ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4047/01

PAPER 1

11 September 2018
2 hours

Additional Materials: Writing paper, Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1 .$$

$$\sec^2 A = 1 + \tan^2 A .$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A .$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

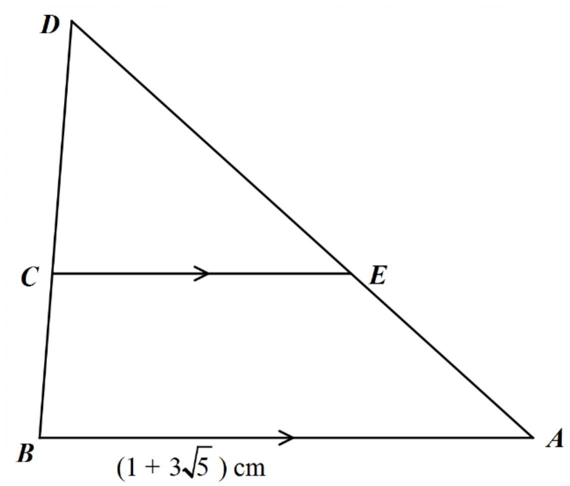
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all the questions

1.



AB is parallel to EC and $AB = (1+3\sqrt{5})$ cm. E is a point on AD such that $AE : ED = \sqrt{5} : 3$.
Find

(i) $\dfrac{EC}{AB}$ in the form of $a + b\sqrt{5}$, where a and b are rational numbers. [3]

(ii) the length of EC in the form of $c + d\sqrt{5}$, where c and d are integers. [3]

2. The equation of a curve is $y = (k+2)x^2 - 10x + 2k + 1$, where k is a constant.

(i) In the case where $k = 1$, sketch the graph of $y = (k+2)x^2 - 10x + 2k + 1$, showing the x- and y- intercepts and its turning point clearly. [3]

(ii) Find the range of values of k for which the curve meets the line $y = 2x + 3$. [5]

3. (a) Express $\dfrac{3x^3 - 5}{x^2 - 1}$ in partial fractions. [5]

(b) Solve the equation $|21 - 18x| - |7 - 6x| = 4x - 1$. [4]

4. The equation of a curve is $y = 2x(x-1)^3$.

(i) Find the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of each of these points using the first derivative test. [3]

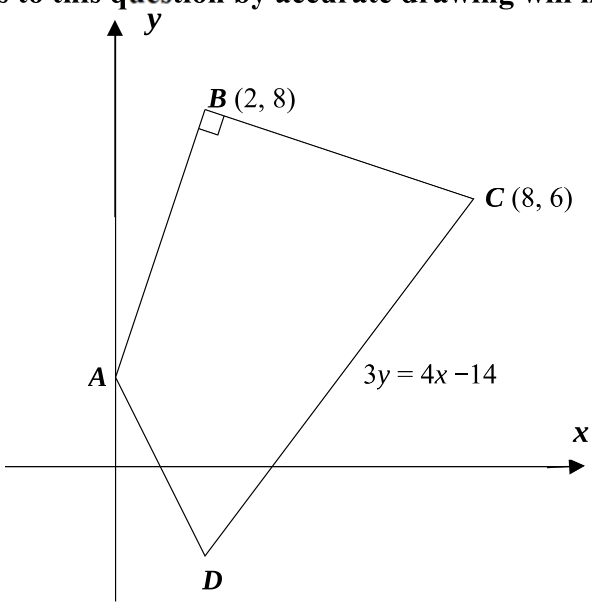
5. (i) On the same diagrams, sketch the graphs $y = \dfrac{4}{x^2}$, $x > 0$ and $y = 3x^{\frac{1}{2}}$, $x \geq 0$. [2]

(ii) Find the value of the constant k for which the $x-$coordinate of the point of intersection of your graphs is the solution to the equation $x^5 = k$. [2]

6. (i) Prove that $\dfrac{1}{3\tan^2\theta+3}=\dfrac{\cos^2\theta}{3}$. [2]

(ii) Show that $\displaystyle\int_0^{\frac{\pi}{3}}\frac{\sec^2\theta\cos 2\theta}{3\tan^2\theta+3}\,\mathrm{d}\theta=\frac{\sqrt{3}}{12}$. [4]

7. **Solutions to this question by accurate drawing will not be accepted.**



The diagram above shows a quadrilateral *ABCD*. Point *B* is (2, 8) and point *C* is (8, 6).

The point *D* lies on the perpendicular bisector of *BC* and the point *A* lies on the *y*-axis.

The equation of *CD* is $3y=4x-14$ and angle $ABC=90^\circ$. Find

(i) the equation of *AB*, [2]

(ii) the coordinates of *A*, [1]

(iii) the equation of the perpendicular bisector of *BC*, [3]

(iv) the coordinates of *D*, [3]

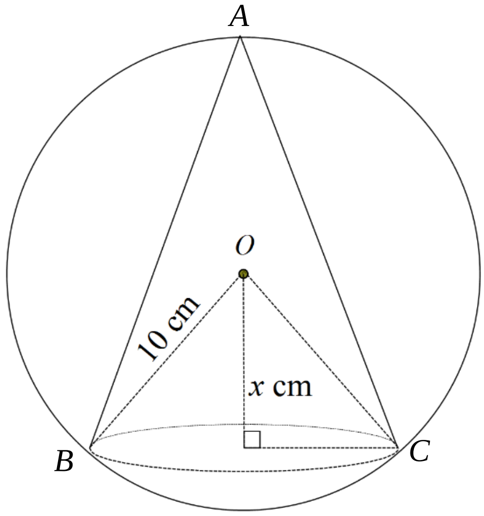
8. (i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^2\ln x-3x)=x+2x\ln x-3$. [2]

(ii) Evaluate $\displaystyle\int_1^4 x\ln x\,\mathrm{d}x$. [4]

9. A curve is such that the gradient function is $1+\dfrac{1}{2x^2}$. The equation of the tangent at point *P* on the curve is $y=3x+1$. Given that the *x*-coordinate of *P* is positive, find the equation of the curve. [7]

[Turn over

10.



A right circular cone, *ABC*, is inscribed in a sphere of radius 10 cm and centre *O*.

The perpendicular distance from *O* to the base of the cone is *x* cm.

$$\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$$

(i) Show that volume, *V*, of the cone is $V = \frac{1}{3}\pi(100 - x^2)(10 + x)$. [2]

(ii) If *x* can vary, find the value of *x* for which *V* has a stationary value. [3]

(iii) Find this stationary volume. [1]

(iv) Determine whether the volume is a maximum or minimum. [2]

11. (a) Find, in radians, the two principal values of *y* for which $2\tan^2 y + \tan y - 6 = 0$. [4]

(b) The height, *h* m, above the ground of a carriage on a carnival ferris wheel is modelled by the equation $h = 7 - 5\cos(8t)$, where *t* in the time in minutes after the wheel starts moving.

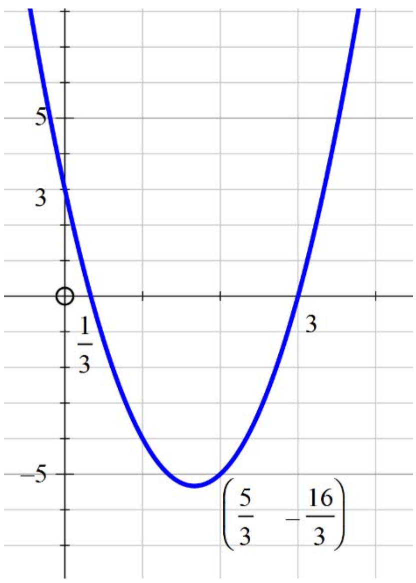
(i) State the initial height of the carriage above ground. [1]

(ii) Find the greatest height reached by the carriage. [1]

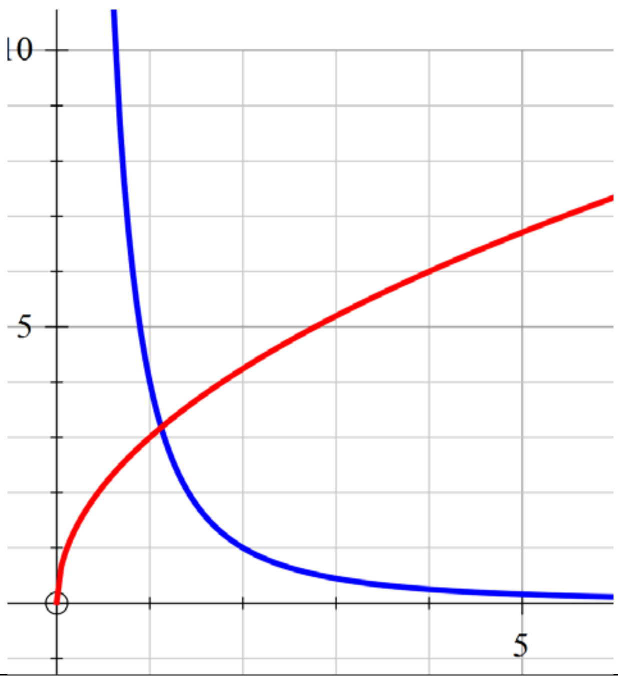
(iii) Calculate the duration of time when the carriage is 9 m above the ground. [3]

END OF PAPER

4E5N 2018 Prelim AMath paper 1 Marking Scheme

1i	ΔABD is similar to ΔECD. $\therefore \frac{CE}{BA}=\frac{DE}{DA}$ $\frac{CE}{BA}=\frac{3}{3+\sqrt{5}}$ [M1] ratio seen $=\frac{3}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}$ [B1] correct conjugate surd $=\frac{9-3\sqrt{5}}{3^2-5}$ $=\frac{9-3\sqrt{5}}{4}$ [A1]	
ii	$EC=\frac{9-3\sqrt{5}}{4}\times\left(1+3\sqrt{5}\right)$ [M1] $=\frac{1}{4}\left[9\left(1+3\sqrt{5}\right)-3\sqrt{5}\left(1+3\sqrt{5}\right)\right]$ $=\frac{1}{4}\left(9+27\sqrt{5}-3\sqrt{5}-9\times 5\right)$ [M1] expansion seen $=\frac{1}{4}\left(-36+24\sqrt{5}\right)$ $=-9+6\sqrt{5}$ [A1]	
2i	When $k=1$, $y=3x^2-10x+3$ $=3\left(x^2-\frac{10}{3}\right)+3$ $=3\left[\left(x-\frac{10}{6}\right)^2-\left(\frac{10}{6}\right)^2\right]+3$ $=3\left(x-\frac{5}{3}\right)^2-\frac{25}{3}+3$ $=3\left(x-\frac{5}{3}\right)^2-\frac{16}{3}$ Turning point $\left(\frac{5}{3},\ -\frac{16}{3}\right)$ When $y=0$, $x=3$ or $\frac{1}{3}$  [B1] y-intercept [B1] x-intercepts [B1] turning point	

ii	$(k+2)x^2-10x+2k+1=2x+3$ [M1] substitution $(k+2)x^2-12x+2k-2=0$ $b^2-4ac\ge 0$ [B1] $(-12)^2-4(k+2)(2k-2)\ge 0$ $144-8(k^2+k-2)\ge 0$ $-8k^2-8k+160\ge 0$ $k^2+k-20\le 0$ $(k+5)(k-4)\le 0$ [M1] factorisation $-5\le k\le 4$ and $k\ne -2$ [A1] [A1]	
3i	By long division [M1] $\dfrac{3x^3-5}{x^2-1}=3x+\dfrac{3x-5}{x^2-1}$ [A1] $\dfrac{3x-5}{(x+1)(x-1)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}$ $3x-5=A(x-1)+B(x+1)$ [M1] any acceptable method to find A and B $x=1$: $3(1)-5=2B$ $B=-1$ $x=-1$: $-3-5=-2A$ $A=4$ [A1] correct A and B $\therefore\dfrac{3x^3-5}{x^2-1}=3x+\dfrac{4}{x+1}-\dfrac{1}{x-1}$ [A1]	
ii	$\lvert 21-18x\rvert-\lvert 7-6x\rvert=4x-1$ $\lvert 3(7-6x)\rvert-\lvert 7-6x\rvert=4x-1$ [B1] factorise 3 $3\lvert 7-6x\rvert-\lvert 7-6x\rvert=4x-1$ $2\lvert 7-6x\rvert=4x-1$ $\lvert 7-6x\rvert=\dfrac{4x-1}{2}$ $7-6x=\dfrac{4x-1}{2}$ or $7-6x=\dfrac{-4x+1}{2}$ [B1] either one seen $x=\dfrac{15}{16}$ or $x=\dfrac{13}{8}$ [A1] [A1]	

4i	$y = 2x(x-1)^3$ $\frac{dy}{dx} = 2x\left[3(x-1)^2\right] + 2(x-1)^3$ [M1] product rule $= 6x(x-1)^2 + 2(x-1)^3$ [A1] $= 2(x-1)^2(3x + x - 1)$ $= 2(x-1)^2(4x-1)$ For $\frac{dy}{dx} = 0$ $2(x-1)^2(4x-1) = 0$ [M1] $x = 1$ or $x = \frac{1}{4}$ $y = 0$ or $y = -\frac{27}{128}$ $(1, 0)$ and $\left(\frac{1}{4}, -\frac{27}{128}\right)$ [A1] [A1]	
ii	By first derivative test, [M1] $(1, 0)$ is a point of inflexion and $\left(\frac{1}{4}, -\frac{27}{128}\right)$ is a min. point [A1], [A1]	
5i	 $y = 3x^{\frac{1}{2}}$ [B1] $y = \frac{4}{x^2}$ [B1]	

ii	$3x^{\frac{1}{2}} = \frac{4}{x^2}$ [M1] substitution $x^{\frac{1}{2}}.\,x^2 = \frac{4}{3}$ $x^{\frac{5}{2}} = \frac{4}{3}$ $x^5 = \left(\frac{4}{3}\right)^2$ [M1] squaring $= \frac{16}{9}$ $\therefore k = \frac{16}{9}$ [A1]	
6i	$\text{LHS} = \frac{1}{3\tan^2\theta + 3}$ [B1] apply correct identity $= \frac{1}{3\left(\sec^2\theta - 1\right) + 3}$ $= \frac{1}{3\sec^2\theta}$ [B1] able to simplify $= \frac{\cos^2\theta}{3}$ $= \text{RHS}$	
ii	$\int_0^{\frac{\pi}{3}} \frac{\sec^2\theta\cos 2\theta}{3\tan^2\theta + 3}\,\mathrm{d}\theta = \int_0^{\frac{\pi}{3}} \frac{\cos^2\theta}{3}\left(\frac{1}{\cos^2\theta}\right)\cos 2\theta\,\mathrm{d}\theta$ [M1] substitution of $\frac{1}{3\tan^2\theta + 3}$ $= \frac{1}{3}\int_0^{\frac{\pi}{3}} \cos 2\theta\,\mathrm{d}\theta$ [B1] $\sec^2\theta = \frac{1}{\cos^2\theta}$ $= \frac{1}{3}\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{3}}$ [B1] correct integration of $\cos 2\theta$ $= \frac{1}{6}\left(\sin\frac{2\pi}{3} - \sin 0\right)$ $= \frac{1}{6}\left(\sin\frac{\pi}{3} - 0\right)$ $= \frac{1}{6}\left(\frac{\sqrt{3}}{2}\right)$ [B1] $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $= \frac{\sqrt{3}}{12}$ (shown)	

7i	Grad. BC $=\frac{8-6}{2-8}$ $=-\frac{1}{3}$ Grad. $AB=3$ [B1] Eqn AB is $\frac{y-8}{x-2}=3$ $\therefore y=3x+2$ [B1]	
ii	When $x=0$, $y=2$ $A\ (0,2)$ [B1]	
iii	Grad. of perpendicular bisector = 3 Midpt. $BC=\left(\frac{2+8}{2},\ \frac{8+6}{2}\right)$ [M1] midpoint formula $=(5,\ 7)$ Eqn is $\frac{y-7}{x-5}=3$ [M1] $y=3x-8$ [A1]	
iv	$3y=4x-14$ $3(3x-8)=4x-14$ [M1] substitution $9x-24=4x-14$ $5x=10$ $x=2$ [A1] $y=3(2)-8$ $=-2$ $D\ (2,-2)$ [A1]	
8i	$\frac{d}{dx}(x^2\ln x-3x)=x^2\left(\frac{1}{x}\right)+2x\ln x-3$ [B1] $\frac{1}{x}$ seen $=x+2x\ln x-3$ [B1] product seen	
ii	$\int_1^4 x+2x\ln x-3\ dx=\left[x^2\ln x-3x\right]_1^4$ [M1] reverse differentiation $\int_1^4 x-3\ dx+\int_1^4 2x\ln x\ dx=4^2\ln 4-3(4)-(0-3)$ $\left[\frac{x^2}{2}-3x\right]_1^4+2\int_1^4 x\ln x\ dx=16\ln 4-12+3$ [A1] $\left[\frac{x^2}{2}-3x\right]$ seen $2\int_1^4 x\ln x\ dx=16\ln 4-9-\left[\frac{4^2}{2}-3(4)-\frac{1}{2}+3\right]$ [A1] simplification $=16\ln 4-\frac{15}{2}$ or -14.7 (3s.f.) [A1]	

9	$\frac{dy}{dx}=1+\frac{1}{2x^2}$ $=1+\frac{1}{2}x^{-2}$ $y=\int\left(1+\frac{1}{2}x^{-2}\right)dx$ [M1] $=x+\frac{1}{2}\left(\frac{x^{-1}}{-1}\right)+c$ $=x-\frac{1}{2x}+c$ [A1] Since $\frac{dy}{dx}=3$ $1+\frac{1}{2x^2}=3$ [M1] $\frac{1}{2x^2}=2$ $x^2=\frac{1}{4}$ $x=\pm\frac{1}{2}$ (reject $-\frac{1}{2}$) [A1] When $x=\frac{1}{2}$, $y=3\left(\frac{1}{2}\right)+1$ $=\frac{5}{2}$ [A1] At $\left(\frac{1}{2},\frac{5}{2}\right)$, $\frac{5}{2}=\frac{1}{2}-\frac{1}{2(0.5)}+c$ [M1] attempt to find c $c=3$ $y=x-\frac{1}{2x}+3$ [A1]	
10i	Radius of cone = $\sqrt{10^2-x^2}$ $=\sqrt{100-x^2}$ [B1] Volume of cone $=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(\sqrt{100-x^2}\right)^2(x+10)$ [B1] application of formula and substitution $=\frac{1}{3}\pi\left(100-x^2\right)(x+10)$	

ii	$\frac{dV}{dx}=\frac{1}{3}\pi\left[-2x\left(x+10\right)+100-x^2\right]$ [M1] product rule $=\frac{1}{3}\pi\left[-20x-2x^2+100-x^2\right]$ $=\frac{1}{3}\pi\left(-3x^2-20x+100\right)$ For stationary V, $\frac{dV}{dx}=0$ [M1] $\frac{1}{3}\pi\left(-3x^2-20x+100\right)=0$ $3x^2+20x-100=0$ $(x+10)(3x-10)=0$ $x=-10$ (rejected), $x=\frac{10}{3}$ [A1]	
iii	$V=\frac{1}{3}\pi\left(100-\frac{100}{9}\right)\left(\frac{10}{3}+10\right)$ $=1241.123$ $=1240\text{ cm}^3$ (3s.f.) [B1]	
iv	$\frac{d^2V}{dx^2}=\frac{1}{3}\pi(-6x-20)$ [M1] Since $\frac{d^2V}{dx^2}<0$, V is a maximum. [A1]	
11a	$2\tan^2 y+\tan y-6=0$ $\left(2\tan y-3\right)\left(\tan y+2\right)=0$ [M1] factorisation $\tan y=\frac{3}{2}$ or $\tan y=-2$ [A1] either one $y=\tan^{-1}\left(\frac{3}{2}\right)$ $y=\tan^{-1}\left(-2\right)$ $=0.9827$ $=-1.1071$ ≈ 0.983 (3s.f.) ≈ -1.11 (3s.f.) [A1] [A1]	
bi	Initial height = 2 m [B1]	
ii	Greatest height = 7 – 5(–1) = 12 m [A1]	

iii	$7-5\cos 8t=9$ [M1] $\cos 8t=-\frac{2}{5}$ $\alpha=1.1592$ $8t=1.9823,\ 4.300$ $t=0.2477,\ 0.5375$ [A1] $\text{Duration}=0.5375-0.2477$ $=0.2898$ ≈ 0.290 minutes (3s.f.) [A1]	

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4047/02

PAPER 2

14 September 2018
2 hours 30 minutes

Additional Materials: Writing paper, Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use
100

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. (i) Given that $u = 4^x$, express $4^x = 9 - 5 \times 4^{1-x}$ as a quadratic equation in u. [2]

   (ii) Hence find the values of x for which $4^x = 9 - 5 \times 4^{1-x}$, giving your answer, where appropriate, to 1 decimal place. [4]

   (iii) Determine the values of k for which $4^x = k - 5 \times 4^{1-x}$ has no solution. [3]

2. (i) By using long division, divide $2x^4 + 5x^3 - 8x^2 - 8x + 3$ by $x^2 + 3x - 1$. [2]

   (ii) Factorise $2x^4 + 5x^3 - 8x^2 - 8x + 3$ completely. [2]

   (iii) Hence find the exact solutions to the equation [4]

$$32p^4 + 40p^3 - 32p^2 - 16p + 3 = 0.$$

3. The roots of the quadratic equation $8x^2 - 4x + 1 = 0$ are $\frac{1}{\alpha^2\beta}$ and $\frac{1}{\alpha\beta^2}$. Find a quadratic equation with roots α^3 and β^3. [7]

4. (i) Write down the general term in the binomial expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$, where p is a constant. [1]

.

   (ii) Given that the coefficient of x^8 in the expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$ is negative $\frac{10}{3}$ times the coefficient of x^5. Show that the value of p is $\frac{1}{2}$. [5]

   (iii) Showing all your working, use the value of p in part (ii), to find the constant term in the expansion of $(2x-1)\left(2x^2 - \frac{p}{x}\right)^{10}$. [5]

5. (a) (i) Show that $\sin 3x = \sin x(4\cos^2 x - 1)$ [3]

   (ii) Solve the equation $3\sin 3x = 16\cos x \sin x$ for $0 \le x \le 2\pi$ [5]

   (b) Differentiate $\cos 2x\,(\tan^2 x - 1)$ with respect to x. No simplification is required. [3]

6 The equation of a curve is $y = x^3 - 4x^2 + px + q$ where p and q are constants. The equation of the tangent to the curve at the point $A(-1, 5)$ is $15x - y + 20 = 0$.

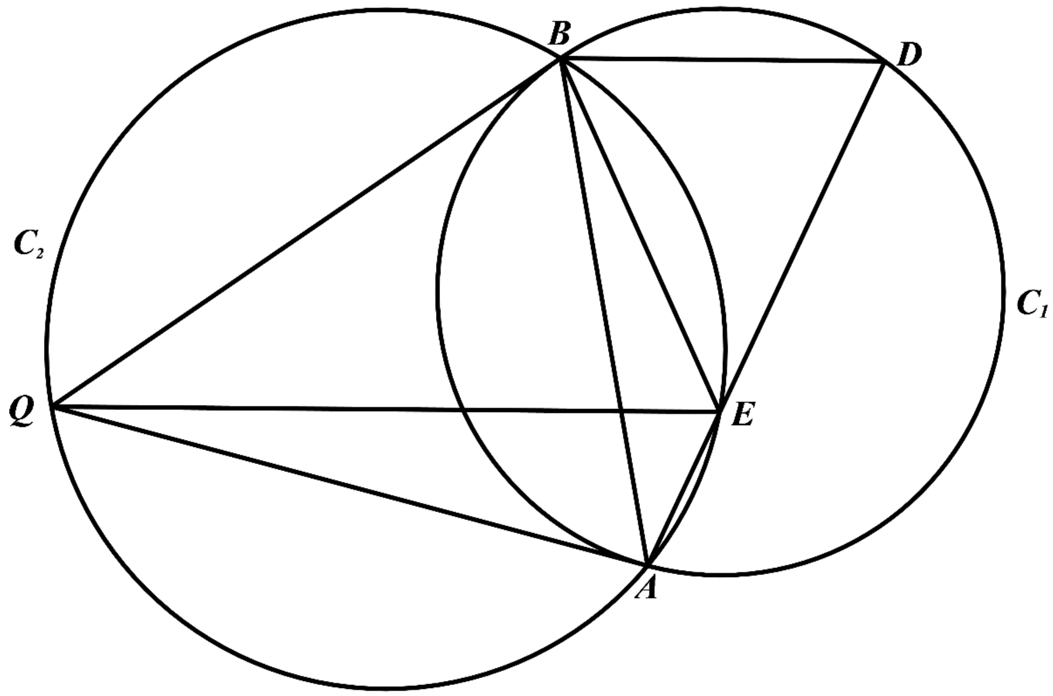
(i) Find the values of p and of q. [4]

(ii) Determine the values of x for which y is an increasing function. [3]

(iii) Find the range of values of x for which the gradient is decreasing. [2]

(iv) A point P moves along the curve in such a way that the x-coordinate of P increases at a constant rate of 0.02 units per second. Find the possible x-coordinates of P at the instant that the y-coordinate of P is increasing at 1.9 units per second. [4]

7.



The diagram shows two intersecting circles, C_1 and C_2. C_1 passes through the vertices of the triangle ABD. The tangents to C_1 at A and B intersect at the point Q on C_2. A line is drawn from Q to intersect the line AD at E on C_2.

Prove that

(i) QE bisects angle AEB, [4]

(ii) $EB = ED$, [2]

(iii) BD is parallel to QE. [2]

8. The number, N, of E. Coli bacteria increases with time, t minutes. Measured values of N and t are given in the following table.

t	2	4	6	8	10
N	3215	3446	3693	3959	4243

It is known that N and t are related by the equation $N = N_o\,(2)^{kt}$, where N_o and k are constants.

(i) Plot lg N against t and draw a straight line graph. The vertical lg N axis should start at 3.40 and have a scale of 2 cm to 0.02. [3]

(ii) Use your graph to estimate the values of N_o and k. [4]

(iii) Estimate the time taken for the number of bacteria to increase by 25%. [2]

9. A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, v m/s, of the car after he applied the brakes is given by $v = 40e^{-\frac{1}{3}t} - 15$, where t, the time after he applied the brakes, is measured in seconds.

(i) Calculate the initial acceleration of the car. [2]

(ii) Calculate the time taken to stop the car. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the car, t seconds after the brakes has been applied. [3]

(iv) Calculate the braking distance. [1]

10. The points $P(4,6)$, $Q(-3,5)$ and $R(4,-2)$ lie on a circle.

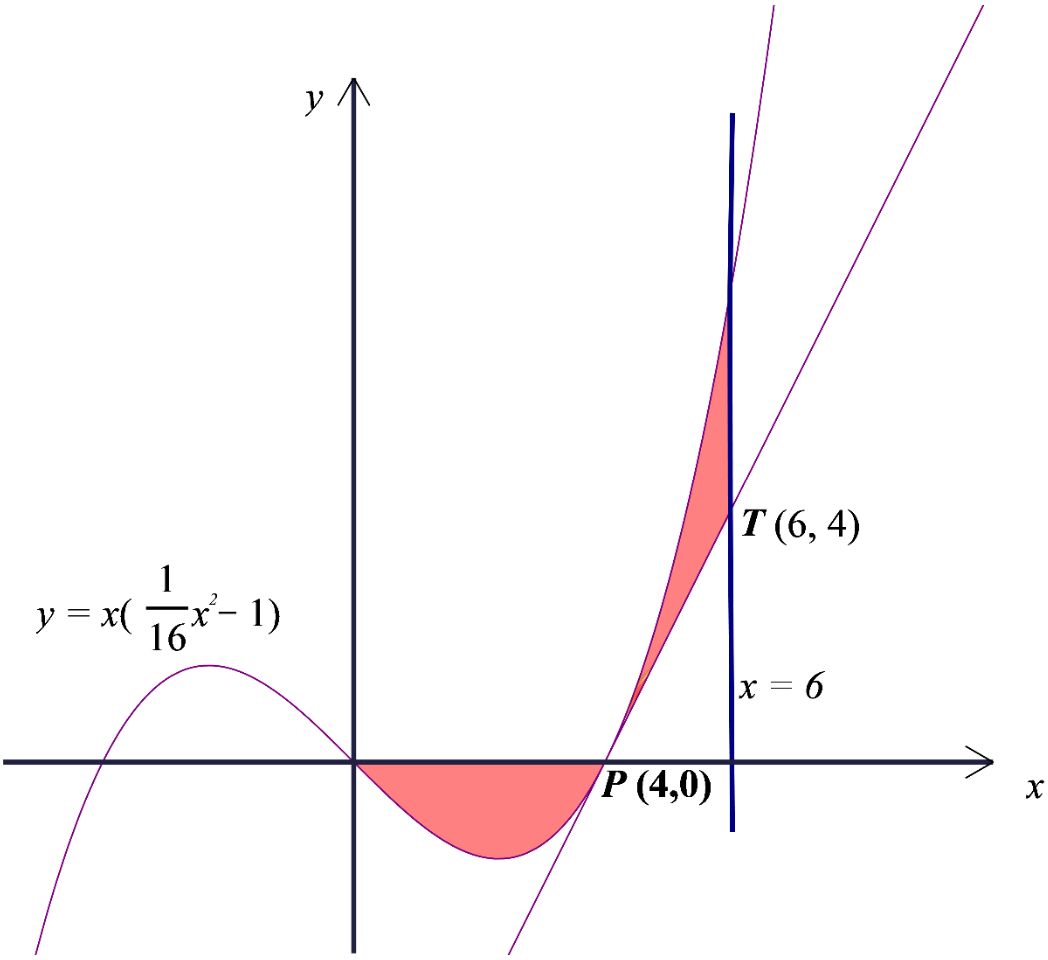
(i) Find the equation of the perpendicular bisector of PQ. [3]

(ii) Show that the centre of the circle is $(1,2)$ and find the radius of the circle. [3]

(iii) State the equation of the circle. [1]

(iv) Find the equation of the tangent to the circle at R. [3]

11. The diagram shows part of the curve $y = x\left(\frac{1}{16}x^2 - 1\right)$. The curve cuts the x-axis at P(4, 0). The tangent to the curve at P meets the vertical line $x = 6$ at $T(6, 4)$. Showing all your workings, find the total area of the shaded regions. [6]



End of paper

1	(i)	$u^2 - 9u + 20 = 0$
	(ii)	$x = 1$
		$x = 1.2$`
	(iii)	$-\sqrt{80} < k < \sqrt{80}$
2	(i)	$\boldsymbol{2x^2 - x - 3}$
	(ii)	$(x^2+3x-1)(2x-3)(x+1)$
	(iii)	$p = \dfrac{-3 \pm \sqrt{13}}{4}$ M1 $p = \dfrac{3}{4}$ or $p = -\dfrac{1}{2}$
3		$x^2 + 4x + 8 = 0$
4	(i)	$\binom{10}{r}(2x^2)^{10-r}\left(-\dfrac{p}{x}\right)^r$
	(ii)	$\dfrac{\binom{10}{4}2^6}{\binom{10}{5}2^5} \times \dfrac{3}{10} = p$ $p = \dfrac{1}{2}$ AG
	(iii)	-15
5a	(ii)	$x = 0,\ \pi,\ 2\pi$ or $x = 1.74$ or 4.54
5b		$2\cos 2x \tan x \sec^2 x - 2\sin 2x\left(\tan^2 x - 1\right)$
6	(i)	$p = 4$ $q = 14$
	(ii)	$x < \frac{2}{3}$ or $x > 2$
	(iii)	$x < \frac{4}{3}$
	(iv)	$x = -\dfrac{13}{3}$ or $x = 7$

8	(ii)	$N_o = 2992$ accept also 2990 $k = 0.05$
	(iii)	time taken= 6.4 mins
9	(i)	$-\frac{40}{3}\text{ m/s}^2$
	(ii)	2.94s
	(iii)	$s = -120e^{-\frac{1}{3}t} - 15t + 120$
	(iv)	30.9m
10	(i)	$y = -7x + 9$
	(ii)	$r = 5$ units
	(iii)	$(x-1)^2 + (y-2)^2 = 25$
	(iv)	$y = \frac{3}{4}x - 5$
11		$\frac{25}{4}$ units^2

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY 4E/5N

Candidate's Name	Class	Register Number
Marking Scheme		

ADDITIONAL MATHEMATICS 4047/02

PAPER 2

14 September 2018
2 hours 30 minutes

Additional Materials: Writing paper, Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use
100

Setter: Mrs Koh SH
Vetted by: Mrs See YN, Mr Poh WB

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions

Given that

1.	(i)	Given that $u = 4^x$, express $4^x = 9 - 5 \times 4^{1-x}$ as a quadratic equation in u.	[2]
	(ii)	Hence find the values of x for which $4^x = 9 - 5 \times 4^{1-x}$, giving your answer, where appropriate, to 1 decimal place.	[4]
	(iii)	Determine the values of k for which $4^x = k - 5 \times 4^{1-x}$ has no solution.	[3]

1	Solutions	Remarks
(i)	(i) $u = 9 - 5 \times \frac{4}{u}$	M1
[2]	$u^2 - 9u + 20 = 0$	A1
(ii)	(ii) $(u-4)(u-5) = 0$	M1
[4]	$u = 4$ or $u = 5$	
	$4^x = 4$ or $4^x = 5$	
	$x = 1$ A1 or $x\lg 4 = \lg 5$	M1 taking lg
	$x = \frac{\lg 5}{\lg 4} = 1.16$	A1
(iii)	(iii) $u = k - \frac{5\times 4}{u}$	
[3]	$u^2 - ku + 20 = 0$	
	For no real roots, $(-k)^2 - 4(1)(20) < 0$	B1
	$\left(k - \sqrt{80}\right)\left(k + \sqrt{80}\right) < 0$	M1
	$-\sqrt{80} < k < \sqrt{80}$	A1

2.	(i)	By using long division, divide $2x^4+5x^3-8x^2-8x+3$ by x^2+3x-1.	[2]

2	(i)	$2x^2 - x - 3$	M1 A1
	[2]	$x^2+3x-1 \overline{)\; 2x^4+5x^3-8x^2-8x+3}$	
		$-\ \underline{(2x^4+6x^3-2x^2)}$	
		$-x^3-6x^2-8x$	
		$\underline{-(-x^3-3x^2+x)}$	
		$-3x^2-9x+3$	
		$\underline{-(-3x^2-9x+3)}$	
		0	

	(ii)	Factorise $2x^4+5x^3-8x^2-8x+3$ completely.	[2]

2	(ii)	$2x^4+5x^3-8x^2-8x+3 = (x^2+3x-1)(2x^2-x-3)$	B1
[2]		$= (x^2+3x-1)(2x-3)(x+1)$	A1

	(iii)	Hence find the exact solutions to the equation $32p^4+40p^3-32p^2-16p+3=0.$	[4]

2	(iii)	Let $x=2p$	
[4]		$2(2p)^4+5(2p)^3-8(2p)^2-8(2p)+3=0$	
		$\left((2p)^2+3(2p)-1\right)\left(2(2p)-3\right)(2p+1)=0$ either B1	
		$(4p^2+6p-1)(4p-3)(2p+1)=0$ or	
		$(4p^2+6p-1)=0$ or $(4p-3)=0$ or $(2p+1)=0$	
		$p=\dfrac{-6\pm\sqrt{36-4(4)(-1)}}{2(4)}$ M1 $\quad p=\dfrac{3}{4}$ or $p=-\dfrac{1}{2}$ [A1 for both ans]	
		$=\dfrac{-3\pm\sqrt{13}}{4}$ A1	

3. The roots of the quadratic equation $8x^2 - 4x + 1 = 0$ are $\frac{1}{\alpha^2\beta}$ and $\frac{1}{\alpha\beta^2}$. Find a quadratic equation with roots α^3 and β^3 .	[7]

3. [7]

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{1}{2} \text{ ---- (1)}$$

$$\frac{1}{\alpha^3\beta^3} = \frac{1}{8} \text{ --------(2)}$$

$\boxed{\text{B1}}$

From (2), $\alpha\beta = \sqrt[3]{8} = 2$ B1

From (1), $\frac{\beta+\alpha}{\alpha^2\beta^2} = \frac{1}{2}$

$\alpha + \beta = \frac{1}{2} \times 4$

$= 2$ B1

$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ B1

$= (\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta)\right]$

B1

$= 2\left[2^2 - 3\times 2)\right]$

$= -4$ B1

$\alpha^3\beta^3 = 8$

Equation is $x^2 + 4x + 8 = 0$ A1

4. (i) Write down the general term in the in the binomial expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$.	[1]

4 [1] (i) General term $= \binom{10}{r}\left(2x^2\right)^{10-r}\left(-\frac{p}{x}\right)^r$ A1

	(ii)	Given that the coefficient of x^8 in the expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$ is negative $\frac{10}{3}$ times the coefficient of x^5 . Show that the value of p is $\frac{1}{2}$.	[5]

4 (ii) For x^8, $x^{20-2r-r} = x^8$,

[5] $20 - 3r = 8$

$r = 4$ x^{20-3r} seen or any method (M1)

For x^5, $x^{20-2r-r} = x^5$,

$20 - 3r = 5$

$r = 5$ A1 for any correct value of r

$$\binom{10}{4}(2)^{10-4}\left(-\frac{1}{2}\right)^4 = -\frac{10}{3}\binom{10}{5}(2)^{10-5}\left(-\frac{1}{2}\right)^5$$

B1 B1

$$\frac{\binom{10}{4}2^6}{\binom{10}{5}2^5} \times \frac{3}{10} = p \quad \text{M1}$$

$p = \frac{1}{2}$ AG

4	(iii)	Showing all your working, use the value of p found in part (i), find the constant term in the expansion of $(2x-1)\left(2x^2-\frac{p}{x}\right)^{10}$.	[5]

4 (iii) [5] $\left(2x^2-\frac{1}{2x}\right)^{10}$

For x^0, $20-3r=0$

$r=\frac{20}{3}$ (not an integer)

No constant term in $\left(2x^2-\frac{1}{2x}\right)^{10}$

B1

4(ii) For x^{-1}, $20-3r=-1$

$r=7$

M1

$$(2x+1)\left(\binom{10}{7}\left(2x^2\right)^3\left(-\frac{1}{2x}\right)^7+\ldots\ldots\right)$$

B1

constant term $=2x\binom{10}{7}\left(2x^2\right)^3\left(-\frac{1}{2x}\right)^7$ M1

$=-15$ A1

5.(a)	(i)	Show that $\sin 3x = \sin x(4\cos^2 x - 1)$	[3]

5 (a) (i) [3] LHS $=\sin(x+2x)$ Addition formula M1

$=\sin x\cos 2x+\cos x\sin 2x$

$=\sin x(2\cos^2 x-1)+\cos x\times 2\sin x\cos x$

using $\cos 2x = 2\cos^2 x - 1$
or $\sin 2x = 2\sin x\cos x$ B1

$=\sin x\left(2\cos^2 x-1+2\cos^2 x\right)$

Factorisation B1

$=\sin x\left(4\cos^2 x-1\right)$

	(ii)	Solve the equation $3\sin 3x = 16\cos x\sin x$ for $0 \le x \le 2\pi$	[5]

5(a) (ii) [5] $3\sin 3x = 16\cos x\sin x$

$3\sin x\left(4\cos^2 x - 1\right) = 16\cos x\sin x$

$\sin x\left(12\cos^2 x - 16\cos x - 3\right) = 0$ factorisation with $\sin x$ seen M1

$\sin x\left(6\cos x + 1\right)\left(2\cos x - 3\right) = 0$ correct factorisation of quad exp B1

$\sin x = 0$ or $\cos x = -\dfrac{1}{6}$ or $\cos x = \dfrac{3}{2}$(rejected) A1

$x = 0,\ \pi,\ 2\pi$ or $x = \pi - 1.40335,\ \pi + 1.40335$

$= 1.74$ or 4.54

A1 A1

5(b)	Differentiate $\cos 2x\,(\tan^2 x - 1)$ with respect to x. No simplification is required		[3]

5(b) [3] $\dfrac{d}{dx}\left[\cos 2x\left(\tan^2 x - 1\right)\right]$

$= \cos 2x\left(2\tan x\sec^2 x\right) + \left(\tan^2 x - 1\right)\left(-2\sin 2x\right)$ M1 product rule

B1 B1

$= 2\cos 2x\tan x\sec^2 x - 2\sin 2x\left(\tan^2 x - 1\right)$

6	The equation of a curve is $y = x^3 - 4x^2 + px + q$ where p and q are constants. The		
	equation of the tangent to the curve at the point $A(-1, 5)$ is $15x - y + 20 = 0$.		
	(i)	Find the values of p and of q.	[4]

6 (i) [4] $\frac{dy}{dx} = 3x^2 - 8x + p$ B1

At $A(-1, 5)$, equation of the tangent is $y = 15x + 20$

gradient = 15

$3(-1)^2 - 8(-1) + p = 15$ M1

$11 + p = 15$

$p = 4$ A1

substitute $p = 4$, $x = -1$, $y = 5$ into equation of curve

$5 = -1 - 4 - 4 + q$

$q = 14$ A1

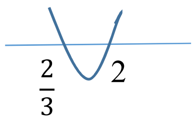
	(ii)	Determine the values of x for which y is an increasing function.	[3]

6(ii) [3] For y to be an increasing function,

$\frac{dy}{dx} > 0$

$3x^2 - 8x + 4 > 0$ B1(with value of p substituted)

$(3x - 2)(x - 2) > 0$ M1



$x < \frac{2}{3}$ or $x > 2$ A1

6	(iii)	Find range of values of x for which the gradient is decreasing.	[2]

6(iii) [2] For decreasing gradient,

$\frac{d^2y}{dx^2} < 0$ either

or M1

$6x - 8 < 0$

$x < \frac{4}{3}$ A1

6	(iv)	A point P moves along the curve in such a way that the x-coordinate of P increases	
		at a constant rate of 0.02 units per second. Find the possible x-coordinates of P at the instant that the y-coordinate of P is increasing at 1.9 units per second.	[4]

6(*iv*) [4]

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$1.9 = \frac{dy}{dx} \times (0.02) \qquad \text{M1}$$

$$\frac{dy}{dx} = \frac{1.9}{0.02}$$

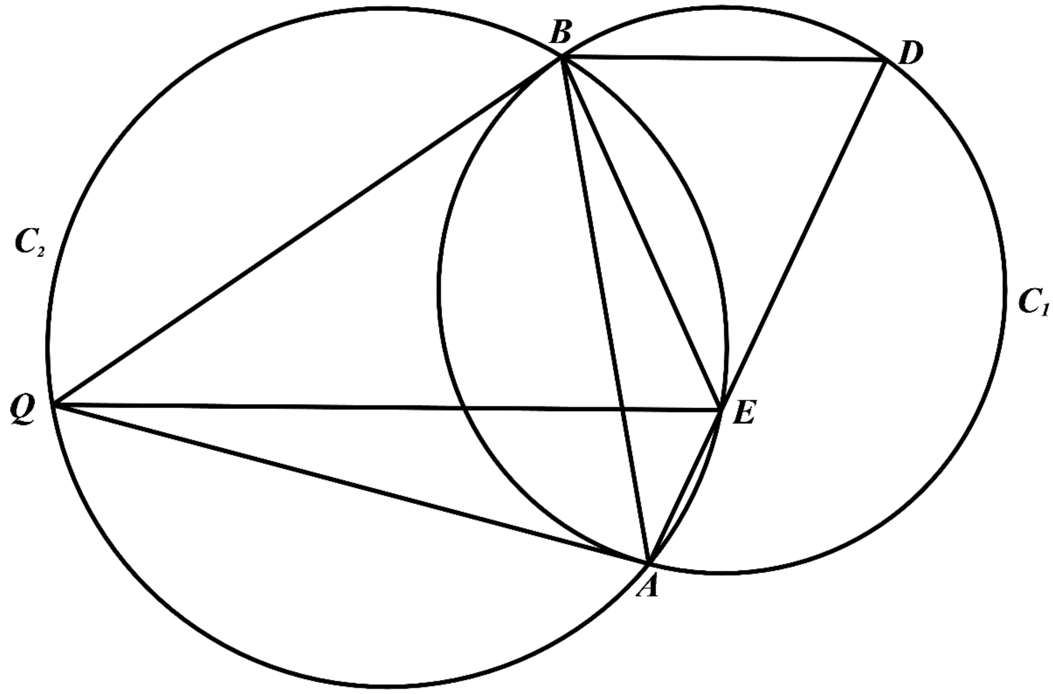
$$= 95$$

$$3x^2 - 8x + 4 = 95 \qquad \text{M1 (quadratic equation in } x\text{)}$$

$$3x^2 - 8x - 91 = 0$$

$$(3x+13)(x-7) = 0$$

$$x = -\frac{13}{3} \text{ or } x = 7 \qquad \text{A2}$$

7.	 The diagram shows two intersecting circles, C_1 and C_2. C_1 passes through the vertices of the triangle ABD. The tangents to C_1 at A and B intersect at the point Q on C_2. A line is drawn from Q to intersect the line AD at E on C_2. Prove that		
	(i)	QE bisects angle AEB	[4]
	(i)	$EB = ED$.	[2]
	(ii)	BD is parallel to QE.	[2]

7.(i)[4] *Let* $\angle QEA = x^\circ$

$\angle QBA = \angle QEA$ (angles in same segment in C_2) B1

$= x^\circ$

$QB = QA$ (tangents to C_1 from external point Q) B1

$\angle QAB = \angle QBA$ (base angles of isosceles triangle) B1

$= x^\circ$

$\angle QEB = \angle QAB$ (angles in the same segment in C_2)

$= x^\circ$

$\therefore \angle QEB = \angle QEA$

Hence QE bisects angle *AEB*.

B1

7(ii) $\angle QBA = x^\circ$ (from (i))

$\angle ADB = \angle QBA$ (angles in alternate segment in C_1) either

$= x^\circ$

$\angle AEB = 2x^\circ$ (from (i))

$\angle DBE = \angle AEB - \angle ADB$ (exterior angle of triangle BDE) or B1

$= 2x^\circ - x^\circ$

$= x^\circ$

$\therefore$ $\angle ADB = \angle EDB = \angle DBE = x^\circ$(base angles of isosceles triangle BDE) B1

Hence $EB = ED$

(iii) [2] From (i) $\angle EBD = \angle QEB = x^\circ$ B1

$\therefore$ $\angle EBD$ and $\angle QEB$ are alternate angles of parallel lines. (alternate angles are equal) B1

BD is parallel to QE

8.	The number, N, of E. Coli bacteria increases with time, t minutes. Measured values of N and t are given in the following table.		

t	2	4	6	8	10
N	3215	3446	3693	3959	4243

It is known that N and t are related by the equation $N = N_o\,(2)^{kt}$, where N_o and k are constants.

(i)	Plot lg N against t and draw a straight line graph. The vertical lg N axis should start at 3.40 and have a scale of 2 cm to 0.02.	[3]
(ii)	Use your graph to estimate the values of N_o and k.	[4]
(iii)	Estimate the time taken for the number of bacteria to increase by 25%.	[2]

8. (i) [3] On graph paper

8(ii) [4] $N = N_o(2)^{kt}$

$\lg N = \lg N_o + kt\lg 2$

$\lg N$-intercept = 3.476 M1

$\lg N_o = 3.476$

$N_o = 2992$ accept also 2990 A1

gradient = $\dfrac{3552-3476}{5-0}$ M1(with points used to find gradient labelled on graph)

= 0.0152

$k\lg 2 = 0.0152$

$k = \dfrac{0.0152}{\lg 2}$

= 0.05 A1

(iii) [2] when $N = 125\%$ of 2992

= 3740 (to 4 sf)

$\lg N = \lg 3740$

= 3.573(M1)

From graph, time taken= 6.4 mins A1

9.	A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, v m/s, of the car after he applied the brakes is given by $v = 40e^{-\frac{1}{3}t} - 15$, where t is the time after he applied the brakes, is measured in seconds.		
	(i)	Calculate the initial acceleration of the car.	[2]
	(ii)	Calculate the time taken to stop the car.	[3]
	(iii)	Obtain an expression, in term of t , for the displacement of the car, t seconds after the brakes has been applied.	[3]
	(iv)	Calculate the braking distance.	[1]

9 [9]

(i) $v = 40e^{-\frac{1}{3}t} - 15$

$a = \dfrac{dv}{dt} = -\dfrac{40}{3}e^{-\frac{1}{3}t}$ B1

Initial acceleration = $-\dfrac{40}{3}$ m/s^2 A1

(ii) when $v = 0$

$40e^{-\frac{1}{3}t} - 15 = 0$ M1

$e^{-\frac{1}{3}t} = \dfrac{3}{8}$

$-\dfrac{t}{3} = \ln\dfrac{3}{8}$ (M1 taking logarithm)

$t = -3\ln\dfrac{3}{8}$

$= 2.94$s (A1)

(iii) $s = \int \left(40e^{-\frac{1}{3}t} - 15 \right) dt$ M1

$= -120e^{-\frac{1}{3}t} - 15t + c$ B1

when $t = 0$, $s = 0$, where s is the displacement from the point where the brakes was applied.

$c = 120$

$s = -120e^{-\frac{1}{3}t} - 15t + 120$ A1

(iv) Substitute $t = -3\ln\dfrac{3}{8}$, Braking distance = $-120\left(\dfrac{3}{8}\right) - 15\left(-3\ln\dfrac{3}{8}\right) + 120$

$= 30.9$m (to 3 sf) A1

10.	The points $P(4,6)$, $Q(-3,5)$ and $R(4,-2)$ lie on a circle.		
	(i)	Find the equation of the perpendicular bisector of PQ.	[3]
	(ii)	Show that the centre of the circle is $(1,2)$ and find the radius of the circle.	[3]
	(iii)	State the equation of the circle.	[1]
	(iv)	Find the equation of the tangent to the circle at R.	[3]

10. [10] (i) midpoint of $PQ = \left(\frac{1}{2}, \frac{11}{2}\right)$ B1

gradient of $PQ = \frac{1}{7}$

gradient of perpendicular bisector of $PQ = -7$ B1

Equation of perpendicular bisector of PQ is

$$y - \frac{11}{2} = -7\left(x - \frac{1}{2}\right)$$

$y = -7x + 9$ A1

(ii) Equation of perpendicular bisector of PR is $y = 2$

B1

Alternatively use :Equation of perpendicular bisector of QR is $y = x + 1$

Since perpendicular bisector of chords passes through centre of circle,

for centre of circle, substitute $y = 2$ into $y = -7x + 9$

$2 = -7x + 9$ M1 solving simultaneous equations

$7x = 7$

$x = 1$

centre = (1, 2) AG

Alternative method : centre = $(a,\ -7a + 9)$ B1

$RC = PC$ M1 forming an equation in a

r = distance between centre and P

$= \sqrt{(4-1)^2 + (6-2)^2}$

= 5 units A1

(iii) Equation of circle is $(x-1)^2 + (y-2)^2 = 25$ A1

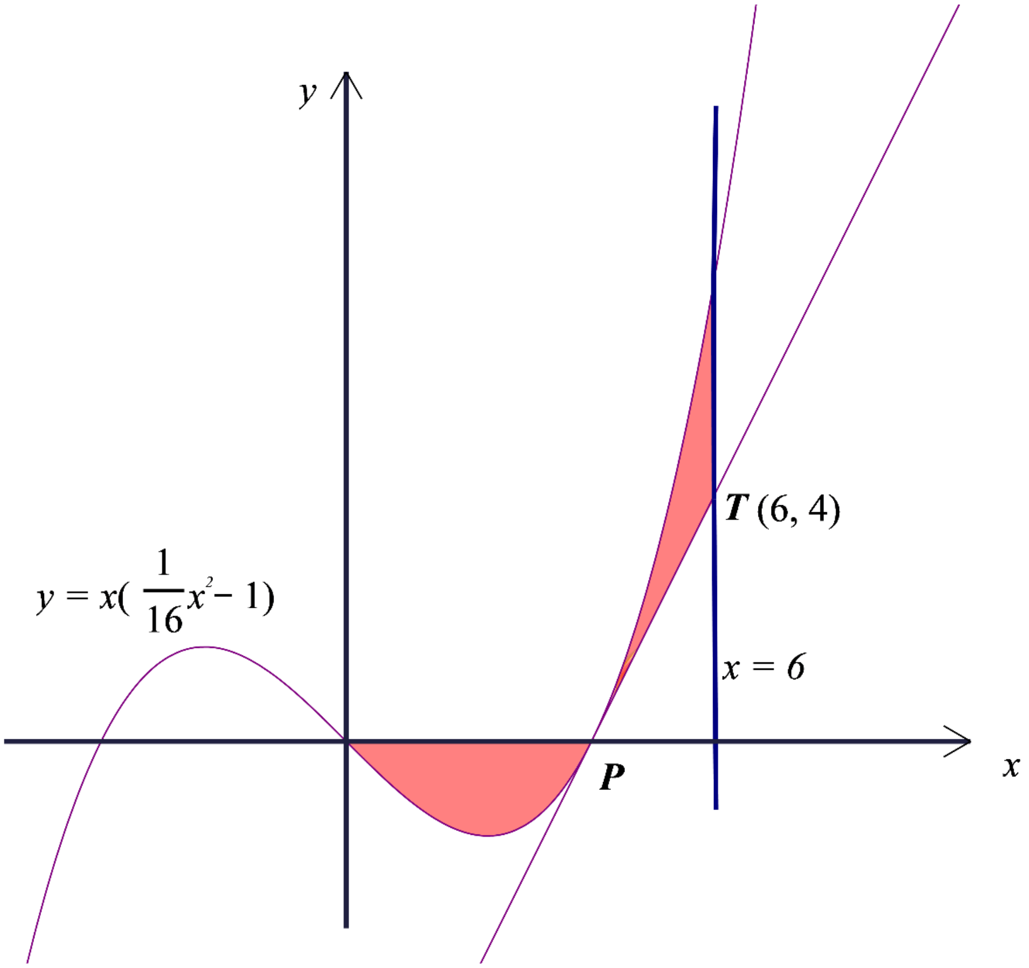
(iv) gradient of normal at $R = \frac{2-(-2)}{1-4} = -\frac{4}{3}$ M1

gradient of tangent at $R = \frac{3}{4}$ M1

Equation of tangent at R is $y + 2 = \frac{3}{4}(x - 4)$

$y = \frac{3}{4}x - 5$ A1

11.	The diagram shows part of the curve $y = x\left(\frac{1}{16}x^2 - 1\right)$. The curve cuts the x-axis at P(4, 0). The tangent to the curve at P meets the vertical line $x = 6$ at $T(6, 4)$. Showing all your workings, find the total area of the shaded regions.	[6]



Area of total shaded regions $= -\int_0^4 \left(\frac{x^3}{16} - x\right) dx + \int_4^6 \left(\frac{x^3}{16} - x\right) dx - \frac{1}{2} \times 2 \times 4$

B1 B1 B1

$= \left[-\frac{1}{16} \times \frac{x^4}{4} + \frac{x^2}{2}\right]_0^4 + \left[\frac{1}{16} \times \frac{x^4}{4} - \frac{x^2}{2}\right]_4^6 - 4$ M1 correct integration

$= -\frac{1}{64} \times 4^4 + \frac{1}{2} \times 4^2 + \left(\frac{6^4}{64} - \frac{6^2}{2}\right) - \left(\frac{4^4}{64} - \frac{4^2}{2}\right) - 4$

M1 correct substitution of upper and lower limits

$= \frac{25}{4}$ units^2 A1

End of paper