JUNYUAN SECONDARY SCHOOL
MID YEAR EXAMINATION 2017
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

CANDIDATE NAME

CLASS

INDEX NUMBER

MATHEMATICS   **4048/01**

Paper 1   **3 May 2017**

                                         2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the
answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

This document consists of **16** printed pages (including the Cover Sheet).

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab \sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1   (a)   Express 126 as the product of its prime factors.

Answer .... [1]

(b)   Using your calculator, find the value of $\pi^{3.14}$ correct to two decimal places.

Answer .... [1]

2   Showing your working clearly, find the fraction exactly halfway between $\frac{7}{8}$ and $\frac{9}{10}$.

Answer .... [2]

3   (a)   Write down $\frac{4}{11}$ in recurring decimal form.

Answer .... [1]

(b)   Express $2\frac{3}{5}$ as a percentage.

Answer .... % [1]

4   Given that $a : b = 5 : 4$, calculate the value of $\frac{2a}{7b}$.

Answer .... [2]

5   Given that $-3 \leq x \leq 2$ and $-7 \leq y \leq 3$, calculate

(a)   the largest possible value of $x - y$,

Answer .... [1]

(b)   the smallest possible value of x^2.

Answer .... [1]

6   (a)   A train left a station at 21:47 and arrived at its destination at 03:56 the following day.

How many minutes did the journey take?

Answer .... minutes [1]

(b)   Jenny runs 2.88 km in 30 minutes.

Calculate her speed in metres per second.

Answer .... m/s [2]

7   The cost of a circular ornament, $C, is directly proportional to the square of its radius, r metres.

(a)   Given that $C = 24$ when $r = 0.2$, form an equation connecting C and r.

Answer .... [2]

(b)   Hence, calculate the cost of a circular ornament with $r = 0.7$.

Answer $ .... [1]

8   For his holiday, Alex changed 2 400 Malaysian Ringgits (MYR) to Singapore Dollars (SGD) when the exchange rate was 1 SGD = 3.20 MYR.
At the end of his holiday, he had 25 SGD left.

(a)   How much did he spend in Singapore Dollars?

Answer .... SGD [2]

(b)   He changed his remaining 25 SGD for 77 MYR at the end of his holiday.

What was the exchange rate of MYR to 1 SGD at the time of exchange?

Answer .... MYR [1]

[Turn over

9   A scale model of a steam engine has a length of 60 cm.
The actual engine is 9.6 m long.

(a)   Find the scale of the model engine to the actual engine in the form 1: n.

Answer 1 : .... [1]

(b)   Given that the name plate on the model measures 4 cm by 5 cm, calculate the actual area of the name plate on the engine in square metres.

Answer .... m^2 [2]

10   (a)   Find the gradient of the straight line L_1 which has the equation $2y - 5x = 4$.

Answer .... [1]

(b)   Another straight line L_2 has the equation $3y + 2x - 4 = 0$.
It passes through the point $(3k, k + 2)$.

Calculate the value of k.

Answer $k = $ .... [2]

11 A metal beam structure weighs 13.2 megagrams and has a volume of $1.1 \times 10^5 \, cm^3$.
(mega $= 10^6$)

(a) Express 13.2 megagrams in grams, giving your answer in standard form.

Answer .... g [1]

(b) Hence find the density of the structure in g/cm^3, giving your answer in standard form.
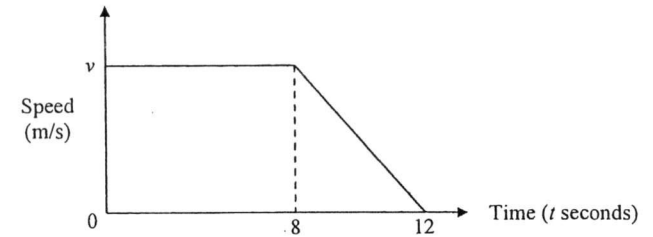
Answer .... g/cm^3 [2]

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12 Solve the inequalities $3x - 2 \le 5x + 1 < 7 - x$, and represent your solution on the number line below.

Answer .... [3]

[Turn over

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13 The diagram shows a speed-time graph for part of a car's journey.

The retardation of the car between $t = 8$ and $t = 12$ is 3.75 m/s^2.

(a) Find the value of v.

Answer $v =$ .... [1]

(b) Calculate the total distance travelled by the car in the 12 seconds.

Answer .... m [2]

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14 Given that $6^x = 9$, where x is a positive integer, find the value of

(a) 6^{-x},

Answer .... [1]

(b) $6^{\frac{x}{2}}$,

Answer .... [1]

(c) 6^{3x+1}.

Answer .... [2]

15   Solve

(a) $\dfrac{5x+6}{4}=x$,

Answer x = ... [2]

(b) $3y^2 = 2y$.

Answer y = ... or .... [2]

16   (a)   Simplify $\dfrac{(2a^{-2}b)^3}{\sqrt{16a^2b^8}}$, giving your answer in positive index.

Answer   .... [2]

(b)   Given that $8^h = \dfrac{16}{4^k}$, express h in terms of k.

Answer h = ... [2]

17   (a) $\xi = \{ \, x : x$ is an integer, where $40 \le x \le 50 \, \}$

$P = \{ \, x : x$ is a prime number $\}$

$Q = \{ \, x : x$ is a multiple of 6 $\}$

(i)   List the elements of Q.

Answer   .... [1]

(ii)   List the elements of $(P \cup Q)'$.

Answer   .... [1]

(b)   On the Venn diagrams below, draw sets A and B when
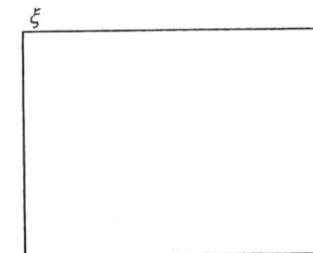
(i) $A \cap B = \emptyset$,

Answer (b)(i)

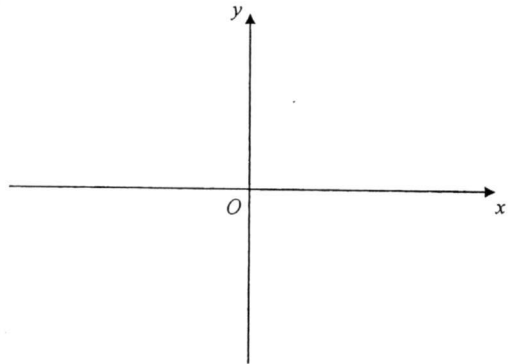[1]

(ii) $A \subset B$.

Answer (b)(ii)

[1]

[Turn over
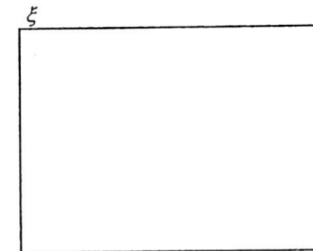
22 Calculate the exact principal amount if the difference between simple interest and compound interest compounded yearly for a period of 3 years at 5% per annum is $305.

Answer $ .... [4]

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23 **(a)** Sketch the graph of $y = -(x-3)^2 + 1$.

Answer (a)

[2]

(b) State the coordinates of the turning point of the graph $y = -(x-3)^2 + 1$.

Answer (.................. , ...) [1]

(c) Write down the equation of the line of symmetry.

Answer .... [1]

24
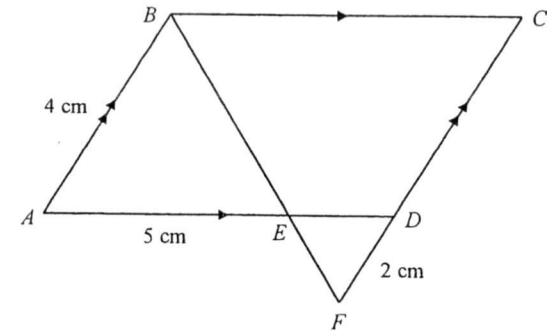
ABCD is a parallelogram.
BEF and *CDF* are straight lines.
$AB = 4$ cm, $DF = 2$ cm and $AE = 5$ cm.

(a) By stating your reasons, show that triangle *ABE* is similar to triangle *CFB*.

Answer (a)

[2]

(b) Calculate *BC*.

Answer .... cm [2]
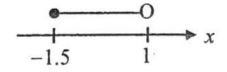
(c) Triangle *DFE* is also similar to triangle *ABE*.
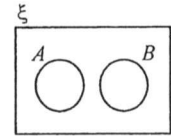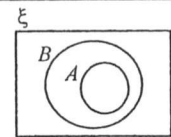
Given that the area of triangle *DFE* is 1.5 cm², find the area of triangle *ABE*.
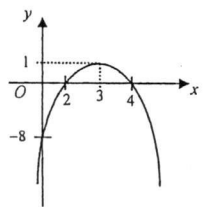
Answer .... cm² [2]

End of Paper

Mid-Year Exam 2017 Secondary 4E/5N EM P1
(MARKING SCHEME)

1	(a)	$2 \times 3^2 \times 7$ (Ans)	B1
	(b)	36.40 (Ans)	B1
2		$\dfrac{1}{2}\left(\dfrac{7}{8} + \dfrac{9}{10}\right)$	M1
		$= \dfrac{1}{2}\left(\dfrac{35+36}{40}\right) \qquad = \dfrac{1}{2}\left(\dfrac{71}{40}\right) \qquad = \dfrac{71}{80}$ (Ans)	A1
3	(a)	$0.3636363636\ldots\ldots = 0.\overset{\bullet}{3}\overset{\bullet}{6}$ (Ans)	B1
	(b)	$\dfrac{13}{5} \times 100\% \qquad = 260\%$ (Ans)	B1
4		$\dfrac{2}{7} \times \dfrac{a}{b} \qquad = \dfrac{2}{7} \times \dfrac{5}{4}$	M1
		$= \dfrac{5}{14}$ (Ans)	A1
5	(a)	$(2) - (-7) \qquad = 9$ (Ans)	B1
	(b)	$(0)^2 \qquad = 0$ (Ans)	B1
6	(a)	21:47 to 03:47 = 6 hours = 360 mins, 03:47 to 03:56 = 9 mins. So total time taken is 369 mins (Ans)	B1
	(b)	$\dfrac{2.88 \times 1000 \text{ m}}{30 \times 60 \text{ s}}$	M1
		$= 1.6$ m/s (Ans)	A1
7	(a)	$C = kr^2$, where k is a constant. $24 = k \times (0.2)^2 \qquad \Rightarrow \qquad k = 600$	M1 A1
		$\therefore C = 600\,r^2$ (Ans)	
	(b)	$C = 600 \times (0.7)^2 \qquad \Rightarrow \qquad C = \$294$ (Ans)	B1√
8	(a)	$\$\dfrac{2400}{3.20} \qquad = \$750$	M1
		$\therefore \$750 - \$25 \qquad = \$725$ (Ans)	A1
	(b)	$\dfrac{77}{25}$ MYR $\qquad = 3.08$ MYR (Ans)	B1
9	(a)	60 cm : 9.6 m $\Rightarrow$ 1 cm : 0.16 m $\Rightarrow$ 1 cm : 16 cm $\therefore n = 16$ (Ans)	B1
	(b)	Area of model = 20 cm^2 Area scale = (1 cm)^2 : (0.16m)^2 $\Rightarrow$ 1 cm^2 : 0.0256 m^2 $\therefore$ 20 cm^2 represents $20 \times 0.0256 \qquad = 0.512$ m^2 (Ans)	M1 A1
10	(a)	$2y = 5x + 4 \quad \Rightarrow \quad y = \dfrac{5}{2}x + 2 \quad \Rightarrow \quad$ gradient $= \dfrac{5}{2}$ (Ans)	B1
	(b)	$3(k+2) + 2(3k) = 4 \quad \Rightarrow \quad 3k + 6 + 6k = 4$	M1
		$\Rightarrow \quad 9k = -2 \quad \Rightarrow \quad k = -\dfrac{2}{9}$ (Ans)	A1
11	(a)	13.2×10^6 g $= 1.32 \times 10^7$ g (Ans)	B1
	(b)	Density $= \dfrac{1.32 \times 10^7 \text{ g}}{1.1 \times 10^5 \text{ cm}^3}$	M1
		$= 1.2 \times 10^2$ g/cm^3 (Ans) (accept if 1.20×10^2 is written)	A1

12		$3x - 2 \le 5x + 1$ and $5x + 1 < 7 - x$	M1
		$-2x \le 3 \qquad\qquad\qquad 6x < 6$	B1√
		$x \ge -1.5 \qquad\qquad\qquad x < 1$	A1
		$\therefore -1.5 \le x < 1$ (Ans)	
13	(a)	$\dfrac{v}{4} = 3.75 \qquad \Rightarrow \qquad v = 15$ m/s (Ans)	B1
	(b)	Total dist. traveled = Area under graph $\qquad = \dfrac{1}{2} \times (8 + 12) \times 15$	M1√
		$= 150$ m (Ans)	A1
14	(a)	$\dfrac{1}{6^x} \qquad = \dfrac{1}{9}$ (Ans)	B1
	(b)	$\left(6^x\right)^{\frac{1}{2}} = \left(9\right)^{\frac{1}{2}} \qquad = 3$ (Ans)	B1
	(c)	$6^{3x} \times 6^1 \qquad = \left(6^x\right)^3 \times 6$	M1
		$= (9)^3 \times 6 \qquad = 4\,374$ (Ans)	A1
15	(a)	$5x + 6 = 4x$	M1
		$\therefore x = -6$ (Ans)	A1
	(b)	$3y^2 - 2y = 0$	M1
		$y(3y - 2) = 0$	
		$\therefore y = 0$ or $y = \dfrac{2}{3}$ (both Ans)	A1
16	(a)	$\dfrac{8a^{-6}b^3}{4ab^4} \qquad = 2a^{-7}b^{-1}$ (for removing root and bringing in power)	M1
		$= \dfrac{2}{a^7 b}$ (Ans)	A1
	(b)	$2^{3h} = \dfrac{2^4}{2^{2k}} \qquad \Rightarrow \qquad 2^{3h} = 2^{4-2k}$	M1
		$\Rightarrow \qquad 3h = 4 - 2k \qquad \Rightarrow \qquad \therefore h = \dfrac{4 - 2k}{3}$ (Ans)	A1
17	(a)(i)	$Q = \{42, 48\}$ (Ans) (1 mark for both elements)	B1
	(a)(ii)	$(P \cup Q)' = \{40, 44, 45, 46, 49, 50\}$ (Ans) (1 mark for all 6 elements)	B1
	(b)(i)	ξ 	B1
	(b)(ii)	ξ 	B1
18	(a)	$(0 \times 2) + (1 \times 5) + (2 \times 1) + (3 \times 4) + (4 \times 6) + (5 \times 2) \qquad = 53$ (Ans)	B1
	(b)	Mean = $(53 \div 20) \qquad = 2.65$ (Ans)	B1
	(c)	Modal number means 'mode' = 4 (Ans)	B1
	(d)	Position $= \dfrac{20 + 1}{2} \qquad = 10.5 \qquad \Rightarrow \qquad$ 10th and 11th $\Rightarrow$ median = 3 (Ans)	B1

19	(a)	$55 + 75 + 23(n-2) = 360$	M1
		$130 + 23n - 46 = 360 \quad \Rightarrow \quad 23n = 276 \quad \Rightarrow \quad n = 12$ (Ans)	A1
	(b)	$\angle DCB = 105° \quad \Rightarrow \quad 2x + 110 + 75 + 105 = 540$	M1
		$\Rightarrow \quad 2x = 250 \quad \Rightarrow \quad x = 125°$ (Ans)	A1
20	(a)	$80\% \times 200 = 160$ From graph,	M1
		$x = 31.6$ mins (Ans)	A1
	(b)	From graph, when $x = 39$ mins, the cumulative frequency $y = 182$. Thus, no. of people who spends more than 39 mins is $200 - 182 = 18$.	M1
		$\therefore$ percentage of people $= \dfrac{18}{200} \times 100\% \quad = 9\%$ (Ans)	A1

21	Let x be the number of days Jim can repair a car alone.	
	Jim alone takes: x days = 1 car $\Rightarrow$ 1 day $= \dfrac{1}{x}$ car	
	Max alone takes: $(x+6)$ days = 1 car $\Rightarrow$ 1 day $= \dfrac{1}{x+6}$ car	M1
	Jim and Max both takes: 4 days = 1 car $\Rightarrow$ 1 day $= \dfrac{1}{4}$ car	
	Thus, $\dfrac{1}{x} + \dfrac{1}{x+6} = \dfrac{1}{4}$	M1
	$\Rightarrow \dfrac{(x+6)+x}{x(x+6)} = \dfrac{1}{4} \quad \Rightarrow \quad x^2 + 6x = 4(2x+6)$	
	which reduces to $\quad x^2 - 2x - 24 = 0$ $(x-6)(x+4) = 0$	M1
	$\therefore \ x = 6 \ $ or $\ x = -4$ (reject)	
	Hence, <u>Jim takes 6 days</u> to repair a car alone, and <u>Max takes 12 days</u> to repair a car alone. **(both Ans)**	A1
	(1 mark can be respectively awarded for equivalent method)	

22	Let the principal amount be \$$x$.	
	Compound Interest $-$ Simple Interest $= x\left(1 + \dfrac{5}{100}\right)^3 - \left(x + \dfrac{x \times 5 \times 3}{100}\right)$	M1
	$305 = x(1.05)^3 - \left(\dfrac{100x + 15x}{100}\right)$	M1
	$305 = 1.157625x - 1.15x$	
	$305 = 0.007625x$	M1
	$\therefore \ x = 40\,000$ (Ans)	A1

23	(a)		
		• correct symmetrical shape = 1 mark	B1
		• correct intersection of x and y-axes = 1 mark	B1
	(b)	From sketch graph, max. point $= (3, 1)$ (Ans)	B1
	(c)	$x = 3$ (Ans)	B1

24	(a)	$\angle ABE = \angle CFB$ (alt. $\angle$s, AB is parallel FC) $\angle BAE = \angle FCB$ (opp. $\angle$s of parallelogram)	B1
		$\therefore$ triangle ABE is similar to triangle CFB (AA property)	B1
	(b)	$\dfrac{AB}{CF} = \dfrac{AE}{CB} \quad \Rightarrow \quad \dfrac{4}{6} = \dfrac{5}{CB} \quad \Rightarrow \quad 4 \times CB = 30$	M1
		$\Rightarrow \quad \therefore \ BC = 7.5$ cm (Ans)	A1
	(c)	$\left(\dfrac{DF}{AB}\right)^2 = \dfrac{1.5}{A_2} \quad \Rightarrow \quad \left(\dfrac{2}{4}\right)^2 = \dfrac{1.5}{A_2}$	M1
		$\Rightarrow \quad A_2 = 6$ cm^2 (Ans)	A1

--- END OF MARKING SCHEME ---

1 (a) Expand and simplify $-(3p-2q)(2q+5p)$. [1]

(b) Given $4r^2 - h = 2f - ghr^2$, express r in terms of f, g and h. [2]

(c) Express $\dfrac{3x^2 + x - 10}{2x^2 - 8}$ as a single fraction in its simplest form. [2]

(d) Solve $\dfrac{1}{y-1} - \dfrac{2}{5} = \dfrac{7}{4y}$. [3]

(e) Cupcakes are sold at 3 for $5.
Louisa bought m cupcakes and was given a 5% discount.

Write an expression, in terms of m, for the amount she paid in dollars. [2]

2 Jeria trekked 10 kilometres to a waterfall at an average speed of x kilometres per hour.

(a) Write down an expression, in terms of x, for the time taken in hours. [1]

(b) For her return journey from the waterfall, Jeria trekked the same 10 kilometres at an average speed of $(x+1)$ kilometres per hour.
The time taken for her return journey is 30 minutes faster than the time taken for her journey to the waterfall.

Write down an equation in x and show that it simplifies to $x^2 + x - 20 = 0$. [4]

(c) Solve the equation $x^2 + x - 20 = 0$. [3]

(d) Marianne has 300 minutes to trek to the waterfall and back.
The ratio of Jeria's to Marianne's trekking time to the waterfall is 2 : 3.

Find the maximum time, in minutes, Marianne could take for her return journey. [2]

[Turn Over

3 The first four terms in a sequence of numbers are given below.

$$T_1 = \qquad\qquad 1 \quad = \quad 1$$

$$T_2 = \qquad\qquad 3 + 5 \quad = \quad 8$$

$$T_3 = \qquad\qquad 7 + 9 + 11 \quad = \quad 27$$
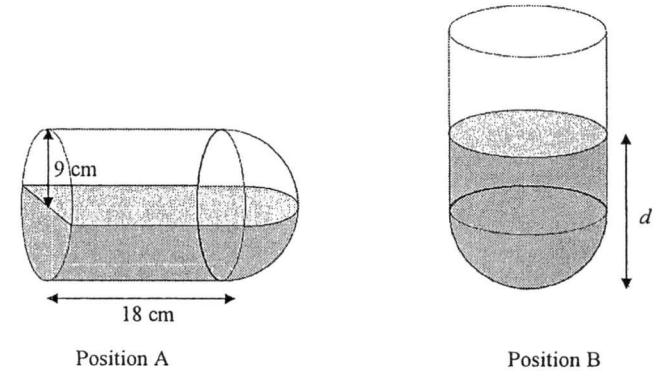
$$T_4 = \qquad 13 + 15 + 17 + 19 \quad = \quad 64$$

(a) Find T_5 and T_6. [2]

(b) What is the name given to the numbers 1, 8, 27, 64, ...? [1]

(c) How many numbers are added up to give the value of T_n? [1]

(d) Find an expression, in terms of n, for the value of T_n. [1]

(e) Explain why the value of T_n is odd when n is odd and even when n is even. [2]

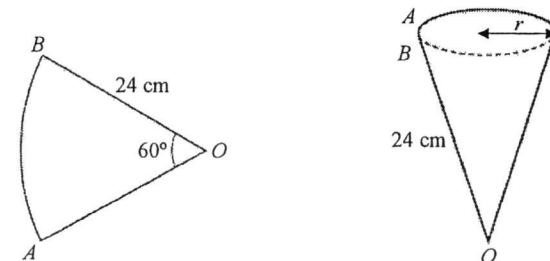**4** **(a)** **Diagram is not drawn to scale.**

A closed container, made by joining a hemisphere of radius 9 cm and a cylinder of length 18 cm, is placed in two rest positions as shown below.

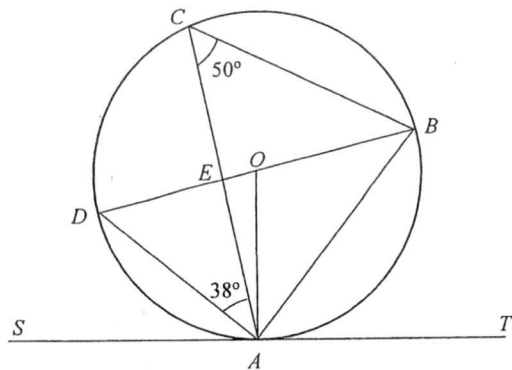In Position A, the container is horizontal and exactly half filled with water.

In Position B, the half-filled container is held with its axis vertical and the hemisphere at the bottom.

Position A

Position B

(i) Find the volume of water in the container, leaving your answer in π. [3]

(ii) Show that the depth of the water, d, in Position B is 15 cm. [3]

(iii) Find the total surface area of the container in contact with the water in Position B. [2]

(b) The wrapper of a giant ice-cream cone is formed from joining points A and B of the sector AOB.

(i) Find the area of sector AOB in terms of π. [1]

(ii) Show that the radius, r, of the ice-cream cone wrapper is 4 cm. [2]

(iii) Find the volume of the ice-cream cone. [2]

5

The diagram shows a circle $ABCD$ with centre O.
SAT is a tangent to the circle at point A and the line AC intersects the line BD at E.
Angle $DAC = 38°$ and angle $ACB = 50°$.

(a) Find, giving reasons for each answer,

    (i)   angle OBA,   [2]

    (ii)   angle OAC,   [1]

    (iii)   angle SAD,   [1]

    (iv)   angle CED.   [2]

(b) Given that $AB = 15$ cm, find

    (i)   the diameter of the circle,   [2]

    (ii)   the area of triangle AOB.   [2]

        [Turn Over

6 Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation

$$y = -\frac{x^2}{3} - \frac{11}{x} + 11.$$

Some corresponding values of x and y, correct to two decimal places, are given in the table.

x	1	1.5	2	3	4	5	6
y	-0.33	2.92	q	4.33	2.92	0.47	-2.83

(a) Find the value of q.   [1]

(b) Using the scale of 2 cm to represent 1 unit on each axis, draw a horizontal x-axis for $0 \le x \le 6$ and a vertical y-axis for $-3 \le y \le 5$.

On your axes, plot the points given in the table and join them with a smooth curve.   [3]

(c) By drawing a tangent, find the gradient of the curve at $(1.5, 2.92)$.   [2]

(d) By drawing a suitable straight line on the same axes, find the solutions to the equation $-\frac{x^2}{3} - \frac{11}{x} + 12 = x$.   [2]

(e) Use your graph to find the coordinates of the maximum point of $y = -\frac{x^2}{3} - \frac{11}{x} + 11$ in the range of $1 \le x \le 6$.   [2]

7   Tickets to a Korean-pop concert were sold online by an organiser.
The number of tickets sold for each category on Friday and Saturday respectively are summarised in the table below.

Day	Category 1	Category 2	Category 3	Category 4
Friday	135	120	100	105
Saturday	150	140	125	85

(a)   Write down a 2×4 matrix **T** to represent the number of tickets sold by the organiser.   [1]

(b)   The price per ticket is $228 for Category 1, $168 for Category 2, $128 for Category 3 and $98 for Category 4.

Represent the ticket prices in a column matrix **S**.   [1]

(c)   Evaluate the matrix **K = TS**.   [2]

(d)   State what the elements of **K** represent.   [1]

(e)   The organiser decided to hold another concert on Sunday.
The prices per ticket for all categories were increased by 5%.

The number of tickets sold on Sunday is shown in the table below.

Day	Category 1	Category 2	Category 3	Category 4
Sunday	180	200	110	75

Using matrix multiplication, find the total sales revenue on Sunday.   [3]

8   (a)   In a team of software engineers, there are 12 females and 18 males.
Two engineers are selected at random, one after another, to attend a global conference.

(i)   Draw a tree diagram to show the probabilities of the possible outcomes.   [2]

(ii)   Find, as a fraction in its simplest form, the probability that one female and male are selected.   [2]

(iii)   The manager decided to select a third engineer to attend the same conference.

Find the probability that all three engineers are females.   [2]

(b)   15 fishermen from Town Koney were asked how many fishes they caught in an 8-hour work day.
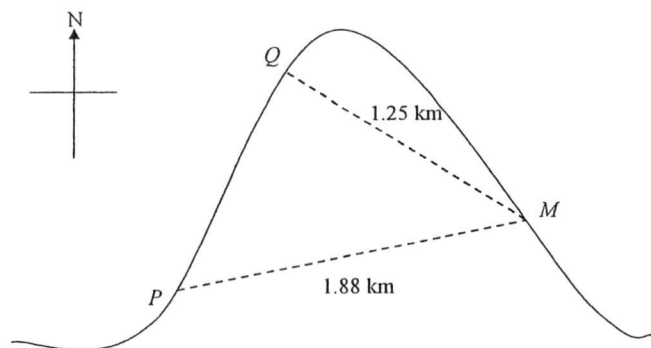The results are shown in the stem-and-leaf diagram.

Stem	Leaf
4	2
5	1 4 8 8 8
6	0 2 x
7	2 5 6 7
8	3
9	9

Key: 4 | 2 = 42 fishes

(i)   The median number of fishes caught is 62.

Form an inequality to represent the range of possible values of x.   [1]

(ii)   Find the percentage of fishermen who caught at most 58 fishes a day.   [1]

(iii)   It is given that $x = 5$.

(a)   Calculate the standard deviation of the number of fishes.   [1]

(b)   Determine and explain whether the mean is a good indicator of average in this distribution.   [2]

(c)   In Town Hona, the mean and standard deviation of the number of fishes are 68 and 13.2 respectively.

Use this information to comment on one difference between the two towns.   [1]

9   Three jetties M, Q and P are situated around a coastal bay at the same ground level.
     Jetty M is 1.88 km away at a bearing of 72° from Jetty P.
     Jetty Q is 1.25 km away at a bearing of 300° from Jetty M.

(a)   Find the straight line distance between Jetty Q and P.   [2]

(b)   Find the bearing of Jetty Q from Jetty P.   [2]

(c)   Ryan rides a water jet ski from Jetty Q towards PM.
      He wishes to reach PM in the shortest time possible.

      Find the distance of the route which allows him to reach in the shortest time.   [2]

(d)   A helicopter flies at a constant altitude of 950 m directly above PM.

      (i)   Find the angle of depression of the helicopter to Jetty Q when it is nearest
             to Q.   [2]

      (ii)   A skydiver jumps off the helicopter when the helicopter is nearest to Q.

             Determine whether the angle of elevation of the skydiver is smaller at point P
             or point M.
             Show your workings clearly.   [2]

10   Jerel will commence his Bachelor's Degree in Computing at the National University of
     Singapore (NUS) this August 2017.

     Information that Jerel needs is on the opposite page.

     Jerel is planning ahead for his annual cost of living.
     He decides to stay on-campus to save the daily transportation time of 2 hours every day.

(a)   Calculate his maximum estimated annual cost of living, excluding the vacation
      period.   [1]

(b)   Jerel receives $800 allowance from his parents every month.
      To cover all his expenses. he works as a part-time tutor in a learning centre.
      He is paid $20 per hour.

      On average, find the minimum number of hours he will need to work monthly to
      cover his cost of living.
      Give your answer in whole number.   [3]

(c)   Jerel has to complete four years of education in NUS for his degree.
      He plans to take a POSB bank loan for his tuition fee payment.

      Suggest whether it is more practical for him to take a 5 or 10 year bank loan.
      Justify the decision you make and show your calculations clearly.   [6]

COST OF LIVING
Full-Time Undergraduate Studies (2017)

The annual costs below have been derived based on a conservative estimate for a reasonably comfortable lifestyle, excluding the vacation period.

Item	Annual Estimated Cost
On-Campus accommodation (single/double occupancy)	$2 625 to $7 000
Meals	$2 600
Personal expenses	$2 200
Transportation within Singapore	$800
Average cost of books/supplies	$400

Notes:
- All amounts stated are in Singapore dollars.
- The costs are calculated on an annual basis, excluding the vacation period.

ANNUAL TUITION FEES
For New Students enrolled in Academic Year 2017/2018

National University of Singapore Fees payable by students per academic year			
Course	Arts & Social Sciences	Business / Accountancy	Engineering / Computing	Law
Annual Tuition Fees	$8 050	$9 450	$8 050	$12 500

POSB BANK LOAN
Education Loan Exclusive Promotion!

Key Features
- Lower interest rates of 4.6% per annum with 2% processing fee
- Up to 10 years repayment period which starts after course completion

END OF PAPER

No.		Answer	Workings	Marks	*Remarks
1	(a)	$4pq - 15p^2 + 4q^2$	$-(3p-2q)(2q+5p)$ $= -(6pq + 15p^2 - 4q^2 - 10pq)$ $= 4pq - 15p^2 + 4q^2$	B1	
	(b)	$r = \pm\sqrt{\dfrac{2f+h}{4+gh}}$	$4r^2 - h = 2f - ghr^2$ $4r^2 + ghr^2 = 2f + h$ $r^2(4+gh) = 2f + h$ $r^2 = \dfrac{2f+h}{4+gh}$ $r = \pm\sqrt{\dfrac{2f+h}{4+gh}}$	M1 A1	
	(c)	$\dfrac{(3x-5)}{2(x-2)}$	$\dfrac{3x^2 + x - 10}{2x^2 - 8}$ $= \dfrac{(3x-5)(x+2)}{2(x^2-4)}$ $= \dfrac{(3x-5)(x+2)}{2(x+2)(x-2)}$ $= \dfrac{(3x-5)}{2(x-2)}$	M1 A1	*1m for correct expansion of either numerator or denominator.
	(d)	$y = 1.70 \text{ or } -2.57$	$\dfrac{1}{y-1} - \dfrac{2}{5} = \dfrac{7}{4y}$ $\dfrac{5 - 2(y-1)}{5(y-1)} = \dfrac{7}{4y}$ $\dfrac{7 - 2y}{5y - 5} = \dfrac{7}{4y}$ $35y - 35 = 28y - 8y^2$ $8y^2 + 7y - 35 = 0$ $y = \dfrac{-7 \pm \sqrt{7^2 - 4(8)(-35)}}{2(8)}$ $y = \dfrac{-7 \pm \sqrt{1169}}{16}$ $y = 1.70 \text{ or } y = -2.57$	M1 M1 A1	
	(e)	$\$\dfrac{19}{12}m$	m cupcakes cost $\$\dfrac{5}{3}m$ Price after discount $= \dfrac{95}{100} \times \dfrac{5}{3}m$ $= \$\dfrac{19}{12}m$	M1 A1	Accept $\$1\dfrac{7}{12}m$

No.		Answer	Workings	Marks	*Remarks
2	(a)	$\dfrac{10}{x}$ h	$\dfrac{10}{x}$ h	B1	
	(b)	$x^2 + x - 20 = 0$ (shown)	Time taken for return journey $= \dfrac{10}{x+1}$ h $\dfrac{10}{x} - \dfrac{10}{x+1} = \dfrac{1}{2}$ $\dfrac{10x + 10 - 10x}{x(x+1)} = \dfrac{1}{2}$ $x^2 + x = 20$ $x^2 + x - 20 = 0$ (shown)	M1 M1 M1 A1	
	(c)	$x = 4 \text{ or } x = -5$	$x^2 + x - 20 = 0$ $(x-4)(x+5) = 0$ $x = 4 \text{ or } x = -5$	M1 A2	*Deduct 1m if student rejects $x = -5$.
	(d)	75 min	Time taken for Jeria's journey to waterfall $= \dfrac{10}{4} = 2.5$ h Time taken for Marianne's journey to waterfall $= \dfrac{2.5}{2} \times 3$ $= 3.75$ h $= 225$ min Maximum time for return journey $= 300 - 225 = 75$ min	M1 A1	
3	(a)	$T_5 = 21 + 23 + 25 + 27 + 29 = 125$ $T_6 = 31 + 33 + 35 + 37 + 39 + 41 = 216$		B1 B1	
	(b)	Perfect Cubes		B1	
	(c)	n		B1	
	(d)	n^3		B1	
	(e)	When n is odd, n^2 will be the product of two odd numbers, which will be odd. $n^3 = n^2 \times n = $ odd x odd = odd. When n is even, n^2 will be the product of two even numbers, which will be even. $n^3 = n^2 \times n = $ even x even = even.		B1 B1	Or equivalent reasonings.

No.		Answer	Workings	Marks	*Remarks
4	(a)(i)	972π cm^3	Volume of cylinder $= \pi(9)^2(18)$ $= 1458\pi$	M1	
			Volume of hemisphere $= \frac{2}{3}\pi(9)^3$ $= 486\pi$	M1	
			Volume of water $= \frac{1}{2}(1458\pi + 486\pi)$ $= 972\pi$ cm^3	A1	
	(a)(ii)	$d = 6$ cm (shown)	Volume of cylinder $= 972\pi - 486\pi$ $= 486\pi$ cm^3	M1	
			$\pi(9)^2 h = 486\pi$ $h = 6$ cm $d = 6 + 9 = 15$ cm (shown)	M1 A1	
	(a)(iii)	848 cm^2	Total surface area in contact with water $= 2\pi(9)^2 + 2\pi(9)(6)$ $= 848.23$ $= 848$ cm^2	M1 A1	*1m for either curved surface area of cylinder or hemisphere
	(b)(i)	96π cm^2	area of sector AOB $= \frac{1}{2}(24)^2(\frac{\pi}{3})$ $= 96\pi$ cm^2	B1	*In degree: $\pi(24)^2(\frac{60^o}{360^o})$ $= 96\pi$ cm^2
	(b)(ii)	4 cm (shown)	Circumference $=$ arc length of sector $= 24(\frac{\pi}{3})$ [or in degree] $= 8\pi$ cm $2\pi r = 8\pi$ $r = 4$ cm	M1 A1	Alternatively, $\pi r l = 96\pi$ $\pi(r)(24) = 96\pi$ $r = 4$ cm
	(b)(iii)	586 cm^3	Height of cone $= \sqrt{24^2 - 4^2}$ $= \sqrt{560}$ cm Volume of cone $= \frac{1}{3}\pi(\sqrt{560})(4)^2$ $= 396$ cm^3	M1 A1	*Accept 397 when using $\pi = 3.142$.

No.		Answer	Workings	Marks	*Remarks
5	(a)(i)	40°	$\angle AOB = 50^o \times 2 = 100^o$ ($\angle$ at centre $= 2\angle$ at circumference) $\angle ABO = (180^o - 100^o) \div 2 = 40^o$ (base $\angle$ of isos triangle)	M1 A1	
	(a)(ii)	12°	$\angle OAC = 90^o - 38^o - 40^o = 12^o$ ($\angle$ in a semicircle)	B1	Accept alternative method
	(a)(iii)	40°	$\angle SAD = 90^o - 38^o - 12^o = 40^o$ (tan $\perp$ rad)	B1	Accept alternative method
	(a)(iv)	88°	$\angle CBE = 38^o$ ($\angle$ in same segment) $\angle CED = 38^o + 50^o = 88^o$ (ext angle of triangle)	M1 A1	Accept alternative method
	(b)(i)	19.6 cm	$\cos 40^o = \frac{15}{DB}$ $DB = 19.581$ cm $= 19.6$ cm	M1 A1	
	(b)(ii)	47.2 cm^2	Radius $= 19.581 \div 2 = 9.7905$ Area of AOB $= \frac{1}{2} \times (9.7905)^2 \times \sin 100^o$ $= 47.2$ cm^2	M1 A1	

No.		Answer	Workings	Marks	*Remarks
6	(a)	$q = 4.17$		B1	
	(b)	(as attached) 1m – correct plotting of point 1m – correct scale 1m – smooth curve		B3	
	(c)	Gradient $= \frac{4.65 - 1.2}{2 - 1}$ $= 3.9 \pm 0.5$ (Accept values from 3 to 4) *sample a few scripts before confirming the acceptable range.		M1 A1	*1m – drawing of line *1m – correct value
	(d)	$-\frac{x^2}{3} - \frac{11}{x} + 12 = x$ $y + 1 = x$ Draw $y = x - 1$ Solutions: $x = 1.05$ or $x = 3.95$ (Accept values ± 0.1)		M1 A1	*M1 – drawing of line $y = x - 1$ (not accepted if equation is found but line is not drawn)
	(e)	(2.55, 4.5) (Accept values ± 0.1)		B2	*1m each for x and y coordinates *deduct 1m for non coordinates form answer

No.		Answer	Workings	Marks	*Remarks
7	(a)	$T = \begin{pmatrix} 135 & 120 & 100 & 105 \\ 150 & 140 & 125 & 85 \end{pmatrix}$		B1	
	(b)	$S = \begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$		B1	
	(c)	$K = \begin{pmatrix} 74030 \\ 82050 \end{pmatrix}$	$K = TS$ $$= \begin{pmatrix} 135 & 120 & 100 & 105 \\ 150 & 140 & 125 & 85 \end{pmatrix} \begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$$ $$= \begin{pmatrix} 74030 \\ 82050 \end{pmatrix}$$	B2	B1 – 74030 B1 – 82050
	(d)	K represents the total ticket sales for all categories on Friday and Saturday respectively.		B1	
	(e)	$100873.50	Prices of tickets of each category on Sunday $$= 1.05 \begin{pmatrix} 228 \\ 168 \\ 128 \\ 98 \end{pmatrix}$$ $$= \begin{pmatrix} 239.40 \\ 176.40 \\ 134.40 \\ 102.90 \end{pmatrix}$$ Total sales revenue $$= (180 \quad 200 \quad 110 \quad 75) \begin{pmatrix} 239.40 \\ 176.40 \\ 134.40 \\ 102.90 \end{pmatrix}$$ $$= (100873.50)$$ Total sales revenue = $100873.50	M1 M1^ A1	*0m if no matrix multiplication is used. *M2 can be awarded if at least 1 matrix multiplication is used. *Not to penalize for 1d.p. within matrix ^can accept: $1.05(228 \ 168 \ 128 \ 98)\begin{pmatrix}180\\200\\110\\75\end{pmatrix}$ or $1.05(180 \ 200 \ 110 \ 75)\begin{pmatrix}228\\168\\128\\98\end{pmatrix}$ or find $(180 \ 200 \ 110 \ 75)\begin{pmatrix}228\\168\\128\\98\end{pmatrix}$ then $\times 1.05$ etc.

No.		Answer	Workings	Marks	*Remarks
8	(a)(i)			B2	Deduct 1m for each wrong probability. Do not penalize marks if fractions are not reduced to simplest form.
	(a)(ii)	$\dfrac{72}{145}$	P(one female one male) $= P(F, M) + P(M, F)$ $= \dfrac{2}{5} \times \dfrac{18}{29} + \dfrac{3}{5} \times \dfrac{12}{29}$ $= \dfrac{72}{145}$	M1 A1	
	(a)(iii)	$\dfrac{11}{203}$	P(F, F, F) $= \dfrac{2}{5} \times \dfrac{11}{29} \times \dfrac{10}{28}$ $= \dfrac{11}{203}$	M1 A1	
	(b)(i)	$62 \le x \le 69$		B1	
	(b)(ii)	40%	$\dfrac{6}{15} \times 100 = 40\%$	B1	
	(b)(iii)(a)	13.9	SD $= \sqrt{\dfrac{\sum fx^2}{\sum f} - \left(\dfrac{\sum fx}{\sum f}\right)^2}$ $= \sqrt{\dfrac{68246}{15} - \left(\dfrac{990}{15}\right)^2}$ $= 13.9$ fishes (3 sf)	B1	
	(b)(iii)(b)	There is an outlier in the data (99 fish). Hence the mean is not a good indicator of average in this distribution.		M1 A1	1m – yes / no 1m – reason *0 m given if no / wrong reason is given.
	(b)(iii)(c)	The mean is higher in Town Hona, hence the fishermen in Town Hona caught more fishes in a day. or The standard deviation is lower in Town Hona, hence the number of fishes caught by the fishermen in Town Hona is more consistent.		B1 √	Follow through their SD in part (b)(iii)(a)

No.		Answer	Workings	Marks	*Remarks
9	(a)	1.40 km	$\angle QMP = 48°$ By Cosine Rule $QP = \sqrt{1.25^2 + 1.88^2 - 2 \times 1.25 \times 1.88 \times \cos 48°}$ $= 1.39713$ km $= 1.40$ km	M1 A1	
	(b)	030.3°	By Sine Rule $\dfrac{1.25}{\sin QPM} = \dfrac{1.39713}{\sin 48°}$ $\angle QPM = 41.6735° = 41.7°$ Bearing of Q from P $= 72° - 41.6735°$ $= 30.3265°$ $= 030.3°$	M1 A1	
	(c)	0.929 km	$\sin 48° = \dfrac{QX}{1.25}$ Shortest distance QX $= 0.92893$ km $= 0.929$ km	M1 A1	
	(d) (i)	45.6°	$\tan \theta = \dfrac{0.95}{0.92893}$ $\theta = 45.6°$	M1 A1	
	(d) (ii)		*X is the point of shortest distance from Q to PM. Distance of PX (1.044 km) is <u>longer</u> than the distance of XM (0.833 km). Hence, <u>angle of elevation from Point P to the skydiver is smaller.</u>	M1 A1	1m – reason 1m – Point P *Award 0 m if only calculations given without any explanations or justifications

No.		Answer	Workings	Marks	*Remarks
10	(a)	$13000	Maximum estimated annual cost of living $7000 + $2600 + $2200 + $400 + $800 $= $13000	B1	
	(b)	15 hours	Annual expenses that he has to pay on his own $= $13000 - 800×12 $= 3400 Per month $= $3400 \div 12 = 283.333 Maximum number of hours $= $283.333 \div 20 = 14.167$ ≈ 15 hours	M1 M1 A1	*Accept reasonable calculations continued from part (a)
	(c)		Total Tuition fee (4 years) $= $8050 \times 4 = 32200 Including 2% processing fee $= $32200 \times \dfrac{102}{100} = 32844 **5 year loan** Interest incurred $I = \dfrac{PRT}{100} = \dfrac{32844 \times 4.6 \times 5}{100} = 7554.12 Total amount he has to pay $= $7554.12 + $32844 = 40398.12 Amount per month $= $40398.12 \div 5 \div 12 = 673.302 } Not needed if reasoning is ① **10 year loan** Interest incurred $I = \dfrac{PRT}{100} = \dfrac{32844 \times 4.6 \times 10}{100} = 15108.24 Total amount he has to pay $= $15108.24 + $32844 = 47952.24 Amount per month $= $47952.24 \div 10 \div 12 = 399.602 } Not needed if reasoning is ① **Conclusion:** ① Jerel will pay lesser per month if he goes for a 10-year loan. However, <u>the total interest for a 5-year loan is $7554.12 lower than that of a 10-year loan.</u> It will be <u>more practical for him to go for a 5-year loan.</u> **Also accept:** ② Although the total interest for a 10-year loan is $7554.12 higher than that of a 5-year loan, he gets to pay a comfortable amount of $400 a month for 10 years. It will be <u>more practical for him to go for a 10-year loan.</u>	M1 M1 M1 M1 M1 A1	o.e.