CANDIDATE NAME ______

CLASS ______ INDEX NUMBER ______

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2019
Secondary 4

Monday	ADDITIONAL MATHEMATICS	4047/1
19 August 2019	PAPER 1	2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This question paper consists of 19 printed pages and 1 blank page.

ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

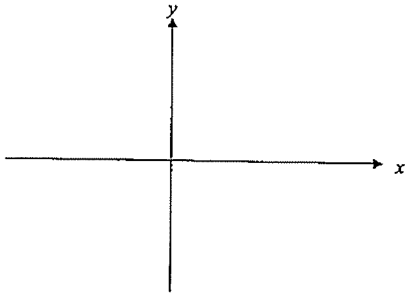
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

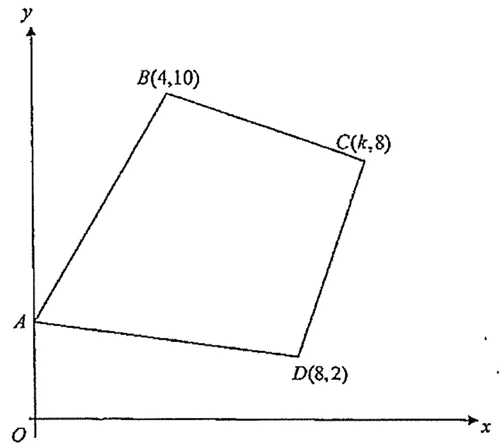
$$\Delta = \frac{1}{2} bc \sin A.$$

1. (a) Sketch the graph $y = 4x^{\frac{1}{2}}$ and $y = \frac{4}{\sqrt{x}}$ where $x \geq 0$. [2]



(b) Find the value of k for which the x-coordinate of the point of intersection satisfies the equation $x = k$. [2]

2. The diagram shows a kite $ABCD$ where $AB = AD$ and $BC = DC$. A lies on the y-axis. The coordinates of point B, C and D are $(4,10)$, $(k,8)$ and $(8,2)$ respectively.



Find

(i) the coordinates of point A, [2]

(ii) the value of k, [1]

(iii) the area of the kite $ABCD$. [2]

3 (a) State the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, giving your answer in radians in exact form. [1]

(b) Given that A is obtuse such that $\sin A = \frac{1}{\sqrt{3}}$, find the exact value of $\cos(A + 60°)$. [4]

4. (i) Given that $y=\dfrac{2}{(1+\sin\theta)^2}$, find $\dfrac{dy}{d\theta}$. [1]

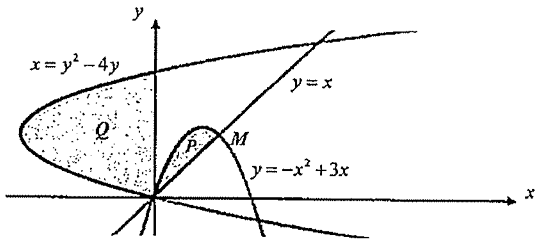
(ii) Hence, find the value of k for which $\int_0^k \dfrac{\cos\theta}{(1+\sin\theta)^3}\,d\theta=\dfrac{5}{18}$ and $k<2$. [5]

5. The curve $y = f(x)$ is such that $f'(x) = (k - 2)e^{3x}$.

(i) For y to be an increasing function of x, what condition must be applied to the constant k? [2]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at P is 4, find an expression for $f(x)$. [4]

6.



(i) M is the point of interection of $y = x$ and $y = -x^2 + 3x$.
Show that the coordinates of M is (2, 2). [1]

(ii) Find the area P, bounded by the curve $y = -x^2 + 3x$ and the line $y = x$. [3]

(iii) Find the area Q, enclosed by the curve $x = y^2 - 4y$ and the y-axis . [3]

7. (i) In an electrical circuit, the voltage, V volts, is given by the formula $V = IR$, where I amperes is the current. Given that $R = \frac{1}{10}\left(6\sqrt{2}+7\sqrt{3}\right)$ and $I = 5\sqrt{6}$, find V in the form of $a\sqrt{3}+b\sqrt{2}$ where a and b are constants to be determined. [2]

(ii) Two resistors, whose resistances are R_1 ohms and R_2 ohms respectively, are connected in parallel in the electrical circuit. The total resistance, R ohms, is given by the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$.

Given that $R=\frac{1}{10}\left(6\sqrt{2}+7\sqrt{3}\right)$ and $R_1=\sqrt{3}$, find $\frac{1}{R_2}$ in the form of $\frac{a\sqrt{2}+b\sqrt{3}}{5}$ where a and b are constants to be determined. [4]

8. (i) Express $y = 2\sin x + 4\cos x$ in the form of $R\sin(x + \alpha)$ and find the minimum value of $y = 2\sin x + 4\cos x$, stating the value of x between $0°$ and $360°$. [4]

(ii) Hence, solve $3 = 2\sin x + 4\cos x$ for $0° \le x \le 360°$. [3]

9. The equation of a circle of centre C is $x^2+y^2+8x-12y+27=0$.

(i) Find the radius of the circle and the coordinates of C. [3]

(ii) The point A is $(0,\ 6)$. Determine if A is inside the circle or outside the circle. [2]

(iii) Find the equation of the line that touches the circle at $B(-7,10)$. [3]

10. (a) Given that the coefficient of x in the expansion of $\left(x+\frac{k}{2x^2}\right)^{16}$ is 4368, find the value of k. [3]

(b) (i) Find the value of n, given that the coefficients of x^4 and x^6 in the expansion of $\left(1+\frac{1}{3}x^2\right)^n$ are in the ratio of $3:2$. [4]

(ii) Hence, find the coefficient of x^6 in the expansion of $(1-6x+9x^2)\left(1+\frac{1}{3}x^2\right)^n$. [2]

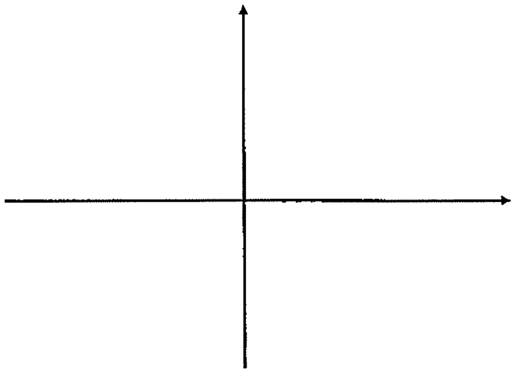
11. (a) Factorise completely $27a^3 - 125b^3$. [2]

(b) It is given that $3x^3 + 3x^2 - 11x - 6$ when divided by $x + a$ has a remainder that is half the remainder when it is divided by $x - a$.

(i) Show that $3a^3 - a^2 = 11a - 2$. [3]

(ii) Solve $3a^3 - a^2 = 11a - 2$, giving your answer to two decimal places where necessary. [4]

12. (a) (i) Sketch the graph of $y = \ln(x-3)$. [2]



(ii) Find the equation of a suitable straight line that can be inserted to solve the equation $2 = (x-3)e^{3x}$. [2]

(b) Find the coordinates of the stationary point of $y=\frac{x^2}{e^{x-1}}$, for $x>0$, leaving your answer in exact form and determine the nature of this stationary point [4]

End of Paper.

CANDIDATE NAME: ANSWERS

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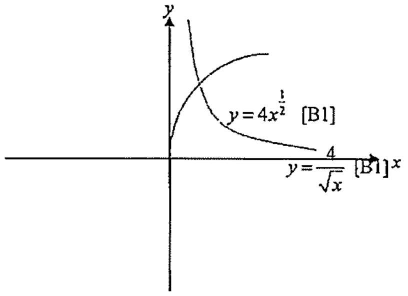


(b) Find the value of k for which the x-coordinate of the point of intersection satisfies the equation $x = k$. [2]

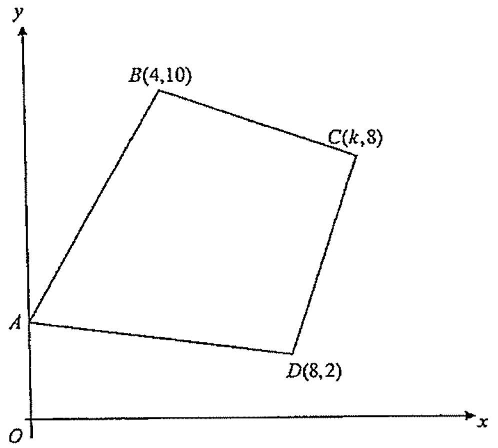
$4x^{\frac{1}{2}} = \frac{4}{\sqrt{x}}$

$x^{\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}}$ [M1]

$x = 1$

$k = 1$ [A1]

2. The diagram shows a kite $ABCD$ where $AB = AD$ and $BC = DC$. A lies on the y-axis. The coordinates of point B, C and D are $(4,10)$, $(k,8)$ and $(8,2)$ respectively.



Find
(i) the coordinates of point A, [2]

METHOD 1: Midpoint of BD $=\left(\frac{4+8}{2},\frac{10+2}{2}\right)=(6,6)$

Gradient of BD $=\frac{10-2}{4-8}=-2$

Gradient of AC $=\frac{1}{2}$

Equation of AC: $y-6=\frac{1}{2}(x-6)$ [M1]

$y=\frac{1}{2}x+3$

$A(0,3)$ [A1]

METHOD 2:

$AB = AD$ let coordinates of A be (0, y)

$\sqrt{(4-0)^2+(10-y)^2}=\sqrt{(8-0)^2+(2-y)^2}$ [M1]

$16+100-20y+y^2=64+4-4y+y^2$

$48=16y$

$y=3$

$A(0,3)$ [A1]

(ii) the value of k, [1]

At $C(k, 8)$,

$8 = \frac{1}{2}k + 3$

$k = 10$ [B1]

(iii) the area of the kite $ABCD$. [2]

$\text{Area} = \frac{1}{2}\begin{vmatrix} 4 & 0 & 8 & 10 & 4 \\ 10 & 3 & 2 & 8 & 10 \end{vmatrix}$

$= \frac{1}{2}[12 + 64 + 100 - 32 - 20 - 24]$ [M1]

$= 50 \text{ units}^2$ [A1]

3 (a) State the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, giving your answer in radians in exact form. [1]

$y = \cos y$ when $-1 \le y \le 1$

Principal value of $\cos^{-1} y$ are $0 \le \cos^{-1} y \le \pi$

principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ [B1]

(b) Given that A is obtuse such that $\sin A = \frac{1}{\sqrt{3}}$, find the exact value of $\cos(A + 60°)$. [4]

$\cos(A + 60°)$

$= \cos A \cos 60° - \sin A \sin 60°$

$= \left(-\frac{\sqrt{2}}{\sqrt{3}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right)$ [M1 each for $\cos A, \cos 60° \sin 60°$]

$= -\frac{\sqrt{2}}{2\sqrt{3}} - \frac{1}{2}$

$= -\frac{\sqrt{2}\sqrt{3}}{6} - \frac{1}{2}$

$= -\frac{1}{6}\left(\sqrt{2}\sqrt{3} + 3\right)$ [A1]

4. (i) Given that $y=\dfrac{2}{(1+\sin\theta)^2}$, find $\dfrac{dy}{d\theta}$. [1]

$y=2(1+\sin\theta)^{-2}$

$\dfrac{dy}{dx}=-4(1+\sin\theta)^{-3}(\cos\theta)$

$=\dfrac{-4\cos\theta}{(1+\sin\theta)^3}$ [A1]

(ii) Hence, find the value of k for which $\int_0^k \dfrac{\cos\theta}{(1+\sin\theta)^3}d\theta=\dfrac{5}{18}$ and $k<2$. [5]

$\int_0^k \dfrac{\cos\theta}{(1+\sin\theta)^3}d\theta=\dfrac{5}{18}$

$-\dfrac{1}{4}\int_0^k \dfrac{-4\cos\theta}{(1+\sin\theta)^3}d\theta=\dfrac{5}{18}$ [M1]

$\left[\dfrac{2}{(1+\sin\theta)^2}\right]_0^k=-4\left(\dfrac{5}{18}\right)$[M1]

$\dfrac{2}{(1+\sin k)^2}-\dfrac{2}{1}=-\dfrac{10}{9}$ [M1]

$\dfrac{2}{(1+\sin k)^2}=\dfrac{8}{9}$

$(1+\sin k)^2=\dfrac{9}{4}$

$1+\sin k=\dfrac{3}{2}$ $\quad\quad$ $1+\sin k=-\dfrac{3}{2}$ [M1]

$\sin k=\dfrac{1}{2}$ $\quad\quad$ $\sin k=-\dfrac{5}{2}$ (rej)

$k=\dfrac{\pi}{6}$ [A1]

(or 0.524)

5. The curve $y = f(x)$ is such that $f'(x) = (k - 2)e^{3x}$.

(i) For y to be an increasing function of x, what condition must be applied to the constant k? [2]

$e^{3x} > 0$ [M1]

$k - 2 > 0$

$k > 2$ [A1]

(ii) Given that $P(0,3)$ is a point on the curve and the gradient of the tangent to the curve at P is 4, find an expression for f(x). [4]

At P (0, 3),

$4 = (k - 2)e^{3(0)}$

$k - 2 = 4$

$k = 6$ [M1]

$f'(x) = 4e^{3x}$

$f(x) = \frac{4e^{3x}}{3} + c$ [M1]

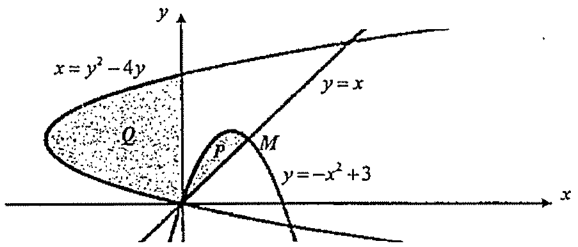
At $P(0, 3)$,

$3 = \frac{4e^{0}}{3} + c$

$c = \frac{5}{3}$ [M1]

$f(x) = \frac{4e^{3x}}{3} + \frac{5}{3}$ [A1]

6.



(i) M is the point of interection of $y = x$ and $y = -x^2 + 3x$.
Show that the coordinates of M is (2, 2). [1]

$x = -x^2 + 3x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \text{ or } x = 2$

When $x = 2, y = 2$ Therefore $M(2, 2)$

(ii) Find the area P, bounded by the curve $y = -x^2 + 3x$ and the line $y = x$. [3]

Area $P = \int_0^2 \left(-x^2 + 3x - x\right) dx$ [M1]

$= \int_0^2 \left(-x^2 + 2x\right) dx$

$= \left[-\frac{x^3}{3} + \frac{2x^2}{2} \right]_0^2$ [M1]

$= 4 - 2\frac{2}{3} = 1\frac{1}{3}$ units² [A1]

(iii) Find the area Q, enclosed by the curve $x = y^2 - 4y$ and the y-axis. [3]

$0 = y^2 - 4y$

$0 = y(y-4)$

$y = 0 \text{ or } y = 4$ [M1]

Area $Q = \left| \int_0^4 \left(y^2 - 4y\right) dy \right| = \left| \left[\frac{y^3}{3} - \frac{4y^2}{2} \right]_0^4 \right|$ [M1] $= 10\frac{2}{3}$ units² [A1]

7. (i) In an electrical circuit, the voltage, V volts, is given by the formula $V = IR$, where I amperes is the current. Given that $R=\frac{1}{10}\left(6\sqrt{2}+7\sqrt{3}\right)$ and $I=5\sqrt{6}$, find V in the form of $a\sqrt{3}+b\sqrt{2}$ where a and b are constants to be determined. [2]

$$V=5\sqrt{6}\left[\frac{1}{10}\left(6\sqrt{2}+7\sqrt{3}\right)\right]$$

$$=\frac{\sqrt{6}}{2}\left[\left(6\sqrt{2}+7\sqrt{3}\right)\right] \text{ [M1]}$$

$$=3\sqrt{12}+\frac{7}{2}\sqrt{18}$$

$$=6\sqrt{3}+\frac{21}{2}\sqrt{2} \text{ [A1]}$$

(ii) Two resistors, whose resistances are R_1 ohms and R_2 ohms respectively, are connected in parallel in the electrical circuit. The total resistance, R ohms, is given by the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$. Given that $R=\frac{1}{10}\left(6\sqrt{2}+7\sqrt{3}\right)$ and $R_1=\sqrt{3}$, find $\frac{1}{R_2}$ in the form of $\frac{a\sqrt{2}+b\sqrt{3}}{5}$ where a and b are constants to be determined. [4]

$$\frac{1}{R_2}=\frac{1}{R}-\frac{1}{R_1}$$

$$\frac{1}{R_2}=\frac{10}{6\sqrt{2}+7\sqrt{3}}-\frac{1}{\sqrt{3}} \text{ [M1]}$$

$$\frac{1}{R_2}=\frac{10\left(6\sqrt{2}-7\sqrt{3}\right)}{36(2)-49(3)}-\frac{\sqrt{3}}{3} \text{ [M1]}$$

$$\frac{1}{R_2}=-\frac{2\left(6\sqrt{2}-7\sqrt{3}\right)}{15}-\frac{\sqrt{3}}{3}$$

$$\frac{1}{R_2}=\frac{-12\sqrt{2}+14\sqrt{3}-5\sqrt{3}}{15} \text{ [M1]}$$

$$\frac{1}{R_2}=\frac{-12\sqrt{2}+9\sqrt{3}}{15}$$

$$\frac{1}{R_2}=\frac{-4\sqrt{2}+3\sqrt{3}}{5} \text{ [M1]}$$

8. (i) Express $y = 2\sin x + 4\cos x$ in the form of $R\sin(x+\alpha)$ and find the minimum value of $y = 2\sin x + 4\cos x$, stating the value of x between $0°$ and $360°$. [4]

$R = \sqrt{(2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$ [M1]

$\alpha = \tan^{-1}\left(\frac{4}{2}\right) = 63.4349°$

$y = 2\sqrt{5}\sin(x + 63.4°)$ (1dp) [A1]

Min Value = $-2\sqrt{5}$ [M1]

$\sin(x + 63.4349°) = -1$

$x + 63.4349° = 270°$

$x = 206.6°$ (1dp) [A1]

(ii) Hence, solve $3 = 2\sin x + 4\cos x$ for $0° \le x \le 360°$. [3]

$3 = 2\sqrt{5}\sin(x + 63.4349°)$

$\sin(x + 63.4349°) = \frac{3}{2\sqrt{5}}$ [M1]

Basic angle $= 42.1304°$

$x + 63.4349° = 180° - 42.1304°, 360° + 42.1304°$

$x = 74.4°, 338.7°$ [A1 each]

9. The equation of a circle of centre C is $x^2+y^2+8x-12y+27=0$.

(i) Find the radius of the circle and the coordinates of C. [3]

Let centre be (-g, -f)

$2gx=8x$ $\quad 2fy=-12y$

$g=4$ $\quad f=-6$

Therefore $C(-4,6)$ [A1]

Radius $=\sqrt{(4)^2+(-6)^2-27}$ [M1]

$=5$ units [A1]

(ii) The point A is $(0, 6)$. Determine if A is inside the circle or outside the circle. [2]

Length of AC $=\sqrt{[0-(-4)]^2+(6-6)^2}$ =4 units [M1]

Since the length of AC is less than the length of the radius, hence A is inside the circle. [A1]

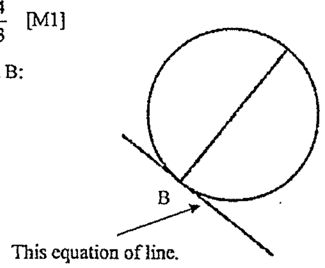
(iii) Find the equation of the line that touches the circle at $B(-7,10)$. [3]

Gradient of line BC $=\frac{10-6}{-7-(-4)}=-\frac{4}{3}$ [M1]

Equation of line that touches circle at B:

$y-10=\frac{3}{4}(x+7)$ [M1]

$y=\frac{3}{4}x+\frac{61}{4}$ [A1]



10. (a) Given that the coefficient of x in the expansion of $\left(x+\frac{k}{2x^2}\right)^{16}$ is 4368, find the value of k. [3]

General term $= \binom{16}{r}(x)^{16-r}\left(\frac{k}{2x^2}\right)^r$ [M1]

$= \binom{16}{r}(x)^{16-r}\left(\frac{k}{2}\right)^r x^{-2r}$

$= \binom{16}{r}\left(\frac{k}{2}\right)^r x^{16-3r}$

$16-3r=1$

$r=5$ [M1]

$4368=\binom{16}{5}\left(\frac{k}{2}\right)^5$

$4368=4368\left(\frac{k}{2}\right)^5$

$\left(\frac{k}{2}\right)^5=1$

$k^5=32$

$k=2$ [A1]

(b) (i) Find the value of n, given that the coefficients of x^4 and x^6 in the expansion of $\left(1+\frac{1}{3}x^2\right)^n$ are in the ratio of $3:2$ [4]

$$\left(1+\frac{1}{3}x^2\right)^n = 1+\binom{n}{1}\left(\frac{1}{3}x^2\right)+\binom{n}{2}\left(\frac{1}{3}x^2\right)^2+\binom{n}{3}\left(\frac{1}{3}x^2\right)^3+..$$

$= 1+n\left(\frac{1}{3}x^2\right)+\binom{n}{2}\frac{1}{9}x^4+\binom{n}{3}\frac{1}{27}x^6+..$ [Showing the coeff of $x^4 = \binom{n}{2}\frac{1}{9}$ M1]

$$\frac{\binom{n}{2}\frac{1}{9}}{\binom{n}{3}\frac{1}{27}}=\frac{3}{2}$$ [Showing the coeff of $x^6 = \binom{n}{3}\frac{1}{27}$ M1]

$$\frac{\binom{n}{2}}{\binom{n}{3}}=\frac{1}{2}$$

$$2\binom{n}{2}=\binom{n}{3}$$

$\frac{2n(n-1)}{2}=\frac{n(n-1)(n-2)}{6}$ [M1]

$n-2=6$

$n=8$ [A1]

If use guess and check to get n = 8, no method mark.

(ii) Hence, find the coefficient of x^6 in the expansion of $(1-6x+9x^2)\left(1+\frac{1}{3}x^2\right)^n$. [2]

Coefficient of $x^6 = \binom{8}{2}\left(\frac{1}{9}\right)\times 9+\binom{8}{3}\left(\frac{1}{27}\right)\times 1$ [M1]

$=30\frac{2}{27}$ [A1]

11. (a) Factorise completely $27a^3 - 125b^3$. [2]

$27a^3 - 125b^3$

$= (3a)^3 - (5b)^3$

$= (3a - 5b)[(3a)^2 + (3a)(5b) + (5b)^2]$ [M1]

$= (3a - 5b)(9a^2 + 15ab + 25b^2)$ [A1]

(b) It is given that $3x^3 + 3x^2 - 11x - 6$ when divided by $x + a$ has a remainder that is half the remainder when it is divided by $x - a$.

(i) Show that $3a^3 - a^2 = 11a - 2$. [3]

Let $f(x) = 3x^3 + 3x^2 - 11x - 6$

$f(-a) = 3(-a)^3 + 3(-a)^2 - 11(-a) - 6$

$= -3a^3 + 3a^2 + 11a - 6$

$f(a) = 3(a)^3 + 3(a)^2 - 11(a) - 6$ [M1]

$-3a^3 + 3a^2 + 11a - 6 = \frac{1}{2}\left(3a^3 + 3a^2 - 11a - 6\right)$

$0 = 9a^3 - 3a^2 - 33a + 6$

$0 = 3a^3 - a^2 - 11a + 2$ [M1]

$3a^3 - a^2 = 11a - 2$

(ii) Solve $3a^3 - a^2 = 11a - 2$, giving your answer to two decimal places where necessary. [4]

$3a^3 - a^2 - 11a + 2 = 0$

[M1] for showing the linear factor.

$(a - 2)$ is a factor.

$(a-2)(3a^2 + pa - 1) = 0$

$-2p - 1 = -11$

$2p = 10$

$p = 5$

[M1] for showing how the quadratic factor is obtained.

$(a-2)(3a^2 + 5a - 1) = 0$

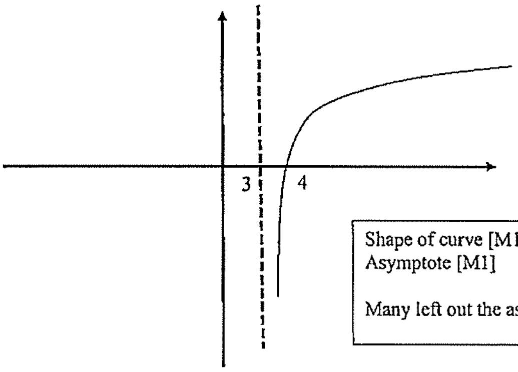
[M1] for showing the linear and quadratic factor equal to zero

$a - 2 = 0 \quad 3a^2 + 5a - 1 = 0$

$a = 2 \quad a = \dfrac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)} = 0.18$ or -1.85 (2dp)

Answer [A1]

12. (a) (i) Sketch the graph of $y=\ln(x-3)$. [2]



Shape of curve [M1]
Asymptote [M1]

Many left out the asymptote line.

(ii) Find the equation of a suitable straight line that can be inserted to solve the equation $2=(x-3)e^{3x}$. [2]

$$\frac{2}{e^{3x}}=(x-3)$$
$$\ln\frac{2}{e^{2x}}=\ln(x-3) \quad [M1]$$
$$\ln 2-\ln e^{3x}=\ln(x-3)$$
$$\ln 2-3x=\ln(x-3)$$
$$y=-3x+\ln 2 \quad [A1]$$

(b) Find the coordinates of the stationary point of $y=\frac{x^2}{e^{x-1}}$, for $x>0$, leaving your answer in exact form and determine the nature of this stationary point [4]

$$\frac{dy}{dx}=\frac{e^{x-1}(2x)-x^2e^{x-1}}{\left(e^{x-1}\right)^2} \quad \text{[M1]}$$

$$=\frac{xe^{x-1}(2-x)}{\left(e^{x-1}\right)^2}$$

$$=\frac{x(2-x)}{e^{x-1}}$$

$$0=\frac{x(2-x)}{e^{x-1}} \quad \text{[M1]}$$

$0=x(2-x)$

$2x=0 \quad 2-x=0$

$x=0 \quad x=2$

When $x=2$, $y=\frac{4}{e}$

For $(2, \frac{4}{e})$, [M1]

x	$x<2$	$x=2$	$x>2$
shape	/	—	\

Or

$$\frac{d^2y}{dx^2}=\frac{e^{x-1}(2-2x)-(2x-x^2)e^{x-1}}{\left(e^{x-1}\right)^2}$$

$$=\frac{2-2x-2x+x^2}{e^{x-1}}$$

$$=\frac{x^2-4x+2}{e^{x-1}}$$

$(2, \frac{4}{e})$ is a max point [A1]

End of Paper.

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PRELIMINARY EXAMINATION 2019
Secondary 4

Wednesday	ADDITIONAL MATHEMATICS	4047/2
21 August 2019	PAPER 2	2 hours 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

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Write in dark blue or black pen.
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Answer all questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This question paper consists of 22 printed pages and 2 blank pages.

ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

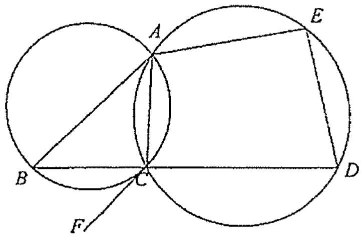
1. A curve has the equation $y=(x-9)\sqrt{2x+5}$. Find,

(i) $\frac{dy}{dx}$, [3]

(ii) the rate of change of x when $x = 5.5$, given that y is increasing at a constant rate of 6.25 units/s. [2]

(iii) If the normal to the curve $y=(x-9)\sqrt{2x+5}$ at the point P (a, b) is parallel to the line $2y+3x=2$, find the integral value of a. [3]

2.



The diagram shows two circles that intersect each other at points A and C. The points E and D lie on the circumference of the larger circle. The point B lies on the circumference of the smaller circle such that BCD is a straight line. Line CF is a tangent to the smaller circle at C. $AC = BC$ and $AE = ED$.

(i) Prove that AB and CF are parallel. [3]

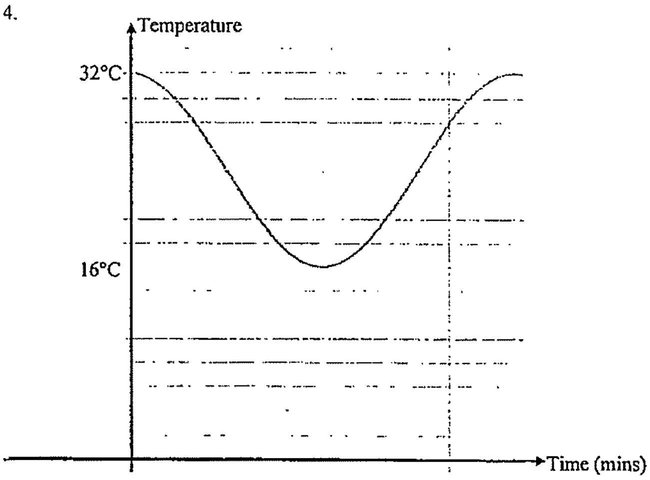
(ii) Prove that ΔABC is similar to ΔADE and hence show that $AB \times DE = AD \times BC$. [3]

3. (i) Differentiate $\ln\left(2x^2+1\right)$ with respect to x. [1]

(ii) Express $\dfrac{6x^2-12x+5}{(x+3)(2x^2+1)}$ in terms of its partial fractions. [3]

(iii) Using the results of part (i) and (ii), evaluate $\int_1^2 \frac{12x^2-24x+10}{(x+3)(2x^2+1)}\,dx$. [4]

4.



For decades, air conditioners had single-speed compressors that were either on or off. With a single-speed compressor, when the temperature inside the room reaches above a certain temperature, the compressor suddenly switches on. And when the temperature drops below a certain value, the compressor will be cut off. The above graph shows how the temperature in a room changes with time for Model A, a single-speed compressor air conditioner. The equation of the curve is given as $T = a\cos\left(\frac{\pi}{30}t\right) + b$ in degree Celsius where t is the time in minutes and T is the temperature in the room.

Find

(i) the time taken to reach the lowest temperature of 16°C, [1]

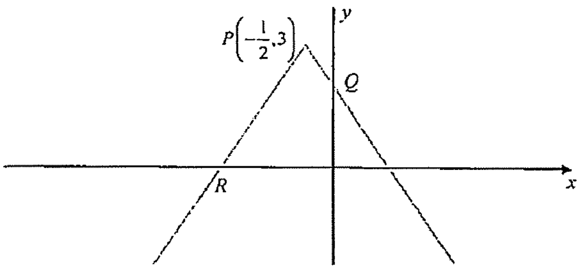
(ii) the values of a and b. [2]

With technological advancements, Model B was designed for Singapore climate. For this Model B, the equation is given by $T_1 = -4\sin\left(\frac{\pi}{40}t\right) + 24°C$.

(iii) Draw the new graph on the same diagram for the first hour. [2]

(iv) State the time duration for Model B to maintain below $24°C$.
Hence, determine which air conditioner model can maintain a temperature below $24°C$ in the room for a longer duration. [2]

5.



The figure shows part of the graph of $y = h - |kx + 1|$, where $P\left(-\frac{1}{2},3\right)$ is the maximum point of the graph. Find

(i) the value of h and of k, [2]

(ii) the coordinates of the points Q and R. [3]

Hence, in each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = h - |kx + 1|$, justifying your answer.

(iii) $m = -2$ and $c < 1$. [2]

(iv) $m = 1$ and $c < 3$. [2]

6. A cup of hot coffee was put on the table to cool at 3 pm. The temperature of the coffee, T°C, after x minutes, is given $T = 20 + ae^{-kx}$ where a and k are constants. The table shows that values of T and x taken at different timings. It is believed that an error was made in recording one of the values of T.

x	5	10	15	20
T	68.5	60.1	52.6	37.1

(i) Using a scale of 4 cm to 5 minutes for x and 4 cm to 1 unit for $\ln(T-20)$, plot $\ln(T-20)$ against x and draw a straight line graph. [2]

(ii) Determine which value of T, in the table above, is the incorrect recording and use your graph to estimate its correct value. [2]

(iii) Use your graph to estimate,

(a) the value of a and the value of k. [3]

(b) the time when the temperature of the coffee is 50°C. [1]

(iv) Explain why the temperature of the coffee is always more than 20°C. [1]

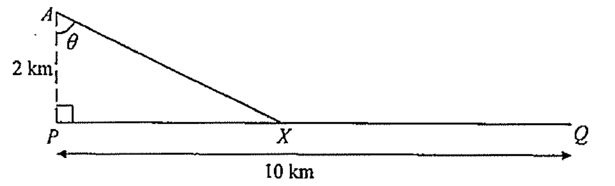
Answer for Question 6(i)

7. (i) Prove the identity $\dfrac{4\cos 2x}{1+\cos 2x} = 4 - 2\sec^2 x$. [3]

(ii) Hence, or otherwise, solve, for $0 < x < 2\pi$, the equation $\dfrac{4\cos 2x}{1+\cos 2x} = 3\tan x - 7$. [4]

(iii) Show that $\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}\,dx = \frac{1}{4}(\pi - 2)$. [3]

8.



The diagram shows a straight road PQ, of length 10 km. A man is at point A, where AP is perpendicular to PQ and AP is 2 km. He travels in a straight line to meet the road at point X, where angle $PAX = \theta$ radians. The man travels at 3 km/h along AX and 5 km/h along XQ.
He takes T hours to travel from A to Q.

(i) Show that $T = \dfrac{2\sec\theta}{3} + 2 - \dfrac{2\tan\theta}{5}$. [4]

(ii) Given that θ can vary, show that T has a minimum value when $PX = 1.5\text{km}$. [6]

9. A particle P moves in a straight line from a fixed point O so that its velocity, v m/s, is given by $v = t^2 - 10t + 24$, where t is the time in seconds after leaving O. Find

(i) the time when the particle is instantaneously at rest, [2]

(ii) the acceleration of P when $t = 5$, [2]

(iii) the distance of P from O at $t = 8$ seconds, [2]

(iv) the average speed of the particle during the first 8 seconds. [3]

10. (a) Given that α and β are the roots of the equation $x^2 - 16x + 48 = 0$,

(i) write down the value of $\alpha + \beta$ and $\alpha\beta$, [2]

(ii) find an equation whose roots are $3\alpha + \beta$ and $\alpha + 3\beta$. [4]

(b) Show that $2x^2 - 8x + 11$ is always positive for all real values of x. [3]

(c) Find the range of values of k such that $y + 5 = kx$ intersects $y + 1 = x^2$ at two distinct points. [3]

11. (a) Solve the equation $\ln(3x^2 - x - 6) = 2\ln x$. [3]

(b) Solve the following simultaneous equations

$e\sqrt{e^x} = e^{2y}$

$\log_4(x+2) = 1 + \log_2 y$, [5]

(c) Given that $\frac{x^{3m}}{y^{2-m}} \times \frac{y^{n}}{(x^{2n-1})^{2}} = \frac{1}{xy^{3}}$, find the values of m and n. [4]

End of Paper.

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1. A curve has the equation $y=(x-9)\sqrt{2x+5}$. Find,

(i) $\frac{dy}{dx}$, [3]

M1 M1

$$\frac{dy}{dx}=(x-9)\left(\frac{1}{2}\right)(2x+5)^{-\frac{1}{2}}(2)+(2x+5)^{\frac{1}{2}}(1)$$

$$=(2x+5)^{-\frac{1}{2}}(x-9+2x+5)$$

$$=\frac{3x-4}{\sqrt{(2x+5)}} \quad \text{A1}$$

(ii) the rate of change of x when $x = 5.5$, given that y is increasing at a constant rate of 6.25 units/s. [2]

$$\frac{dy}{dt}=6.25$$

$$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt} \quad \text{M1}$$

$$6.25=\left(\frac{3x-4}{\sqrt{2x+5}}\right)\frac{dx}{dt}$$

$$6.25=\left(\frac{3(5.5)-4}{\sqrt{2(5.5)+5}}\right)\frac{dx}{dt}$$

$$6.25=\frac{25}{8}\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt}=2$$

Hence the rate of change of $x = 2$ units/s A1

(iii) If the normal to the curve $y=(x-9)\sqrt{2x+5}$ at the point P (a, b) is parallel to the line $2y+3x=2$, find the integral value of a. [3]

Gradient of $2y+3x=2$, $y=-\frac{3}{2}x+1$,

Gradient of the normal $= -\frac{3}{2}$ M1

Gradient of the tangent $= \frac{2}{3}$

$$\frac{3x-4}{\sqrt{2x+5}}=\frac{2}{3}$$

$$9x-12=2\sqrt{2x+5} \quad \text{M1}$$

$$81x^2-216x+144=4(2x+5)$$

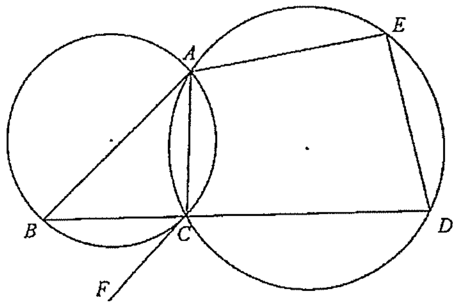
$$81x^2-224x+124=0$$

$$x=\frac{224\pm\sqrt{(224)^2-4(81)(124)}}{2(81)}$$

$$=2 \quad \text{or} \quad \frac{62}{81} \text{ NA}$$

A1

2.



The diagram shows two circles that intersect each other at points A and C. The points E and D lie on the circumference of the larger circle. The point B lies on the circumference of the smaller circle such that BCD is a straight line. Line CF is a tangent to the smaller circle at C. $AC = BC$ and $AE = ED$.

(i) Prove that AB and CF are parallel. [3]

Let $\angle ABC = x$

$\angle CAB = \angle ABC = x$ $(AC = BC)$ [1]

$\angle CAB = \angle FCB = x$ (tangent chord theorem) [1]

Since $\angle FCB = \angle ABC = x$, AB and CF are parallel, alternate angles. [1]

(ii) Prove that ΔABC is similar to ΔADE and hence show that $AB \times DE = AD \times BC$. [3]

$\angle ACB = 180° - 2x$ (angle sum of triangle)
$\angle ACD = 2x$ (Supplementary angle)
$\angle AED = 180° - 2x$ (angle in opposite segment) [1]
$= \angle ACB$
Since $\angle EAD = x$ (angle sum of isosceles triangle) $= \angle ABC$ [1]
Triangle ABC is similar to triangle ADE

$$\frac{AB}{AD} = \frac{BC}{DE} \quad [1]$$

$$AB \times DE = BC \times AD$$

3. (i) Differentiate $\ln(2x^2+1)$ with respect to x. [1]

$$\frac{d}{dx}\left[\ln\left(2x^2+1\right)\right]=\frac{4x}{2x^2+1} \qquad [1]$$

(ii) Express $\dfrac{6x^2-12x+5}{(x+3)(2x^2+1)}$ in terms of its partial fractions. [3]

$$\frac{6x^2-12x+5}{(x+3)(2x^2+1)}=\frac{A}{x+3}+\frac{Bx+C}{2x^2+1}$$

$$6x^2-12x+5=A\left(2x^2+1\right)+\left(x+3\right)\left(Bx+C\right)$$

Subs $x=-3$, $95=A\,(19)$

$A=5$ [1]

Subs $x=0$, $5=A\,(1)+3(C)$

$C=0$ [1]

Subs $x=1$, $6-12+5=A\,(3)+B\,(4)$

$B=-4$ [1]

$$\frac{6x^2-12x+5}{(x+3)(2x^2+1)}=\frac{5}{x+3}-\frac{4x}{2x^2+1}$$

(iii) Using the results of part (i) and (ii), evaluate $\int_1^2 \frac{12x^2-24x+10}{(x+3)(2x^2+1)}\,dx$. [4]

$$\int_1^2 \frac{12x^2-24x+10}{(x+3)(2x^2+1)}\,dx = 2\int_1^2 \frac{6x^2-12x+5}{(x+3)(2x^2+1)}\,dx \quad \text{M1}$$

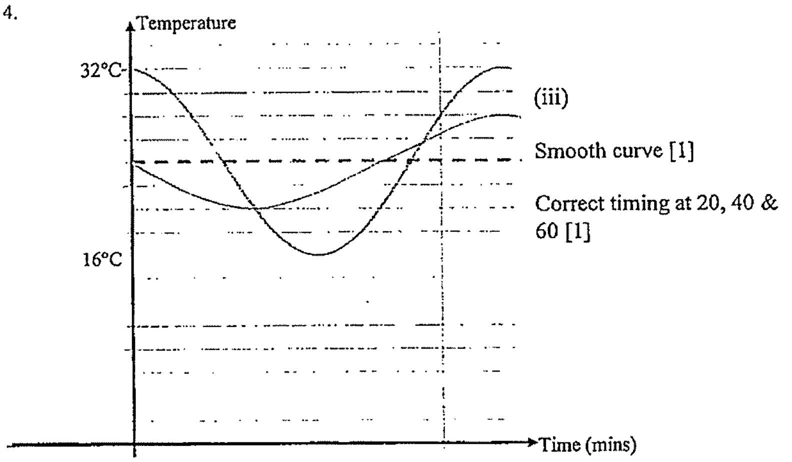
$$= 2\int_1^2 \frac{5}{x+3} - \frac{4x}{2x^2+1}\,dx$$

$$= 2\Big[5\ln(x+3) - \ln(2x^2+1)\Big]_1^2 \quad \text{M1, M1}$$

$$= 2\Big[(5\ln 5 - \ln 9) - (5\ln 4 - \ln 3)\Big]$$

$$= 0.0342 \quad \text{A1}$$

4.



For decades, air conditioners had single-speed compressors that were either on or off. With a single-speed compressor, when the temperature inside the room reaches above a certain temperature, the compressor suddenly switches on. And when the temperature drops below a certain value, the compressor will be cut off. The above graph shows how the temperature in a room changes with time for Model A, a single-speed compressor air conditioner. The equation of the curve is given as $T = a\cos\left(\frac{\pi}{30}t\right) + b$ in degree Celsius where t is the time in minutes and T is the temperature in the room.

Find

(i) the time taken to reach the lowest temperature of 16°C, [1]

$\frac{\pi}{30}t = \pi$ [A1]

$t = 30$ mins

(ii) the values of a and b. [2]

$a = \frac{32-16}{2}$ [A1] $b = 16 + 8 = 24$ [A1]

$= 8$

With technological advancements, Model B was designed for Singapore climate. For this Model B, the equation is given by $T_1 = -4\sin\left(\frac{\pi}{40}t\right) + 24°C$.

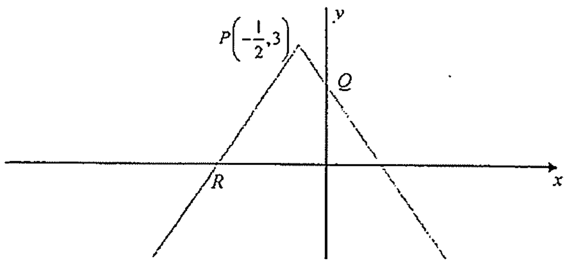
(iii) Draw the new graph on the same diagram for the first hour. [2]

(iv) State the time duration for Model B to maintain below $24°C$.
Hence, determine which air conditioner model can maintain a cooler temperature in the room for a longer duration? [2]

Time duration for Model B is 40 mins. [1]

Air conditioner model B can maintain for a longer duration. [1]

5.



The figure shows part of the graph of $y = h - |kx+1|$, where $P\left(-\frac{1}{2},3\right)$ is the maximum point of the graph. Find

(i) the value of h and of k, [2]

$h = 3$ [1]

$kx + 1 = 0$

$k\left(-\frac{1}{2}\right)+1=0$ [1]

$k = 2$

(ii) the coordinates of the points Q and R. [3]

$y = 3 - |2x+1|$

when $x = 0, y = 2$ [1]

$Q(0, 2)$

When $y = 0$, $0 = 3 - |2x+1|$

$3 = |2x+1|$

$3 = 2x+1$ or $-3 = 2x+1$ [1]

$2x = 2$ $x = -2$

$x = 1$

Therefore R (–2, 0) [1]

Hence, in each of the following cases, determine the number of intersections of the line $y = mx + c$ with $y = h - |kx+1|$, justifying your answer.

(iii) $m = -2$ and $c < 1$. [2]

$y = -2x + c$, where $c < 1$

A1

The line is parallel to the right arm of the modulus graph and hence only 1 point of intersection.

A1

(iv) $m = 1$ and $c < 3$. [2]

$y = -2x + c$, where $c < 3$

A1

Line is not parallel to any of the arms of the modulus graph and $c < 3$,

Therefore, the line will intersect at 2 points.

A1

6. A cup of hot coffee was put on the table to cool at 3 pm. The temperature of the coffee, after x minutes, is given $T = 20 + ae^{-kx}$ where a and k are constants. The table shows that values of T and x taken at different timings. It is believed that an error was made in recording one of the values of T.

x	5	10	15	20
T	68.5	60.1	52.6	37.1

(i) Using a scale of 4 cm to 5 minutes for x and 4 cm to 1 unit for $\ln(T-20)$, plot $\ln(T-20)$ against x and draw a straight line graph. [2]

(ii) Determine which value of T, in the table above, is the incorrect recording and use your graph to estimate its correct value. [2]

Incorrect value of $T = 37.1$ A1

$\ln(T-20) = 3.25$

Correct value: $T - 20 = 25.790$

$T = 45.79 \approx 45.8$ A1

(iii) Use your graph to estimate,

(a) the value of a and the value of k. [3]

$T - 20 = ae^{-kx}$

$\ln(T-20) = \ln a - kx$

$\text{Gradient} = -k = \dfrac{4.15-3.25}{-20} = -0.045$

$k = 0.045$ $[0.04 \le k \le 0.045]$ A1

$\ln a = 4.15$ $[4.05 \le \ln a \le 4.15]$ A1

$a = 63.4$ $[57.4 \le a \le 63.4]$ A1

(b) the time when the temperature of the coffee is 50°C. [1]

$\ln(50-20) = \ln 30 = 3.40119$

From the graph, $x = 16.25$ $[16.25 \le x \le 16.9]$ A1

Time: 3.16pm or 3.17pm

(iv) Explain why the temperature of the coffee is always more than 20°C. [1]

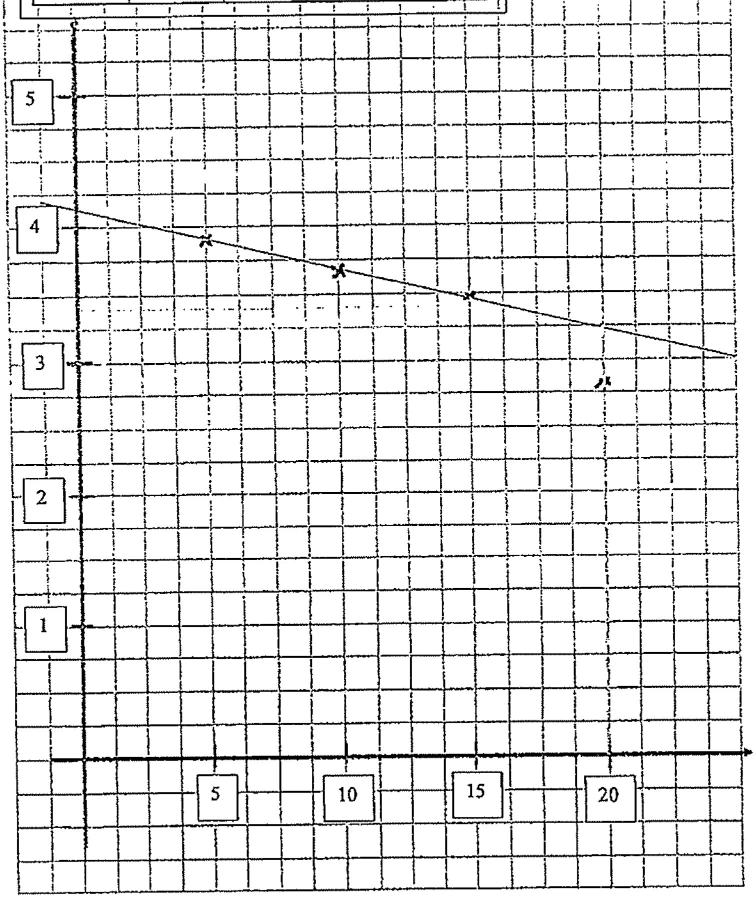
$T - 20 = ae^{-kx}$

$T = 20 + ae^{-kx}$ A1

As $ae^{-kx} > 0$, $T > 20$ and hence the lowest temperature of coffee always more than 20°C.

Answer for Question 6(i)

x	5	10	15	20
$ln(T-20)$	3.88	3.69	3.48	2.83



7. (i) Prove the identity $\frac{4\cos 2x}{1+\cos 2x} = 4 - 2\sec^2 x$. [3]

$$\frac{4\cos 2x}{1+\cos 2x} = \frac{4(2\cos^2 x - 1)}{1+(2\cos^2 x - 1)}$$ M1 M1

$$= \frac{8\cos^2 x - 4}{2\cos^2 x}$$

$$= 4 - \frac{2}{\cos^2 x}$$ A1

$$= 4 - 2\sec^2 x$$

(ii) Hence, or otherwise, solve, for $0 < x < 2\pi$, the equation $\frac{4\cos 2x}{1+\cos 2x} = 3\tan x - 7$. [4]

$$\frac{4\cos 2x}{1+\cos 2x} = 3\tan x - 7$$

$$4 - 2\sec^2 x = 3\tan x - 7$$

$$4 - 2(1+\tan^2 x) = 3\tan x - 7$$ M1

$$2\tan^2 x + 3\tan x - 9 = 0$$

$$(\tan x + 3)(2\tan x - 3) = 0$$ M1

$\tan x = -3$ or $\tan x = \frac{3}{2}$

$x = 1.89, 5.03$ A1 $\quad x = 0.983, 4.12$ A1

(iii) Show that $\int_0^{\frac{\pi}{4}} \frac{\cos 2x}{1+\cos 2x}\,dx = \frac{1}{4}(\pi - 2)$. [3]

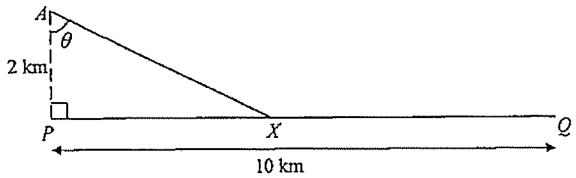
$$\frac{1}{4}\int_0^{\frac{\pi}{4}} \frac{4\cos 2x}{1+\cos 2x}\,dx = \frac{1}{4}\int_0^{\frac{\pi}{4}} \left(4 - 2\sec^2 x\right)dx \quad \boxed{\text{M1}}$$

$$= \frac{1}{4}\left[4x - 2\tan x\right]_0^{\frac{\pi}{4}} \quad \boxed{\text{M1}}$$

$$= \frac{1}{4}\left[4\left(\frac{\pi}{4}\right) - 2\tan\left(\frac{\pi}{4}\right) - 0\right]$$

$$= \frac{1}{4}(\pi - 2) \quad \boxed{\text{A1}}$$

8.



The diagram shows a straight road PQ, of length 10 km. A man is at point A, where AP is perpendicular to PQ and AP is 2 km. He travels in a straight line to meet the road at point X, where angle $PAX = \theta$ radians. The man travels at 3 km/h along AX and 5 km/h along XQ. He takes T hours to travel from A to Q.

(i) Show that $T = \frac{2\sec\theta}{3} + 2 - \frac{2\tan\theta}{5}$. [4]

$$AX = \frac{2}{\cos\theta}$$ M1

Total time taken to travel $AX = \frac{1}{3} \times \frac{2}{\cos\theta} = \frac{2\sec\theta}{3}$ M1

$$\frac{PX}{2} = \tan\theta$$
$$PX = 2\tan\theta$$
$$XQ = 10 - 2\tan\theta$$ M1

Time for $XQ = \frac{10 - 2\tan\theta}{5} = 2 - \frac{2\tan\theta}{5}$

$$T = \frac{2\sec\theta}{3} + 2 - \frac{2\tan\theta}{5}$$ A1

(ii) Given that θ can vary, show that T has a minimum value when $PX = 1.5$ km. [6]

$$T = \frac{2\sec\theta}{3} + 2 - \frac{2\tan\theta}{5}$$

$$\frac{dT}{d\theta} = \frac{d}{d\theta}\left(\frac{2}{3\cos\theta}\right) - \frac{2\sec^2 x}{5}$$ M1

$$= \frac{2}{3}\left[-1(\cos\theta)^{-2}(-\sin\theta)\right] - \frac{2}{5\cos^2\theta}$$

$$= \frac{2\sin\theta}{3\cos^2\theta} - \frac{2}{5\cos^2\theta}$$ M1

$$\frac{dT}{d\theta} = 0$$

$$\frac{2\sin\theta}{3\cos^2\theta} = \frac{2}{5\cos^2\theta}$$

For minimum value, $5\cos^2\theta\sin\theta = 3\cos^2\theta$ M1

$$\cos^2\theta(5\sin\theta - 3) = 0$$

$\cos^2\theta = 0$ (NA) or $\sin\theta = \frac{3}{5}$ M1

$PX = 1.5$, $2\tan\theta = 1.5$, $\tan\theta = \frac{3}{4}$; $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$ M1

$\theta < 0.6435$	$\theta = 0.6435$	$\theta > 0.6435$
$-$ve	0	$+$ve

M1

Hence, T has a minimum value when $PX = 1.5$ km

9. A particle P moves in a straight line from a fixed point O so that its velocity, v m/s, is given by $v = t^2 - 10t + 24$, where t is the time in seconds after leaving O. Find

(i) the time when the particle is instantaneously at rest, [2]

$v = t^2 - 10t + 24 = 0$ M1

$(t-4)(t-6) = 0$

$t = 4$ or $t = 6$ A1

(ii) the acceleration of P when $t = 5$, [2]

$a = 2t - 10$ M1

at $t = 5$, acceleration = 0 A1

(iii) the distance of P from O at $t = 8$ seconds, [2]

$s = \int t^2 - 10t + 24 \; dt$

$= \frac{t^3}{3} - 5t^2 + 24t + c$ where c is an arbitrary constant M1

At $t = 0$, $s = 0$, $c = 0$

$s = \frac{t^3}{3} - 5t^2 + 24t$

at $t = 8$, $s = \frac{128}{3} = 42\frac{2}{3}$ m A1

(iv) the average speed of the particle during the first 8 seconds. [3]

at $t = 0$, $s = 0$

$t = 4$, $s = 37\frac{1}{3}$

$t = 6$, $s = 36$ M1

$t = 8$, $s = 42\frac{2}{3}$ M1

average speed $= \dfrac{37\frac{1}{3} + \left(37\frac{1}{3} - 36\right) + \left(42\frac{2}{3} - 36\right)}{8} = 5\frac{2}{3}$ A1

10. (a) Given that α and β are the roots of the equation $x^2-16x+48=0$,

(i) Write down the value of $\alpha+\beta$ and $\alpha\beta$. [2]

$\alpha+\beta=16$ A1

$\alpha\beta=48$ A1

(ii) Find an equation whose roots are $3\alpha+\beta$ and $\alpha+3\beta$. [4]

$3\alpha+\beta+\alpha+3\beta=4(\alpha+\beta)=4(16)=64$ M1

$$(3\alpha+\beta)(\alpha+3\beta)=3\alpha^2+3\beta^2+10\alpha\beta$$
$$=3\left[(\alpha+\beta)^2-2\alpha\beta\right]+10\alpha\beta$$ M1
$$=3(\alpha+\beta)^2+4\alpha\beta$$
$$=3(16)^2+4(48)$$
$$=960$$ M1

The equation is

$$x^2-64x+960=0$$ A1

(b) Show that $2x^2-8x+11$ is always positive for all real values of x. [3]

Method 1

$$2x^2-8x+11=2\left(x^2-4x\right)+11$$

$$=2\left[\left(x-2\right)^2-4\right]+11 \quad \boxed{\text{M1}}$$

$$=2\left(x-2\right)^2+3 \quad >0 \quad \boxed{\text{A1}}$$

Hence $2x^2-8x+11$ is always positive. A1

Method 2

$$D=\left(-8\right)^2-4(2)(11)$$

$$=-24 \quad <0 \quad \boxed{\text{M1}}$$

A1

Coefficient of $x^2>0$, the curve is entirely above the x-axis. Therefore, $2x^2-8x+11$ is always positive. A1

(c) Find the range of values of k such that $y+5=kx$ intersects $y+1=x^2$ at two distinct points. [3]

$$y=kx-5$$

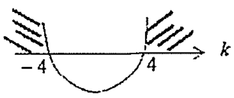
$$kx-5+1=x^2$$

$$x^2-kx+4=0 \quad \boxed{\text{M1}}$$

$$b^2-4ac>0$$

$$k^2-4(4)(1)>0$$

$$k^2-16>0 \quad \boxed{\text{M1}}$$



$k<-4$ or $k>4$ A1

11. (a) Solve the equation $\ln(3x^2-x-6)=2\ln x$. [3]

$$\ln(3x^2-x-6)=2\ln x$$
$$3x^2-x-6=x^2 \quad \boxed{\text{M1}}$$
$$2x^2-x-6=0$$
$$(2x+3)(x-2)=0$$
$$\boxed{\text{A1}} \quad x=-\frac{3}{2} \text{ (NA)} \quad \text{or} \quad x=2 \quad \boxed{\text{A1}}$$

(b) Solve the following simultaneous equations

$e\sqrt{e^x}=e^{2y}$

$\log_4(x+2)=1+2\log_2 y$, [5]

$$e\sqrt{e^x}=e^{2y}$$
$$e^{1+\frac{x}{2}}=e^{2y} \quad \boxed{\text{M1}}$$
$$1+\frac{x}{2}=2y \quad \text{---- (1)}$$
$$\log_4(x+2)=1+\log_2 y$$
$$\frac{\log_2(x+2)}{\log_2 4}=\log_2 2+\log_2 y \quad \boxed{\text{M1}}$$
$$\frac{\log_2(x+2)}{2}=\log_2 2y$$
$$\log_2(x+2)=2\log_2 2y$$
$$(x+2)=4y^2 \quad \boxed{\text{M1}} \quad \text{---- (2)}$$

$$(x+2)=(2y)^2$$
$$x+2=\left(1+\frac{x}{2}\right)^2$$
$$x+2=\left(\frac{2+x}{2}\right)^2 \quad \boxed{\text{M1}}$$
$$4(x+2)=4+4x+x^2$$
$$4x+8=4+4x+x^2$$
$$x^2=4 \quad \boxed{\text{M1}}$$
$$x=\pm 2$$

At $x=2, y=1$ $\boxed{\text{A1}}$

$x=-2, y=0$ (NA)

(c) Given that $\frac{x^{3m}}{y^{2-m}} \times \frac{y^n}{(x^{2n-1})^2} = \frac{1}{xy^3}$, find the values of m and n. [4]

$$\frac{x^{3m}}{y^{2-m}} \times \frac{y^n}{(x^{2n-1})^2} = \frac{1}{xy^3}$$

$$x^{3m-4n+2} y^{n+m-2} = x^{-1} y^{-3}$$ M1

$$3m-4n+2=-1\cdots(1)$$
$$n+m-2=-3\cdots(2)$$ M1
$$3m-4n=-3\cdots(1)$$
$$n+m=-1\cdots(2)$$
$$4m+4n=-4\cdots(2)$$
$$7m=-7$$
$$m=-1$$ A1
$$n=0$$ A1

Nan Chiau High School 2019

1 In a triangle ABC, given that angle $A = 60°$, find the exact value of $\sin(45° - B - C)$. [4]

2 (i) By using the substitutions $2^x = a$ and $3^x = b$,show that the equation $\frac{4^x - 9^x}{6^x + 4^x} = \frac{1}{3}$ can be simplified to $2a = 3b$, where $a \neq -b$. [4]

(ii) Hence, find the value of x.

3 Given that $\log_2 x + 2\log_4 y = 12$, show that $\log_8(xy) = 4$. [4]

4 (i) Show that $2\cos^2\left(\frac{\pi}{4}-x\right) = 1 + \sin 2x$. [2]

(ii) Hence, sketch the graph of $y = 2\cos^2\left(\frac{\pi}{4}-x\right)$ for $0 \leq x \leq \frac{3\pi}{2}$. [3]

5 Given that $a + b\sqrt{3} = \sqrt{151 + 28\sqrt{3}}$, where a and b are positive integers, calculate the value of a and of b. [6]

6 **(i)** On the same diagram, sketch the curves $y^2 = -\frac{1}{3}x$ and $y = x^{-\frac{1}{3}}$. [3]

(ii) Find the exact x-coordinate of the point of intersection of the two curves. [3]

7 (i) Express $\frac{5x-3}{(2x-1)(4x^2-1)}$ in partial fractions. [5]

(ii) Hence evaluate $\int_1^5 \frac{5x-3}{(2x-1)(4x^2-1)}\,dx$. [5]

8 The profit of a company, $P(x)$, in thousand dollars, is related to the number of workers, x, in hundreds, employed at the manufacturing plant. It is given that

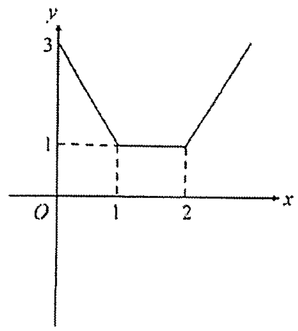
$$P'(x)=6x^2-54x+108.$$

(i) Find the range of values of x for which $P(x)$ is an increasing function. [3]

(ii) By considering $P''(x)$, explain whether $P'(x)$ is an increasing function for $x>4.5$. [2]

(iii) Given that the profit is \$ 78 000 when there are 100 workers employed at the manufacturing plant, obtain an expression for $P(x)$, in terms of x. [3]

9 The diagram shows the graph of $y = |x-1| + |x-2|$.

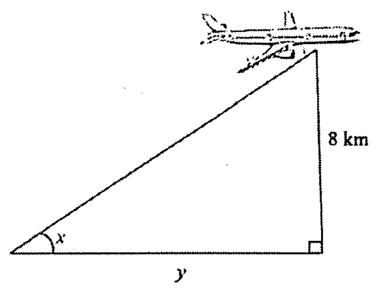


(i) Explain with clear working, why $y = |x-1| + |x-2|$ remains at the constant value of 1 for $1 \leq x \leq 2$. [2]

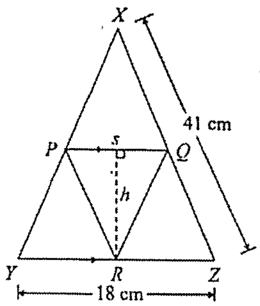
(ii) On the same diagram above, add in the graph of $y = -|2x-3| + 2$. [3]

(iii) Hence, state the number of solution(s) to the equation $|2-2x| + |4-2x| + |6-4x| = 4$. [1]

10 An aircraft flies at a constant height along a straight path of 8 km above the ground. [5]
The angle of elevation from a fixed point on the ground to the aircraft is marked as x radians, which varies with time. The horizontal distance along the ground of the aircraft from the fixed point is marked as y km.
The aircraft is flying at a constant speed of 500 km/h.
Find the rate of change of x, in radian per hour, when x is $\frac{\pi}{6}$ radians ?



11 A triangle XYZ has sides $XY = XZ = 41$ cm and $YZ = 18$ cm. The triangle PQR is inscribed in the triangle XYZ so that PQ is parallel to YZ. R is the midpoint of YZ. Given that $PQ = s$ cm, the perpendicular height of R from PQ is h cm and the area of triangle PQR is A cm^2.



(i) Express h in terms of s. [3]

(ii) Show that the area of triangle PQR is given by $A = 20s - \frac{10}{9}s^2$. [2]

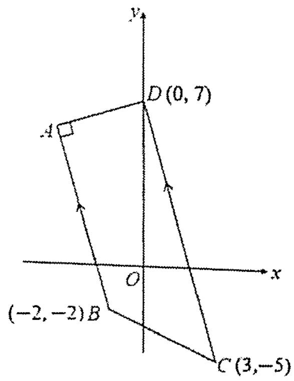
Given that s can vary,

(iii) find the value of s for which A has a stationary value. [2]

(iv) determine the nature of this stationary value, [1]

(v) name the type of quadrilateral $XQRP$ is when A takes on this stationary value. [1]

12 The diagram shows a trapezium $ABCD$ in which the coordinates of B, C and D are $(-2,-2)$, $(3,-5)$ and $(0,7)$ respectively. AB is parallel to DC and AB is perpendicular to AD.



(i) Calculate the acute angle, in degrees, that line segment AB makes with respect to the x- axis. [1]

(ii) Find the coordinates of A. [5]

(iii) Given that a point E lies on DC such that the area of triangle CBE is $\frac{2}{3}$ of the area of triangle CBD, find the coordinates of E. [3]

(iv) A point F lies on AB produced such that $BFCE$ is a parallelogram. Find the coordinates of F. [2]

~~~End of paper~~~
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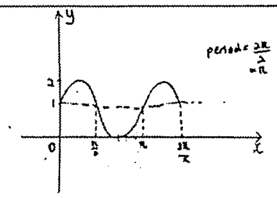
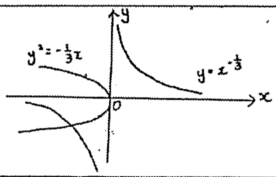
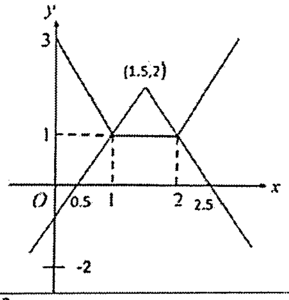
Answer key

1		$-\frac{\sqrt{2}(1+\sqrt{3})}{4}$
2	ii	$x = -1$
4	ii	
5		$b = 7(rej\ -7\ as\ b\ is\ positive)$ $a = \frac{14}{7} = 2$
6	i	
	ii	$x = (-3)^{\frac{3}{5}}\ or\ \sqrt[5]{(-3)^3}\ or\ \sqrt[5]{-27}\ or\ \left(-\frac{1}{3}\right)^{-\frac{3}{5}}$
7	i	$-\frac{1}{4(2x-1)^2} + \frac{11}{8(2x-1)} - \frac{11}{8(2x+1)}$
	ii	0.506
8	i	$0 < x < 3, x > 6$
	ii	$P''(x) > 0$ $\therefore$ *increasing function*
	iii	$P(x) = 2x^3 - 27x^2 + 108x - 5$
9	i	$\lvert x-1 \rvert = x-1$ $\lvert x-2 \rvert = -(x-2)$ $y = x - 1 - (x-2)$ $= x - 1 - x + 2$ 1

	ii	
	iii	2
10		$-15\frac{5}{8}$ or $-\frac{125}{8}$ or -15.625 radian per hour
11	i	$h = 40 - \frac{20}{9}s$
	iii	$s = 9$
	iv	$\frac{d^2A}{dx^2} = -\frac{20}{9} < 0$ A is a maximum when s = 9
	v	rhombus
12	i	76.0°
	ii	$A(-4,6)$
	iii	$\frac{CE}{CD} = \frac{2}{3}$
	iv	$F(0,-10)$