$

$(p-4)(p+1) > 0$



$p < -1$ or $p > 4$
(NA)

4) Volume $= \left(\frac{x}{\sqrt{3}} + \frac{\sqrt{12}}{3}\right) \times 3$

$x\sqrt{45} = \frac{3x}{\sqrt{3}} + \sqrt{12}$

$3x\sqrt{5} - \frac{3x}{\sqrt{3}} = 2\sqrt{3}$

$x\left[3\sqrt{5} - \sqrt{3}\right] = 2\sqrt{3}$

$x = \frac{2\sqrt{3}}{(3\sqrt{5} - \sqrt{3})} \frac{(3\sqrt{5} + \sqrt{3})}{(3\sqrt{5} + \sqrt{3})}$

$= \frac{6\sqrt{15} + 6}{45 - 3}$

$= \frac{6\sqrt{15} + 6}{42}$

$= \frac{1}{7}\sqrt{15} + \frac{1}{7}$

$\therefore a = 1,\ b = 15$ #

5 i) $4x^3 + 16x^2 + 13x + 3 = (x-k)f(x)$

By observation,
$x = -3$
$x = -\frac{1}{2}$

$\therefore$ Since k is an integer,
$k = -3$

ii) $4x^3 + 16x^2 + 13x + 3 = (x+3)f(x)$

By synthetic division,

	4	16	13	3
$x=-3$		-12	-12	-3
	4	4	1	0

$f(x) = 4x^2 + 4x + 1$
At x-axis, $f(x) = 0$

$4x^2 + 4x + 1 = 0$
$b^2 - 4ac$

Since $b^2 - 4ac = 0$,
x-axis is tangent to curve

5iii) $4x^3+16x^2+13x+3 = (x+3)f(x)$

$= (x+3)(4x^2+4x+1)$

$= (x+3)(2x+1)^2$

$4x^6+16x^4+13x^2+3=0$

$4(x^2)^3+16(x^2)^2+13x^2+3=0$

Replace x by x^2,

$x^2+3=0$ $\quad$ $2(x^2)+1=0$

$x^2=-3$ $\quad$ $x^2=-1/2$

$\therefore$ There is no real soln.

6i) $(\operatorname{cosec}\theta-\cot\theta)(1+\sec\theta)$

$= \left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)\left(1+\frac{1}{\cos\theta}\right)$

$= \frac{(1-\cos\theta)(\cos\theta+1)}{(\sin\theta)(\cos\theta)}$

$= \frac{1-\cos^2\theta}{(\sin\theta)(\cos\theta)}$

$= \frac{\sin^2\theta}{(\sin\theta)(\cos\theta)}$

$= \tan\theta$ (shown)

6ii) $(\operatorname{cosec}\theta-\cot\theta)(1+\sec\theta) = \frac{1}{2}\sec^2\theta$

$\tan\theta = \frac{1}{2}\sec^2\theta$

$\tan\theta=\frac{1}{2}(1+\tan^2\theta)$

Let $y=\tan\theta$,

$y=\frac{1}{2}(1+y^2)$

$2y=1+y^2$

$y^2-2y+1=0$

$(y-1)^2=0$

$y=1$

$\tan^2\theta=1$

$\tan\theta=\pm1$

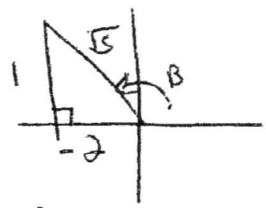
$\alpha=\tan^{-1}(1)$

$=\frac{\pi}{4}$

$\theta=\frac{\pi}{4}$ #

7) $\tan(2A+B)=4$

$\frac{\tan 2A+\tan B}{1-\tan 2A\tan B}=4$



$\frac{\tan 2A+\left(-\frac{1}{2}\right)}{1+\frac{\tan 2A}{2}}=4$

$\tan 2A-\frac{1}{2}=4+2\tan 2A$

$-4\frac{1}{2}=\tan 2A$

$270\leq 2A\leq 360$ since.

$135°\leq A\leq 180$ # $\quad 0\leq B\leq 90$

8i) By observation,

$p=3$

$|1-2x|=0$

$x=1/2$

$q=1/2$

ii) $y=|1-2x|+3$

At A, $x=0$,

$y=|1-2(0)|+3$

$y=4$ $\quad$ A(0,4) #

iii) $y=|1-2x|+3$ —①

$y=3x$ —②

① = ②

$|1-2x|+3=3x$

$|1-2x|=3x-3$

$1-2x=3x-3$ OR $1-2x=3-3x$

$4=5x$ $\quad$ $-2=-x$

$x=4/5$ (NA). $\quad$ $x=2$

$y=6$ (2,6)

iv) $c=3$ OR $c>4$

10)

$y = 2\sqrt{e^x} + \frac{4}{\sqrt{e^x}} + x$

$y = 2e^{\frac{x}{2}} + \frac{4}{e^{x/2}} + x$

$y = 2e^{\frac{x}{2}} + 4e^{-\frac{x}{2}} + x$

$\frac{dy}{dx} = e^{\frac{x}{2}} - 2e^{-x/2} + 1$

At stationary point, $\frac{dy}{dx} = 0$

$e^{\frac{x}{2}} - 2e^{-\frac{x}{2}} + 1 = 0$

Let $y = e^{\frac{x}{2}}$,

$y - \frac{2}{y} + 1 = 0$

$y^2 - 2 + y = 0$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2 \quad y = 1$

$e^{\frac{x}{2}} = -2 \quad e^{\frac{x}{2}} = 1$

(NA). $\frac{x}{2} = 0$

$x = 0$

$y = 2\sqrt{e^0} + \frac{4}{\sqrt{e^0}} + 0$

$= 2 + 4$

$y = 6$

$(0,6)$

ii) $\frac{d^2y}{dx^2} = \frac{1}{2}e^{\frac{x}{2}} + e^{-\frac{x}{2}}$

At $x = 0$,

$\frac{d^2y}{dx^2} > 0$ (Minimum)

11) Point C $(0,y)$; $A(6,6)$. $B, (x,y)$

$AB = BC$

$\sqrt{(6-x)^2 + (6-y)^2} = \sqrt{20}$

$(6-x)^2 + (6-y)^2 = 20$ —①

$y + 2x = 18$

$y = 18 - 2x$ —②

Sub ② into ①,

$(6-x)^2 + (6-18+2x)^2 = 20$

$(6-x)^2 + (2x-12)^2 = 20$

$36 - 12x + x^2 + 4x^2 - 48x + 144 = 20$

$5x^2 - 60x + 160 = 0$

$(x-8)(x-4) = 0$

$x = 8$ OR $x = 4$

$y = 2 \qquad y = 10$

$(8,2) \qquad (4,10)$

(NA)

$\therefore$ $B(4,10)$

$C(0,10)$

$A(6,6)$

$m_{BC} = \frac{10-10}{4-0} = 0$.

$l_{BC}: y = 10$ #

ii) $M: \left(\frac{6+0}{2}, \frac{10+6}{2}\right)$

$= (3,8)$

iii) $B(4,10)$ #.

iv) Area of quadrilateral

$= \frac{1}{2}\left| \begin{matrix} 0 & 3 & 4 & 0 & 0 \\ 0 & 8 & 10 & 10 & 0 \end{matrix} \right|$

$= \frac{1}{2}\left| (30+40) - (32) \right| = \frac{1}{2}|38| = 19 \text{ units}^2$ #.

9) $\left(x - \frac{k}{x^2}\right)^7 = \binom{7}{0}(x)^7 + \binom{7}{1}(x)^6\left(\frac{-k}{x^2}\right) + \binom{7}{2}(x)^5\left(\frac{-k}{x^2}\right)^2 + \ldots$

$= x^7 + -7kx^4 + 21k^2x + \ldots$

ii) $T_{r+1} = \binom{7}{r}(x)^{7-r}\left(\frac{-k}{x^2}\right)^r$

$= \binom{7}{r}(-k)^r\left(\frac{x^{7-r}}{x^{2r}}\right)$

$= \binom{7}{r}(-k)^r\left(x^{7-3r}\right)$

$x^{7-3r} = x^0$

$r = 7/3$

Since $r = 7/3 \neq$ integer, there is no term which is independent of x.

iii) $(2+x^3)\left(x - \frac{k}{x^2}\right)^7 = \ldots 7x^4 + \ldots$

$T_{r+1} = \binom{7}{r}(x)^{7-r}\left(\frac{-k}{x^2}\right)^r$

$= \binom{7}{r}(-k)^r\left(\frac{x^{7-r}}{x^{2r}}\right)$

$T_{r+1} = \binom{7}{r}(-k)^r\left(x^{7-3r}\right)$

$x^{7-3r} = x^0$

$7 - 3r = 0$

$r = 7/3$ (NA)

$x^{7-3r} = x^1$

$7 - 3r = 1$

$3r = 6$

$r = 2$

when $r = 2$,

$T_3 = \binom{7}{2}(-k)^2x^1$

$T_3 = (21k^2)x$

$(2+x^3)(\ldots 21k^2x + \ldots)$

$= 21k^2x^4$

By comparing coefficient of x^4:

$7 = 21k^2$

$\frac{1}{3} = k^2 \Rightarrow k = \pm\sqrt{\frac{1}{3}}$

$\Rightarrow k = \sqrt{\tfrac{1}{3}}$ $\quad k = -\sqrt{\tfrac{1}{3}}$ (NA).