1 Express $\frac{8x^2 - 2x + 19}{(1-x)(4+x^2)}$ in partial fractions. [5]

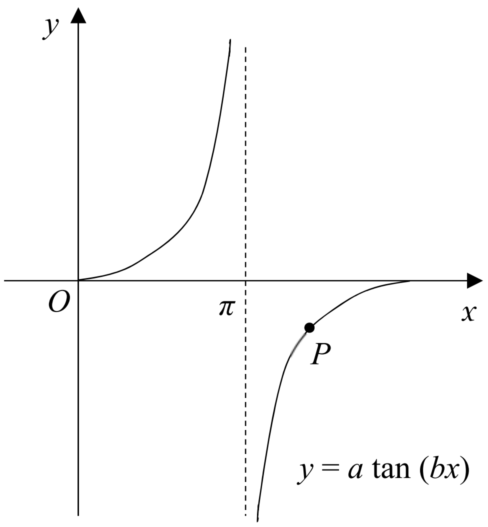
2 **(i)** On the same axes sketch the curves $y = -\sqrt{x}$ and $y = -\sqrt{32}\,x^3$. [2]

(ii) Find the x-coordinates of the points of intersection of the two curves. [2]

3 **(a)** Given that $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, express θ in terms of π.

Hence, find the exact value of $\sin 2\theta + \tan \theta$. [4]

(b)



The figure shows part of the graph of $y = a \tan (bx)$ and a point $P\left(\frac{3\pi}{2}, -2\right)$ marked. Find the value of each of the constants a and b. [2]

4 The equation of a curve is $y = e^x + 2e^{-x}$.

(i) Find the coordinates of the stationary point of the curve, leaving your answer in exact form. [4]

(ii) Determine the nature of this point. [2]

5 **(i)** Sketch the graph of $y = \left|4 - \frac{x}{2}\right| - 1$, indicating clearly the vertex and the intercepts on the coordinate axes. [3]

(ii) State the range of y. [1]

(iii) Find the values of x for $\left|4 - \frac{x}{2}\right| - 1 = 6$. [2]

(iv) The graph $y = \left|4 - \frac{x}{2}\right| - 1$ is reflected in the y-axis.
Write down the equation of the new graph. [1]

6 **(a)** Find the maximum and minimum values of $(1 - \cos A)^2 - 5$ and the corresponding value(s) of A where each occurs for $0° \le A \le 360°$. [4]

(b) A, B and C are angles of a triangle such that $\cos A = -\frac{1}{\sqrt{5}}$ and $\sin B = \frac{5}{13}$.

(i) State the range of values for A. [1]

(ii) Find the exact value of $\cos(A + B)$.

Hence find the exact value of $\cos C$. [4]

7 **(a)** **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{4x}\right) = \frac{1 - \ln x}{4x^2}$. [3]

(ii) Integrate $\frac{\ln x}{x^2}$ with respect to x. [4]

(b) Given that $\int_1^5 f(x)\,\mathrm{d}x = 8$, find $\int_1^2 f(x)\,\mathrm{d}x - \int_5^2 [f(x) + 3x]\,\mathrm{d}x$. [3]

8 **(a)** A curve C is such that $\frac{dy}{dx} = 8\cos 2x$ and $P\left(\frac{\pi}{3}, 2\sqrt{3} - 3\right)$ is a point on C.

(i) The normal to the curve at P crosses the x-axis at Q.

Find the coordinates of Q. [3]

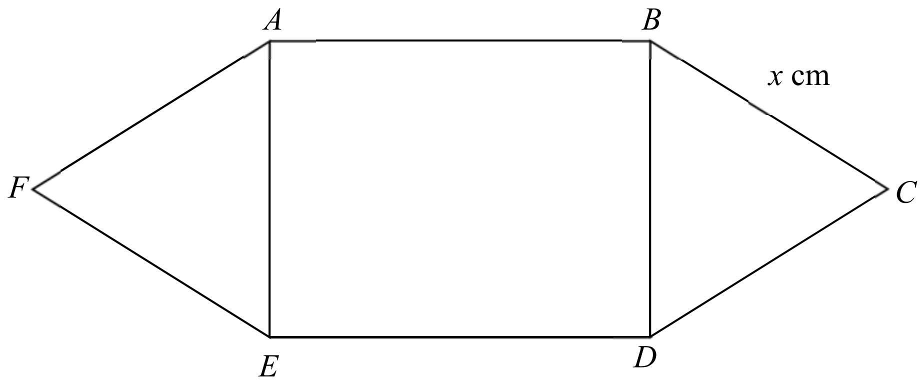
(ii) Find the equation of C. [3]

(b) Given that $y = \sin 4x$, show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx} = -32\sin 8x$. [4]

9 **(a)** Find the range of values of k for which $2x(2x + k) + 6 = 0$ has no real roots.[4]

(b) If p and q are roots of the equation $x^2 + 2x - 1 = 0$ and $p > q$, express $\frac{q}{p^2}$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

10



A hexagon $ABCDEF$ has a fixed perimeter of 210 cm.
BCD and AFE are 2 equilateral triangles and $ABDE$ is a rectangle.
The length of BC is represented as x cm.

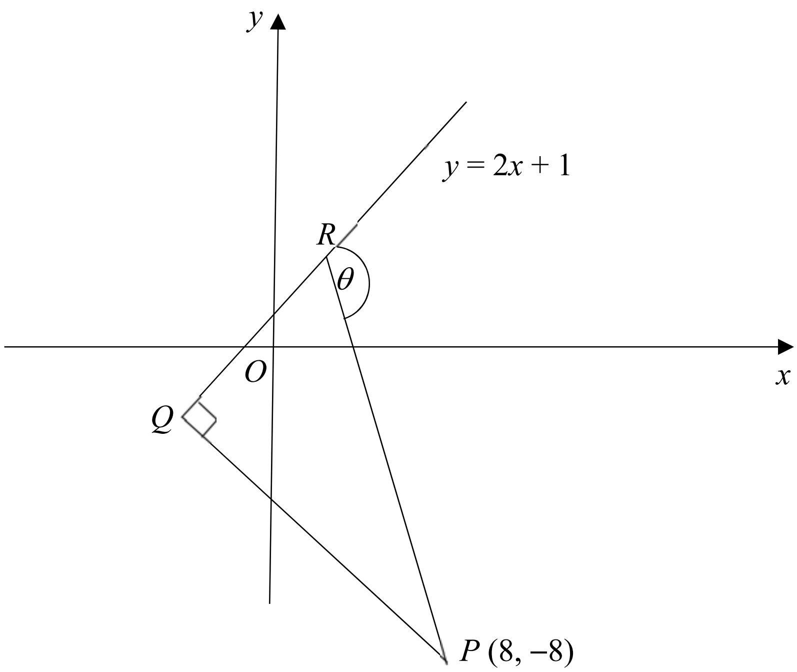
(i) Express AB in terms of x. [1]

(ii) Show that the area of the hexagon, H is given by

$$H = \left(\frac{\sqrt{3}}{2} - 2\right)x^2 + 105x.$$ [2]

(iii) Find the value of x for which H is a maximum. [4]

11



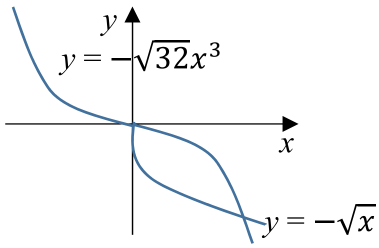
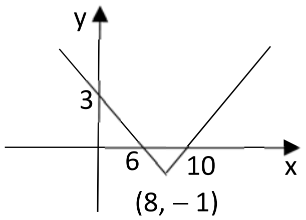
The diagram shows triangle PQR in which the point P is (8, −8) and angle PQR is 90°. The gradient of PR is $-\frac{13}{8}$ and the equation of QR produced is $y = 2x + 1$.

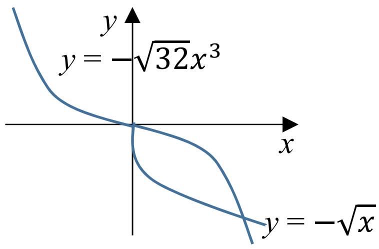
The line PR makes an angle θ with QR produced.

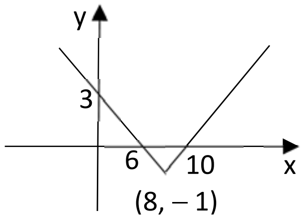
(i) Find the coordinates of Q. [4]

(ii) Find the value of θ. [3]

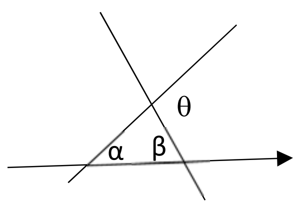
Answers

1	$\dfrac{5}{1-x} - \dfrac{3x+1}{4+x^2}$
2(i)	
2(ii)	$x = 0$ or $\frac{1}{2}$
3(a)	$\theta = -\frac{\pi}{3}$ $\quad$ $2\sin\theta\cos\theta + \tan\theta = -\frac{3}{2}\sqrt{3}$
3(b)	$a = 2;\ \ b = \frac{1}{2}$
4(i)	$(\ln\sqrt{2}, 2\sqrt{2})$ (ii) Minimum point
5(i)	
5(ii)	$y \geq -1$ (iii) $x = -6$ or 22
5(iv)	$y = \left\lvert 4 + \frac{x}{2} \right\rvert - 1$
6(a)	Max value = –1 when A = 180° Min value = – 5 when A = 0°,360°
6(b)(i)	90° < A < 180° or $\frac{\pi}{2} < A < \pi$
6(b)(ii)	$\cos(A+B) = -\frac{22}{13\sqrt{5}}$ $\quad$ $\cos C = \frac{22}{13\sqrt{5}}$
7(a)(ii)	$\int \frac{\ln x}{x^2}\,dx = = -\frac{1}{x} - \frac{\ln x}{x} + c$ $\quad$ (b) $39\frac{1}{2}$
8(a)(i)	$Q(\ 12 - 8\sqrt{3} + \frac{\pi}{3}, 0)$ or $(-0.809, 0)$
8(ii)	$y = 4\sin 2x - 3$
9(a)	$-\sqrt{24} < k < \sqrt{24}$
9(b)	$p = -1 + \sqrt{2}$, $q = -1 - \sqrt{2}$ $\quad$ $\frac{q}{p^2} = -7 - 5\sqrt{2}$
10(i)	$AB = 105 - 2x$
10(iii)	$x = 46.3$ Maximum H
11(i)	$Q(-2, -3)$ (ii) θ = 121.8°

Qn	Working	Marks
1	$\dfrac{8x^2-2x+19}{(1-x)(4+x^2)} = \dfrac{A}{1-x} + \dfrac{Bx+C}{4+x^2}$ $8x^2-2x+19 = A(4+x^2) + (Bx+C)(1-x)$ Sub $x = 1$, $8 - 2 + 19 = 5A$ $\quad A = 5$ Sub $x = 0$, $19 = 4(5) + C$ $\quad C = -1$ Compare coeff of x^2, $8 = A - B$ $\quad B = -3$ $\dfrac{8x^2-2x+19}{(1-x)(4+x^2)} = \dfrac{5}{1-x} - \dfrac{3x+1}{4+x^2}$	B1 correct PF M1 A2 For all 3correct A1 For 2 correct √ A1 Only if B1 awarded
	Total	**5 marks**
2(i)		G1 G1
2(ii)	$x^{\frac{1}{2}} = \sqrt{32}\,x^3$ $x = 32x^6$ $x(1 - 32x^5) = 0$ $x = 0$ or $\frac{1}{2}$	M1 A1
	Total	**4 marks**
3(a)	$\theta = -\frac{\pi}{3}$ $2\sin\theta\cos\theta + \tan\theta = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(-\sqrt{3}\right)$ $= -\frac{3}{2}\sqrt{3}$	B1 B1 value of cos θ B1 value of tan θ B1
3(b)	$a = 2$ Period $= 2\pi = \frac{\pi}{b}$ $\quad b = \frac{1}{2}$	B1 B1
	Total	**6 marks**
4(i)	$\frac{dy}{dx} = e^x - 2e^{-x} = 0$ $e^{2x} = 2$ $x = \ln\sqrt{2}$ $y = e^{ln\sqrt{2}} + 2e^{-ln\sqrt{2}}$ $= \sqrt{2} + \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$ $\quad$ Point is $(\ln\sqrt{2}, 2\sqrt{2})$	M1 $\frac{dy}{dx} = 0$ B1 Differentiate A1 value of x B1 *o.e.*
4(ii)	$\frac{d^2y}{dx^2} = e^x + 2e^{-x}$ $x = \ln\sqrt{2}$, $\frac{d^2y}{dx^2} = 2 + \frac{2}{\sqrt{2}} > 0$ Minimum point	M1 Knowing test Correct concl based on test √A1
	Total	**6 marks**

Qn	Working	Marks
5(i)		G1 vertex G1 *x* ints G1 *y* int
5(ii)	$y \geq -1$	B1
5(iii)	$\left\|4 - \frac{x}{2}\right\| - 1 = 6$ $\left\|4 - \frac{x}{2}\right\| = 7$ $4 - \frac{x}{2} = 7$ or $4 - \frac{x}{2} = -7$ $x = -6$ or 22	 M1 or by counting A1
5(iv)	$y = \left\|4 + \frac{x}{2}\right\| - 1$	B1
	Total	**7 marks**
6(a)	$(1 - \cos A)^2 - 5$ Max value $= (1-(-1))^2 - 5 = -1$ When $\cos A = -1$, $A = 180°$ Min value $= (1-1)^2 - 5 = -5$ When $\cos A = 1$, $A = 0°, 360°$	 B1 B1 B1 B1
6(b)(i)	$90° < A < 180°$ or $\frac{\pi}{2} < A < \pi$	B1
6(b)(ii)	$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $= -\frac{1}{\sqrt{5}}\left(\frac{12}{13}\right) - \frac{2}{\sqrt{5}}\left(\frac{5}{13}\right)$ $= -\frac{22}{13\sqrt{5}}$ $\cos C = \cos(180° - (A + B))$ $= -\cos(A + B)$ $= \frac{22}{13\sqrt{5}}$	B1 value of cos B B1 value of sin A B1 √B1 e
	Total	**9marks**

Qn	Working	Marks
7(a)(i)	$\frac{d}{dx}\left(\frac{\ln x}{4x}\right) = \frac{4x\left(\frac{1}{x}\right) - 4\ln x}{(4x)^2}$ $= \frac{4-4\ln x}{16x^2}$ $= \frac{1-\ln x}{4x^2}$ (shown)	M1 quotient rule M1 diff ln x B1 working seen
7(a)(ii)	$\int \frac{1-\ln x}{4x^2}\,dx = \frac{\ln x}{4x} + c_1$ $\frac{1}{4}\int \frac{\ln x}{x^2}\,dx = \int \frac{1}{4}x^{-2}\,dx - \frac{\ln x}{4x} + c_1$ $= \frac{x^{-1}}{-4} - \frac{\ln x}{4x} + c_1$ $\int \frac{\ln x}{x^2}\,dx = = -\frac{1}{x} - \frac{\ln x}{x} + c$	B1 use integ^{n} as reverse of diff Ignore if +c is missing B1 rearrange terms B1 $\int x^{-2}\,dx$ B1 must have +c
7(b)	$\int_1^2 f(x)\,dx + \int_2^5 [f(x) + 3x]dx$ $= \int_1^2 f(x)\,dx + \int_2^5 f(x)dx + \int_2^5 3x\,dx$ $= 8 + \left[\frac{3x^2}{2}\right]_2^5$ $=8 + [\frac{3}{2}(25) - \frac{3}{2}(4)]$ $= \frac{79}{2} = 39\frac{1}{2}$	M1 switch limits and –ve becomes +ve B1 correct integral A1
	Total	**10 marks**
8(a)(i)	When $x = \frac{\pi}{3}$, $\frac{dy}{dx} = 8\cos\frac{2\pi}{3} = -4$ $\frac{0-(2\sqrt{3}-3)}{x-\frac{\pi}{3}} = \frac{1}{4}$ $Q(12 - 8\sqrt{3} + \frac{\pi}{3}, 0)$ or $(-0.809, 0)$	B1 M1 A1
8(ii)	$y = 4\sin 2x + c$ Sub $\left(\frac{\pi}{3}, 2\sqrt{3}-3\right)$ $2\sqrt{3} - 3 = 4\sin\frac{2\pi}{3} + c$ $2\sqrt{3} - 3 = 4\left(\frac{\sqrt{3}}{2}\right) + c$ $y = 4\sin 2x - 3$	B1 ignore if +c missing M1 A1
8(iii)	$\frac{dy}{dx} = 4\cos 4x$ $\frac{d^2y}{dx^2} = -16\sin 4x$ $\frac{d^2y}{dx^2} \times \frac{dy}{dx} = (-16\sin 4x)(4\cos 4x)$ $= -32(2\sin 4x\cos 4x)$ $= -32\sin 8x$	B1 $\frac{d}{dx}\sin x = \cos x$ B1 $\frac{d}{dx}\cos x = -\cos x$ B1 use of chain rule B1 2sin4xcos4x seen
	Total	**10 marks**

Qn	Working	Marks
9(a)	$2x(2x + k) + 6 = 0$ $4x^2 + 2kx + 6 = 0$ Discriminant < 0 $(2k)^2 - 4(4)(6) < 0$ $k^2 - 24 < 0$ $\quad (k - \sqrt{24})(k + \sqrt{24}) < 0$ $-\sqrt{24} < k < \sqrt{24}$	 B1 For D < 0 M1 correct sub M1 Solve ineq A1 (M0 if $k < \pm\sqrt{24}$)
9(b)	$x^2 + 2x - 1 = 0$ $x = \frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2}$ $p = -1 + \sqrt{2}$, $q = -1 - \sqrt{2}$ $\frac{q}{p^2} = \frac{-1-\sqrt{2}}{(-1+\sqrt{2})^2}$ $= \frac{-1-\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$ $= \frac{-3-2(2)-3\sqrt{2}-2\sqrt{2}}{9-4(2)}$ $= -7 - 5\sqrt{2}$	 M1 A1 p > q M1 rationalise M1 simplify A1
	Total	**9 marks**
10(i)	$4x + 2(AB) = 210$ $AB = 105 - 2x$	 B1
10(ii)	$H = 2\left(\frac{1}{2}\right)x^2\sin 60 + (105 - 2x)x$ $= \frac{\sqrt{3}}{2}x^2 + 105x - 2x^2$ $= \left(\frac{\sqrt{3}}{2} - 2\right)x^2 + 105x$ (shown)	B1 Area of Δ B1 sub & working
10(iii)	$\frac{dH}{dx} = 2\left(\frac{\sqrt{3}}{2} - 2\right)x + 105$ $\frac{dH}{dx} = 0$ $x = 46.3$ $\frac{d^2H}{dx^2} = \sqrt{3} - 4 < 0$ Maximum H	B1 M1 A1 B1 test & concl
	Total	**7marks**
11(i)	Eqn of PQ: $y - (-8) = -\frac{1}{2}(x - 8)$ $y = -\frac{1}{2}x - 4$ ----------(1) QR: $y = 2x + 1$ ----------(2) Solving simultaneously $Q(-2, -3)$	B1 correct m_{PQ} B1 form eqn M1 A1
11(ii)	$\tan\alpha = 2$ $\alpha = 63.43°$ $\tan\beta = \frac{13}{8}$ $\beta = 58.39°$  $\theta = 63.43° + 58.39°$(ext ∠ of Δ) $= 121.8°$	M1 use grads to Find angles M1 manipulate ∠s A1
	Total	**7 marks**

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2018
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4047/02

Paper 2

Tuesday 28 August 2018

2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1).......(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

1 The amount of energy, E erg, generated in an earthquake is given by the equation $E = 10^{a+bM}$, where a and b are constants and M is the magnitude of the earthquake.

The table below shows some corresponding values of M and E.

M	1	2	3	4	5
E (erg)	2.0×10^{13}	6.3×10^{14}	2.0×10^{16}	6.3×10^{17}	2.0×10^{19}

(i) Plot lg E against M. [2]

(ii) Using your graph, find an estimate for the value of a and of b. [3]

(iii) Using your answers from **(ii)**, find the amount of energy generated, in erg, by an earthquake of magnitude 7. [2]

2 **(i)** Write down the expansion of $(3 - x)^3$ in ascending powers of x. [1]

(ii) Expand $(3 + 2x)^8$, in ascending powers of x, up to the term in x^3. [3]

(iii) Write down the expansion of $(3 - x)^3\,(3 + 2x)^8$ in ascending powers of x, up to x^2. [2]

(iv) By letting $x = 0.01$ and your expansion in **(iii)**, find the value of $2.99^3 \times 3.02^8$, giving your answer correct to 3 significant figures.
Show your workings clearly. [2]

(v) Explain clearly why the expansion in **(iii)** is not suitable for finding the value of $2^3 \times 5^8$. [2]

3 **(i)** By writing 3θ as $(2\theta + \theta)$, show that $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$. [3]

(ii) Solve $\sin(3\theta) = 3\sin\theta\cos\theta$ for $0° < \theta < 360°$. [5]

4 The equation $x^2 + bx + c = 0$ has roots α and β, where $b > 0$.

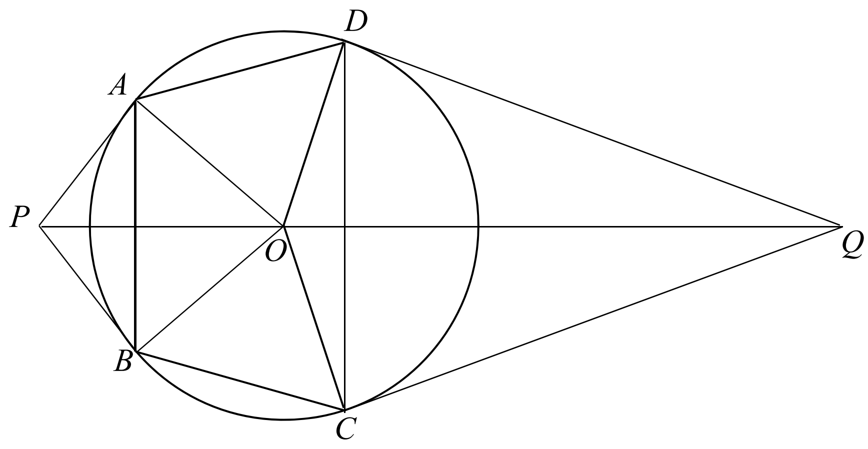
(i) Write down, in terms of b and/or c, the value of $\alpha + \beta$ and of $\alpha\beta$. [1]

(ii) Find a quadratic equation with roots α^2 and β^2, in terms of a and b. [3]

(iii) Find the relation between b and c for which the equation found in **(ii)** has two distinct roots. [2]

(iv) Give an example of values of b and c which satisfy the relation found in **(iii)**. [1]

5 In the diagram, A, B, C and D are points on the circle centre O.
AP and BP are tangents to the circle at A and B respectively.
DQ and CQ are tangents to the circle at D and C respectively.
POQ is a straight line.



(i) Prove that angle $COD = 2 \times$ angle CDQ. [3]

(ii) Make a similar deduction about angle AOB. [1]

(iii) Prove that $2 \times$ angle OAD = angle CDQ + angle BAP. [4]

6 **(i)** Differentiate $y = 2e^{3x}(1 - 2x)$ with respect to x. [3]

(ii) Find the range of values of x for which y is decreasing. [1]

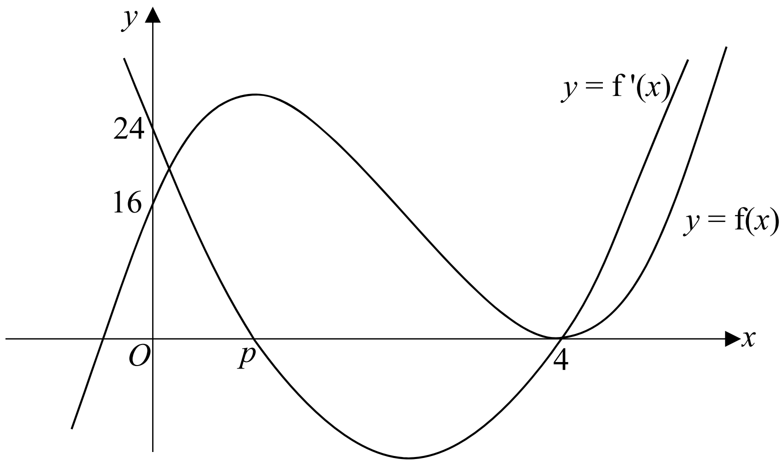
(iii) Given that x is decreasing at a rate of 5 units per second, find the rate of change of y at the instant when $x = -1.5$. [3]

7 **(i)** By using an appropriate substitution, express $2^{3a+1} - 2^{2a+2} + 2^a$ as a cubic function. [3]

(ii) Solve the equation $2^{3a+1} - 2^{2a+2} + 2^a = 0$. [5]

(iii) Find the range of values of k for which $2^{3a+1} - 2^{2a+2} + k(2^a) = 0$ has at least one real solution. [3]

8 The diagram shows the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}'(x)$.



The function $\mathrm{f}(x) = ax^3 + bx^2 + 24x + 16$ has stationary points at $x = p$ and $x = 4$.

(i) Find an expression for $\mathrm{f}'(x)$, in terms of a and b. [1]

(ii) Find the value of a and of b. [3]

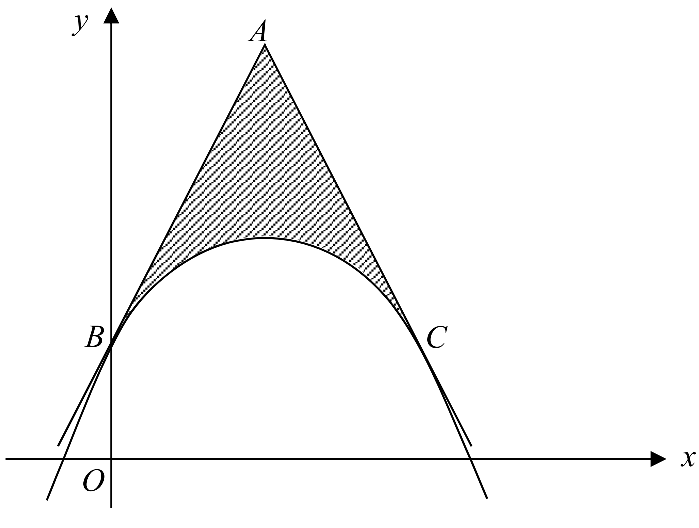
(iii) Find the value of p.
State the range of values of k, where $k > 0$ and $y = \mathrm{f}(x) - k$ has only one real root. [3]

(iv) Find the minimum value of the gradient of $\mathrm{f}(x)$. [2]

9 The diagram shows the graph of $y = -\frac{1}{2}(x-2)^4 + 16$.

AB and AC are tangents to the curve at B and C respectively.

B lies on the y-axis and $AB = AC$.



(i) Find the gradient function of the curve. [1]

(ii) Find the equation of the tangent at B.
Hence, state the coordinates of A. [3]

(iii) Find the area of the shaded region. [6]

10 A particle, P, travels along a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s is given by

$$v = (12e^{kt} + 18), \text{ where } k \text{ is a constant.}$$

(i) Find the initial velocity of the particle. [1]

Two seconds later, its velocity is 40 m/s.

(ii) Show that $k = 0.3031$, correct to 4 significant figures. [3]

(iii) Sketch the graph of $v = 12e^{kt} + 18$, for $0 \le t \le 4$. [3]

(iv) Explain why the distance travelled by P during the 4 seconds does not exceed 180 metres. [2]

(v) Find the maximum acceleration of P during the interval $0 \le t \le 4$. [2]

11 A circle, C_1, with centre A, has equation $x^2 + y^2 - 8x - 4y - 5 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

(ii) Show that (1, 6) lies on the circle. [1]

(iii) Find the equation of the tangent to the circle at (1, 6). [3]

The equation of the tangent to the circle at (1, 6) cuts the x-axis at B.

(iv) Find the coordinates of B. [2]

Another circle, C_2, has centre at B and radius r.

(v) Find the exact value of r given that circle C_2 touches circle C_1. [3]

End of Paper

Answers:

1 (i) $a = 11.7$ to 11.9, $b = 1.49$ to 1.51 (iii) $E = 2.0 \times 10^{22}$ Erg

2 (i) $27 - 9x + 3x^2 - x^3$ (ii) $6\,561 + 34\,992x + 81\,648x^2 + 108\,864x^3 + \ldots$
(iii) $177\,147 + 885\,735x + 1\,909\,251x^2 + \ldots$
(iv) 186 000
(v) For $2^3 \times 5^8$, need to use $x = 1$
Since 1 is large in comparison to 0.01, the value is inaccurate because a significantly large value is removed after the 3rd term.

3 (ii) 104.5°, 255.5°, 180°

4 (i) $\alpha + \beta = -b$, $\alpha\beta = c$ (ii) $x^2 - (b^2 - 2c)x + c^2 = 0$ o.e
(iii) $b^2 - 4c > 0$ (iv) $b = 5$, $c = 2$ o.e.

6 (i) $\frac{dy}{dx} = 2e^{3x}(1 - 6x)$ (ii) $x > \frac{1}{6}$ (iii) -1.11 units/sec

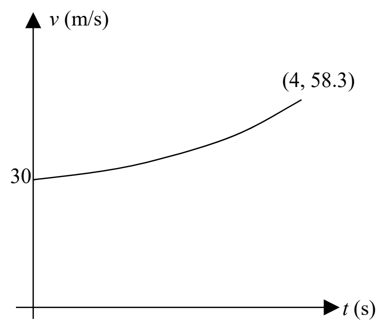
7 (i) $2x^3 - 4x^2 + x$ (o.e.) (ii) $a = 0.7771$ or -1.77 (iii) $k \leq 2$

8 (i) $f'(x) = 3ax^2 + 2bx + 24$ (ii) $a = 2$, $b = -15$
(iii) $p = 1$, $k > 27$ (iv) -13.5

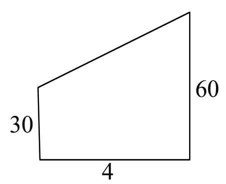
9 (i) $\frac{dy}{dx} = -2(x - 2)^3$ (ii) Eq AB: $y = 16x + 8$, A is (2, 40)
(iii) 38.4 units^2

10 (i) 30 m/s

(iii)



(iv) area of trapezium < 0.5(30 + 60) × 4 = 180

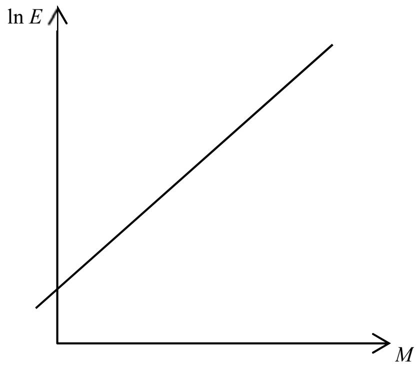


∴ distance travelled < 180 m

(v) max $a = 12.23$ m/s^2

11 (i) A is (4, 2), Radius = 5 units (iii) $4y - 3x = 21$ (o.e.)

(iv) (−7, 0) (v) $r = 5\sqrt{5} - 5$

<table>
<tr><th>Qn</th><th>Key Steps</th><th>Marks / Remarks</th><th></th></tr>
<tr><td>1(i)</td><td><table><tr><td>M</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$\ln E$</td><td>13.3</td><td>14.8</td><td>16.3</td><td>17.8</td><td>19.3</td></tr></table>

</td><td>B1 TOV

B1 Line passes through pts</td><td></td></tr>
<tr><td>(ii)</td><td>$\lg E = a + bM$
a = vertical intercept = 11.8
b = gradient (their rise/run)
= 1.5</td><td>
B1 11.7 to 11.9
M1 working for gradient
A1 1.49 to 1.51</td><td></td></tr>
<tr><td>(iii)</td><td>$\lg E = 11.8 + 1.5(7) = 22.3$
$E = 2.0 \times 10^{22}$ Erg</td><td>M1
A1 1.34×10^{22} to 2.95×10^{22}</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>2(i)</td><td>$(3 - x)^3 = 27 - 27x + 9x^2 - x^3$</td><td>B1</td><td></td></tr>
<tr><td>(ii)</td><td>$(3 + 2x)^8$
$= 3^8 + \binom{8}{1}(3)^7(2x) + \binom{8}{2}(3)^6(2x)^2 + \binom{8}{3}(3)^5(2x)^3$
$= 6\,561 + 34\,992x + 81\,648x^2 + 108\,864x^3 + \ldots$</td><td>B3 1m for each term (2nd to 4th)
−1m if 1st term missing
B0 is all not evaluated</td><td></td></tr>
<tr><td>(iii)</td><td>$(3 - x)^3\,(3 + 2\text{x})^8$
= their (i) × their (ii)
$= 177\,147 + 767\,637x + 2\,854\,035x^2 + \ldots$</td><td>
M1 choosing correct pairs
A1</td><td></td></tr>
<tr><td>(iv)</td><td>$2.99^3 \times 3.02^8$
$= 177\,147 + 767\,637(0.01) + 2\,854\,035(0.01)^2$
= 185108.7735
= 185 000</td><td>
B1 Subn must be seen

B1 reject 184 956</td><td></td></tr>
<tr><td>(v)</td><td>For $2^3 \times 5^8$, need to use $x = 1$

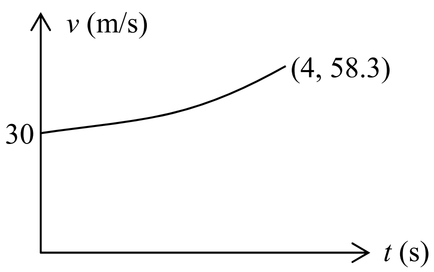
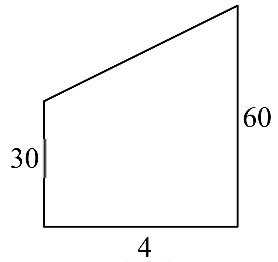
Since 1 is large in comparison to 0.01, the value is inaccurate because a significantly large value is removed after the 3rd term</td><td>B1 $x = 1$ seen

B1 o.e. "big" or "large" seen</td><td>10</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

Qn	Key Steps	Marks / Remarks	
3(i)	$\sin(\theta + 2\theta)$ $= \sin\theta\cos 2\theta + \cos\theta\sin 2\theta$ $= \sin\theta(1 - 2\sin^2\theta) + \cos\theta(2\sin\theta\cos\theta)$ $= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta\cos^2\theta$ $= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta(1 - \sin^2\theta)$ $= 3\sin\theta - 4\sin^3\theta$	B1 Use compound angle B1 Any double angle seen B1 Use identity AG	
(ii)	$\sin(3\theta) = 3\sin\theta\cos\theta$ $3\sin\theta - 4\sin^3\theta = 3\sin\theta\cos\theta$ $\sin\theta(3 - 4\sin^2\theta - 3\cos\theta) = 0$ $\sin\theta = 0 \quad \therefore \theta = 180^\circ$ or $3 - 4\sin^2\theta - 3\cos\theta = 0$ $3 - 4(1 - \cos^2\theta) - 3\cos\theta = 0$ $4\cos^2\theta - 3\cos\theta - 1 = 0$ $(4\cos\theta + 1)(\cos\theta - 1) = 0$ $\cos\theta = -\frac{1}{4}$ or $\cos\theta = 1$ (NA) Hence, $\theta = 104.5^\circ, 255.5^\circ$	B1 $\theta = 180^\circ$ seen M1 Solve a quadratic B1 Use identity B2 −1m for extra answer	**8**
4(i)	$\alpha + \beta = -b \qquad \alpha\beta = c$	B1 Both correct	
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= b^2 - 2c$ $\alpha^2\beta^2 = c^2$ Eqn: $x^2 - (b^2 - 2c)x + c^2 = 0$ ✓	B1 Correct sum B1 Correct product B1 Equation seen	
(iii)	For 2 distinct roots, $(b^2 - 2c)^2 - 4c^2 > 0$ ✓ $b^2(b^2 - 4c) > 0$ Since $b^2 > 0$, hence $b^2 - 4c > 0$	B1 Correct D ok if $[-(b^2 - 2c)]^2$ or $(b^2 - 2c)^2$ B1 o.e.	
(iv)	$b = 5, c = 2$ ✓	B1 o.e.	**7**

Qn	Key Steps	Marks / Remarks	
5(i)	Let $\angle CDQ = a$ $\angle ODQ = 90°$ (tan ⊥ rad) $\therefore\ \angle ODC = 90° - a$ $\therefore\ \angle COD = 180° - 2(90° - a)$ ($\angle$sum, ΔCOD)	B1 with reason B1 B1 with reason	
(ii)	$\angle AOB = 2 \times \angle BAP$	B1	
(iii)	From (i) and (ii), $2(\angle CDQ + \angle BAP) = \angle COD + \angle AOB$ $\angle CDQ + \angle BAP = \frac{1}{2}(\angle COD + \angle AOB)$ $= \angle AOP + \angle DOQ$ (⊥ prop of chord) $= 180° - \angle AOD$ $= 2\angle OAD$	B1 attempt to use (i) and (ii) B1B1 1m for reason B1	**8**
6(i)	$y = 2e^{3x}(1-2x)$ $\frac{dy}{dx} = 2e^{3x}(-2) + 6e^{3x}(1-2x)$ $= 2e^{3x}(1-6x)$	B1 Product Rule B1 Diff exponential fn B1 Simplify, ok if not factorised	
(ii)	For decreasing function, $\frac{dy}{dx} < 0$ $\therefore\ 1 - 6x < 0$ $x > \frac{1}{6}$ ✓	B1	
(iii)	Given that $\frac{dy}{dx} = -5$ units/s $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= 2e^{3x}(1-6x)(-5)$ $= 2e^{3(-1.5)}(1 + 6 \times 1.5)(-5)$ $= -1.11$ units/sec	B1 with negative seen B1 with subs seen B1	**7**

Qn	Key Steps	Marks / Remarks	
7(i)	$2^{3a+1} - 2^{2a+2} + 2^a$ Let $2^a = x$ $= 2 \times 2^{3a} - 4 \times 2^{2a} + 2^a$ $= 2x^3 - 4x^2 + x$	B1 Use of: $2^{p+q} = 2^p \times 2^q$ B1 Use of: $(2^p)^q = 2^{pq}$ B1	
(ii)	$2x^3 - 4x^2 + x = 0$ $x(2x^2 - 4x + 1) = 0$ $x = 0$, $\therefore 2^a = 0$ (rej) or $2x^2 - 4x + 1 = 0$ $x = \dfrac{-4 \pm \sqrt{16 - 4 \times 2 \times 1}}{4}$ $= 1.707$ or 0.2929 $2^a = 1.707$ or 0.2929 $a = \dfrac{\lg 1.707}{\lg 2}$ or $\dfrac{\lg 0.2929}{\lg 2}$ $= 0.7771$ or -1.77	B1 $x = 0$ seen M1 Solving quad with working seen A1 Both x M1 Using log (any base) A1 Both a	
(iii)	$2^{3a+1} - 2^{2a+2} + (k)2^a = 0$ has at least one root $\therefore 2x^2 - 4x + k = 0$ has at least one root $\therefore 16 - 4 \times 2 \times k \geq 0$ $k \leq 2$	M1 Using quad part of eqn B1 Correct D with subs A1	**11**
8(i)	$f(x) = ax^3 + bx^2 + 24x + 16$ $f'(x) = 3ax^2 + 2bx + 24$	B1	
(ii)	Sub (4, 0) into $f'(x) = 0$ $3a(16) + 2b(4) + 24 = 0$ $\therefore 48a + 8b + 24 = 0$ ……………………(1) Sub (4, 0) into $f(x)$ $a(64) + 16b + 24(4) + 16 = 0$ $\therefore 64a + 16b + 96 + 16 = 0$ …………….(2) $a = 2$, $b = -15$	B1 Sub into their $f'(x)$ and $f(x)$ M1 Solve simul eqn A1 Both	
(iii)	$f'(x) = 6x^2 - 30x + 24$ $= 6(x^2 - 5x + 4)$ $= 6(x - 1)(x - 4)$ $\therefore p = 1$ At $x = 1$, $f(x) = 2(1) - 15(1) + 24(1) + 16 = 27$ Hence, $k > 27$	B1 M1 Using their p A1	
(iv)	Min value of $f'(x) = 6(2.5)^2 - 30(2.5) + 24$ $= -13.5$	M1 Use $x = 2.5$ A1	**9**

Qn	Key Steps	Marks / Remarks	
9(i)	$y = -\frac{1}{2}(x-2)^4 + 16, \quad \therefore \frac{dy}{dx} = -2(x-2)^3$	B1 o.e.	
(ii)	Grad of $AB = -2(-8) = 16$ At B, $x = 0$, $\therefore y = 8$ Eqn AB: $y = 16x + 8$ $\therefore A$ is (2, 40)	B1 Grad AB seen B1 Eqn AB seen B1	
(iii)	Area $OBACD$ $= (8 + 40) \times 2$ $= 96$ units^2 Area bounded by curve and axes $= \int_0^4 \left(-\frac{1}{2}(x-2)^4 + 16\right) dx$ $= \left(-\frac{1}{10}(x-2)^5 + 16x\right)_0^4$ $= (-\frac{1}{10} \times 32 + 64) - (\frac{1}{10} \times 32)$ $= 57.6$ $\therefore$ shaded area – 96 – 57.6 = 38.4 units^2	M1 Using composite figures A1 B1 Knowing to use integral for area B1 Correct integration B1 Subs seen B1	**10**
10(i)	$v_0 = 12e^{k(0)} + 18 = 30$ m/s	B1 Sub need not be seen	
(ii)	$v_2 = 40 \quad \therefore 40 = 12e^{k(2)} + 18$ $e^{2k} = \frac{11}{6}$ $2k = \ln(\frac{11}{6})$ $k = 0.303\ 1$	B1 Sub into eqn B1 Using logarithm B1	
(iii)		B1 Shape B1 Label y-intercept B1 Label (4, 58.3)	
(iv)	Area under curve < Area of trapezium Area of trapezium = 0.5(30 + 60) × 4 = 180  $\therefore$ distance travelled < 180 m	B1 Find relevant distance travelled using any suitable method B1 Making conclusion	
(v)	Max accn occurs at $t = 4$ where the gradient is most steep Max accn $= 0.3031 \times 12\ e^{0.3031(4)}$ $= 12.23$ m/s^2	M1 Knowing to differentiate A1	**11**

Qn	Key Steps	Marks / Remarks	
11(i)	$x^2 + y^2 - 8x - 4y - 5 = 0$ A is (4, 2) Radius = $\sqrt{4^2 + 2^2 + 5}$ = 5 (units)	B1 M1A1	
(ii)	$1^2 + 6^2 - 8(1) - 4(6) - 5 = 0$ Hence, (1, 6) lies on the circle.	B1 Subs seen and statement	
(iii)	Gradient of line joining (4, 2) and (1, 6) $= -\frac{4}{3}$ Eqn of tangent at (1, 6) is $y - 6 = -\frac{4}{3}(x - 1)$ $4y - 3x = 21$	B1 ⊥ grad seen B1 Find eqn B1 o.e.	
(iv)	At B, $y = 0$ $\therefore x = -7$ $\therefore B$ is (−7, 0)	M1 Finding x A1 Ordered pair seen	
(v)	Distance between centres $= \sqrt{11^2 + 2^2}$ $= \sqrt{125}$ $\therefore$ radius of C_2 $= \sqrt{125} - 5$ $= 5\sqrt{5} - 5$	M1 Find dist between centres M1 Using sum radii = distance A1	**12**