Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2018

SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **17 September 2018**

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

80

This document consists of **6** printed pages.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = ax - 2$, where a is a constant. The curve has a minimum gradient at $x = \frac{1}{3}$.

(i) Show that $a = 6$. [1]

The tangent to the curve at $(1,\ 4)$ is $y = 2x + 2$.

(ii) Find the equation of the curve. [6]

2 The roots of the quadratic equation $3x^2 + 2x + 4 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = -\frac{20}{9}$. [3]

(ii) Find a quadratic equation with roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [4]

3 It is given that $\mathrm{f}(x) = (x+h)^2(x-1) + k$, where h and k are constants and $h < k$. When $\mathrm{f}(x)$ is divided by $x + h$, the remainder is 6. It is given that $\mathrm{f}(x)$ is exactly divisible by $x + 5$.

(i) State the value of k and show that $h = 4$. [4]

(ii) Find the range of values of the constant b for which the graph of $y = \mathrm{f}(x) + bx$ is an increasing function for all values of x. [4]

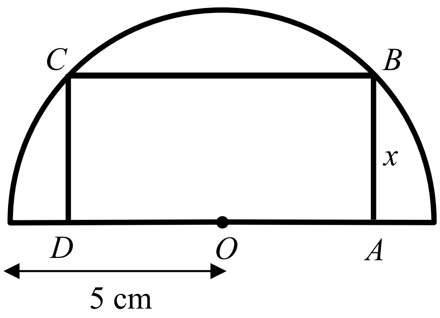
4 Given that $\tan(x+y) = -\frac{120}{119}$ and $\cos x = \frac{5}{13}$, where x and y are acute angles, show that $x = y$ **without** finding the values of x and y. [4]

5 The variables x and y are such that when $\frac{x}{y}$ are plotted against x, a straight line l_1 of gradient 2 is obtained. It is given that $y = \frac{1}{5}$ when $x = 3$.

(i) Express y in terms of x. [3]

(ii) When the graph of $x = 2y$ is plotted on the same axes as the line l_1, the two lines intersect at one point. Find the coordinates of the point of intersection. [2]

6 The figure shows a semicircle of radius 5 cm and centre, O. A rectangle $ABCD$ is inscribed in the semicircle such that the four vertices A, B, C and D touch the edge of the semicircle. The length of $AB = x$ cm.



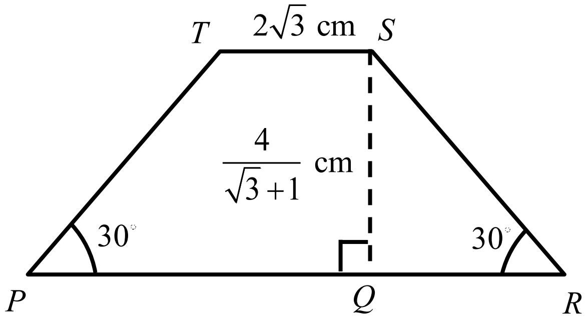
(i) Show that the perimeter, P cm, of rectangle $ABCD$ is given by

$$P = 2x + 4\sqrt{25 - x^2}$$ [2]

(ii) Given that x can vary, find the value of x when the perimeter is stationary. [4]

7 In the diagram below, $PQRST$ is a trapezium where angle QRS = angle TPR = 30°. SQ is the height of the trapezium and the length of SQ is $\frac{4}{\sqrt{3}+1}$ cm. The length of TS is $2\sqrt{3}$ cm.

By rationalising $\frac{4}{\sqrt{3}+1}$, find the area of trapezium $PQRST$ in the form $\left(a\sqrt{3} - 12\right)$ cm^2, where a is an integer. [5]



8 A particle moving in a straight line passes a fixed point A with a velocity of -8 cms^{-1}. The acceleration, a cms^{-2} of the particle, t seconds after passing A is given by $a = 10 - kt$, where k is a constant. The particle first comes to instantaneous rest at $t = 1$ and reaches maximum speed at T seconds (The particle does not come instantaneously to rest at $1 < t < T$).

(i) Find the value of k. [3]

(ii) Find the total distance travelled by the particle when $t = T$. [5]

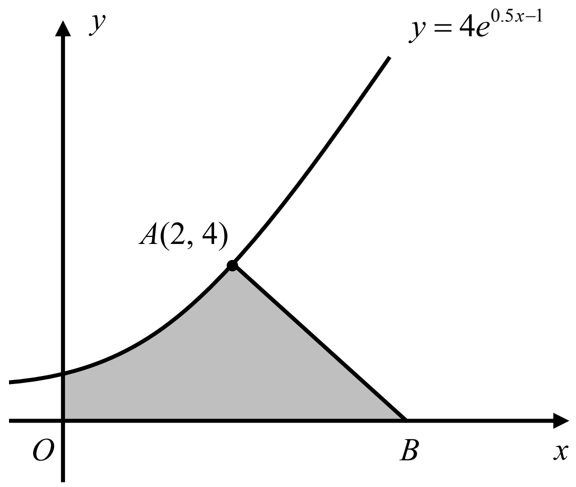
9 It is given that $y = 1 - 3\sin 2x$ for $-\dfrac{\pi}{2} \le x \le \pi$.

(i) State the period of y. [1]

(ii) Sketch the graph of $y = 1 - 3\sin 2x$. [3]

(iii) By drawing a straight line on the same diagram as in part **(ii)**, find the number of solutions to the equation $3\sin 2x + 1\dfrac{1}{2} = \dfrac{3x}{\pi}$ for $-\dfrac{\pi}{2} \le x \le \pi$. [3]

10



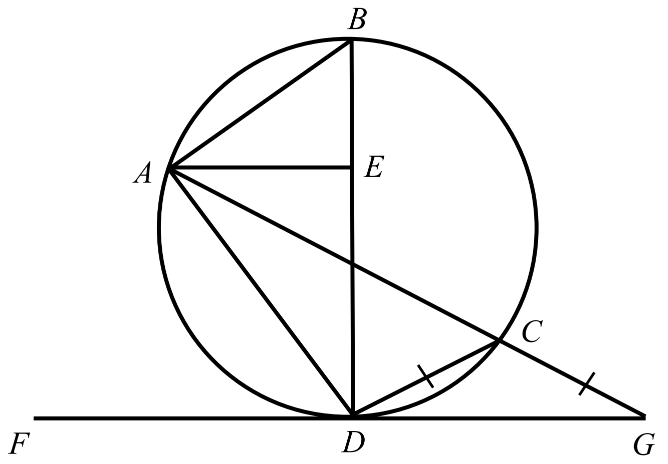
The diagram shows part of the curve $y = 4e^{0.5x-1}$. The normal to the curve at point $A(2, 4)$ cuts the x-axis at point B.

Find

(i) the coordinates of B, [4]

(ii) the area of the shaded region. [3]

11

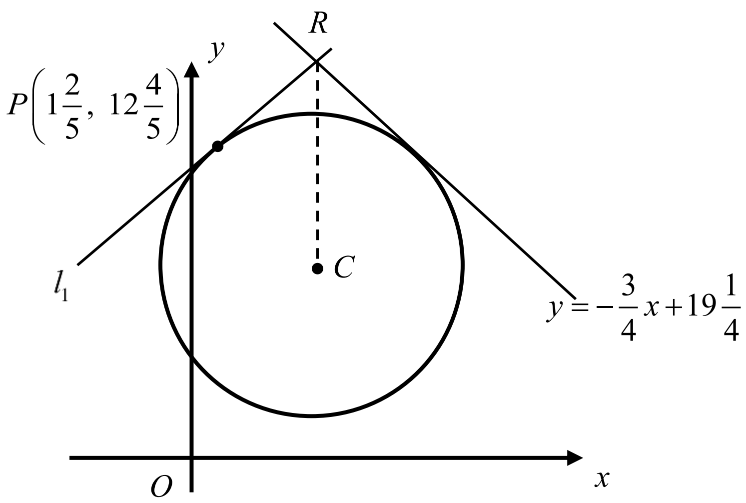


In the diagram, BD and AC are chords of the circle. FD is a tangent to the circle at D. AC and FD are produced to meet at G such that $CG = CD$. E is a point along BD. Triangle BAE is similar to triangle ADE.

(i) By showing that triangle BAD and triangle AED are similar, prove that AB is perpendicular to AD. [4]

(ii) Show that angle $ADB = 90° - 2 \times$ (angle CGD) . [4]

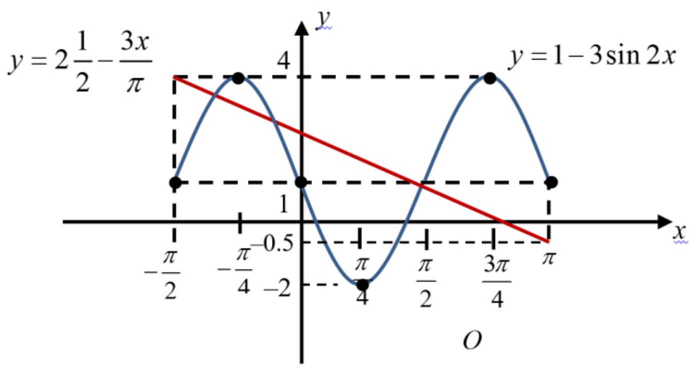
12 The line $y = -\frac{3}{4}x + 19\frac{1}{4}$ is a tangent to the circle, centre C. Another line, l_1 is tangent to the circle at point $P\left(1\frac{2}{5},\ 12\frac{4}{5}\right)$. The two tangents intersect at point R, which is directly above the centre of the circle.



(i) Show that the coordinates of R are $\left(5,\ 15\frac{1}{2}\right)$. [4]

(ii) Find the equation of the circle. [4]

Answer Key

1	**(i)**	Show question
	(ii)	$y = x^3 - x^2 + x + 3$
2	**(i)**	Show question
	(ii)	$x^2 - \frac{16}{9}x + \frac{4}{3} = 0$ or any other equivalent equation
3	**(i)**	$k = 6$; $h = 4$ (show question)
	(ii)	$b > 8\frac{1}{3}$
4		Show question
5	**(i)**	$y = \frac{x}{2x+9}$
	(ii)	$\left(-3\frac{1}{2}, 2\right)$
6	**(i)**	Show question
	(ii)	$x = \sqrt{5}$ or 2.24 (3 s.f.)
7		$\left(12\sqrt{3} - 12\right)$ cm^2
8	**(i)**	$k = 4$
	(ii)	$8\frac{1}{6}$ m
9	**(i)**	π
	(ii)	
	(iii)	3 solutions
10	**(i)**	$B(10, 0)$
	(ii)	$\left(24 - \frac{8}{e}\right)$ units^2 or 21.1 units^2 (3 s.f)
11	**(i), (ii)**	Show question
12	**(ii)**	$(x-5)^2 + (y-8)^2 = 36$

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School
Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School Chung Cheng High School

PRELIMINARY EXAMINATION 2018

SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **17 September 2018**

2 hours

Additional Materials: Answer Paper

MARK SCHEME

This document consists of **6** printed pages.

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = ax - 2$, where a is a constant. The curve has a minimum gradient at $x = \frac{1}{3}$.

(i) Show that $a = 6$. [1]

The tangent to the curve at $(1,\ 4)$ is $y = 2x + 2$.

(ii) Find the equation of the curve. [6]

Marking Scheme

(i) At minimum gradient, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$

$$a\left(\frac{1}{3}\right) - 2 = 0$$

$$\frac{a}{3} = 2$$

$$a = 6$$

(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = \int (6x - 2)\ \mathrm{d}x$

$= 3x^2 - 2x + c$ where c is an arbitrary constant

$y = 2x + 2$

Gradient of tangent $= 2$

$$3(1)^2 - 2(1) + c = 2$$

$$c = 1$$

$y = \int (3x^2 - 2x + 1)\ \mathrm{d}x$

$= x^3 - x^2 + x + c_1$ where c_1 is an arbitrary constant

Sub. (1, 4)

$$4 = 1^3 - 1^2 + 1 + c_1$$

$$c_1 = 3$$

Equation of curve is $y = x^3 - x^2 + x + 3$

2. The roots of the quadratic equation $3x^2+2x+4=0$ are α and β.

(i) Show that $\alpha^2+\beta^2=-\frac{20}{9}$. [3]

(ii) Find a quadratic equation with roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [4]

<u>Marking Scheme</u>

(i) $\alpha+\beta=-\frac{2}{3}$

$\alpha\beta=\frac{4}{3}$

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$\alpha^2\beta^2=\left(-\frac{2}{3}\right)^2-2\left(\frac{4}{3}\right)$$

$$=\frac{4}{9}-\frac{8}{3}$$

$$=-\frac{20}{9}\text{ (shown)}$$

(ii) Sum of roots $=\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

$$=\frac{\alpha^3+\beta^3}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$$

$$=\frac{\left(-\frac{2}{3}\right)\left(-\frac{20}{9}-\frac{4}{3}\right)}{-\frac{4}{3}}$$

$$=\frac{16}{9}$$

Product of roots $=\left(\frac{\alpha^2}{\beta}\right)\left(\frac{\beta^2}{\alpha}\right)$

$$=\alpha\beta$$

$$=\frac{4}{3}$$

The quadratic equation is $x^2-\frac{16}{9}x+\frac{4}{3}=0$

OR $9x^2-16x+12=0$

3. It is given that $\mathrm{f}(x)=(x+h)^2(x-1)+k$, where h and k are constants and $h<k$. When $\mathrm{f}(x)$ is divided by $x+h$, the remainder is 6. It is given that $\mathrm{f}(x)$ is exactly divisible by $x+5$.

(i) State the value of k and show that $h=4$. [4]

(ii) Find the range of values of the constant b for which the graph of $y=\mathrm{f}(x)+bx$ is an increasing function for all values of x. [4]

Marking Scheme

(i) $k=6$ $\boxed{\text{B1}}$

$\mathrm{f}(-5)=0$

$(-5+h)^2(-5-1)+6=0$

$(h-5)^2(-6)=-6$

$(h-5)^2=1$

$h-5=-1$ or 1

$h=4$ or 6 (rejected as $h<k$)

(ii) $y=(x+4)^2(x-1)+6+bx$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=2(x+4)(x-1)+(x+4)^2+b$$
$$=(x+4)\left[2(x-1)+(x+4)\right]+b$$
$$=(x+4)(3x+2)+b$$

For increasing function, $\frac{\mathrm{d}y}{\mathrm{d}x}>0$

$(x+4)(3x+2)+b>0$

$3x^2+14x+8+b>0$

Discriminant <0

$(14)^2-4(3)(8+b)<0$

$196-96-12b<0$

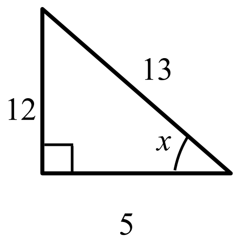
$12b>100$

$b>8\frac{1}{3}$

4. Given that $\tan(x+y)=-\frac{120}{119}$ and $\cos x=\frac{5}{13}$, where x and y are acute angles, show that $x=y$ **without** finding the values of x and y. [4]

<u>Marking Scheme</u>

$$\tan(x+y)=-\frac{120}{119}$$
$$\frac{\tan x+\tan y}{1-\tan x\tan y}=-\frac{120}{119}$$



$$\tan x=\frac{12}{5}$$

$$\frac{\frac{12}{5}+\tan y}{1-\frac{12}{5}\tan y}=-\frac{120}{119}$$
$$\frac{12}{5}+\tan y=-\frac{120}{119}+\frac{288}{119}\tan y$$
$$\frac{2028}{595}=\frac{169}{119}\tan y$$

$$\tan y=\frac{12}{5}$$

Since $\tan x=\tan y$ and x and y are both acute, $x=y$.

5. The variables x and y are such that when $\frac{x}{y}$ are plotted against x, a straight line l_1 of gradient 2 is obtained. It is given that $y = \frac{1}{5}$ when $x = 3$.

(i) Express y in terms of x. [3]

(ii) When the graph of $x = 2y$ is plotted on the same axes as the line l_1, the two lines intersect at one point. Find the coordinates of the point of intersection. [2]

<u>Marking Scheme</u>

(ii) $\frac{x}{y} = 2x + c$

$\frac{3}{\frac{1}{5}} = 2(3) + c$

$c = 9$

$\frac{x}{y} = 2x + 9$

$\frac{y}{x} = \frac{1}{2x+9}$

$y = \frac{x}{2x+9}$

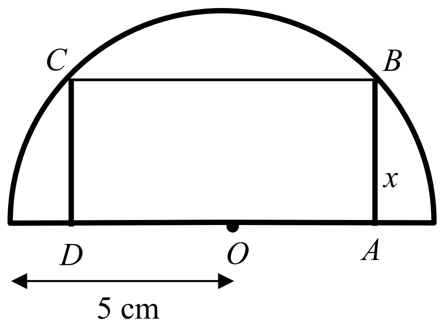
(iii) $x = 2y \Rightarrow \frac{x}{y} = 2$

$2x + 9 = 2$

$x = -3\frac{1}{2}$

The point of intersection is $\left(-3\frac{1}{2},\ 2\right)$.

6. The figure shows a semicircle of radius 5 cm and centre, O. A rectangle $ABCD$ is inscribed in the semicircle such that the four vertices A, B, C and D touch the edge of the semicircle. The length of $AB = x$ cm.



(i) Show that the perimeter, P cm, of rectangle $ABCD$ is given by

$$P = 2x + 4\sqrt{25 - x^2}$$ [2]

(ii) Given that x can vary, find the value of x when the perimeter is stationary. [4]

Marking Scheme

(i) $OB = 5$ cm (radius of circle)

$OB^2 = OA^2 + AB^2$

$25 = OA^2 + x^2$

$OA = \sqrt{25 - x^2}$

$P = AB + CD + AD + BC$

$= 2AB + 4OA$

$= 2x + 4\sqrt{25 - x^2}$ (shown)

(ii) $P = 2x + 4\sqrt{25 - x^2}$

$$\frac{dP}{dx} = 2 + 4\left(\frac{1}{2}\right)\left(25 - x^2\right)^{-\frac{1}{2}}\left(-2x\right)$$

$$= 2 - \frac{4x}{\sqrt{25 - x^2}}$$

At stationary P, $\frac{dP}{dx} = 0$

$$2 - \frac{4x}{\sqrt{25 - x^2}} = 0$$

$$\frac{4x}{\sqrt{25 - x^2}} = 2$$

$$\frac{16x^2}{25 - x^2} = 4$$

$4x^2 = 25 - x^2$

$5x^2 = 25$

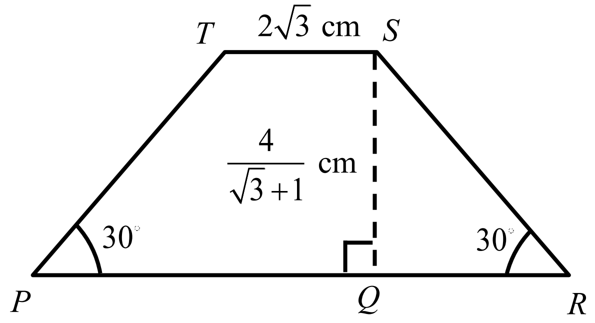
$x^2 = 5$

$x = \sqrt{5}$ or $-\sqrt{5}$ (rejected)

or 2.24 (3 s.f)

7. In the diagram below, *PQRST* is a trapezium where angle *QRS* = angle *TPR* = 30°. *SQ* is the height of the trapezium and the length of *SQ* is $\frac{4}{\sqrt{3}+1}$ cm. The length of *TS* is $2\sqrt{3}$ cm.

By rationalising $\frac{4}{\sqrt{3}+1}$, find the area of trapezium *PQRST* in the form $\left(a\sqrt{3}-12\right)$ cm^2, where *a* is an integer. [5]



Marking Scheme

$$\frac{4}{\sqrt{3}+1} = \frac{4}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{4\sqrt{3}-4}{3-1}$$
$$= 2\sqrt{3}-2$$

$$\tan 30^\circ = \frac{2\sqrt{3}-2}{QR}$$
$$\frac{1}{\sqrt{3}} = \frac{2\sqrt{3}-2}{QR}$$
$$QR = 2(3) - 2\sqrt{3}$$
$$= \left(6-2\sqrt{3}\right) \text{ cm}$$

$$\text{Area of trapezium} = \frac{1}{2}\left[2\left(6-2\sqrt{3}\right)+2\left(2\sqrt{3}\right)\right]\left(2\sqrt{3}-2\right)$$
$$= \frac{1}{2}\left(12-4\sqrt{3}+4\sqrt{3}\right)\left(2\sqrt{3}-2\right)$$
$$= \frac{1}{2}(12)\left(2\sqrt{3}-2\right)$$
$$= 6\left(2\sqrt{3}-2\right)$$
$$= \left(12\sqrt{3}-12\right) \text{ cm}^2$$

8. A particle moving in a straight line passes a fixed point A with a velocity of $-8\ \text{cms}^{-1}$. The acceleration, $a\ \text{cms}^{-2}$ of the particle, t seconds after passing A is given by $a = 10 - kt$, where k is a constant. The particle first comes to instantaneous rest at $t = 1$ and reaches maximum speed at T seconds (The particle does not comes instantaneous to rest at $1 < t < T$).

(i) Find the value of k. [3]

(ii) Find the total distance travelled by the particle when $t = T$. [5]

Marking Scheme

(i) $a = 10 - kt$

$v = \int (10 - kt)\ \mathrm{d}t$

$= 10t - \dfrac{kt^2}{2} + c$ where c is an arbitary constant

When $t = 0$, $v = -8$

$-8 = c$

$\therefore v = 10t - \dfrac{kt^2}{2} - 8$

When $t = 1$, $v = 0$

$0 = 10 - \dfrac{k}{2} - 8$

$k = 4$

(ii) $a = 10 - 4t$

At maximum speed, $a = 0$

$10 - 4t = 0$

$t = 2\dfrac{1}{2}$

$s = \int (10t - 2t^2 - 8)\ \mathrm{d}t$

$= 5t^2 - \dfrac{2t^3}{3} - 8t + c_1$ where c_1 is an arbitary constant

When $t = 0$, $s = 0$, $c_1 = 0$

$\therefore s = 5t^2 - \dfrac{2t^3}{3} - 8t$

When $t = 0$, s = 0

When $t = 1,\ s = -\frac{11}{3}$

When $t = 2\frac{1}{2},\ s = \frac{5}{6}$

$$\text{Total distance travelled} = \left(\frac{11}{3}\right) \times 2 + \frac{5}{6}$$

$$= 8\frac{1}{6}\ \text{m}$$

9. It is given that $y = 1 - 3\sin 2x$ for $-\frac{\pi}{2} \le x \le \pi$.

(i) State the period of y. [1]

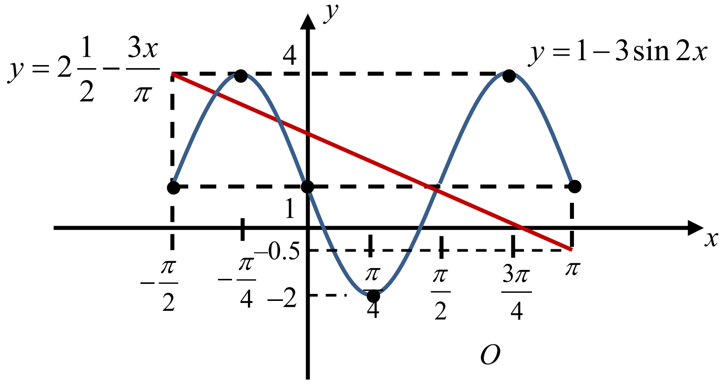
(ii) Sketch the graph of $y = 1 - 3\sin 2x$. [3]

(iii) By drawing a straight line on the same diagram as in part **(ii)**, find the number of solutions to the equation $3\sin 2x + 1\frac{1}{2} = \frac{3x}{\pi}$ for $-\frac{\pi}{2} \le x \le \pi$. [3]

Marking Scheme

(i) 180° or π

(ii)



$$3\sin 2x + 1\frac{1}{2} = \frac{3x}{\pi}$$

$$3\sin 2x = \frac{3x}{\pi} - 1\frac{1}{2}$$

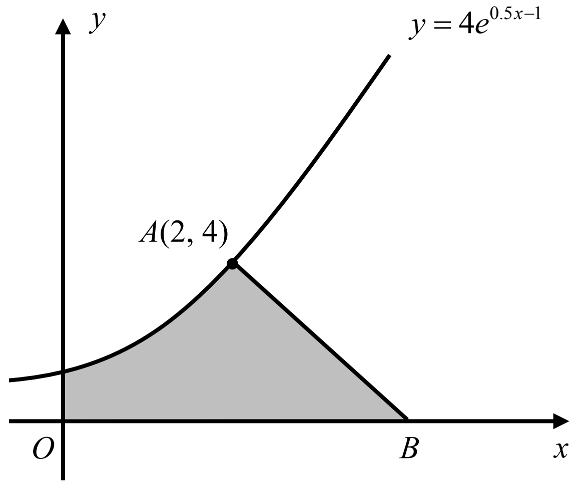
$$3\sin 2x - 1 = \frac{3x}{\pi} - 2\frac{1}{2}$$

$$1 - 3\sin 2x = 2\frac{1}{2} - \frac{3x}{\pi}$$

Draw the line of $y = 2\frac{1}{2} - \frac{3x}{\pi}$.

From the graph, there are 3 points of intersections, thus there are 3 solutions

10.



The diagram shows part of the curve $y = 4e^{0.5x-1}$. The normal to the curve at point $A(2, 4)$ cuts the x-axis at point B.

Find

(i) the coordinates of B, [4]

(ii) the area of the shaded region. [3]

<u>Marking Scheme</u>

(i) $y = 4e^{0.5x-1}$

$$\frac{dy}{dx} = 4(0.5)e^{0.5x-1}$$
$$= 2e^{0.5x-1}$$

When $x = 2$, $\frac{dy}{dx} = 2$

Gradient of normal $= -\frac{1}{2}$

Let $B(x, 0)$.

$$\frac{4-0}{2-x} = -\frac{1}{2}$$
$$8 = -2 + x$$
$$x = 10$$
$$\therefore B(10, 0)$$

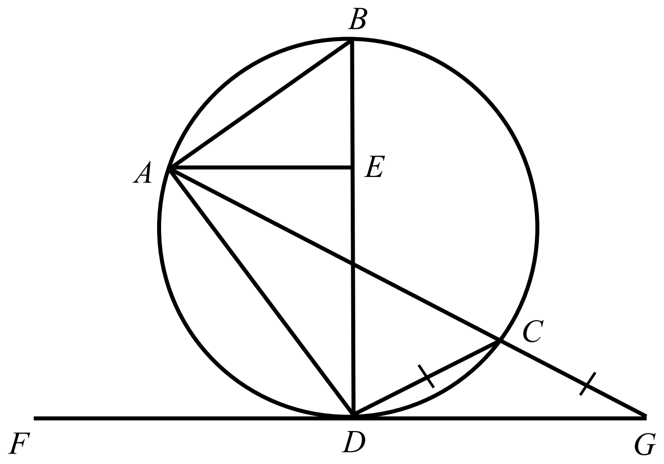
(ii) Area of shaded region $= \int_0^2 4e^{0.5x-1}\ \mathrm{d}x + \frac{1}{2}(10-2)(4)$

$$= \left[\frac{4e^{0.5x-1}}{0.5}\right]_0^2 + 16$$

$$= 8e^0 - 8e^{-1} + 16$$

$$= \left(24 - \frac{8}{e}\right) \text{ units}^2 \text{ or } 21.1 \text{ units}^2 \text{ (3 s.f)}$$

11.



In the diagram, *BD* and *AC* are chords of the circle. *FD* is a tangent to the circle at *D*. *AC* and *FD* are produced to meet at *G* such that *CG* = *CD*. *E* is a point along *BD*. Triangle *BAE* is similar to triangle *ADE*.

(i) By showing that triangle *BAD* and triangle *AED* are similar, prove that *AB* is perpendicular to *AD*. [4]

(ii) Show that angle *ADB* = 90° – 2 × (angle *CGD*) [4]

Marking Scheme

(i) $\angle ABE = \angle DAE$ (corresponding angles of similar triangles *BAE* and *ADE*)

$\angle ADE = \angle BDA$ (common angle)

By AA similarity rule, triangles *BAD* and *AED* are similar.

$\angle BEA = \angle AED$ (corresponding angles of similar triangles *BAE* and *ADE*)

$= 90^\circ$ (adjacent $\angle$s on straight line)

$\therefore \angle BAD = \angle AED$ (corresponding angles of similar triangles *BAD* and *AED*)

$= 90^\circ$

$AB \perp AD$ (shown)

(ii) Let $\angle CGD = a$.

$\angle CDG = \angle CGD$ (base $\angle$s of isosceles Δ)

$= a$

BD is a diameter (right-angle in a semicircle)

$\therefore \angle EDG = 90^\circ$ (tangent $\perp$ radius)

$\angle DAC = \angle CDG$ ($\angle$s in alternate segment)

$= a$

Consider ΔADG,

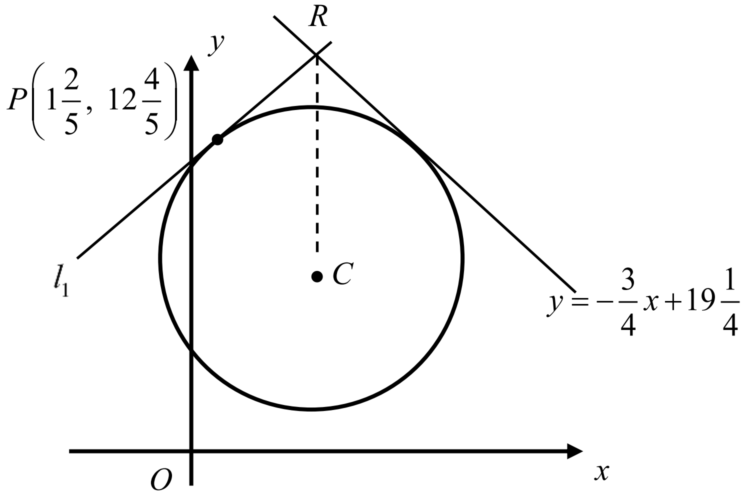
$\angle ADB = 180^\circ - \angle DAC - \angle CGD - \angle EDG$ (sum of $\angle$s in Δ)

$= 180^\circ - a - a - 90^\circ$

$= 90^\circ - 2a$

$= 90^\circ - 2 \times \angle CGD$ (shown)

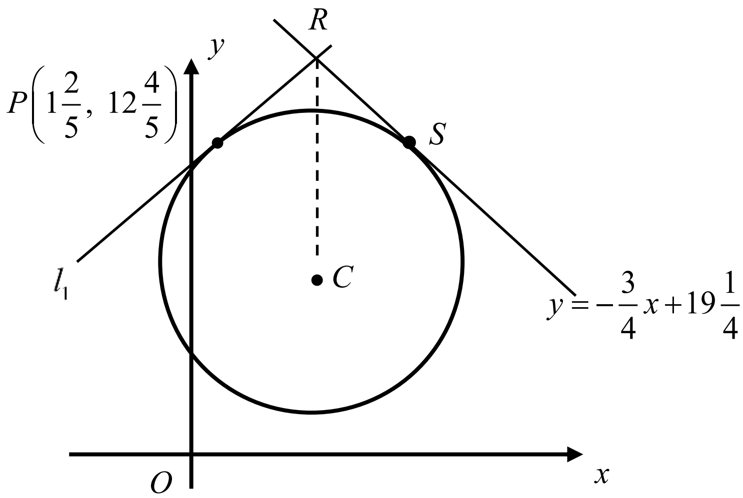
12. The line $y = -\frac{3}{4}x + 19\frac{1}{4}$ is a tangent to the circle, centre C. Another line, l_1 is tangent to the circle at point $P\left(1\frac{2}{5},\ 12\frac{4}{5}\right)$. The two tangents intersect at point R, which is directly above the centre of the circle.



(i) Show that the coordinates of R are $\left(5,\ 15\frac{1}{2}\right)$. [4]

(ii) Find the equation of the circle. [4]

Marking Scheme



y-coordinate of $S = 12\frac{4}{5}$

$12\frac{4}{5} = -\frac{3}{4}x + 19\frac{1}{4}$

$x = 8\frac{3}{5}$

$\therefore S\left(8\frac{3}{5},\ 12\frac{4}{5}\right)$

$$x_C = \frac{8\frac{3}{5}+1\frac{2}{5}}{2}$$
$$= 5$$

$$y = -\frac{3}{4}(5) + 19\frac{1}{4}$$
$$= 15\frac{1}{2}$$

$\therefore R\left(5, 15\frac{1}{2}\right)$ (shown)

Alternative Method

Gradient of $l_1 = \frac{3}{4}$

Equation of l_1 is $y - 12\frac{4}{5} = \frac{3}{4}\left(x - 1\frac{2}{5}\right)$ --------- (1)

$y = -\frac{3}{4}x + 19\frac{1}{4}$ ----------- (2)

Sub. (2) into (1),

$-\frac{3}{4}x + 19\frac{1}{4} - 12\frac{4}{5} = \frac{3}{4}\left(x - 1\frac{2}{5}\right)$

$-\frac{3}{4}x + \frac{129}{20} = \frac{3}{4}x - \frac{21}{20}$

$-\frac{3}{2}x = -\frac{15}{2}$

$x = 5$ sub. into (2)

$y = 15\frac{1}{2}$

$\therefore R\left(5, 15\frac{1}{2}\right)$ (shown)

(ii) Gradient of normal at $S = \frac{4}{3}$

Equation of normal is $y - 12\frac{4}{5} = \frac{4}{3}\left(x - 8\frac{3}{5}\right)$

When $x = 5$,

$y - 12\frac{4}{5} = \frac{4}{3}\left(5 - 8\frac{3}{5}\right)$

$y = 8$

$\therefore C(5,8)$

Radius $= \sqrt{\left(5 - 8\frac{3}{5}\right)^2 + \left(8 - 12\frac{4}{5}\right)^2}$

$= 6$ units

Equation of circle is $(x-5)^2 + (y-8)^2 = 36$.

<u>Alternative Method</u>

Gradient of normal at $P = -\frac{4}{3}$

Gradient of normal at P is $y - 12\frac{4}{5} = -\frac{4}{3}\left(x - 1\frac{2}{5}\right)$

Sub. $x = 5$,

$y = 8$

Centre of circle is $(5,8)$

Radius of circle $= \sqrt{\left(5 - 1\frac{2}{5}\right)^2 + \left(8 - 12\frac{4}{5}\right)^2}$

= 6 units

Equation of circle is $(x-5)^2 + (y-8)^2 = 36$

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

Parent's Signature

PRELIMINARY EXAMINATION 2018
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **18 September 2018**

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/ 100

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 An empty, inverted cone has a height of 600 cm. The radius of the top of the cone is 200 cm. Water is poured into the cone at a constant rate.

(i) When the depth of the water in the cone is h cm, find the volume of the water in the cone in terms of π and h. [4]

The water level is rising at a rate of 3 cm per minute when the depth of the water is 120 cm.

(ii) Find the rate at which water is being poured into the cone, leaving your answer in terms of π. [3]

2 It is given that $y = x - \ln(\sec x + \tan x),\ 0 < x < \frac{\pi}{2}$.

(i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$. [3]

(ii) Hence, express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the form $a + b\sec x$, where a and b are integers. [3]

(iii) Deduce that y is a decreasing function. [2]

3 (a) Prove that $\dfrac{1+\sin 2x+\cos 2x}{\cos x+\sin x} = 2\cos x$. [3]

(b) Given that $\dfrac{\sec^2 x}{2\tan^2 x+1} = \dfrac{3}{4}$, where $180° < x < 270°$, find the exact value of $\sin x$. [5]

4 (a) Solve, for x and y, the simultaneous equations

$$2^x = 8\left(2^y\right),$$
$$\lg(2x+y) = \lg 63 - \lg 3.$$
[4]

(b) Express $\log_{\sqrt{2}} y = 3 - \log_2(y-6)$ as a cubic equation. [4]

5 (i) Express $\dfrac{2x^2-7}{(x+1)(x^2-x-6)}$ in partial fractions. [4]

(ii) Hence, find $\displaystyle\int_4^5 \frac{8x^2-28}{(x+1)(x^2-x-6)}\,\mathrm{d}x$. [4]

6 **(a)** **(i)** Sketch the graph of $y = |(x-1)(x-5)|$. [3]

(ii) Determine the set of values of a for which the line $y = a$ intersects the graph of $y = |(x-1)(x-5)|$ at four points. [2]

(b) Find the range of values of k for which the line $y = kx - 3$ does not intersect the curve $y = 2x^2 - 6x + 5$. [4]

7 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 3x}{2x^2}\right) = \frac{1}{2x^3} - \frac{\ln 3x}{x^3}$. [3]

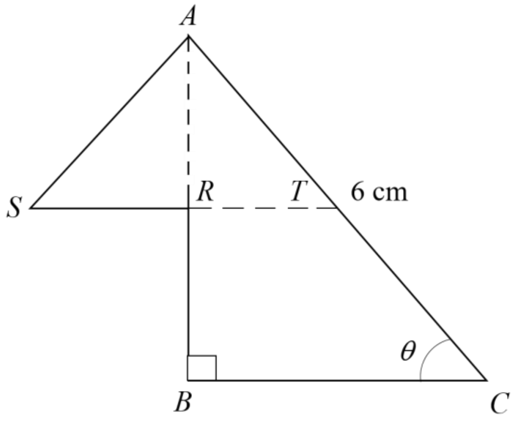
(ii) Hence, integrate $\frac{\ln 3x}{x^3}$ with respect to x. [3]

(iii) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{1}{3}, \frac{3}{4}\right)$ and is such that $\mathrm{f}'(x) = \frac{\ln 3x}{x^3}$, find $\mathrm{f}(x)$. [2]

8 **(i)** Find the coefficient of x^4 in the expansion of $\left(6 - x^2\right)^5\left(2x^2 + \frac{1}{3}\right)$. [4]

(ii) In the expansion of $(2 + x)^n$, the ratio of the coefficients of x and x^2 is $2:3$. Find the value of n. [5]

9 In the diagram, triangle ABC is a right angle triangle where angle $ACB = \theta$ and $AC = 6$ cm. R is a point on AB and T is the mid-point of AC. RT is parallel to BC and AR is a line of symmetry of triangle AST.

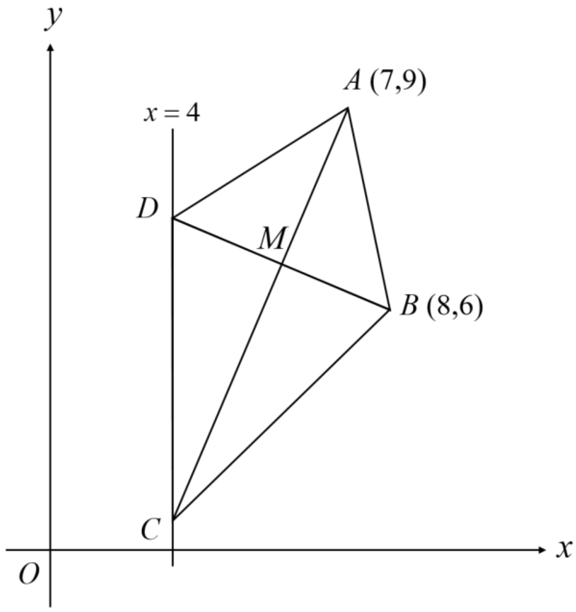


(a) Show that the perimeter, P cm, of the above diagram is $P = 9\cos\theta + 3\sin\theta + 9$. [2]

(b) **(i)** By expressing P in the form $m + n\cos(\theta - \alpha)$, find the value of θ for which $P = 15$. [6]

(ii) Hence, state the maximum value of P and find the corresponding value of θ. [3]

10 The diagram shows a quadrilateral $ABCD$ where the coordinates of vertices A and B are $(7,9)$ and $(8,6)$ respectively. Both vertices C and D lie on the line $x = 4$. AC passes through M, the midpoint of BD.



(i) Given that $AB = AD$, find the coordinates of C and D. [7]

(ii) Hence or otherwise, prove that quadrilateral $ABCD$ is a kite. [2]

(iii) Find the area of the kite $ABCD$. [2]

11 **(a)** The amount of caffeine, C mg, left in the body t hours after drinking a certain cup of coffee is represented by $C = 100\mathrm{e}^{-kt}$.

(i) Given that the amount of caffeine left in the body is 20 mg after 2.5 hours, find the value of k. [2]

(ii) Find the number of hours, correct to 3 significant figures, for half the initial amount of caffeine to be left in the body. [3]

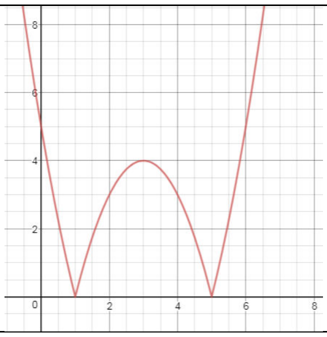
(b) The curve $y = ax^4 + bx^3 + 7$, where a and b are constants, has a minimum point at $(1, 6)$.

Find

(i) the value of a and of b, [4]

(ii) the coordinates of the other stationary point on the curve and determine the nature of this stationary point. [4]

Answer Key

1	**(i)**	$v=\dfrac{\pi h^3}{27}$
	(ii)	$4800\pi\ \text{cm}^3/\text{min}$
2	**(ii)**	$1-\sec x$
3	**(b)**	$\sin x=-\dfrac{\sqrt{3}}{3}$
4	**(a)**	$x=8,\ y=5$
	(b)	$y^3-6y^2-8=0$
5	**(i)**	$\dfrac{2x^2-7}{(x+1)(x^2-x-6)}=\dfrac{5}{4(x+1)}+\dfrac{11}{20(x-3)}+\dfrac{1}{5(x+2)}$
	(ii)	2.56
6	**(ai)**	
	(aii)	$0<a<4$
	(b)	$-14<k<2$
7	**(ii)**	$-\dfrac{1}{4x^2}-\dfrac{2\ln 3x}{x^2}+c$
	(iii)	$f(x)=\dfrac{\ln 3x}{-2x^2}-\dfrac{1}{4x^2}+3$
8	**(i)**	-12440
	(ii)	$n=7$
9	**(bi)**	$P=9+\sqrt{90}\cos(\theta-18.43495°)$; $\theta=69.2°$
	(bii)	maximum value of $P=9+\sqrt{90}$, corresponding value of $x=18.4°$
10	**(i)**	$D(4,8)$, $C(4,3)$
	(ii)	Since $\text{M}_{AC}\bullet\text{M}_{BD}=-1$, diagonals AC and BD are perpendicular to each other. AC bisects BD. $\therefore$ quadrilateral $ABCD$ is a kite.
	(iii)	15 units^2
11	**(ai)**	$k=0.644$
	(aii)	$t=1.08$ hours
	(bi)	$a=3$ and $b=-4$
	(bii)	$(0,7)$, point of inflexion

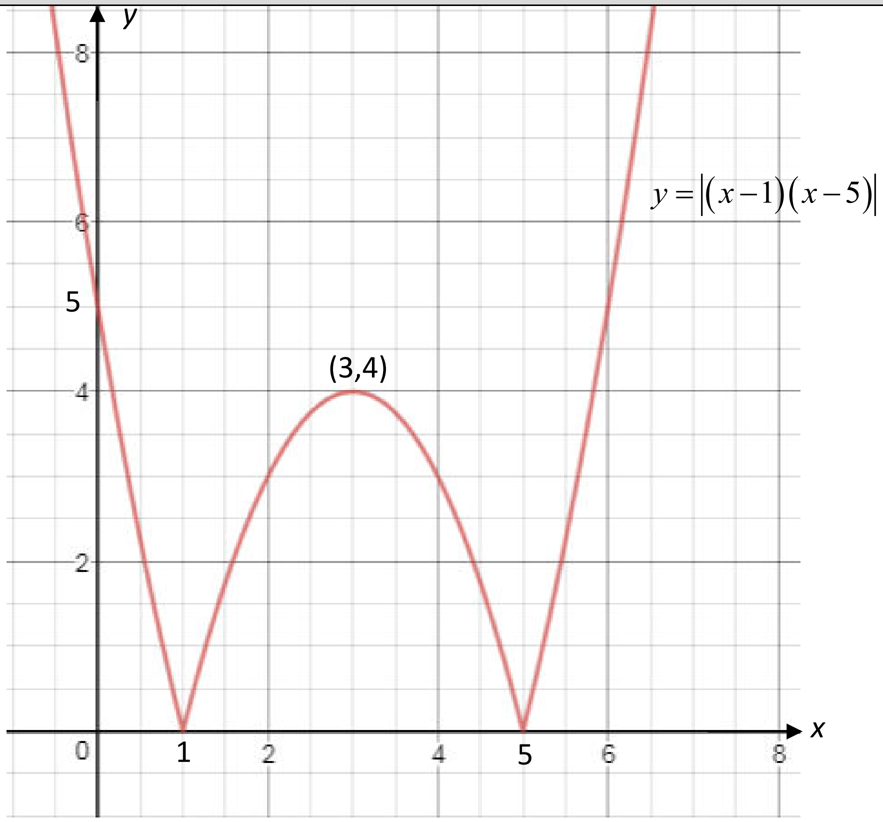
	Working	**Common Issues**
1 (i)	$\frac{h}{600}=\frac{r}{200}$ (ratio of corresponding sides are equal) $r=\frac{h}{3}$ $V=\frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$ $v=\frac{\pi h^3}{27}$	
(ii)	$\frac{\mathrm{d}h}{\mathrm{d}t}=3$ cm/s $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{27}\left(3h^2\right)$ $=\frac{\pi h^2}{9}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $=\frac{\pi h^2}{9}\times 3$ $=4800\pi\text{ cm}^3/\text{min}$	

	Working	Common Issues
2 (i)	$y = x - \ln(\sec x + \tan x)$ $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$ $= \frac{(\cos x)(0)-(1)(-\sin x)}{\cos^2 x}$ $= \frac{\sin x}{\cos^2 x}$ $= \sec x \tan x$	
(ii)	$\frac{dy}{dx} = 1 - \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$ $= 1 - \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$ $= 1 - \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$ $= 1 - \sec x$	
(iii)	$\frac{dy}{dx} = 1 - \sec x$ $= 1 - \frac{1}{\cos x}$ $= \frac{\cos x - 1}{\cos x}$ Numerator: $0 < \cos x < 1$ $\therefore \cos x - 1$ will always be negative. Denominator: $0 < \cos x < 1$ $\therefore \cos x$ will always be positive. $\therefore \frac{dy}{dx} < 0$, y is a decreasing function.	

	Working	Common Issues
3 **(a)**	$\text{LHS} = \dfrac{1+\sin 2x+\cos 2x}{\cos x+\sin x}$ $= \dfrac{1+(2\sin x\cos x)+(2\cos^2 x-1)}{\cos x+\sin x}$ $= \dfrac{2\cos^2 x+2\sin x\cos x}{\cos x+\sin x}$ $= \dfrac{2\cos x(\cos x+\sin x)}{\cos x+\sin x}$ $= 2\cos x$ $= \text{RHS (proven)}$	
(b)	$\dfrac{\sec^2 x}{2\tan^2 x+1} = \dfrac{3}{4}$ $\dfrac{4}{\cos^2 x} = 6\tan^2 x+3$ $\dfrac{4}{\cos^2 x} = \dfrac{6\sin^2 x}{\cos^2 x}+\dfrac{3\cos^2 x}{\cos^2 x}$ $4 = 6\sin^2 x+3\cos^2 x$ $4 = (3\sin^2 x+3\cos^2 x)+3\sin^2 x$ $4 = 3+3\sin^2 x$ $\sin^2 x = \dfrac{1}{3}$ $\sin x = \sqrt{\dfrac{1}{3}}$ (reject as $180° < x < 270°$) or $\sin x = -\dfrac{\sqrt{3}}{3}$ **<u>Alternative:</u>** $\dfrac{1+\tan^2 x}{2\tan^2 x+1} = \dfrac{3}{4}$ $4+4\tan^2 x = 6\tan^2 x+3$ $\tan^2 x = \dfrac{1}{2}$ $\dfrac{\sin^2 x}{\cos^2 x} = \dfrac{1}{2}$ $\dfrac{\sin^2 x}{(1-\sin^2 x)} = \dfrac{1}{2}$ $2\sin^2 x = 1-\sin^2 x$ $\sin^2 x = \dfrac{1}{3}$ $\sin x = \sqrt{\dfrac{1}{3}}$ (reject as $180° < x < 270°$) or $\sin x = -\dfrac{\sqrt{3}}{3}$	

	Working	Common Issues
4 (a)	$2^x = 8\left(2^y\right)$ ----- (1) $\lg\left(2x+y\right) = \lg 63 - \lg 3$ ----- (2) From (1), $2^x = 2^3 \times 2^y$ $x = 3 + y$ ----- (3) From (2), $\lg\left(2x+y\right) = \lg\left(\frac{63}{3}\right)$ $2x + y = 21$ ----- (4) Sub (3) into (4), $2\left(3+y\right) + y = 21$ $y = 5$ $x = 8$	
4 (b)	$\log_{\sqrt{2}} y = 3 - \log_2\left(y-6\right)$ $\log_{2^{\frac{1}{2}}} y = 3 - \log_2\left(y-6\right)$ $\frac{\lg y}{\lg 2^{\frac{1}{2}}} = 3 - \frac{\lg\left(y-6\right)}{\lg 2}$ $\frac{\lg y}{\frac{1}{2}\lg 2} + \frac{\lg\left(y-6\right)}{\lg 2} = 3$ $2\lg y + \lg\left(y-6\right) = 3\lg 2$ $\lg y^2 + \lg\left(y-6\right) = \lg 2^3$ $\lg\left[y^2\left(y-6\right)\right] = \lg 8$ $y^3 - 6y^2 - 8 = 0$	

	Working	Common Issues
5 (i)	$$\frac{2x^2-7}{(x+1)(x^2-x-6)}=\frac{2x^2-7}{(x+1)(x-3)(x+2)}$$ $$=\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{x+2}$$ $$A(x-3)(x+2)+B(x+1)(x+2)+C(x+1)(x-3)=2x^2-7$$ When $x=1$, $$A(-4)(1)=2(-1)^2-7$$ $$A=\frac{5}{4}$$ When $x=-2$, $$C(-1)(-5)=2(-2)^2-7$$ $$C=\frac{1}{5}$$ When $x=3$, $$B(4)(5)=2(3)^2-7$$ $$B=\frac{11}{20}$$ $$\frac{2x^2-7}{(x+1)(x^2-x-6)}=\frac{5}{4(x+1)}+\frac{11}{20(x-3)}+\frac{1}{5(x+2)}$$	
(ii)	$$\int_4^5\frac{8x^2-28}{(x+1)(x^2-x-6)}\mathrm{d}x$$ $$=4\int_4^5\left[\frac{5}{4(x+1)}+\frac{11}{20(x-3)}+\frac{1}{5(x+2)}\right]\mathrm{d}x$$ $$=4\left[\frac{5}{4}\ln(x+1)+\frac{11}{20}\ln(x-3)+\frac{1}{45}\ln(x+2)\right]_4^5$$ $$=\left[\left(5\ln 6+\frac{11}{5}\ln 2+\frac{4}{5}\ln 7\right)-\left(5\ln 5+\frac{11}{5}\ln 1+\frac{4}{5}\ln 6\right)\right]$$ $$=2.56\ (3\text{sf})$$	

	Working	Common Issues
6 (a i)		
(aii)	$0<a<4$	
(b)	$2x^2-6x+5=kx-3$ $2x^2-(6+k)x+8=0$ $b^2-4ac<0$ $\because$ no intersection $\left[-(6+k)\right]^2-4(2)(8)<0$ $36+12k+k^2-64<0$ $k^2+12k-28<0$ $(k+14)(k-2)<0$ $-14<k<2$	

	Working	**Common Issues**
7 (i)	$\frac{d}{dx}\left(\frac{\ln 3x}{2x^2}\right)=\frac{1}{2}\frac{d}{dx}\left(\frac{\ln 3x}{x^2}\right)$ $=\frac{1}{2}\left[\frac{x^2\left(\frac{3}{3x}\right)-(2x)(\ln 3x)}{x^4}\right]$ $=\frac{1}{2}\left[\frac{x-2x(\ln 3x)}{x^4}\right]$ $=\frac{x-2x(\ln 3x)}{2x^4}$ $=\frac{1}{2x^3}-\frac{\ln 3x}{x^3}$ (shown)	
(ii)	$\int \frac{1}{2x^3}-\frac{\ln 3x}{x^3}\,dx=\frac{\ln 3x}{2x^2}+c$ $\int \frac{\ln 3x}{x^3}\,dx=\frac{1}{2}\int x^{-3}dx-\frac{\ln 3x}{2x^2}+c$ $=\frac{1}{2}\left(\frac{x^{-2}}{-2}\right)-\frac{\ln 3x}{2x^2}+c$ $=-\frac{1}{4x^2}-\frac{2\ln 3x}{x^2}+c$	
(iii)	$f'(x)=\frac{\ln 3x}{x^3}$ $f(x)=\frac{\ln 3x}{-2x^2}-\frac{1}{4x^2}+c$ Given $f\left(\frac{1}{3}\right)=\frac{3}{4}$, $\frac{3}{4}=0-\frac{1}{4\left(\frac{1}{3}\right)^2}+c$ $c=3$ $\therefore f(x)=\frac{\ln 3x}{-2x^2}-\frac{1}{4x^2}+3$	

	Working	Common Issues
8 (i)	$\left(6-x^2\right)^5=\binom{5}{0}(6)^5\left(x^2\right)^0-\binom{5}{1}(6)^4\left(x^2\right)^1+\binom{5}{2}(6)^3\left(x^2\right)^2+....$ $=7776-6480x^2+2160x^4+....$ Coefficient of $x^4=(-6480)(2)+(2160)\left(\frac{1}{3}\right)$ $=-12960+720$ $=-12240$	
(ii)	For x term, $r=1$ $T_2=\binom{n}{1}\left(2^{n-1}\right)x$ $=\frac{2^n(n)}{2}x$ For x^2 term, $r=2$ $T_3=\binom{n}{2}\left(2^{n-2}\right)x^2$ $=\frac{2^n(n)(n-1)}{8}x^2$ $\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=\frac{\frac{2^n(n)}{2}}{\frac{2^n(n)(n-1)}{8}}=\frac{2}{3}$ $\frac{2^n(3n)}{2}=\frac{2^n(n)(n-1)}{4}$ $2^n(6n)=2^n(n)(n-1)$ $2^n(n)(n-1)-2^n(6n)=0$ $2^n(n)\left[(n-1)-6\right]=0$ $2^n=0$ (reject as $2^n>0$) $n=0$ (reject as $n\neq 0$) $n=7$	

		Working	Common Issues
9	**(a)**	$AB = 6\sin x$ $BC = 6\cos x$ $RB = \dfrac{6\sin x}{2} = 3\sin x$ (ratio of corresponding sides are equal) $SR = RT = \dfrac{6\cos x}{2} = 3\cos x$ (ratio of corresponding sides are equal) $P = 6 + 6\cos x + 3\sin x + 3\cos x + 3$ $= 9 + 9\cos x + 3\sin x$ (shown)	
	(bi)	$P = 9 + 9\cos\theta + 3\sin\theta$ $= 9 + n\cos(\theta - \alpha)$ $= 9 + n(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ Comparing coefficients, $n\cos\alpha = 9, \qquad n\sin\alpha = 3$ $\tan\alpha = \dfrac{1}{3}$ $\alpha = \tan^{-1}\dfrac{1}{3}$ $= 18.43495°$ $n^2 = 9^2 + 3^2$ $n = \sqrt{90}$ $\therefore P = 9 + \sqrt{90}\cos(\theta - 18.43495°)$ $9 + \sqrt{90}\cos(\theta - 18.43495°) = 15$ $\cos(\theta - 18.43495°) = \dfrac{15-9}{\sqrt{90}}$ Basic angle $= \cos^{-1}\left(\dfrac{6}{\sqrt{90}}\right) = 50.7685°$ $\theta - 18.43495° = 50.7685°$ or $\theta - 18.43495° = 360° - 50.7685°$ (reject) $\theta = 69.2°$	
	(bii)	Maximum value of P is when $\cos(x - 18.43495°) = 1$ $\therefore$ maximum value of $P = 9 + \sqrt{90}$ corresponding value of $x = 18.4°$	

	Working	Common Issues
10 (i)	Length of $AD=\sqrt{(y-9)^2+(4-7)^2}$ Length of $AB=\sqrt{(6-9)^2+(8-7)^2}$ $(y-9)^2+9=9+1$ $(y-9)^2=1$ $y-9=1$ or $y-9=-1$ $y=10$ (reject) or $y=8$ $\therefore$ coordinates of $D(4,8)$. Coordinates of $M=\left(\frac{8+4}{2},\frac{6+8}{2}\right)$ $=(6,7)$ Gradient of AM $=\frac{9-7}{7-6}$ $=2$ Equation of AC: $9=2(7)+c$ $c=-5$ $y=2x-5$ When $x=4,\ y=3$ $\therefore$ coordinates of $C(4,3)$.	
(ii)	$\text{M}_{AC}=\text{M}_{AM}=2$ $\text{M}_{BD}=\frac{6-8}{8-4}=-\frac{1}{2}$ Since $\text{M}_{AC}\bullet\text{M}_{BD}=-1$, diagonals AC and BD are perpendicular to each other. $\therefore$ quadrilateral $ABCD$ is a kite.	
(iii)	Area of $ABCD=\frac{1}{2}\begin{vmatrix}7&4&4&8&7\\9&8&3&6&9\end{vmatrix}$ $=\frac{1}{2}\{[(7\times8)+(4\times3)+(4\times6)+(8\times9)]-[(9\times4)+(8\times4)+(3\times8)+(6\times7)]\}$ $=\frac{1}{2}\times30$ $=15\text{ units}^2$	

	Working	Common Issues
11 (ai)	$20 = 100e^{-k(2.5)}$ $\ln\frac{1}{5} = -2.5k$ $k = 0.644$	
(aii)	$100e^{-0.643775t} = \frac{1}{2}100e^{0}$ $-0.643775t = \ln\frac{1}{2}$ $t = 1.08$ hours	
(bi)	$y = ax^4 + bx^3 + 7$ $\frac{dy}{dx} = 4ax^3 + 3bx^2$ Sub $x = 1$ into $\frac{dy}{dx}$, $4a + 3b = 0$ ----- (1) Sub $(1,6)$ into curve y, $6 = a + b + 7$ $a = -b - 1$ ----- (2) Sub (2) into (1), $4(-b-1) + 3b = 0$ $b = -4,\quad a = 3$	
(bii)	$\frac{dy}{dx} = 4(3)x^3 + 3(-4)x^2$ $= 12x^3 - 12x^2$ When $\frac{dy}{dx} = 0$, $12x^3 - 12x^2 = 0$ $12x^2(x-1) = 0$ $x = 0$ or $x = 1$ When $x = 0,\ y = 7$ $\therefore$ the other stationary point is $(0,7)$.	

<table>
<tr><th></th><th>Working</th><th>Common Issues</th></tr>
<tr><td></td><td>

$\frac{d^2y}{dx^2} = 36x^2 - 24x$

When $x = 0$, $\frac{d^2y}{dx^2} = 0$ (not conclusive)

	x = -0.1	x = 0	x = 0.1
$\frac{dy}{dx}$	negative	0	negative

Using the first derivative test, the gradient changes from negative to negative, thus $(0,7)$ is a point of inflexion.

</td><td></td></tr>
</table>

End of Paper