Name: ___________________________ (   )   Class: _____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS   4047/01

Paper 1   **Thursday 16 August 2018**

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q6		Q11	
Q2		Q7			
Q3		Q8			
Q4		Q9			
Q5		Q10			

/ 80

This document consists of **5** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)........(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 Express $\dfrac{3x^3+2x^2+4x-1}{x^3+x^2}$ in partial fractions. [4]

2 A cylinder has a radius of $(1+2\sqrt{2})$ cm and its volume is $\pi(84+21\sqrt{2})\,\text{cm}^3$.
Find, **without using a calculator**, the exact length of the height of the cylinder in the form $(a+b\sqrt{2})$ cm, where a and b are integers. [5]

3 **(i)** Sketch the graph of $y=4-3\sin 2x$ for $0\le x\le\pi$. [3]

(ii) State the range of values of k for which $4-3\sin 2x=k$ has two roots for $0\le x\le\pi$. [2]

4 **Solutions to this question by accurate drawing will not be accepted.**
PQRS is a parallelogram in which the coordinates of the points P and R are $(-5, 8)$ and $(6,-2)$ respectively. Given that PQ is perpendicular to the line $y=-\dfrac{1}{2}x+3$ and QR is parallel to the x axis, find

(i) the coordinates of Q and of S, [5]

(ii) the area of *PQRS*. [2]

5 **(i)** Differentiate $\dfrac{\ln x}{x}$ with respect to x. [3]

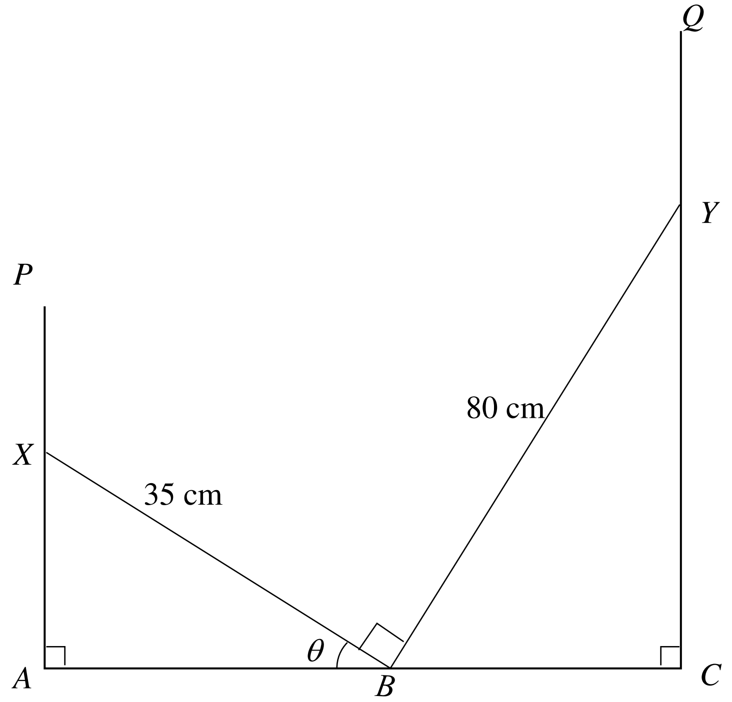
(ii) Hence find $\displaystyle\int\frac{\ln x^2}{x^2}\,\mathrm{d}x$. [4]

6 **(i)** Show that $\dfrac{2}{\tan\theta + \cot\theta} = \sin 2\theta$. [3]

(ii) Hence find the value of p, giving your answer in terms of π, for which

$\int_0^p \dfrac{4}{\tan 2x + \cot 2x}\,dx = \dfrac{1}{4}$, where $0 < p < \dfrac{\pi}{4}$. [4]

7



In the diagram XBY is a structure consisting of a beam XB of length 35 cm attached at B to another beam BY of length 80 cm so that angle $XBY = 90^{\circ}$. Small rings at X and Y enable X to move along the vertical wire AP and Y to move along the vertical wire CQ. There is another ring at B that allows B to move along the horizontal line AC. Angle $ABX = \theta$ and θ can vary.

(i) Show that $AC = (35\cos\theta + 80\sin\theta)$ cm. [2]

(ii) Express AC in the form of $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Tom claims that the length of AC is 89cm. Without measuring, Mary said that this was not possible. Explain how Mary came to this conclusion. [1]

8 **(a)** Find the range of values of p for which $px^2 + 4x + p > 3$ for all real values of x. [5]

(b) Find the range of values of k for which the line $5y = k - x$ does not intersect the curve $5x^2 + 5xy + 4 = 0$. [5]

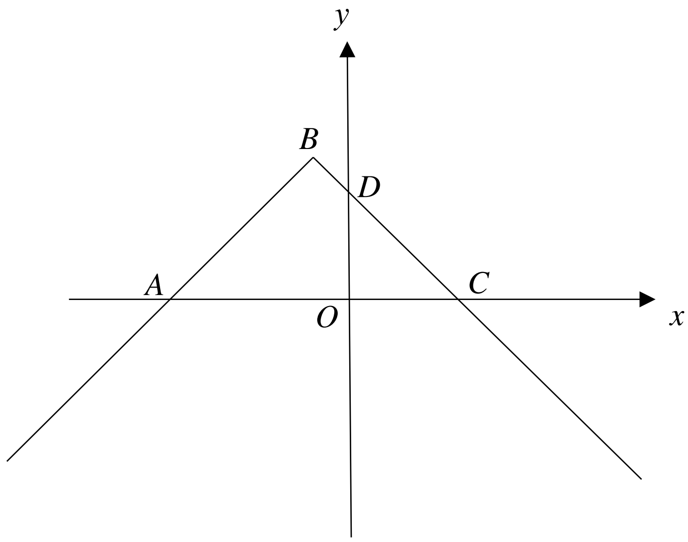
9 The diagram shows part of the graph of $y = 4 - |x+1|$.

(i) Find the coordinates of the points *A*, *B*, *C* and *D*. [5]

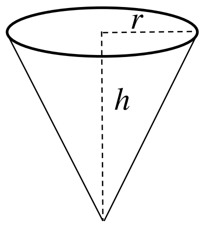
(ii) Find the number of solutions of the equation $4 - |x+1| = mx + 3$ when

(a) $m = 2$ **(b)** $m = -1$ [2]

(iii) State the range of values of *m* for which the equation $4 - |x+1| = mx + 3$ has two solutions. [1]



10 The diagram shows a cone of radius *r* cm and height *h* cm. It is given that the volume of the cone is 10π cm^3.



(i) Show that the curved surface area, *A* cm^2, of the cone, is $A = \dfrac{\pi\sqrt{r^6 + 900}}{r}$. [3]

(ii) Given that *r* can vary, find the value of *r* for which *A* has a stationary value. [4]

(iii) Determine whether this value of *A* is a maximum or a minimum. [2]

11 The equation of a curve is $y = x(2-x)^3$.

(i) Find the range of values of *x* for which *y* is an increasing function. [5]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Hence, sketch the graph of $y = x(2-x)^3$. [3]

St Nicholas Girls School Additional Mathematics Preliminary Examination Paper I 2018

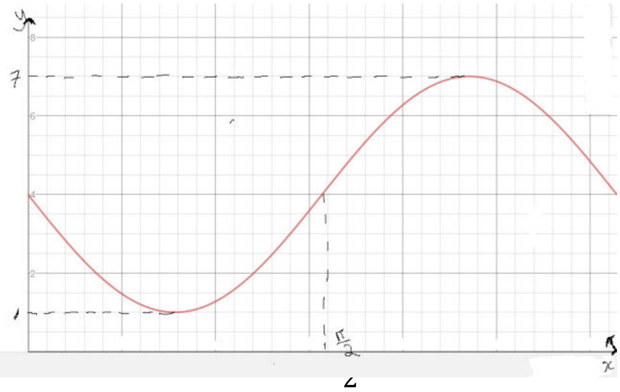
Answers

Paper 1

1. $3+\frac{5}{x}-\frac{1}{x^2}-\frac{6}{x+1}$

2. $(12-3\sqrt{2}\ )$ cm

3 (i)



(ii) $1<k<4$ or $4<k<7$

4 (i) $Q(-10,\ -2)$, $S(11,8)$ (ii) 160 units^2

5 (i) $\frac{1-\ln x}{x^2}$ (ii) $2\left(-\frac{1}{x}-\frac{\ln x}{x}\right)+c$

6 (ii) $\frac{\pi}{12}$

7 (ii) $5\sqrt{305}\sin(\theta+23.6^\circ)$ cm or $87.3\sin(\theta+23.6^\circ)$ cm

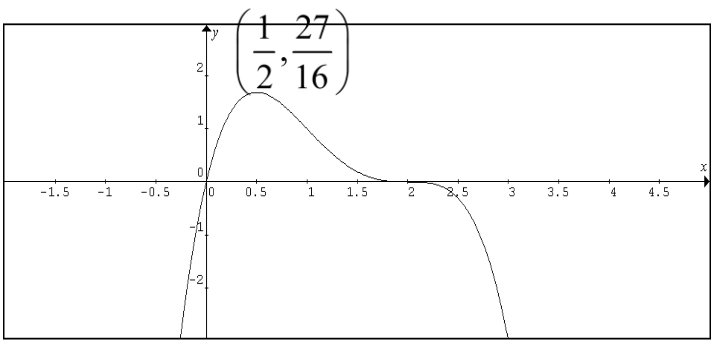
(iii) The maximum value of AC=87.3cm <89 cm

8 (a) $p>4$ (b) $-8<k<8$

9 (i) $A(-5,0)$, B(−1,4), $C(3,0)$, D(0,3) (ii) (a) 1 (b) infinite (iii) 2

10 (ii) 2.77 (iii) minimum

11 (i) $x<\frac{1}{2}$ (ii) $(2,0)$ $\left(\frac{1}{2},\frac{27}{16}\right)$ (iii)



Name: ____________________________ (   )   Class: _____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **Marking Scheme** **Thursday 16 August 2018**

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$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{3x^3+2x^2+4x-1}{x^3+x^2}$ in partial fractions. [4]

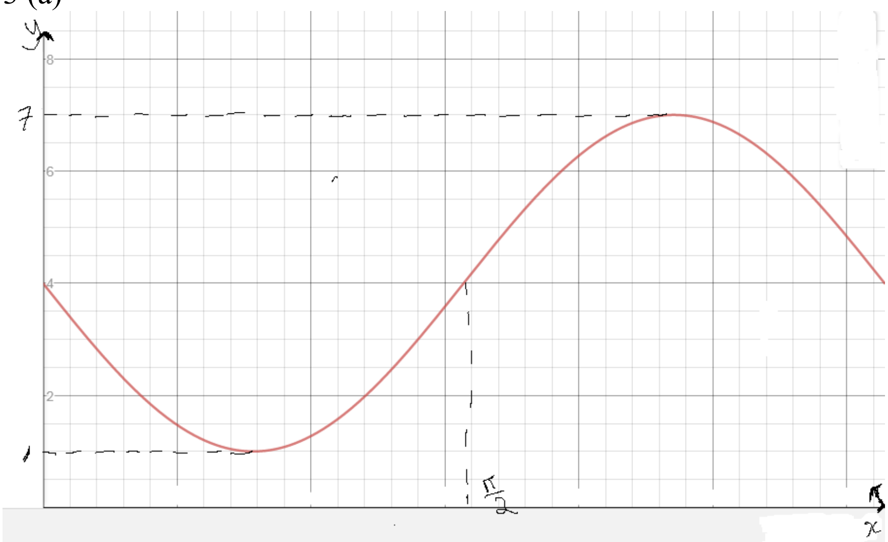
1 $x^3+x^2 \overline{\big) \begin{array}{l} 3x^3+2x^2+4x-1 \\ 3x^3+3x^2 \\ \hline -x^2+4x-1 \end{array}}$ quotient 3 $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=3+\frac{-x^2+4x-1}{x^2(x+1)}$	
$\dfrac{-x^2+4x-1}{x^2(x+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{c}{x+1}$	M1√
$-x^2+4x-1=Ax(x+1)+B(x+1)+cx^2$	M1√
Let $x=-1$ $\quad -1-4-1=c$ $c=-6$ Let $x=0$ $\quad$ B= -1	M1√
$-x^2+4x-1=Ax(x+1)-1(x+1)-6x^2$ Let $x=1$ $\quad -1+4-1=2A-2-6$ $A=5$	
$\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=3+\frac{5}{x}-\frac{1}{x^2}-\frac{6}{x+1}$ **[4]**	A1
If	
• $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=\frac{A}{x}+\frac{B}{x^2}+\frac{c}{x+1}$	Max 3m
• $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=3+\frac{Ax+B}{x^2}+\frac{c}{x+1}$	3m
• $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=\frac{Ax+B}{x^2}+\frac{c}{x+1}$	2m

2 A cylinder has a radius of $(1+2\sqrt{2})$ cm and its volume is $\pi(84+21\sqrt{2})\text{ cm}^3$.
Find, **without using a calculator**, the exact length of the height of the cylinder in the form $(a+b\sqrt{2})$ cm, where a and b are integers. [5]

2. $\pi(84+21\sqrt{2}) = \pi(1+2\sqrt{2})^2 \times h$		
$h = \dfrac{84+21\sqrt{2}}{(1+2\sqrt{2})^2}$	B1	
$h = \dfrac{84+21\sqrt{2}}{1+4\sqrt{2}+8}$	M1	expansion
$h = \dfrac{(84+21\sqrt{2})(4\sqrt{2}-9)}{(4\sqrt{2}+9)(4\sqrt{2}-9)}$	M1√	Conjugate surd
$h = \dfrac{756-336\sqrt{2}+189\sqrt{2}-168}{81-32}$	M1√	For either expansion
$h = \dfrac{588-147\sqrt{2}}{49}$		
$h = (12-3\sqrt{2}\ )$ cm **[5]**	A1	No unit, overall -1m

3 **(i)** Sketch the graph of $y = 4-3\sin 2x$ for $0 \le x \le \pi$. [3]

(ii) State the range of values of k for which $4-3\sin 2x = k$ has two roots for $0 \le x \le \pi$. [2]

3 (a)  [3]	B1 any one pt B1 2nd pt The 2 pts must be differernt nature B1 perfect	• sine shape • -ve sine shape • 1 cycle • Amplitude • shift +4 up ignoring no labelling of axes
3 (b) $1<k<4$ or $4<k<7$ [2] **Alternative** **[5]** $1<k<7, k \ne 4$	B1 + B1 B1 + B1	

[Turn over

4 Solutions to this question by accurate drawing will not be accepted.

PQRS is a parallelogram in which the coordinates of the points *P* and *R* are $(-5, 8)$ and $(6, -2)$ respectively. Given that *PQ* is perpendicular to the line $y = -\frac{1}{2}x + 3$ and *QR* is parallel to the *x* axis, find

(i) the coordinates of *Q* and of *S*, [5]

(ii) the area of *PQRS*. [2]

1(i)	Since *QR* parallel to the *x* axis, $y_Q = -2$. Since *PQ* is perpendicular to the line $y = -\frac{1}{2}x + 3$, gradient of *PQ* = 2 $\frac{(-2)-(8)}{x_Q-(-5)} = 2$ $-10 = 2x_Q + 10$ $x_Q = -10$ $Q(-10, \ -2)$ Midpoint of *PR* = Midpoint of *QS* or by inspection $\left(\frac{(-5)+(6)}{2}, \frac{(8)+(-2)}{2}\right) = \left(\frac{(-10)+x_s}{2}, \frac{(-2)+y_s}{2}\right)$ $1 = -10 + x_s$ $6 = -2 + y_s$ $x_s = 11$ $y_s = 8$ *S*(11, 8) [5]	B1 B1 ($\perp$ gradient) M1 A1 B1
(ii)	Area of *PQRS* $= \frac{1}{2}\begin{vmatrix} -5 & -10 & 6 & 11 & -5 \\ 8 & -2 & -2 & 8 & 8 \end{vmatrix}$ $= \frac{1}{2}\lvert(10+20+48+88)-(-80-12-22-40)\rvert$ or (5+11)(8+2) $= \frac{1}{2}\lvert 320 \rvert$ [2] $= 160 \text{ units}^2$ **[7]**	 √M1 A1 no unit overall -1m

5 **(i)** Differentiate $\dfrac{\ln x}{x}$ with respect to x. [3]

(ii) Hence find $\displaystyle\int \frac{\ln x^2}{x^2}\,\mathrm{d}x$. [4]

(i)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\ln x}{x}\right)=\dfrac{x\left(\dfrac{1}{x}\right)-\ln x}{x^2}$ $=\dfrac{1-\ln x}{x^2}$ [3]	B1 +B1 A1	Either $v\frac{du}{dx}$ $or u\frac{dv}{dx}$ with the use of quotient rule /product rule perfect
(ii)	$\displaystyle\int \frac{1-\ln x}{x^2}\,\mathrm{d}x=\frac{\ln x}{x}$ $\displaystyle\int \frac{1}{x^2}\,\mathrm{d}x-\int \frac{\ln x}{x^2}\,\mathrm{d}x=\frac{\ln x}{x}$ $\displaystyle\int x^{-2}\,\mathrm{d}x-\frac{\ln x}{x}=\int \frac{\ln x}{x^2}\,\mathrm{d}x$ $\displaystyle\frac{x^{-1}}{-1}-\frac{\ln x}{x}=\int \frac{\ln x}{x^2}\,\mathrm{d}x$ $\displaystyle\int \frac{\ln x}{x^2}\,\mathrm{d}x=-\frac{1}{x}-\frac{\ln x}{x}$ $\displaystyle\int \frac{\ln x^2}{x^2}\,\mathrm{d}x=2\int \frac{\ln x}{x^2}\,\mathrm{d}x$ $\displaystyle=2\left(-\frac{1}{x}-\frac{\ln x}{x}\right)+c$ [4] **[7]**	M1 M1 B1 A1	Integration is the reverse process of differentiation Making $\int \frac{\ln x}{x^2}\,\mathrm{d}x$ the subject or split the expression Integration of x^{-2} With c

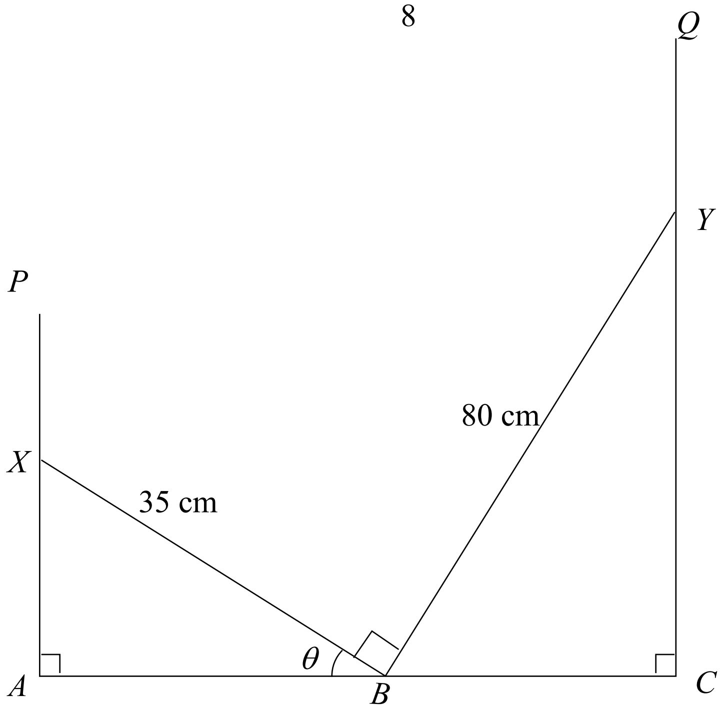
6 **(i)** Show that $\dfrac{2}{\tan\theta+\cot\theta}=\sin 2\theta$. [3]

(ii) Hence find the value of p, giving your answer in terms of π, for which

$\displaystyle\int_0^p \frac{4}{\tan 2x+\cot 2x}\,dx=\frac{1}{4}$, where $0< p<\dfrac{\pi}{4}$. [4]

(i)	$\dfrac{2}{\tan\theta+\cot\theta}=2\div\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\right)$	B1	change to sin and cos
	$=2\div\left(\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$	M1	combine terms
	$=2\div\left(\dfrac{1}{\cos\theta\sin\theta}\right)$ $=2\sin\theta\cos\theta$ $=\sin 2\theta$ [3]	M1	for identity …to the end. (must show "1")
(ii)	$\displaystyle\int_0^p \frac{4}{\tan 2x+\cot 2x}\,\mathrm{dx}$ $\displaystyle=2\int_0^p \sin 4x\,\mathrm{dx}$	B1	
	$=2\left[-\dfrac{\cos 4x}{4}\right]_0^p$	M1	integrate their sinkx
	$=\left(-\dfrac{1}{2}\cos 4p\right)-\left(-\dfrac{1}{2}\cos 0\right)$	M1	for substitution in their integral
	$=-\dfrac{1}{2}\cos 4p+\dfrac{1}{2}$ $\displaystyle\int_0^p \frac{4}{\tan 2x+\cot 2x}\,dx=\frac{1}{4}$ $-\dfrac{1}{2}\cos 4p+\dfrac{1}{2}=\dfrac{1}{4}$ $-\dfrac{1}{2}\cos 4p=-\dfrac{1}{4}$ $\cos 4p=\dfrac{1}{2}$ $4p=\dfrac{\pi}{3}$		
	$p=\dfrac{\pi}{12}$ [4] **[7]**	A1	

7

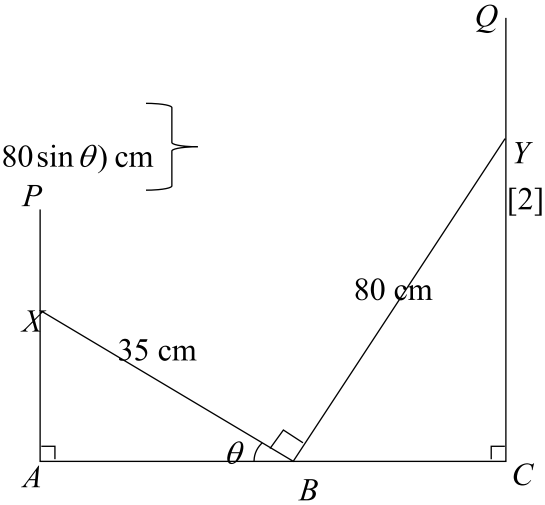


In the diagram XBY is a structure consisting of a beam XB of length 35 cm attached at B to another beam BY of length 80 cm so that angle $XBY = 90^\circ$. Small rings at X and Y enable X to move along the vertical wire AP and Y to move along the vertical wire CQ. There is another ring at B that allows B to move along the horizontal line AC. Angle $ABX = \theta$ and θ can vary.

(i) Show that $AC = (35\cos\theta + 80\sin\theta)$ cm. [2]

(ii) Express AC in the form of $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iii) Tom claims that the length of AC is 89cm. Without measuring, Mary said that this was not possible. Explain how Mary came to this conclusion. [1]

7 (i)	$AB = 35\cos\theta$ $\angle YBC = 90^\circ - \theta$ $\angle BYC = \theta$ $BC = 80\sin\theta$ $AC = (35\cos\theta + 80\sin\theta)$ cm  [2]	B1 either AB or BC B1 -1m overall for no unit
7 (ii)	$R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $AC = 35\cos\theta + 80\sin\theta$ $R\sin\alpha = 35$ $R\cos\alpha = 80$ $R^2\cos^2\alpha + R^2\sin^2\alpha = 80^2 + 35^2$ $R = \sqrt{80^2 + 35^2}$ $R^2 = 7625$ $R = 87.3 \quad or \quad 5\sqrt{305}$ $\frac{R\sin\alpha}{R\cos\alpha} = \frac{35}{80}$ $\tan\alpha = \frac{35}{80}$ $\alpha = 23.6^\circ$ $AC = 35\cos\theta + 80\sin\theta$ $35\cos\theta + 80\sin\theta = 5\sqrt{305}\sin(\theta + 23.6^\circ)$ cm or $87.3\sin(\theta + 23.6^\circ)$ cm [4]	B1 M1 for R M1 for $\tan\alpha = \frac{35}{80}$ A1
7 (iii)	The maximum value of AC=87.3cm Therefore it is not possible for the length to be more than that. **Alternative** $5\sqrt{305}\sin(\theta + 23.6^\circ) = 89$ $\sin(\theta + 23.6^\circ) = \frac{89}{5\sqrt{305}}$ No Solution Therefore it is not possible for the length to be more than that. [1] **[7]**	DB1 DB1

8 **(a)** Find the range of values of p for which $px^2+4x+p>3$ for all real values of x. [5]

(b) Find the range of values of k for which the line $5y=k-x$ does not intersect the curve $5x^2+5xy+4=0$. [5]

(a)	$px^2+4x+p>3$ for all real values of x $px^2+4x+p-3>0$ for all real values of x, D<0 $4^2-4(p)(p-3)<0$ $16-4p^2+12p<0$ $4p^2-12p-16>0$ $p^2-3p-4>0$ $(p-4)(p+1)>0$ $p<-1,\ p>4$ NA As $p>0$ [5]		M1 M1 M1 DA1+DA1	D<0 with substitution For b^2-4ac For factorisation Upon correct factorisation Ignore"and" and no p>0
(b)	$5y=k-x$ $5x^2+5xy+4=0$ $5x^2+5x\left(\frac{k-x}{5}\right)+4=0$ $5x^2+kx-x^2+4=0$ $4x^2+kx+4=0$ $k^2-4(4)(4)<0$	$5(k-5y)^2+5(k-5y)y+4=0$ $5k^2-50ky+125y^2+5ky-25y^2+4=0$ $100y^2-45ky+5k^2+4=0$ $(-45k)^2-400(5k^2+4)<0$ $2025k^2-2000k^2-1600<0$ $k^2-64<0$ $(k-8)(k+8)<0$	M1 M1 +M1√ M1	For substitution D<0 with substitution For b^2-4ac factorisation
	$-8<k<8$ [5] **[10]**		DA1	Upon correct factorisation

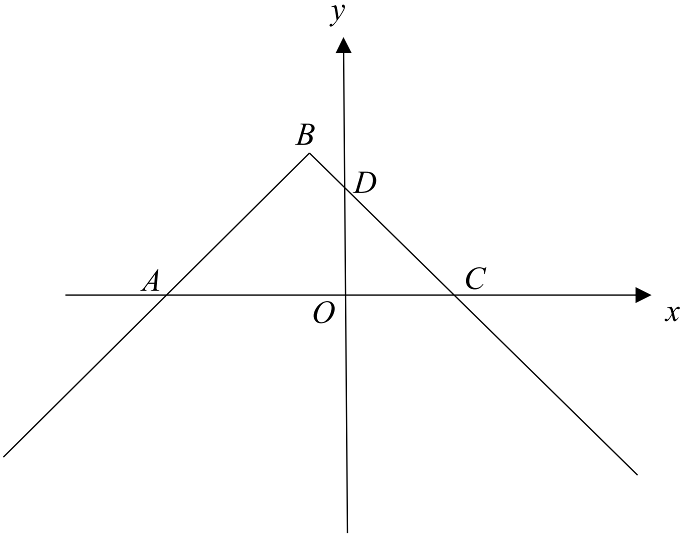
9 The diagram shows part of the graph of $y = 4 - |x+1|$.

(i) Find the coordinates of the points A, B, C and D. [5]

(ii) Find the number of solutions of the equation $4 - |x+1| = mx + 3$ when

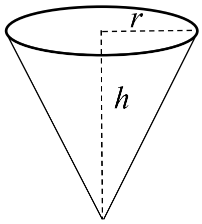
(a) $m = 2$ **(b)** $m = -1$ [2]

(iii) State the range of values of m for which the equation $4 - |x+1| = mx + 3$ has two solutions. [1]



(i)	B(−1,4), D(0,3) $4 - \|x+1\| = 0$ $\|x+1\| = 4$ $x + 1 = \pm 4$ $x + 1 = 4$ or $x + 1 = -4$ $x = 3$ or $x = -5$ $A(-5,0)$ $C(3,0)$ [5]	A1+A1 B1 A1 +A1
(ii) (a) (b)	$4 - \|x+1\| = mx + 3$ When $m = 2$, the number of solutions is 1 When $m = -1$, the number of solutions is infinite [2]	 A1 A1
(iii)	When $-1 < m < 1$, the number of solutions is 2 [1] **[8]**	A1

10 The diagram shows a cone of radius r cm and height h cm. It is given that the volume of the cone is 10π cm^3.



(i) Show that the curved surface area, A cm^2, of the cone, is $A = \dfrac{\pi\sqrt{r^6 + 900}}{r}$. [3]

(ii) Given that r can vary, find the value of r for which A has a stationary value. [4]

(iii) Determine whether this value of A is a maximum or a minimum. [2]

10(i)	Volume = $\frac{1}{3}\pi r^2 h = 10\pi$ $h = \frac{30}{r^2}$ $l^2 = r^2 + h^2$ $= r^2 + \left(\frac{30}{r^2}\right)^2$ $l = \sqrt{r^2 + \frac{900}{r^4}}$ $A = \pi r l = \pi r\sqrt{r^2 + \frac{900}{r^4}}$ $A = \pi r\sqrt{\frac{(r^6+900)}{r^4}}$ $A = \frac{\pi r\sqrt{(r^6+900)}}{r^2}$ $A = \frac{\pi\sqrt{(r^6+900)}}{r}$ [3]	B1 M1 A1	If put cm^2 -1m over all
(ii)	$u = \pi\sqrt{r^6+900}$, $v = r$ $\frac{du}{dr} = \frac{1}{2}\times\pi\times(r^6+900)^{-\frac{1}{2}}\times 6r^5$ $\frac{dv}{dr} = 1$ $\frac{du}{dr} = 3\pi r^5(r^6+900)^{-\frac{1}{2}}$ $\frac{dA}{dr} = \frac{3\pi r^6(r^6+900)^{-\frac{1}{2}} - \pi(r^6+900)^{\frac{1}{2}}}{r^2}$ When $\frac{dA}{dr} = 0$ $\frac{\pi(r^6+900)^{-\frac{1}{2}}[3r^6-r^6-900]}{r^2} = 0$ $\frac{\pi[3r^6-r^6-900]}{r^2\,(r^6+900)^{\frac{1}{2}}} = 0$ $2r^6 - 900 = 0$ $r^6 = 450$ $r = 2.77$ [4]	B1 B1 M1 A1	Either $u\frac{dv}{dx}$ or $v\frac{du}{dx}$ With the use of quotient rule or product rule Perfect $\frac{dA}{dr} = 0$ with substitution With cm -1m overall

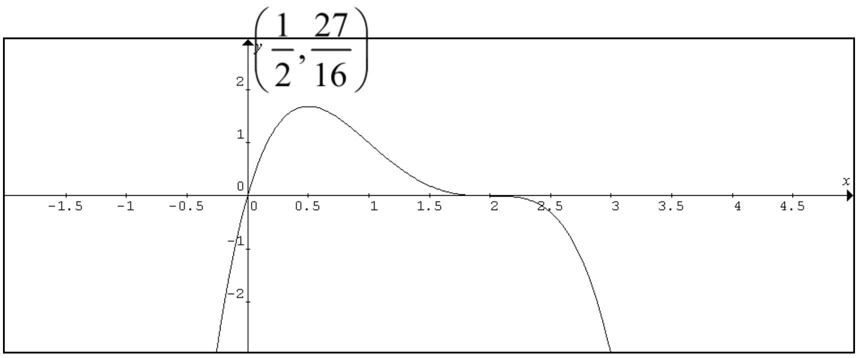
(iii)	<table><tr><td>r</td><td>r < 2.768</td><td>$r = 2.768$</td><td>r > 2.768</td></tr><tr><td>$\frac{dA}{dr}$</td><td>-</td><td>0</td><td>+</td></tr><tr><td>Sketch</td><td>\</td><td>–</td><td>/</td></tr></table> A is a minimum when $r = 2.77$ [2] **[9]**	M1 DA1	For subst with + r Upon correct $\frac{dA}{dr}$

11 The equation of a curve is $y = x(2-x)^3$.

(i) Find the range of values of x for which y is an increasing function. [5]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Hence, sketch the graph of $y = x(2-x)^3$. [3]

$y = x(2-x)^3$ $\frac{dy}{dx} = (2-x)^3(1) - 3x(2-x)^2$ $= (2-x)^2[2-x-3x]$ $= (2-x)^2(2-4x)$ when $\frac{dy}{dx} > 0$, $2-4x > 0$ $-4x > -2$ $x < \frac{1}{2}$ Note: should not see this $2-x>0$ $-x>-2$ $x<2$ No [A1] [5]	B1 +B1 A1 M1 A1	Either $v\frac{du}{dx}$ or $u\frac{dv}{dx}$ and the use of product rule Perfect for $\frac{dy}{dx} > 0$ with substitution
when $\frac{dy}{dx} = 0$, $(2-x)^2(2-4x) = 0$ $x = 2$, $x = \frac{1}{2}$ $y = 2(2-2)^3$ $y = \frac{1}{2}\left(2-\frac{1}{2}\right)^3$ $= 0$ $= \frac{27}{16}$ Ans $(2,0)$ $\left(\frac{1}{2}, \frac{27}{16}\right)$ [3]	M1 A1+A1	$\frac{dy}{dx} = 0$ with substitution If $(2-x)(2-4x) = 0$ don't penalise]
 [3] **[11]**	B1√ B1√ B1	their max pt $\left(\frac{1}{2}, \frac{27}{16}\right)$ (2,0) their pt of inflexion (0,0) -1m for less than perfect

Name: ________________________ (   )   Class: ____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS — 4047/02

Paper 2 — **Friday 17 August 2018**

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8		Q12	

/ 100

圣尼各拉女校
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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)........(n-r+1)}{r!}$

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Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

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$$\sin\,(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

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1 **(i)** On the same axes sketch the curves $y^2 = 64x$ and $y = -x^2$. [2]

(ii) Find the equation of the line passing through the points of intersection of the two curves. [4]

2 The roots of the equation $x^2 + 2x + p = 0$, where p is a constant, are α and β.
The roots of the equation $x^2 + qx + 27 = 0$, where q is a constant, are α^3 and β^3 .
Find the value of p and of q. [6]

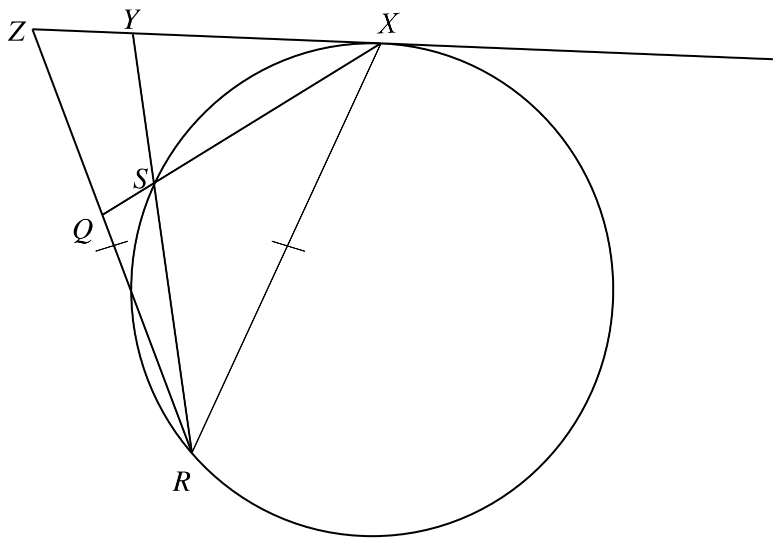
3 **(a)** Given that $3^{2x-2} \times 5^{-2x} = 27^x \div 5^{x+1}$, evaluate the exact value of 15^x. [3]

(b) Given that $\log_x y = 64\log_y x$, express y in terms of x. [4]

4 **(i)** Write down, and simplify, the first three terms in the expansion of $(1-\frac{x^2}{2})^n$,in ascending powers of x, where n is a positive integer greater than 2. [2]

(ii) The first three terms in the expansion, in ascending powers of x, of $(2+3x^2)(1-\frac{x^2}{2})^n$ are $2 - px^2 + 2x^4$, where p is an integer. Find the value of n and of p. [5]

5



In the figure, XYZ is a straight line that is tangent to the circle at X.
XQ bisects $\angle RXZ$ and cuts the circle at S. RS produced meets XZ at Y and $ZR = XR$.
Prove that

(a) $SR = SX$, [3]

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(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part **(i)**, solve the equation $3x^3 + ax^2 + bx + 4 = 0$, expressing non-integer roots in the form $\dfrac{c \pm \sqrt{d}}{3}$, where c and d are integers. [4]

7 **(a)** Prove that $\sec\theta + 1 = \dfrac{\tan\theta\sin\theta}{1-\cos\theta}$. [4]

(b) Hence or otherwise, solve $\dfrac{\tan\theta\sin\theta}{1-\cos\theta} = \dfrac{3}{4}\sec^2\theta$ for $0 \le \theta \le 2\pi$. [4]

8 The temperature, A °C, of an object decreases with time, t hours. It is known that A and t can be modelled by the equation $A = A_0 e^{-kt}$, where A_0 and k are constants.
Measured values of A and t are given in the table below.

t (hours)	2	4	6	8
A (°C)	49.1	40.2	32.9	26.9

(i) Plot $\ln A$ against t for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of A_0 and of k. [4]

(iii) Assuming that the model is still appropriate, estimate the number of hours for the temperature of the object to be halved. [2]

9 The curve $y = \mathrm{f}(x)$ passes through the point $(0,3)$ and is such that $\mathrm{f}'(x) = \left(e^x + \dfrac{1}{e^x}\right)^2$.

(i) Find the equation of the curve. [4]

(ii) Find the value of x for which $\mathrm{f}''(x) = 3$. [4]

10 A circle has the equation $x^2 + y^2 + 4x + 6y - 12 = 0$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The highest point of the circle is A.

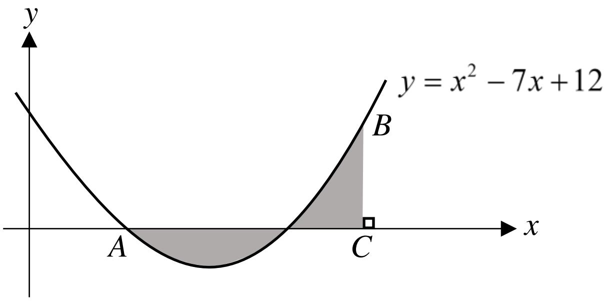
(ii) State the equation of the tangent to the circle at A. [1]

(iii) Determine whether the point $(0, -7)$ lies within the circle. [2]

The equation of a chord of the circle is $y = 7x - 14$.

(iv) Find the length of the chord. [5]

11



The diagram shows part of the curve of $y = x^2 - 7x + 12$ passing through the point B and meeting the x-axis at the point A.

(i) Find the gradient of the curve at A. [4]

The normal to the curve at A intersects the curve at B.

(ii) Find the coordinates of B. [4]

The line BC is perpendicular to the x-axis.

(iii) Find the area of the shaded region. [4]

12 A particle P moves in a straight line, so that, t seconds after passing through a fixed point O, its velocity, v m s^{-1}, is given by $v = \cos t - \sin 2t$, where $0 \le t \le \frac{\pi}{2}$. Find

(i) in terms of π, the values of t, when P is at instantaneous rest, [5]

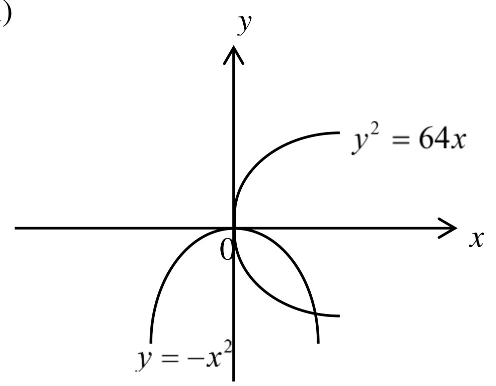
(ii) the distance travelled by P from $t = 0$ to $t = \frac{\pi}{2}$, [6]

(iii) an expression for the acceleration of P in terms of t. [1]

St Nicholas Girls School Additional Mathematics Preliminary Examination Paper II 2018

Answers

1 (i)



(ii) $y = -4x$

2 $p = 3, q = -10$

3 (a) $\frac{5}{9}$ (b) $y = x^8$, $y = x^{-8}$

4 (i) $1 - n\left(\frac{x^2}{2}\right) + \frac{n(n-1)}{8}x^4 + \cdots$ (ii) $n = 8$, $p = 5$

6 (i) $a = -8, b = 2$ (ii) $x = 2$, $x = \frac{1 \pm \sqrt{7}}{3}$

7 (b) $\frac{\pi}{3}, \frac{5\pi}{3}$

8 (ii) $A_0 = 59.7$, $k = 0.1$ (iii) 6.93

9 (i) $y = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + 3$ (ii) $\frac{1}{2}\ln 2$

10 (i) Centre $= (-2, \ -3)$, Radius $= 5$ units (ii) $y = 2$

(iii) The distance of the point from the centre of the cicle $= \sqrt{20} < \sqrt{25}$ radius of the circle, so the point lies within the circle.

(iv) $5\sqrt{2}$ units

11 (i) -1 (ii) $B(5,2)$ (iii) 1 sq unit.

12 (i) $\frac{\pi}{2}, \frac{\pi}{6}$ (ii) $\frac{1}{2}$ m (iii) $-\sin t - 2\cos 2t$

Name: ________________________ (   )   Class: ____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **Friday 17 August 2018**

Marking Scheme

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

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Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8		Q12	

/ 100

This document consists of **5** printed pages.

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[Turn over

Formulae

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1 **(i)** On the same axes sketch the curves $y^2 = 64x$ and $y = -x^2$. [2]

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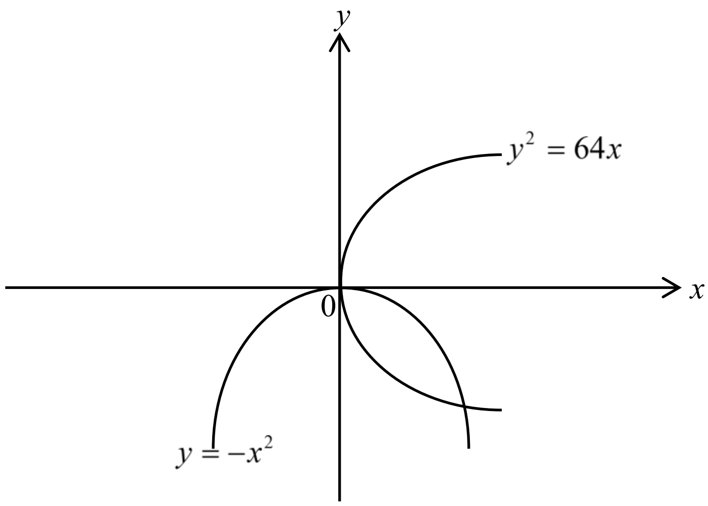
(i)



B1 +B1

-1 mark if no label

[2]

(ii)	$y^2 = 64x$ ------- (1) $y = -x^2$ ------- (2) Sub (2) into (1), $(-x^2)^2 = 64x$ $x^4 = 64x$ $x^4 - 64x = 0$ $x(x^3 - 64) = 0$ $x = 0$ or $x^3 - 64 = 0$ $y = 0$ $x^3 = 64$ $x = 4$ $y = -16$ $m = \dfrac{-16-0}{4-0}$ $= -4$ $y = -4x$ [4] **[6]**	M1 B1+B1 DA1	Solving Simultaneous Equations Either 1 pairs of x values or y values. [or 1m for each pair of x and y values] Must have $(-4,16)$

2 The roots of the equation $x^2+2x+p=0$, where p is a constant, are α and β.

The roots of the equation $x^2+qx+27=0$, where q is a constant, are α^3 and β^3.

Find the value of p and of q. [6]

2	$x^2+2x+p=0$ $\quad$ $x^2+qx+27=0$ $\alpha+\beta=-2$ $\quad$ $\alpha^3+\beta^3=-q$	B1	For both sum of roots or first pair of sum & product of roots.
	$\alpha\beta=p$ $\quad$ $\alpha^3\beta^3=27$	B1	For both product of roots or 2nd pair of product and sum of roots
	$\alpha\beta=3$		
	$p=3$	A1	
	$(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=-q$ or $(\alpha+\beta)^3-3\alpha^2\beta+3\beta^2\alpha=-q$	B1	For $\alpha^3+\beta^3$
	$(\alpha+\beta)[(\alpha+\beta)^2-2\alpha\beta-\alpha\beta]=-q$ or $(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-q$		
	$(-2)[4-9]=-q$ $\quad$ or $(-2)^3-3p(-2)=-q$	M1√	
	$q=-10$ **[6]**	A1	

3 **(a)** Given that $3^{2x-2} \times 5^{-2x} = 27^x \div 5^{x+1}$, evaluate the exact value of 15^x. [3]

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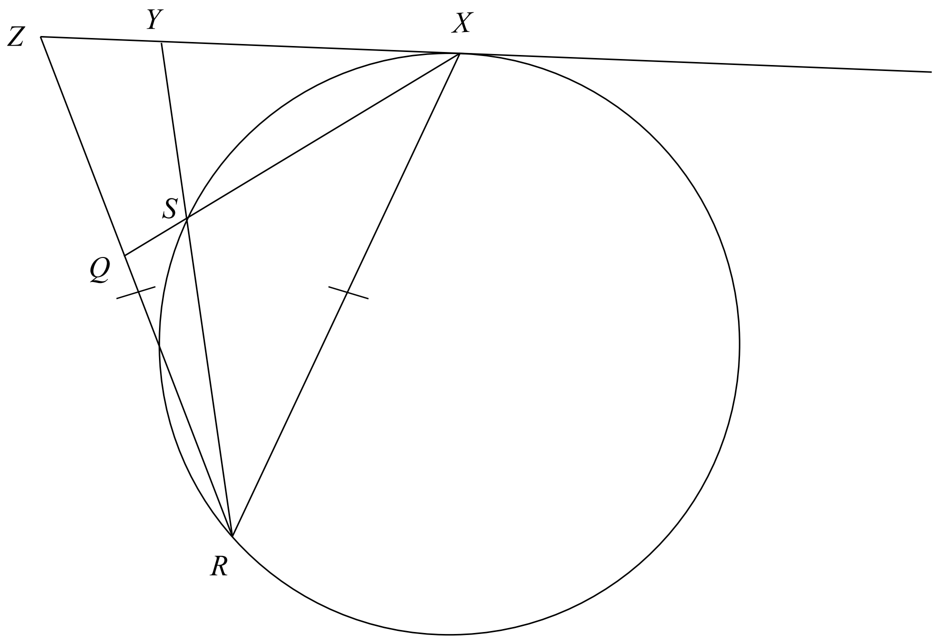
(a)	$3^{2x-2} \times 5^{-2x} = 27^x \div 5^{x+1}$ Method (i) $3^{2x-2} \times 5^{-2x} = 3^{3x} \times 5^{-1-x}$ $\frac{3^{2x-2}}{3^{3x}} = \frac{5^{-1-x}}{5^{-2x}}$ $3^{2x-2-3x} = 5^{-1-x+2x}$ $3^{-x-2} = 5^{x-1}$ $3^{-x} \times 3^{-2} = 5^x \times 5^{-1}$ $3^x \times 5^x = 5^{-1} \div 3^{-2}$	M1 M1√	applying index Law correctly on either LHS or RHS grouping and making power of x on one side
	Method (ii) $3^{2x} \times 3^{-2} \times 5^{-2x} = 3^{3x} \times 5^{-x} \times 5^{-1}$ $3^x \times 5^x = 5^{-1} \div 3^{-2}$	M1 M1√	Applying index law grouping and making power of x on one side
	$15^x = \frac{5}{9}$ [3]	A1	
(b)	$\log_x y = 64\log_y x$ $\log_x y = \frac{64 \log_x x}{\log_x y}$ $(\log_x y)^2 = 64$ $\log_x y = \pm 8$ $y = x^8, \quad y = x^{-8}$ [4] **[7]**	B1 M1√ A1+A1	change of base

4 **(i)** Write down, and simplify, the first three terms in the expansion of $(1-\frac{x^2}{2})^n$,in ascending powers of x, where n is a positive integer greater than 2. [2]

(ii) The first three terms in the expansion, in ascending powers of x, of $(2+3x^2)(1-\frac{x^2}{2})^n$ are $2-px^2+2x^4$, where p is an integer. Find the value of n and of p. [5]

(i)	$\left(1-\frac{x^2}{2}\right)^n = 1-n\left(\frac{x^2}{2}\right)+{}^nC_2\left(\frac{x^4}{4}\right)+\cdots\ldots\ldots\ldots$ $\left(1-\frac{x^2}{2}\right)^n = 1-n\left(\frac{x^2}{2}\right)+\frac{n(n-1)}{8}x^4+\cdots\ldots\ldots\ldots$	M1 B1	 Or any two terms 1m, perfect 2m [2]
(ii)	$(2+3x^2)(1-\frac{x^2}{2})^n = (2+3x^2)(1-\frac{nx^2}{2}+\frac{n(n-1)}{8}x^4+\cdots.)$ $= 2-nx^2+\frac{n(n-1)}{4}x^4+3x^2-\frac{3n}{2}x^4+\cdots\ldots\ldots..$ $= 2-(n-3)x^2+\left(\frac{n^2-7n}{4}\right)x^4+\cdots....$ $= 2-px^2+2x^4+\cdots....$ $\frac{n^2-7n}{4}=2$ $n^2-7n-8=0$ $(n-8)(n+1)=0$ $n=8, n=-1(\text{NA})$ $-n+3=-p$ $-8+3=-p$ $p=5$	 M1√ M1√ DA1 M1√ A1	 factorisation Upon correct factorisation [5] **[7]**

5



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Prove that

(a) $SR = SX$, [3]

(b) a circle can be drawn passing through Z, Y, S and Q. [4]

(a)	$\angle ZXQ = \angle SRX$ (Alternate Segment Theorem) $\angle ZXQ = \angle QXR$ (XQ is the angle bisector of $\angle RXZ$) $\angle QXR = \angle SRX$ By base angles of isosceles triangles, SR=SX [3]	B1 B1 B1
(b)	Let $\angle QXR$ be x $\angle RSX = 180^\circ - 2x$ (Isosceles Triangle) $\angle YSQ = 180^\circ - 2x$ (Vertically Opposite Angles) $\angle RZX = \angle ZXR = 2x$ (Base angles of Isosceles Triangle) $\angle RZX + \angle YSQ = 180^\circ - 2x + 2x = 180^\circ$ Since opposite angles are supplementary in cyclic quadrilaterals, a circle that passes through Z, Y, S and Q can be drawn **Alternative** [4] Similar but use of tangent secant theorem. **[7]**	B1 B1 B1 B1

[Turn over

6 The expression $3x^3 + ax^2 + bx + 4$, where a and b are constants, has a factor of $x - 2$ and leaves a remainder of -9 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part **(i)**, solve the equation $3x^3 + ax^2 + bx + 4 = 0$, expressing non-integer roots in the form $\dfrac{c \pm \sqrt{d}}{3}$, where c and d are integers. [4]

(i)	$f(x) = 3x^3 + ax^2 + bx + 4$ x-2 is a factor $\quad f(2) = 0$ $3(8) + 4a + 2b + 4 = 0$ $4a + 2b + 28 = 0$ $2a + b + 14 = 0$---------(1) $f(-1) = -9$ $-3 + a - b + 4 = -9$ $a - b = -10$---------(2) (1)+(2) $\quad 3a = -24$ $a = -8$ Sub into (2) $\quad -8 - b = -10$ $b = 2$ [4]	 M1 M1 A1 A1
(ii)	$$\begin{array}{r|l} & 3x^2 - 2x - 2 \\ x - 2 & 3x^3 - 8x^2 + 2x + 4 \\ & 3x^3 - 6x^2 \\ \hline & -2x^2 + 2x \\ & -2x^2 + 4x \\ \hline & -2x + 4 \\ & -2x + 4 \\ \hline \end{array}$$ $3x^3 - 8x^2 + 2x + 4 = 0$ $(x - 2)(3x^2 - 2x - 2) = 0$ $x - 2 = 0 \qquad 3x^2 - 2x - 2 = 0$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times -2}}{2 \times 3}$ $x = \frac{2 \pm \sqrt{28}}{6}$ $x = \frac{2(1 \pm \sqrt{7})}{6}$ $x = 2 \qquad x = \frac{1 \pm \sqrt{7}}{3}$ [4] **[8]**	 B1 M1√ A1 +A1

7 **(a)** Prove that $\sec\theta + 1 = \dfrac{\tan\theta\sin\theta}{1-\cos\theta}$. [4]

(b) Hence or otherwise, solve $\dfrac{\tan\theta\sin\theta}{1-\cos\theta} = \dfrac{3}{4}\sec^2\theta$ for $0 \le \theta \le 2\pi$. [4]

(a)	$RHS = \dfrac{\tan\theta\sin\theta}{1-\cos\theta}$ $= \dfrac{\frac{\sin\theta}{\cos\theta}\sin\theta}{1-\cos\theta}$ $= \dfrac{\frac{\sin^2\theta}{\cos\theta}}{1-\cos\theta}$ $= \dfrac{1-\cos^2\theta}{(1-\cos\theta)\cos\theta}$ $= \dfrac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)\cos\theta}$ $= \dfrac{1+\cos\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta} + 1$ $= \sec\theta + 1$ [4]	B1 B1 B1 B1	change *tan* change $\sin^2$ to $\cos^2$ identity $a^2 - b^2$ split and bring to answer
(b)	$\dfrac{\tan\theta\sin\theta}{1-\cos\theta} = \dfrac{3}{4}\sec^2\theta$ $1 + \sec\theta = \dfrac{3}{4}\sec^2\theta$ $3\sec^2\theta - 4\sec\theta - 4 = 0$ $(\sec\theta - 2)(3\sec\theta + 2) = 0$ $\sec\theta = 2$ or $\sec\theta = -\dfrac{2}{3}$ $\cos\theta = \dfrac{1}{2}$ or $\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ $\cos\theta = -\dfrac{3}{2}$ (No Solution) $= 1.05, 5.24$ [4] **[8]**	B1 M1 DA1+ DA1	substitution factorization 1st DA1 for change to cos & no soln Upon correct factorisation

8 The temperature, A °C, of an object decreases with time, t hours. It is known that A and t can be modelled by the equation $A = A_0 e^{-kt}$, where A_0 and k are constants.

Measured values of A and t are given in the table below.

t (hours)	2	4	6	8
A (°C)	49.1	40.2	32.9	26.9

(i) Plot $\ln A$ against t for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of A_0 and of k. [4]

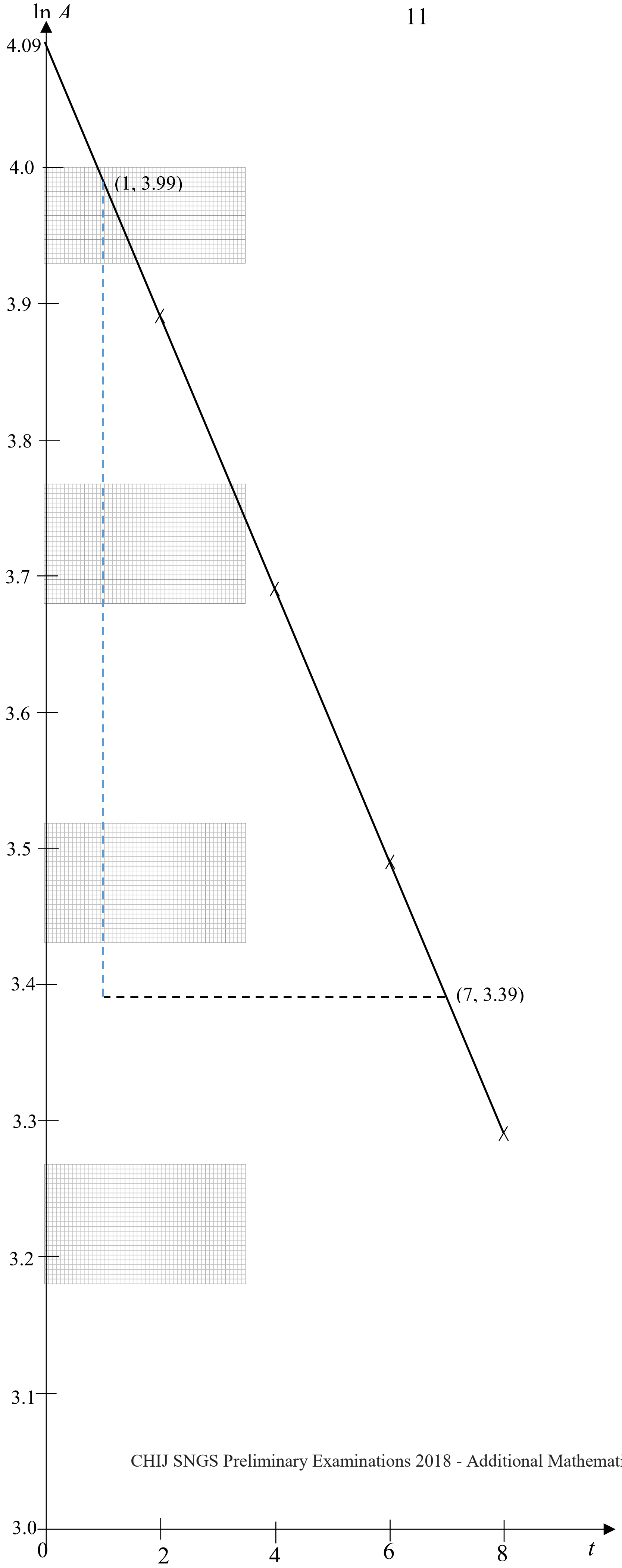
(iii) Assuming that the model is still appropriate, estimate the number of hours for the temperature of the object to be halved. [2]

8 **(i)** B1 for correct points, values & correct axes.

B1 best fit line . [2]

t	2	4	6	8
$\ln A$	3.89	3.69	3.49	3.29

(ii)	$A = A_0 e^{-kt}$ $\ln A = -kt + \ln A_0$ $-k = \text{gradient}$ $-k = \dfrac{3.39 - 3.99}{7 - 1}$ $k = 0.1 \pm 0.02$ $\ln A_0 = 4.09$ $A_0 = e^{4.09}$ $A_0 = 59.7$ (3s.f.) ± 4 [4]	 M1 A1 M1 A1	 gradient vertical intercept
(iii)	$\dfrac{1}{2}A_0 = 29.865$ Or $\frac{1}{2}A_0 = A_0 e^{-kt}$ $\ln 29.865 = 3.396$ OR $\dfrac{1}{2} = e^{-0.1t}$ From the graph, $t = 6.9$ $t = 6.93$ (3s.f.) [2] **[8]**	 √M1 A1 ±0.5	


ln A
4.09
4.0
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3.0
(1, 3.99)
(7, 3.39)
0
2
4
6
8
t

9 The curve $y = \mathrm{f}(x)$ passes through the point $(0, 3)$ and is such that $\mathrm{f}'(x) = \left(e^x + \frac{1}{e^x}\right)^2$.

(i) Find the equation of the curve. [4]

(ii) Find the value of x for which $\mathrm{f}''(x) = 3$. [4]

(i)	$y = \int \left(e^x + \frac{1}{e^x}\right)^2 \mathrm{d}x$	M1	knowing $y = \int \mathrm{f}(x)\,\mathrm{d}x$
	$= \int e^{2x} + 2 + e^{-2x}\,\mathrm{d}x$		
	$= \frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} + c$	B1	ignore no + c
	at $(0,3)$, $3 = \frac{1}{2}e^0 + 2(0) - \frac{1}{2}e^0 + c$	M1	for ubstitution
	$c = 3$		
	$y = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + 3$	A1	
	[4]		
(ii)	$\mathrm{f}'(x) = e^{2x} + 2 + e^{-2x}$ $\quad\quad$ $\mathrm{f}'(x) = \left(e^x + e^{-x}\right)^2$		
	$\mathrm{f}''(x) = 2e^{2x} - 2e^{-2x}$ $\quad\quad$ $\mathrm{f}''(x) = 2\left(e^x + e^{-x}\right)\left(e^x - e^{-x}\right)$	B1	
	when $\mathrm{f}''(x) = 3$, $2e^{2x} - 2e^{-2x} = 3$		
	Let $e^{2x} = a$, $\quad 2a - \frac{2}{a} = 3$		
	$2a^2 - 2 = 3a$		
	$2a^2 - 3a - 2 = 0$		
	$(2a+1)(a-2) = 0$	M1	factorisation
	$a = -\frac{1}{2}$ $\quad a = 2$		
	$e^{2x} = -\frac{1}{2}$ $\quad e^{2x} = 2$	+DA1	Upon correct factorisation
	no solution $\quad 2x = \ln 2$		
	$x = \frac{1}{2}\ln 2 = \ln\sqrt{2} = 0.347$	+DA1	
	[4]		
	[8]		

10 A circle has the equation $x^2 + y^2 + 4x + 6y - 12 = 0$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The highest point of the circle is A.

(ii) State the equation of the tangent to the circle at A. [1]

(iii) Determine whether the point $(0, -7)$ lies within the circle. [2]

The equation of a chord of the circle is $y = 7x - 14$.

(iv) Find the length of the chord. [5]

<table>
<tr><td rowspan="2">(i)</td><td>$x^2+y^2+4x+6y-12=0$
$x^2+y^2+2gx+2fy+c=0$
$2g=4$ $\quad 2f=6$
$g=2$ $\quad f=3$
Centre = $(-2,\ -3)$
Radius $=\sqrt{g^2+f^2-C}$
$=\sqrt{(-2)^2+(-3)^2-(-12)}$</td><td>$(x)^2+2(x)(2)+(2)^2+(y)^2+2(y)(3)+(3)^2$
$=12+(2)^2+(3)^2$
$(x+2)^2+(y+3)^2=25$</td><td>A1

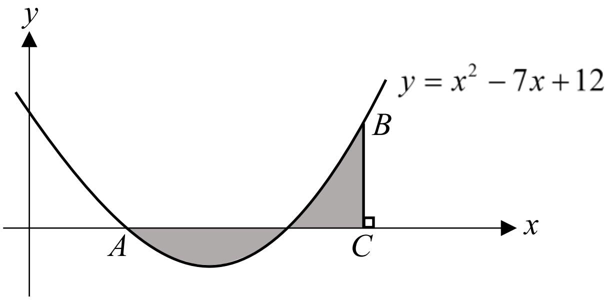
M1</td></tr>
<tr><td colspan="2">Radius $=5$ units [3]</td><td>A1 ignore no unit</td></tr>
<tr><td>(ii)</td><td colspan="2">$y=2$ (y= their y coord of centre +radius) [1]</td><td>B1 √</td></tr>
<tr><td>(iii)</td><td colspan="2">The distance of the point from the centre of the cicle
$=\sqrt{(0-(-2))^2+(-7-(-3))^2}$
$=\sqrt{20}\quad<\sqrt{25}$
Since it is lesser than the radius of the circle, it lies within the circle. [2]</td><td>M1√ their centre

DA1</td></tr>
<tr><td>(iv)</td><td colspan="2">$y=7x-14$ -------- (1)
$x^2+y^2+4x+6y-12=0$ -------- (2)
Sub (1) into (2),
$x^2+(7x-14)^2+4x+6(7x-14)-12=0$
$x^2+49x^2-196x+196+4x+42x-84-12=0$
$50x^2-150x+100=0$
$x^2-3x+2=0$
$(x-1)(x-2)=0$
$x=1$ or $x=2$ Sub into (1),
$y=-7$ or $y=0$
The length of the chord $=\sqrt{(1-2)^2+(-7-0)^2}$
$=\sqrt{50}$
$=5\sqrt{2}$ units [5]
[11]</td><td>M1 Solving simultaneous equations

M1 Factorizing
B1 Either 1 pair correct or both x solutions are correct
√M1

A1 accept 7.07</td></tr>
</table>

11



The diagram shows part of the curve of $y = x^2 - 7x + 12$ passing through the point B and meeting the x-axis at the point A.

(i) Find the gradient of the curve at A. [4]

The normal to the curve at A intersects the curve at B.

(ii) Find the coordinates of B. [4]

The line BC is perpendicular to the x-axis.

(iii) Find the area of the shaded region. [4]

(i)	$y = x^2 - 7x + 12$ $= (x-3)(x-4)$ $\frac{dy}{dx} = 2x - 7$ $when\, x = 3,\ \frac{dy}{dx} = 2(3) - 7$ $= -1$ [4]	M1 B1 M1 A1	using smaller (positive) x value
(ii)	$\perp\ m = 1$ sub $m = 1$ and $(3,0)$ into $y = mx + c$ $0 = 1(3) + c$ $c = -3$ equation of normal: $y = x - 3$ $x^2 - 7x + 12 = x - 3$ or $(x-3)(x-4) = x - 3$ $x^2 - 8x + 15 = 0$ $x - 4 = 1$ $(x-3)(x-5) = 0$ $x = 5$ $x = 3 \quad x = 5$ $y = 2$ $B(5,2)$ [4]	M1 M1 M1 A1	sub $\perp$m and their(3,0) curve and normal factorisation
(iii)	Area $= \left\lvert \int_3^4 x^2 - 7x + 12\, dx \right\rvert + \int_4^5 x^2 - 7x + 12\, dx$ $= \left\lvert \left[\frac{x^3}{3} - \frac{7x^2}{2} + 12x \right]_3^4 \right\rvert + \left[\frac{x^3}{3} - \frac{7x^2}{2} + 12x \right]_4^5$ $= \left\lvert \left(\frac{64}{3} - \frac{7(16)}{2} + 12(4) \right) - \left(\frac{27}{3} - \frac{7(9)}{2} + 12(3) \right) \right\rvert$ $+ \left(\frac{125}{3} - \frac{7(25)}{2} + 12(5) \right) - \left(\frac{64}{3} - \frac{7(16)}{2} + 12(4) \right)$ $= \left\lvert 13\frac{1}{3} - 13\frac{1}{2} \right\rvert + 14\frac{1}{6} - 13\frac{1}{3}$ $= \left\lvert -\frac{1}{6} \right\rvert + \frac{5}{6}$ $= 1$ sq unit [4] **[12]**	M1 B1 M1 A1	Area $= \left\lvert \int y\, dx \right\rvert + \int y\, dx$ √their limits from (i) and (ii) for integration substitution

12 A particle P moves in a straight line, so that, t seconds after passing through a fixed point O, its velocity, v m s^{-1}, is given by $v = \cos t - \sin 2t$, where $0 \leq t \leq \dfrac{\pi}{2}$. Find

(i) in terms of π, the values of t, when P is at instantaneous rest, [5]

(ii) the distance travelled by P from $t = 0$ to $t = \dfrac{\pi}{2}$, [6]

(iii) an expression for the acceleration of P in terms of t. [1]

(i)	$v = \cos t - \sin 2t$ when $v = 0$, $\cos t - \sin 2t = 0$ $\cos t - 2\sin t \cos t = 0$ $\cos t(1 - 2\sin t) = 0$ $\cos t = 0$ $\sin t = \frac{1}{2}$ $t = \frac{\pi}{2}$ $t = \frac{\pi}{6}$ [5]	B1 B1 M1 A1+A1	For v=0 for double angle factorisation
(ii)	$s = \int \cos t - \sin 2t \, \mathrm{d}t$ $= \sin t + \frac{1}{2}\cos 2t + c$ when $t = 0, s = 0$ $0 = \sin 0 + \frac{1}{2}\cos 0 + c$ $c = -\frac{1}{2}$ $s = \sin t + \frac{1}{2}\cos 2t - \frac{1}{2}$ when $t = \frac{\pi}{6}$, $s = \sin\frac{\pi}{6} + \frac{1}{2}\cos\frac{\pi}{3} - \frac{1}{2}$ $= \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}$ $= \frac{1}{4}$ when $t = \frac{\pi}{2}$, $s = \sin\frac{\pi}{2} + \frac{1}{2}\cos\pi - \frac{1}{2}$ [6] $= 1 + \frac{1}{2}(-1) - \frac{1}{2}$ $= 0$ Distance travelled $= 2\left(\frac{1}{4}\right)$ $= \frac{1}{2}$ m	B1 B1+B1 M1 M1 DA1	For $s = \int v \, \mathrm{d}t$ Integration ignore no +c Sub either $t = \frac{\pi}{6}$ or $t = \frac{\pi}{2}$ For both s for $t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$ found
(iii)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = (-\sin t - 2\cos 2t) m / s^2$ [1] **[12]**	B1	