Name _____ ( )	Class 4 _____.

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR

S4

MATHEMATICS **4048/01**
Paper 1 **Monday 10 September 2018**
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10
Marks										
Question	11	12	13	14	15	16	17	18	19	20
Marks										

Table of Penalties		**Qn. No.**		
Presentation	–1			
Units	–1			
Significant Figures	–1		**Parent's/ Guardian's Name/ Signature/ Date**	**80**

This question paper consists of 17 printed pages.

[Turn over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 One solution of $2x^2 + kx - 12 = 0$ is $x = -4$. Find

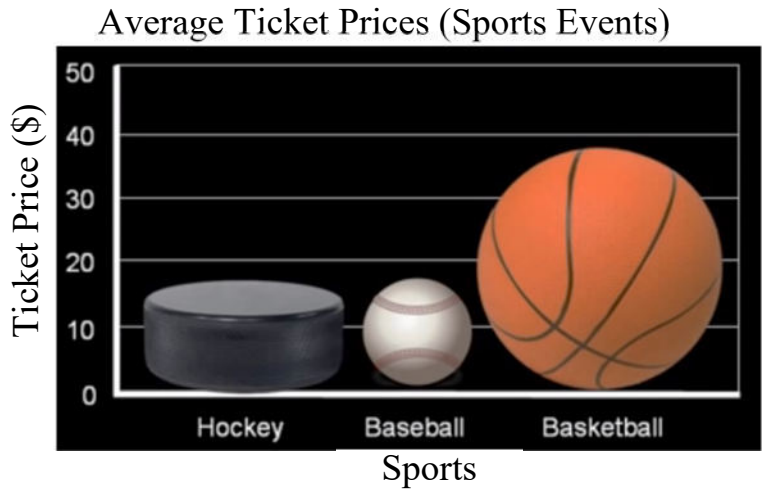
(a) the value of k,

Answer $k =$ ……..…….. [1]

(b) the other solution of the equation.

Answer ……..…… [1]

2



Adapted from https://www.youtube.com/watch?v=ETbc8GIhfHo.

State one aspect of the above graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

Answer ……

……

…….. [2]

---

3 **(a)** Express $\sqrt[5]{121}$ in index form with base 11.

Answer …… [1]

(b) Evaluate $3^{\frac{2}{3}} \times 24^{\frac{1}{3}}$ without using a calculator.

Answer …… [2]

(c) Simplify $ab \div \left(\frac{2}{a}\right)^{-2}$

Answer …… [1]

---

4 **(a)** Solve the inequality $-\frac{17-8x}{4} < 2 - \frac{4-3x}{2} < 5\frac{1}{3}$.

Answer …… [2]

(b) **Hence**, state the smallest prime number that satisfies the inequality.

Answer …… [1]

---

5 It is given that $a = \sqrt[3]{\dfrac{2b+c}{c-b}}$.

(a) Express b in terms of a and c.

Answer ……. [2]

(b) Find the value of b when $a = 2$ and $c = 5$.

Answer b = ……...…… [1]

---

6 **(a)** Simplify $5(3x-5)^2 - 3(5-3x)$.

Answer ……. [2]

(b) Factorise completely $-25y^2 - 5xy + x + 1$.

Answer ……. [2]

---

7 **(a)** It is given that

$\xi = \{x: x \text{ is an integer between 0 and 9 inclusive}\}$
$A \subset \xi$ and $B \subset \xi$
$\{0, 2\} \subset (A' \cap B)$, $7 \in A \cap B$, $\{1, 4, 5, 8\} \subset ((A \cup B) \cap B')$ and $3, 6, 9 \notin (A \cup B)$

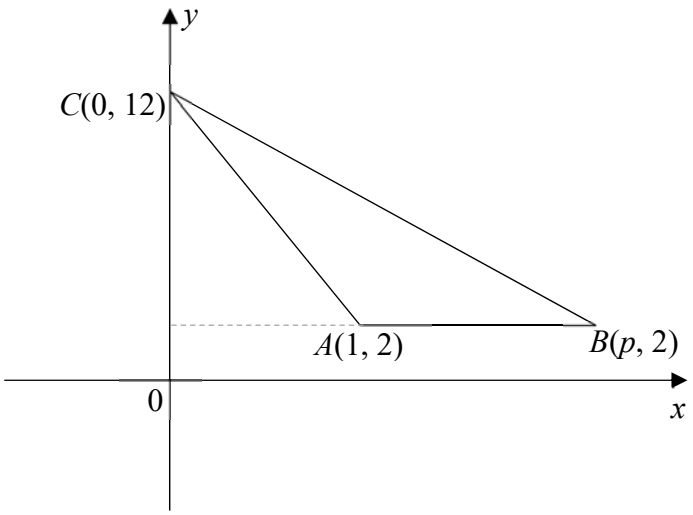
Draw a Venn diagram to represent the information given. [3]

Answer

(b) List down all the proper subsets of the set $\{a, b, c\}$.

Answer…… [1]

8 In the diagram below, not drawn to scale, A is the point (1, 2), B is the point (p, 2) and C is the point (0, 12).



(a) Find the length of the line AC.

Answer …….……units [2]

(b) Write down the value of $\cos \angle BAC$.

Answer ……..…… [1]

(c) Given that the length of the line BC is $5\sqrt{5}$ units, find the value of p.

Answer p = ……..…..…… [2]

9 **(a)** Mr Tan wants to change $4 000 Singapore dollars to US dollars for a holiday trip to the USA.
The exchange rate in Singapore is 1 SGD = 0.736 USD.
The exchange rate in USA is 1 USD = 1.352 SGD.
In which country should he change his money and how much more USD can he get?

Answer…… [2]

(b) In 2017, Matthew earned an annual income of $80 000. He is required to pay tax based on net income. His net income is obtained after deducting CPF contribution of $16 000 and personal expenses relief of $3000 from the annual income. The tax rate is $200 for the first $30 000 of net income and 5% for the remaining net income. Calculate Matthew's

(i) net income,

Answer $ ……..…… [1]

(ii) income tax.

Answer $ ……..…… [2]

10 Amanda wrote down four numbers.

The mean of these numbers is 15, the median is 12 and the mode is 8.

Find the four numbers.

Answer ……., ……., ……., ……. [2]

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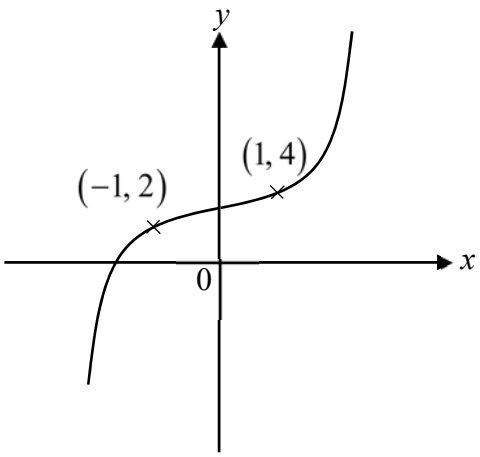
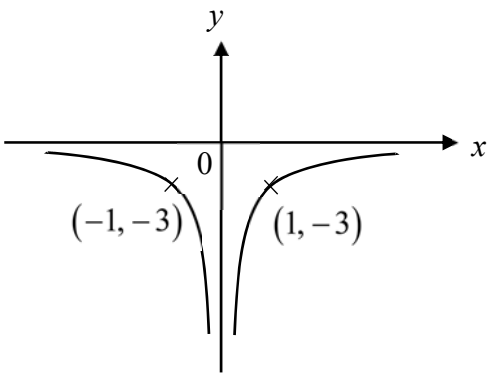
11 Write down a possible equation for each of the graphs shown below.

(a)



Answer …...……..……. [1]

(b)



Answer …...……..……. [1]

---

12 **(a)** Find the greatest integer that will divide both 126 and 2100.

Answer ……..…… [2]

(b) A rectangular field is measured as 49.9 and 24.5 correct to the nearest 0.1 metre. Find,

(i) the least possible perimeter in metres.

Answer ……..…..…..…… m [1]

(ii) the greatest possible area in square metres. Express your answer in standard form correct to 4 significant figures.

Answer ……..…..…… m^2 [2]

---

13 The surface area of a sphere is directly proportional to the square of its radius. If the radius increases by 15%, find the percentage increase in the surface area.

Answer …...…...……..……. % [2]

---

14 In the following sequence,

$$(1\times 2)-2=0$$
$$(2\times 3)-4=2$$
$$(3\times 4)-6=6$$
$$\cdots$$
$$\cdots$$
$$(a\times 13)-b=c$$
$$\cdots$$
$$\cdots$$
$$(d\times e)-f=g$$

(a) Find the values of a, b and c.

Answer a = …..…..…..……

b = …..…..…..……

c = …..…..…… [2]

(b) Express g in terms of d only.

Answer ……..…… [2]

(c) Explain why 343 cannot be the result of an equation in this sequence.

Answer ……...……

……

…..…….

…….

…… [1]

15 At an online supermarket, a 0.5 kg bag of carrots costs \$0.90, a 0.5 kg of onions costs \$1.50 and a 0.5 kg bag of local tomatoes costs \$1.30. On Monday, there were 730 orders for carrots, 421 orders for onions and 279 orders for tomatoes. On Tuesday, there were x orders for carrots, 355 orders for onions and 249 orders for tomatoes.

(a) Write down a 2×3 matrix, $\boldsymbol{M}$, representing the orders over Monday and Tuesday.

Answer $\boldsymbol{M} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

(b) Find, in terms of x, the matrix $\boldsymbol{P} = \boldsymbol{M}\begin{pmatrix} 0.90 \\ 1.50 \\ 1.30 \end{pmatrix}$.

Answer $\boldsymbol{P} =$ [2]

(c) Explain clearly what each element in matrix P represents.

Answer ……...

……... [1]

(d) If the total cost of orders on Tuesday is about 10% less than the total cost of orders on Monday,

(i) calculate the value of x.

Answer $x =$ …..…… [1]

(ii) use a matrix method to compute the total cost of orders on Monday and Tuesday.

Answer \$ …..…… [1]

16 The stem-and-leaf diagram below shows the times of two groups of students, Group A and Group B, doing shuttle run.

Group A		Group B
	9	4 9 9
7	10	0 1 3 4 4 4 5 6 9
1 1 1 2 2 3 5 7	11	0 1
0 2 3 6 7	12	

Key (Group A)
7 | 10 means 10.7 seconds

Key (Group B)
9 | 4 means 9.4 seconds

(a) Write down the modal timing of Group B.

Answer ……..…… seconds [1]

(b) Write down the median of Group A.

Answer …… seconds [1]

(c) Explain briefly which group of students ran faster.

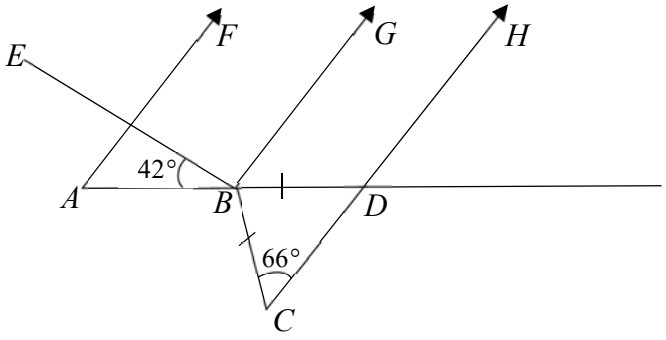
Answer ……..……...

……..……...

……..……...

……..... [1]

17 In the diagram, $BC=BD$, $\angle ABE = 42°$ and $\angle BCD = 66°$. AF, BG and CH are parallel.



Show your working and give reasons, calculate

(a) $\angle CBD$,

Answer …...……..…..…… ° [1]

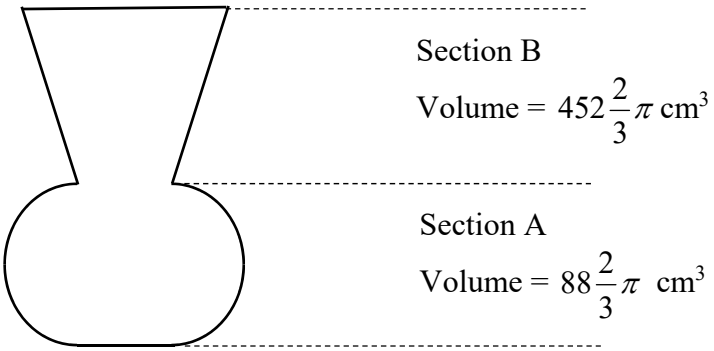
(b) $\angle GBE$,

Answer …...……..…..…… ° [2]

(c) $\angle BAF$.

Answer …...……..…..…… ° [1]

18 The diagram shows the cross section of a vase.



A factory produces 3 geometrically similar vases, Small, Medium and Large. The sketch above shows the dimensions of the Large vase. The volume of the Small vase is $67\frac{2}{3}\pi$ cm^3 with height 14 cm. The Medium vase has a height 50% more than the Small vase.

(a) Calculate

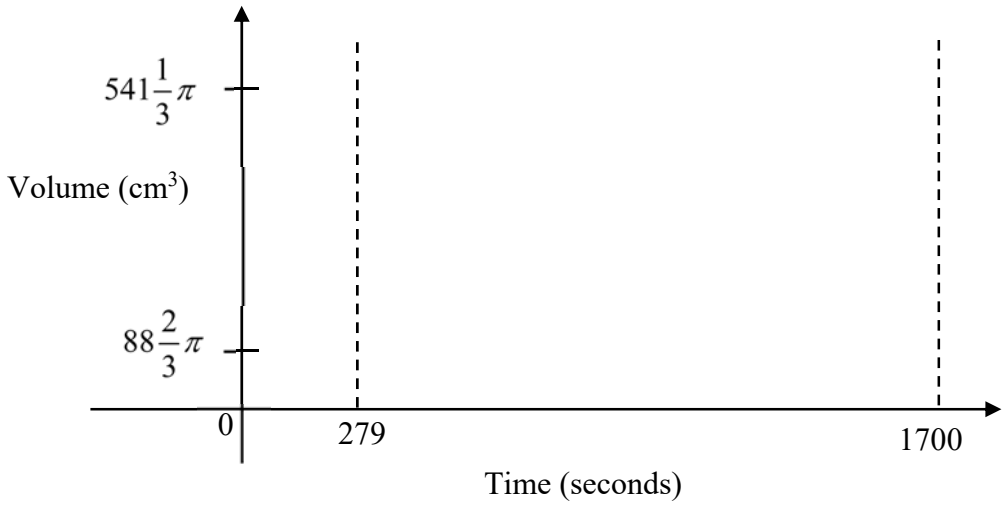
(i) the height of the Large vase, and

Answer …...……...…… cm [2]

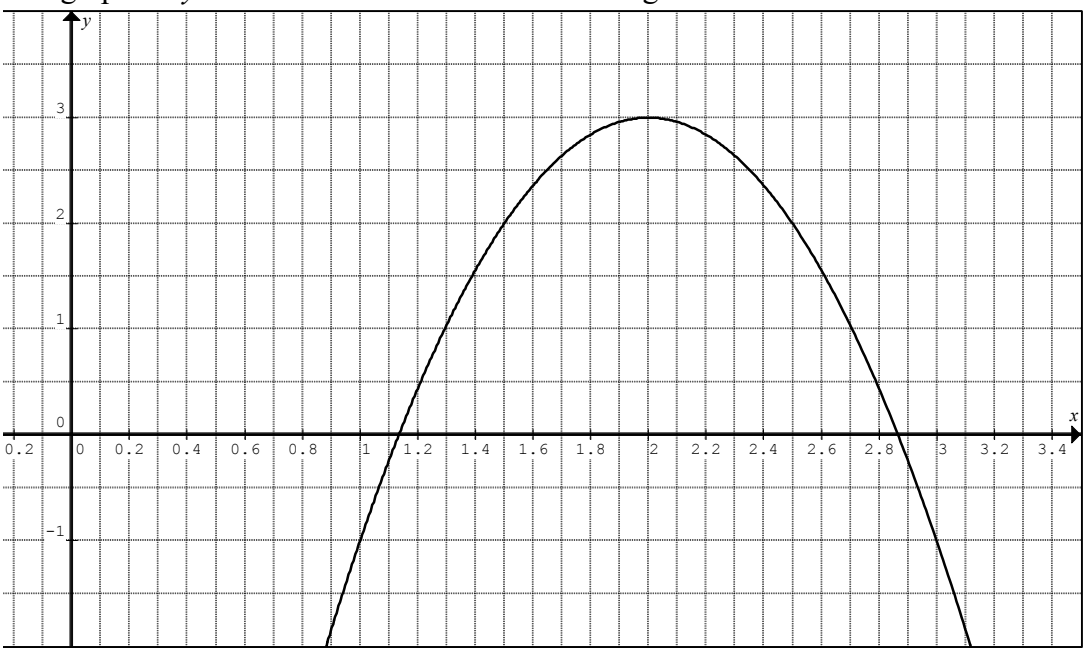
(ii) the volume of the Medium vase, in terms of π.

Answer ……...…… cm^3 [2]

(b) Water is poured into the Large vase at 1 cm^3/s. Sketch the volume-time graph of the Large vase.
[3]



19 The graph of $y = -4x^2 + 16x - 13$ is drawn on the grid.



(a) Explain why $y = 3.7$ has no solution.

Answer ...……

……...…… [1]

(b) The points $(1.5, 2)$ and $(2.5, 2)$ are the intersection points for this curve and another curve, $y = ax^2 + bx + c$. Given that $a > 0$, find a possible equation for this second curve.

Answer ……..…...……... [1]

(c) The equation $-2x^2 + 7x - 5 = 0$ can be solved by adding a straight line to the grid above. Find the equation of this line.

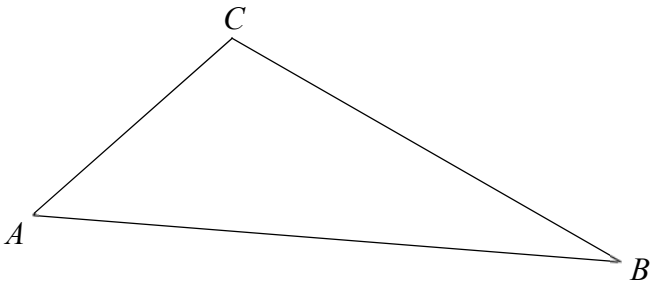
Answer ……..…...……... [1]

(d) By drawing this straight line, solve the equation $-2x^2 + 7x - 5 = 0$.

Answer x = ……. or x = ……... [3]

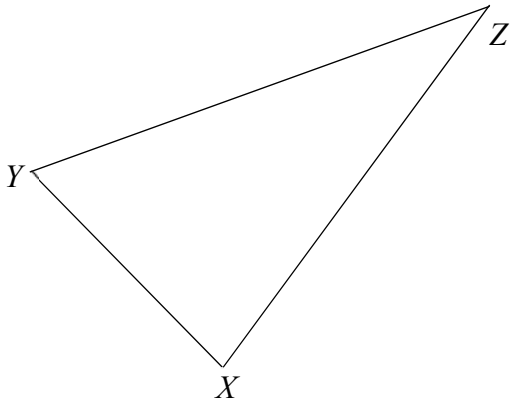
20 **(a)** Find, by construction, the point P, that is equidistant from the points A, B and C.

Hence, draw a circle passing through A, B and C. Measure the radius of the circle.



Answer …..……..…….. [3]

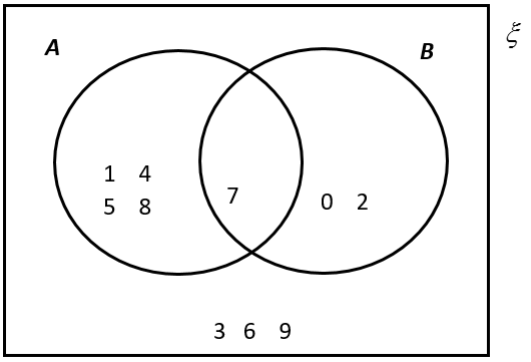
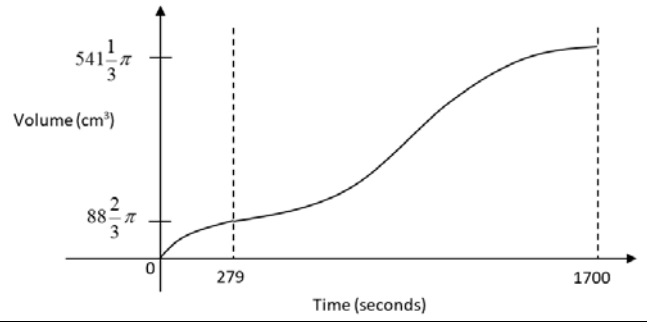
(b) Find, by construction, the point Q, that is equidistant from the lines XY, YZ and XZ.
Hence, or otherwise, draw a circle that is tangent to the lines XY, YZ and XZ.
Measure the radius of the circle.

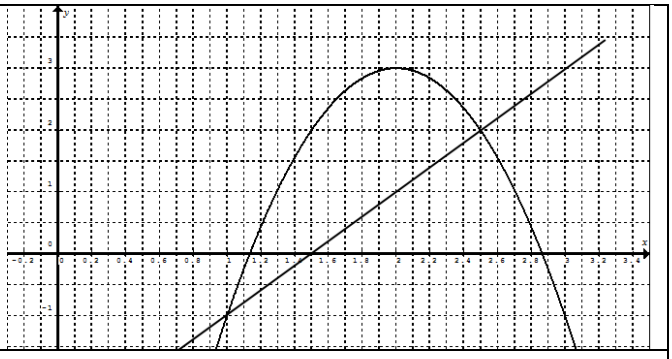
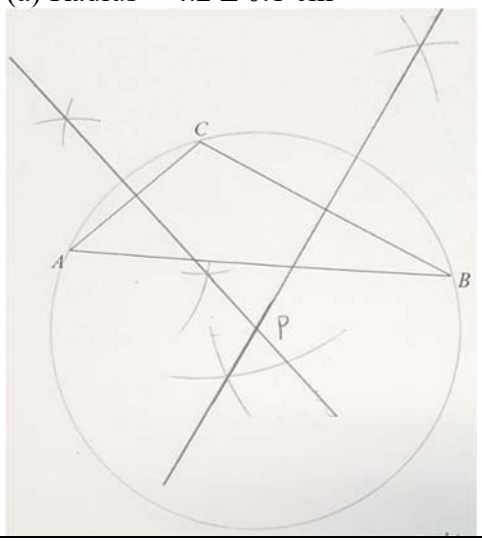
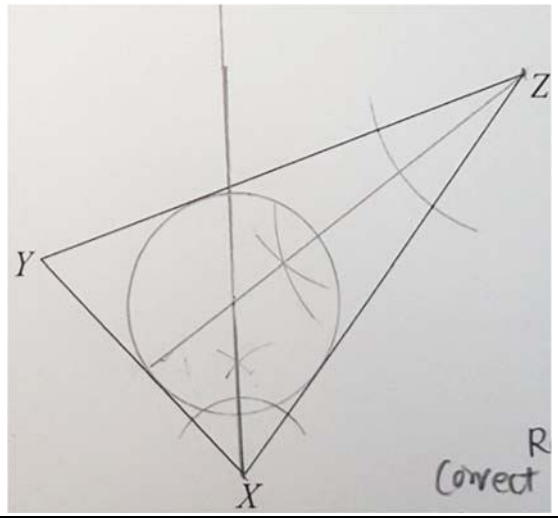


Answer …..……..…….. [3]

End of Paper

Answer key

1	(a) $k = 5$ (b) $x = 1.5$ or $\frac{3}{2}$.	3	(a) $11^{\frac{2}{5}}$ (b) 6 (c) $\frac{4b}{a}$
2	The three objects are all not the same shape. One is a cylinder and the other two are spheres. It is not clear how ticket prices can be determined. For Hockey, one can look at the centre of the top of the cylinder or the top of the diagram. For Baseball and Basketball, one may consider the centre of the objects or the top of the circles. The curved tops make finding the highest point inaccurate.		
4	(a) $x < 3\frac{5}{9}$ (b) Smallest prime number = 2	5	(a) $b = \frac{c\left(a^3 - 1\right)}{2 + a^3}$ (b) $b = 3.5$
6	(a) $45x^2 - 141x + 110$ (b) $(1-5y)(1+5y+x)$	7	(a)  (b) $\{\ \}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{b,c\}$, $\{a,c\}$
8	**(a)** 10.0 units **(b)** ≈ -0.0995 **(c)** $p = 5$ only		
9	(a) He should change in USA. He will get 14.58 US dollars more. (b)(i) \$61 000 (ii) \$1750		
10	8, 8, 16 and 28.		
11	(a) $y = x^n + 3$, where n must be odd (b) $y = -\frac{3}{x^n}$ where n must be even		
12	(a) 42 (b)(i) 148.6m (ii) $1.226 \times 10^3 \text{m}^2$	13	32.25%
14	(a) $a = 12$, $b = 24$, $c = 132$ (b) $g = d(d-1)$ (c) Since d cannot be negative and non-integer, therefore 343 cannot be the result of an equation in this sequence. **Alternatively,** g is the product of an odd and an even number. Therefore g must be even. Since 343 is odd, 343 cannot be the result of an equation. **Alternatively,** From the pattern shown above, g is always even. Therefore 343 cannot be the answer as it is odd		
15	(a) $\begin{pmatrix} 730 & 421 & 279 \\ x & 355 & 249 \end{pmatrix}$ (b) $\begin{pmatrix} 1651.20 \\ 0.9x + 856.20 \end{pmatrix}$ (c) The elements show that on Monday, the total cost of orders was \$1651.20 and on Tuesday, the total cost of orders was $\$0.9x + 856.20$. (d) (i) 700 (ii) \$3137.29	16	(a) 10.4 seconds (b) 11.4 seconds (c) Since Group B has a smaller median than Group A, so Group B ran faster.
		17	(a) 48° (b) 72° (c) 66°
		18	(a)(i) 28cm (ii) $228\frac{3}{8}\pi$ cm^3 (b) 

<table>
<tr>
<td>19</td>
<td>(a) The maximum value for y is 3.
Hence there is no solution for $y = 3.7$.
(b) $y = 4x^2 - 16x + 17$
(c) $y = 2x - 3$
(d) $x = -1$ or $x = 2.5$</td>
<td>(d)</td>
<td>  </td>
</tr>
<tr>
<td>20</td>
<td>(a) Radius = 4.2 ± 0.1 cm
 
</td>
<td>(b)</td>
<td>Radius = 1.3 ± 0.1 cm
 
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</table>

Answer Scheme for Sec 4 Math Prelim Paper 1 2018

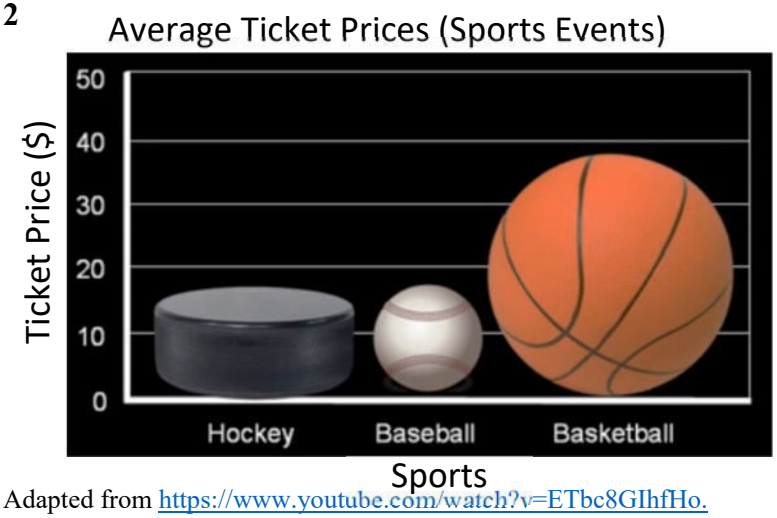
1 One solution of $2x^2 + kx - 12 = 0$ is $x = -4$. Find

(a) the value of k, [1]

(b) the other solution of the equation. [1]

1a	Sub $x = -4$ into $2x^2 + kx - 12 = 0$, $2(-4)^2 + k(-4) - 12 = 0$ $20 - 4k = 0$ $k = 5$	[B1]
1b	$2x^2 + 5x - 12 = 0$ $(x + 4)(2x - 3) = 0$ The other solution is $x = 1.5$ or $\frac{3}{2}$.	[B1]

2



Adapted from https://www.youtube.com/watch?v=ETbc8GIhfHo.

State one aspect of the above graph that may be misleading and explain how this may lead to a misinterpretation of the graph. [2]

2	The three objects are all not the same shape. One is a cylinder and the other two are spheres. It is not clear how ticket prices can be determined. For Hockey, one can look at the centre of the top of the cylinder or the top of the diagram. For Baseball and Basketball, one may consider the centre of the objects or the top of the circles. The curved tops make finding the highest point inaccurate.	B1 B1

3 **(a)** Express $\sqrt[5]{121}$ in index form with base 11. [1]

(b) Evaluate $3^{\frac{2}{3}} \times 24^{\frac{1}{3}}$ without using calculator. [2]

(c) Simplify $ab \div \left(\frac{2}{a}\right)^{-2}$. [1]

3a	$\sqrt[5]{121} = 121^{\frac{1}{5}}$ $= \left(11^2\right)^{\frac{1}{5}}$ $= 11^{\frac{2}{5}}$	B1
3b	$3^{\frac{2}{3}} \times 24^{\frac{1}{3}}$ $= 3^{\frac{2}{3}} \times (3\times 8)^{\frac{1}{3}}$ $= 3^{\frac{2}{3}} \times 3^{\frac{1}{3}} \times 8^{\frac{1}{3}}$ $= 3 \times (2^3)^{\frac{1}{3}}$ $= 3 \times 2$ $= 6$	M1 for any of the first 3 steps. A1
3c	$ab \div \left(\frac{2}{a}\right)^{-2}$ $= ab \times \left(\frac{2}{a}\right)^{2}$ $= ab \times \frac{4}{a^2}$ $= \frac{4b}{a}$	B1

4 **(a)** Solve the inequality $-\frac{17-8x}{4} < 2 - \frac{4-3x}{2} < 5\frac{1}{3}$. [2]

(b) **Hence**, state the smallest prime number that satisfies the inequality. [1]

4a	$-\frac{17-8x}{4} < 2 - \frac{4-3x}{2}$ $-17+8x < 2\times 4 - 2(4-3x)$ $-17 < 8 - 8 + 6x - 8x$ $-17 < -2x$ $x < 8.5$ Therefore, $x < 3\frac{5}{9}$.	or	$2 - \frac{4-3x}{2} < \frac{16}{3}$ $2\times 6 - 3(4-3x) < 32$ $12 - 12 + 9x < 32$ $9x < 32$ $x < 3\frac{5}{9}$	M1 splitting A1
4b	Smallest prime number = 2			B1

5 It is given that $a = \sqrt[3]{\dfrac{2b+c}{c-b}}$.

(a) Express b in terms of a and c. [2]

(b) Find the value of b when $a = 2$ and $c = 5$. [1]

5a	$a = \sqrt[3]{\dfrac{2b+c}{c-b}}$ $a^3 = \dfrac{2b+c}{c-b}$ $a^3c - a^3b = 2b + c$ $a^3c - c = 2b + a^3b$ $b\left(2+a^3\right) = c\left(a^3-1\right)$ $b = \dfrac{c\left(a^3-1\right)}{2+a^3}$	M1 for any first 3 steps A1
5b	$b = \dfrac{c\left(a^3-1\right)}{2+a^3}$ $b = \dfrac{5(8-1)}{2+8}$ $b = \dfrac{35}{10}$ $b = 3.5$	 B1

6 **(a)** Simplify $5(3x-5)^2 - 3(5-3x)$. [2]

(b) Factorise completely $-25y^2 - 5xy + x + 1$. [2]

6a	$5(3x-5)^2 - 3(5-3x) = 5\left(9x^2 - 30x + 25\right) - 15 + 9x$ $= 45x^2 - 150x + 125 - 15 + 9x$ $= 45x^2 - 141x + 110$	M1 A1
6b	$1 - 25y^2 + x - 5xy = (1+5y)(1-5y) + x(1-5y)$ $= (1-5y)(1+5y+x)$	M1 A1

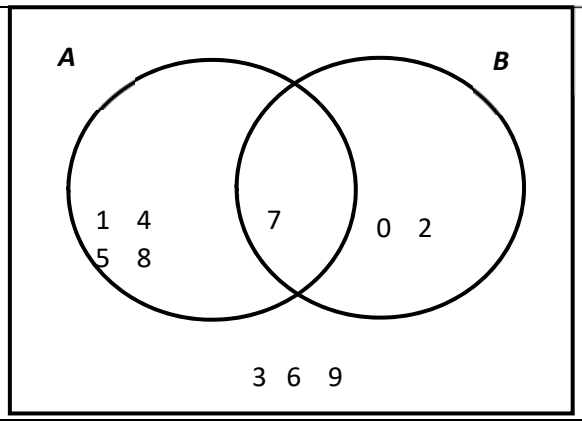
7 **(a)** It is given that
ξ = { x: x is an integer between 0 and 9 inclusive }

$A \subset \xi$ and $B \subset \xi$

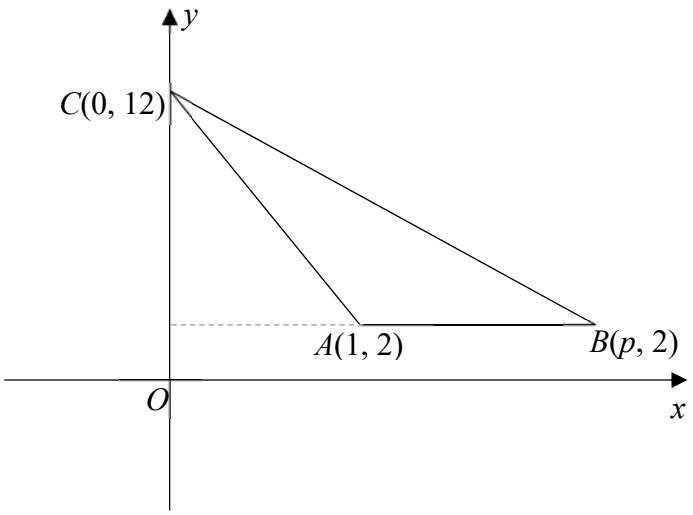
$\{0, 2\} \subset (A' \cap B)$, $7 \in A \cap B$, $\{1, 4, 5, 8\} \subset ((A \cup B) \cap B')$ and
$3, 6, 9 \notin (A \cup B)$

Draw a Venn diagram to represent the information given. [3]

(b) List down all the proper subsets of the set $\{a, b, c\}$. [1]

7a		B3 Deduct one mark for each error.
7b	{ }, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, $\{a, c\}$	B1

8 In the diagram below, not drawn to scale, A is the point (1, 2), B is the point (p, 2) and C is the point (0, 12).



(a) Find the length of the line AC. [2]

(b) Write down the value of $\cos \angle BAC$. [1]

(c) Given that the length of the line BC is $5\sqrt{5}$ units, find the value of p. [2]

8a	$AC^2 = 1^2 + 10^2$ $= 101$ $AC = \sqrt{101}$ $= 10.0499$ ≈ 10.0 units	M1 A1
8b	$\cos \angle BAC = -\cos \alpha$ $= -\frac{1}{\sqrt{101}} \approx -0.0995$	 B1
8c	$(p-0)^2 + (2-12)^2 = \left(5\sqrt{5}\right)^2$ $p^2 + 100 = 125$ $p^2 = 25$ $p = 5$ only	M1 A1

9 **(a)** Mr Tan wants to change \$4 000 Singapore dollars to US dollars for a holiday trip to the USA.
The exchange rate in Singapore is 1 SGD = 0.736 USD.
The exchange rate in USA is 1 USD = 1.352 SGD.
In which country should he change his money and how much more USD can he get? [2]

(b) In 2017, Matthew earned an annual income of \$80 000. He is required to pay tax based on net income. His net income is obtained after deducting CPF contribution of \$16 000 and personal expenses relief of \$3000 from the annual income. The tax rate is \$200 for the first \$30 000 of net income and 5% for the remaining net income. Calculate Matthew's

(i) net income, [1]

(ii) income tax. [2]

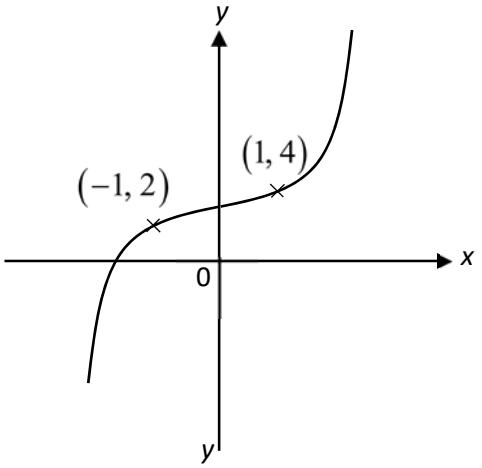
9a	In Singapore, Mr Tan will get $4000 \times 0.736 = 2944$ US dollars In USA, Mr Tan will get $4000 \div 1.352 = 2958.58$ US dollars He should change in USA. He will get 2958.57 - 2944 =14.58 US dollars more.	M1 for changing both into US dollars A1
9bi	Net Income = 80 000 – 16 000 – 3000 = \$61 000	 B1
9bii	Income Tax = 200 + (61 000 – 30 000) x 5% = \$1750	M1 A1

10 Amanda wrote down four numbers.
The mean of these numbers is 15, the median is 12 and the mode is 8.
Find the four numbers. [2]

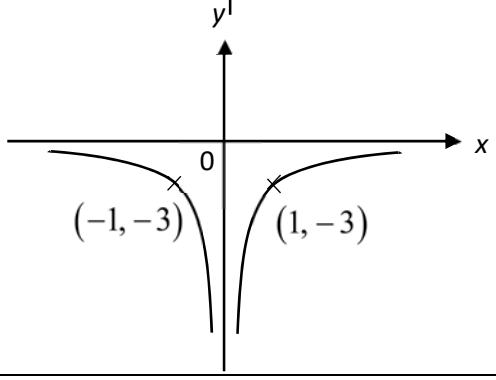
10	Let the four numbers be a, b, c and d in ascending order. Since the mode is smaller than median, so a and b will be 8. Since median is 12, so $\frac{8+c}{2}=12$ $c=24-8$ $c=16$ Since mean is 15, $8+8+16+d=15\times 4$ $d=28$ The four numbers are 8, 8, 16 and 28. Note: Award B2 for 4 correct answers if students do not show any working at all.	M1 for either mode, median or mean A1

11 Write down a possible equation for each of the graphs shown below.

(a)



(b)



11a	$y=x^n+3$, where n must be odd	B1
11b	$y=-\frac{3}{x^n}$ where n must be even	B1

12 **(a)** Find the greatest integer that will divide both 126 and 2100. [2]

(b) A rectangular field is measured as 49.9 and 24.5 correct to the nearest 0.1 metre. Find,

(i) the least possible perimeter in metres. [1]

(ii) the greatest possible area in square metres. Express your answer in standard form correct to 4 significant figures. [2]

12a	$126 = 2\times 3^2 \times 7$ $2100 = 2^2 \times 3 \times 5^2 \times 7$ HCF value is greatest integer to divide both numbers. HCF $= 2\times 3\times 7$ $= 42$	M1 B1
12bi	Least Perimeter $= 2(49.85) + 2(24.45) = 148.6\text{m}$	B1
12bii	Greatest area $= 49.95 \times 24.55$ $= 1226.273$ $= 1.226\times 10^3 \text{m}^2$	M1 A1

13 The surface area of a sphere is directly proportional to the square of its radius. If the radius increases by 15%, find the percentage increase in the surface area. [2]

13	$S = kr^2$ Let $S_1 = k(r_1)^2$ $r_2 = 1.15r_1$ $S_2 = k(r_2)^2$ $= k(1.15r_1)^2$ Percentage Increase $= \dfrac{S_2 - S_1}{S_1}\times 100$ $= \dfrac{k(1.15r_1)^2 - (kr_1)^2}{k(r_1)^2}\times 100$ $= \left((1.15)^2 - 1\right)\times 100$ $= 32.25\%$	 M1 A1

14 In the following sequence,

$$(1\times 2)-2=0$$
$$(2\times 3)-4=2$$
$$(3\times 4)-6=6$$
$$\cdots$$
$$\cdots$$
$$(a\times 13)-b=c$$
$$\cdots$$
$$\cdots$$
$$(d\times e)-f=g$$

(a) Find the values of a, b and c. [2]
(b) Express g in terms of d. [2]
(c) Explain why 343 cannot be the result of an equation in this sequence. [1]

14a	$a = 12,\ b = 24, c = 132$	B2
14b	$e = d+1$ $f = 2d$ $g = d(d+1)-2d$ $= d^2+d-2d$ $= d^2-d$ $= d(d-1)$	M1 A1
14c	$d^2-d=343$ $d^2-d-343=0$ $d=\dfrac{1\pm\sqrt{1-4(1)(-343)}}{2}$ $=\dfrac{1\pm 37.054}{2}$ $=19.03$ or -18.03 Since d cannot be negative and non-integer, therefore 343 cannot be the result of an equation in this sequence. **Alternatively,** g is the product of an odd and an even number. Therefore g must be even. Since 343 is odd, 343 cannot be the result of an equation. **Alternatively,** From the pattern shown above, g is always even. Therefore 343 cannot be the answer as it is odd.	 B1 or B1 or B1

15 At an on-line supermarket, a 0.5 kg bag of carrots costs \$0.90, a 0.5 kg of onions costs \$1.50 and a 0.5 kg bag of local tomatoes costs \$1.30. On Monday, there were 730 orders for carrots, 421 orders for onions and 279 orders for tomatoes. On Tuesday, there were x orders for carrots, 355 orders for onions and 249 orders for tomatoes.

(a) Write down a 2×3 matrix, M, representing the orders over Monday and Tuesday. [1]

(b) Find, in terms of x, the matrix $P = M\begin{pmatrix}0.90\\1.50\\1.30\end{pmatrix}$. [2]

(c) Explain clearly what each element in matrix P represents. [1]

(d) If the total cost of orders on Tuesday is about 10% less than the total cost of orders on Monday,

(i) calculate the value of x. [1]

(ii) use a matrix method to compute the total cost of orders on Monday and Tuesday. [1]

15a	$M = \begin{pmatrix}730 & 421 & 279\\ x & 355 & 249\end{pmatrix}$	B1
15b	$P = \begin{pmatrix}730 & 421 & 279\\ x & 355 & 249\end{pmatrix}\begin{pmatrix}0.90\\1.50\\1.30\end{pmatrix}$ $= \begin{pmatrix}1651.20\\0.9x+856.20\end{pmatrix}$	M1 A1
15c	The elements show that on Monday, the total cost of orders was \$1651.20 and on Tuesday, the total cost of orders was \$ $0.9x+856.20$.	B1
15di	$0.9x+856.20 = 0.9\times1651.20$ $x = 699.88$ $x = 700$	B1
15dii	$\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1651.20\\0.9\times699.88+856.20\end{pmatrix} = \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1651.20\\1486.092\end{pmatrix}$ $= \begin{pmatrix}1\times1651.20+1\times1486.092\end{pmatrix}$ $= \begin{pmatrix}3137.292\end{pmatrix}$ The total cost of orders for Monday and Tuesday was \$3137.29.	B1

16 The stem-and-leaf diagram below shows the times of two groups of students, Group A and Group B, doing shuttle run.

Group A		Group B
	9	4 9 9
7	10	0 1 3 4 4 4 5 6 9
1 1 1 2 2 3 5 7	11	0 1
0 2 3 6 7	12	

Key (Group A)
7 | 10 means 10.7 seconds

Key (Group B)
9 | 4 means 9.4 seconds

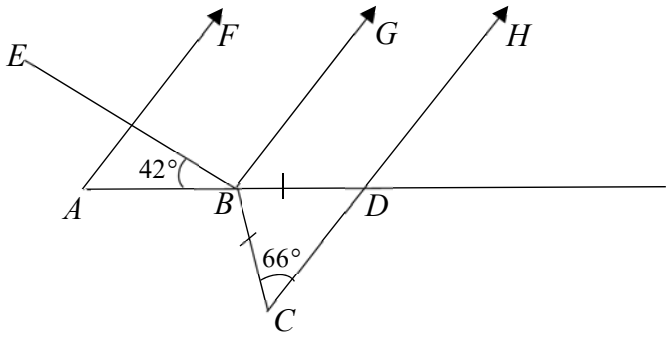
(a) Write down the modal timing of Group B. [1]

(b) Write down the median of Group A. [1]

(c) Explain briefly which group of students ran faster. [1]

16a	Mode = 10.4 seconds	B1
16b	Median = $\frac{11.3+11.5}{2} = 11.4$ seconds	B1
16c	Median for Group A = 11.4 seconds Median for Group B = $\frac{10.4+10.4}{2} = 10.4$ seconds. Since Group B has a smaller median than Group A, so Group B ran faster. Note: Mode is not acceptable in this answer as not many students recorded the modal values.	B1

17 In the diagram, $BC=BD$, $\angle ABE = 42°$ and $\angle BCD = 66°$. AF, BG and CH are parallel.



Showing your working and giving reasons, calculate

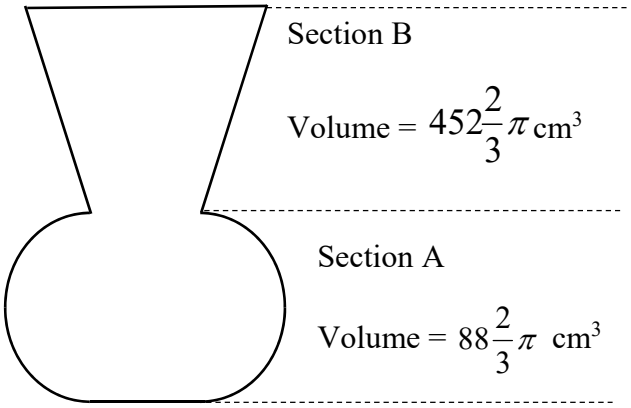
(a) $\angle CBD$, [1]

(b) $\angle GBE$, [2]

(c) $\angle BAF$. [1]

17a	$\angle CBD = 180° - 66° - 66°$(angle sum of isosceles triangle) $= 48°$	B1
17b	$\angle HDB = 180° - 66°$(adjacent angle on a straight line) $= 114°$ $\angle GBE + 42° = \angle HDB$(corr angles, BG / / CH) $\angle GBE = 114° - 42°$ $= 72°$	M1 A1
17c	$\angle BAF = 180° - \angle HDB$ (interior angles, AF / / DH) $= 180° - 114°$ $= 66°$ Deduct 1 mark from the whole of question for not stating reason or incorrect reason.	 B1

18 The diagram shows the cross section of a vase.



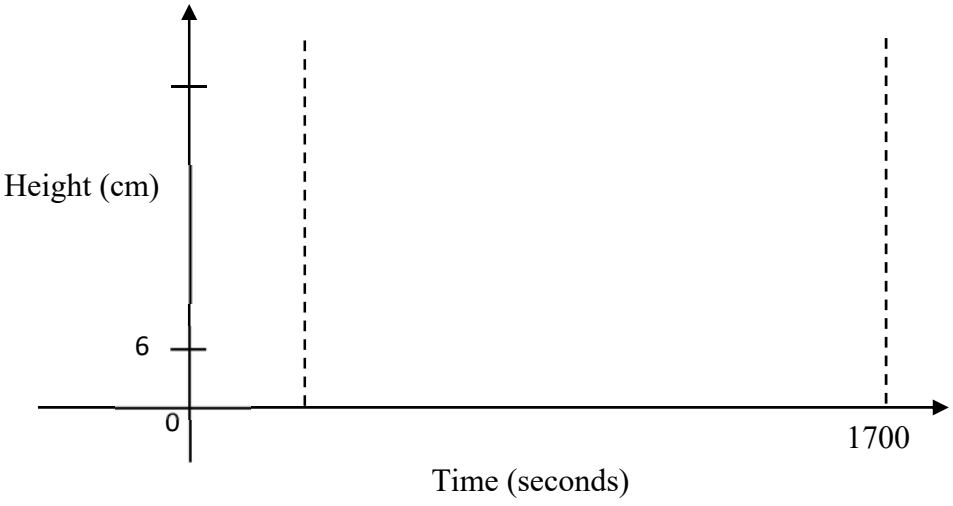
A factory produces 3 geometrically similar vases, Small, Medium and Large. The sketch above shows the dimensions of the Large vase. The volume of the Small vase is $67\frac{2}{3}\pi$ cm^3 with height 14 cm. The Medium vase has a height 50% more than the Small vase.

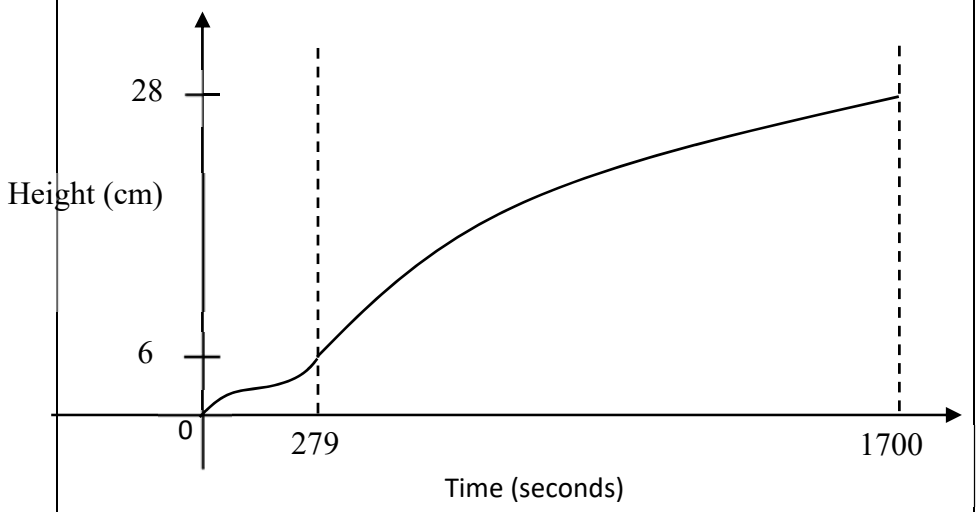
(a) Calculate

(i) the height of the Large vase, and [2]

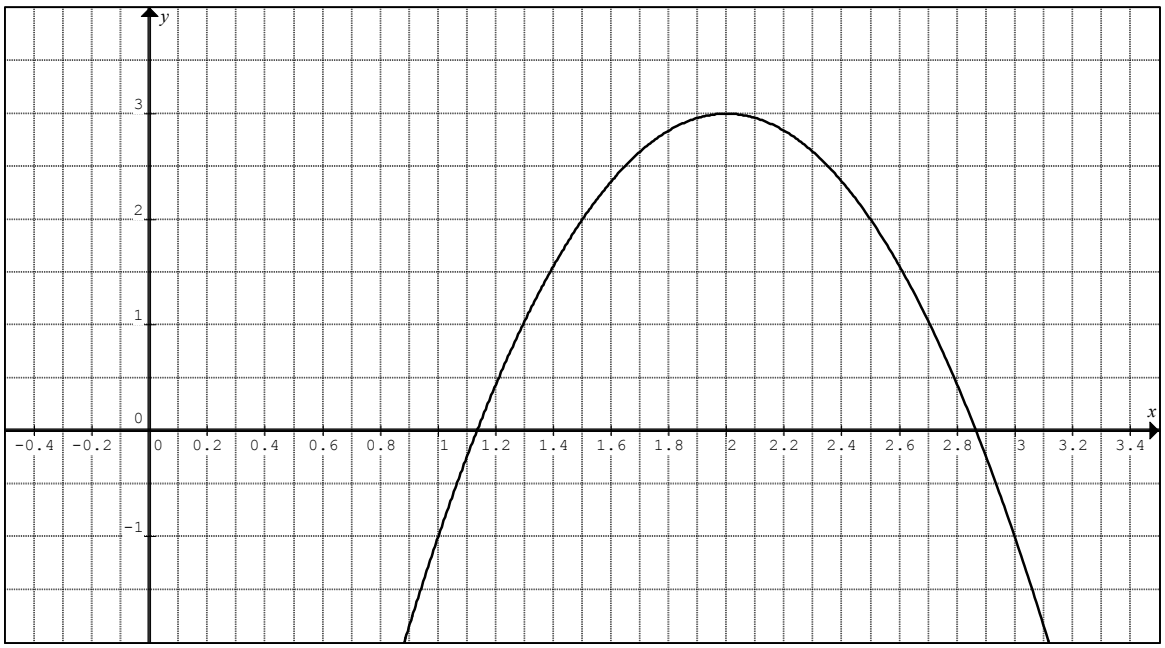
(ii) the volume of the Medium vase, in terms of π. [2]

(b) Water is poured into the Large vase at 1 cm^3/s. The height of Section A is approximately 6 cm. Sketch the height-time graph of the water in the Large vase. [3]



18a	Volume of the Small vase $= 67\frac{2}{3}\pi \text{ cm}^3$ Volume of Large vase $= 452\frac{2}{3}\pi + 88\frac{2}{3}\pi = 541\frac{1}{3}\pi \text{ cm}^3$	
18ai	Height of the Small vase $= 14$ cm $\left(\frac{\text{Height of Large vase}}{14}\right)^3 = \frac{541\frac{1}{3}\pi}{67\frac{2}{3}\pi}$ $= 8$ $\frac{\text{Height of Large vase}}{14} = 2$ Height of Large vase $= 28$ cm	M1 A1
18aii	Height of the Medium vase $= 14 + 7 = 21$ cm $\frac{\text{Volume of Medium vase}}{\text{Volume of Small vase}} = \left(\frac{21}{14}\right)^3$ $\frac{\text{Volume of Medium vase}}{67\frac{2}{3}\pi} = \left(\frac{3}{2}\right)^3$ Volume of Medium vase $= \left(\frac{3}{2}\right)^3 \times 67\frac{2}{3}\pi$ $= 228\frac{3}{8}\pi \text{ cm}^3$	 M1 A1
18b	Time taken to fill Section A $= 88\frac{2}{3}\pi \times 1 = 279$ sec (3 significant figures) Time taken to fill Section B $= 452\frac{2}{3}\pi \times 1 = 1420$ sec (3 significant figures)  	B1 B1 for A B1 for B

19 The graph of $y = -4x^2 + 16x - 13$ is drawn on the grid.



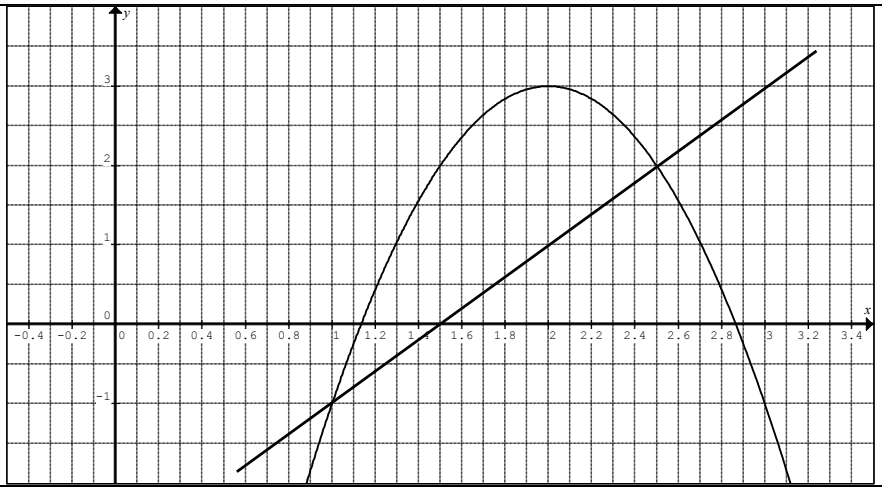
(a) Explain why $y = 3.7$ has no solution. [1]

(b) The points $(1.5, 2)$ and $(2.5, 2)$ are the intersection points for this curve and another curve, $y = ax^2 + bx + c$. Given that $a > 0$, find a possible equation for this second curve. [1]

(c) The equation $-2x^2 + 7x - 5 = 0$ can be solved by adding a straight line to the grid above. Find the equation of this line. [1]

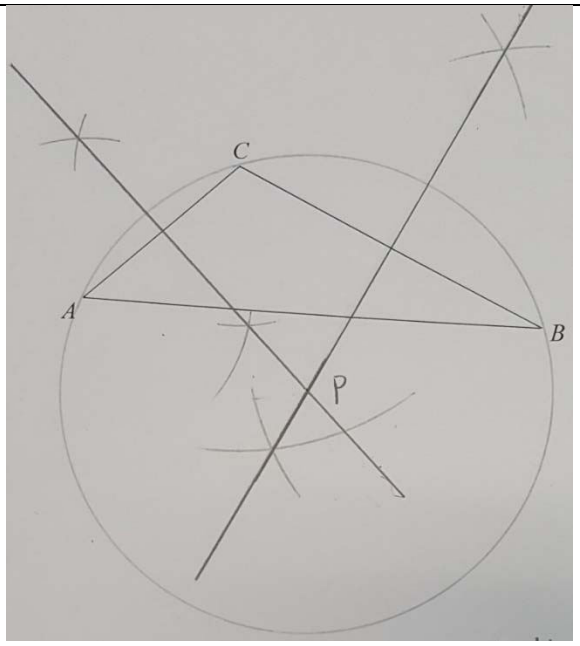
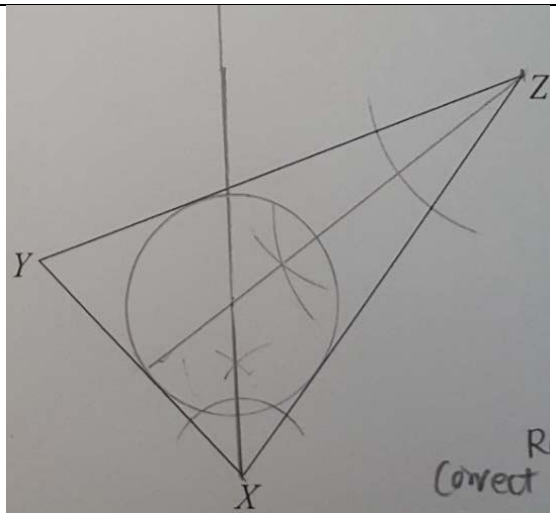
(d) By drawing this straight line, solve the equation $-2x^2 + 7x - 5 = 0$. [3]

19a	The maximum value for y is 3. Hence there is no solution for $y = 3.7$.	B1
19b	The solutions $x = 1.5$ and $x = 2.5$ come from the equation $(x-1.5)(x-2.5) = 0$. $(x-1.5)(x-2.5) = 0$ $\left(x-\frac{3}{2}\right)\left(x-\frac{5}{2}\right) = 0$ $(2x-3)(2x-5) = 0$ $4x^2 - 16x + 15 = 0$ When $x = 1.5$, $y = 2$, when $x = 2.5$, $y = 2$ One possible equation is $y = 4x^2 - 16x + 15 + 2$ $y = 4x^2 - 16x + 17$ Or	B1 or

	Reflect the curve $y=-4x^2+16x-13$ in the $x-$axis. The new equation is $y=-\left(-4x^2+16x-13\right)$ The points become $(1.5,-2)$ and $(2.5,-2)$. To get back $(1.5,2)$ and $(2.5,2)$, translate the graph by 4 units. The equation becomes $y=-\left(-4x^2+16x-13\right)+4$ $y=4x^2-16x+17$	B1
19c	$-2x^2+7x-5=0$ $-4x^2+14x-10=0$ $-4x^2+16x-10=2x$ $-4x^2+16x-13=2x-3$ The equation of the straight line is $y=2x-3$	B1
19d		
19(d)	Drawing of the line $y=2x-3$. $x=-1$ or $x=2.5$	M1A1 A1

20 **(a)** Find, by construction, the point P, that is equidistant from the points A, B and C. Hence, draw a circle passing through A, B and C. Measure the radius of the circle.

(b) Find, by construction, the point Q, that is equidistant from the lines XY, YZ and XZ. Hence, or otherwise, draw a circle that is tangent to the lines XY, YZ and XZ. Measure the radius of the circle.

20a	 Radius = 4.2 ± 0.1 cm Correct construction of two perpendicular bisectors. Circle passing through A, B and C.	 B1 B1 B1
20b	 Radius = 1.3 ± 0.1 cm Correct construction of two angle bisectors Circle tangent to XY, XZ and YZ.	 B1 B1 B1

Name _____ ( )	Class 4 _____

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR

S4

MATHEMATICS **4048/02**
Paper 2 **Thursday 13 September 2018**
2 hours 30 minutes

Additional Materials
Writing Paper × 7
Graph Paper × 1

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Table of Penalties		Qn. No.		
Presentation	–1			
Units	–1			
Significant Figures	–1		**Parent's/ Guardian's Name/ Signature/ Date**	**100**

This question paper consists of 12 printed pages.

Mathematical Formulae

Compound Interest

Total amount = $P\left(1+\frac{r}{100}\right)^n$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

Mean = $\frac{\sum fx}{\sum f}$

Standard deviation = $\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$

1 **(a)** Express as a single fraction in its simplest form $\frac{3x}{2-3y}+\frac{6x}{9y^2-4}$. [2]

(b) Simplify $\frac{21p^2q^3r^0}{2r^5}\div\frac{7pq}{4p^2}$, leaving your answer in positive indices. [2]

(c) **(i)** Simplify $\frac{6x^2-x-12}{3x^2-11x-20}$. [2]

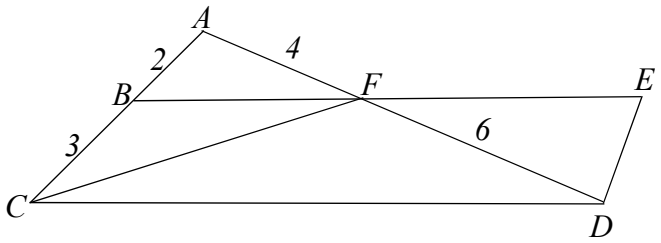
(ii) Hence, or otherwise, solve $\frac{6x^2-x-12}{3x^2-11x-20}=3$. [2]

---

2 **(a)** Each exterior angle of a regular polygon is $24°$. Find the number of sides of the polygon. [1]

(b) Interior angles of a hexagon are $(x+20)°, 120°, 53°, (2x-24)°, 3x°$ and $17°$. Find the value of x. [2]

(c) In the diagram, $AB = 2$ cm, $BC = 3$ cm, $AF = 4$ cm and $FD = 6$ cm.



(i) Show that ΔACD is similar to ΔABF. [2]

(ii) Explain why BE is parallel to CD. [2]

(iii) Given that the area of ΔABF is 20 cm^2, calculate the area of $BCDF$. [2]

---

3 A wardrobe has 3 white, 1 black and 2 pink shirts. Two shirts are drawn at random, one after another, without replacement.

(a) Draw the possibility diagram to show the outcome of the draw. [2]

(b) Find, as a fraction in its simplest form, the probability that

(i) both shirts are white, [1]

(ii) both shirts are of different colours, [1]

(iii) at least one of the shirts is pink. [1]

---

4 **Answer the whole of this question on a sheet of graph paper.**

A population of flies increases according to the formula

$$N = 30 \times 2^t$$

where N is the population of flies after t days.
The table shows some corresponding values of variables N and t.

t	1	1.5	2	2.5	3	3.5	4
N	60	84.9	120	k	240	339	480

(a) Find the value of k. [1]

(b) Determine the initial number of flies. [1]

(c) Using a scale of 2 cm to represent 1 unit, draw a horizontal scale for $0 \le t \le 4$.
Using a scale of 2 cm to represent 100 units, draw a vertical scale for $0 \le N \le 500$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

(d) Use your graph to determine the time when the population reaches 250. [1]

(e) By drawing a tangent, find the gradient of the curve at $t = 2$. Explain what this gradient represents. [2]

(f) Use your graph to determine the time when the population is increasing at 200 flies per day. [2]

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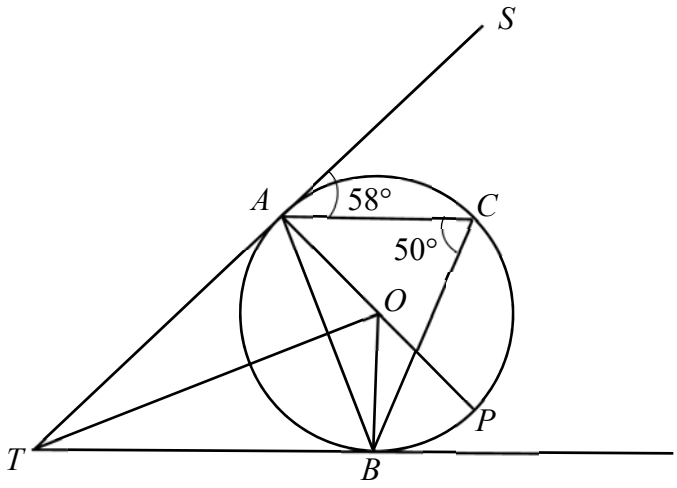
5 In the diagram, O is the centre of the circle. SAT and BT are tangents to the circle. AP is the diameter. $\angle SAC = 58°$ and $\angle ACB = 50°$.



(a) Show that triangle AOT is congruent to triangle BOT. [2]

(b) Find

(i) $\angle CAO$, [1]

(ii) $\angle AOB$, [1]

(iii) $\angle BAO$, [1]

(iv) $\angle ATB$, [1]

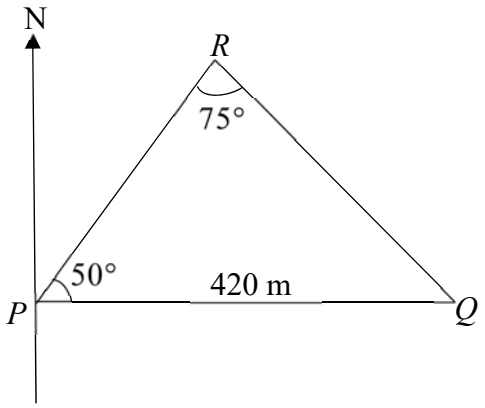
(v) $\angle OBC$, [2]

(vi) $\angle OPB$. [1]

Show your working and give reasons.

(c) A point D is such that $ACBD$ is a quadrilateral where $\angle ADB = 130°$.
Determine whether D lies on the circumference of the circle. [1]

6 In the diagram below, not drawn to scale, P, Q and R are on level ground and Q is due east of P. $PQ = 420$ m, $\angle RPQ = 50°$ and $\angle PRQ = 75°$.



(a) Find the distance PR. [2]

(b) A flag pole is erected at point P such that the angle of depression from the top of the flag pole to point R is $1.93°$. Calculate the height of the flag pole. [2]

(c) **(i)** Find the area of ΔPQR. [1]

(ii) Hence, find the shortest distance from R to PQ. [2]

(d) The bearing of point S from point P is $120°$. Given $PS = 200$ m, find

(i) angle RPS, [1]

(ii) the distance RS. [2]

7 Peter bought m kiwi fruits for \$64.

(a) Find an expression, in terms of m, for the cost of one kiwi fruit in cents. [1]

(b) Given that 5 of the kiwi fruits were rotten and could not be sold, Peter sold each remaining kiwi fruit at 40 cents more than he paid for it.

Without simplifying, write down an expression in terms of m, for the total amount he received from the sale of the kiwi fruits. [1]

(c) He made a profit of \$26 from the sale of the kiwi fruits.
Write down an equation in m to represent this information, and show that it reduces to $m^2 - 70m - 800 = 0$. [3]

(d) Solve the equation $m^2 - 70m - 800 = 0$. [3]

(e) Find the selling price of a kiwi fruit. [2]

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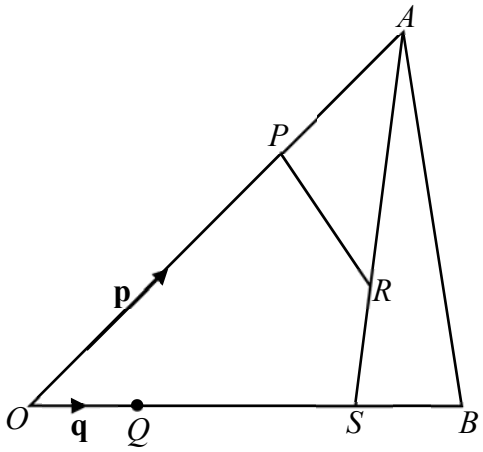
8 In the diagram below, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. It is given that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$, $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OS}$, $OQ = SB$ and $SA = 3SR$.



(a) Express, as simply as possible, in terms of **p** and **q**,

(i) $\overrightarrow{SA}$, [1]

(ii) $\overrightarrow{PB}$, [1]

(iii) $\overrightarrow{PR}$. [2]

(b) Prove that P, R and B are collinear. [2]

(c) Find the numerical value of

(i) $\dfrac{\text{Area of } \Delta APR}{\text{Area of } \Delta ARB}$, [1]

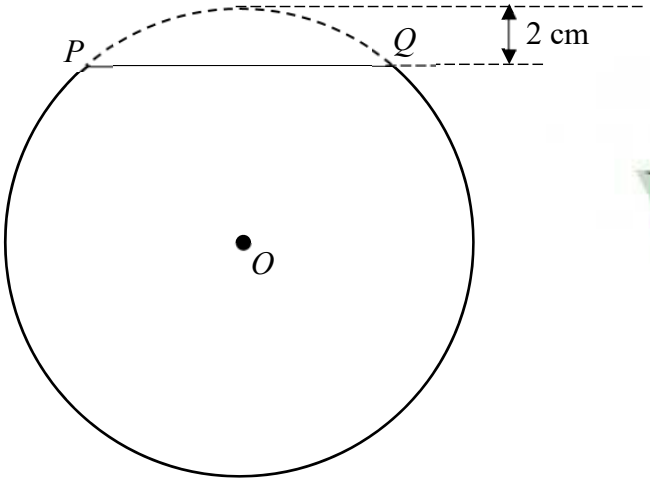
(ii) $\dfrac{\text{Area of } \Delta RSB}{\text{Area of } \Delta APR}$. [2]

9 **(a)** The diagram below shows a circle with centre O. The major arc PQ is 30.4 cm. Given that the straight line PQ is 9.4 cm and the minor segment has a vertical height of 2 cm from the centre of line PQ.

(i) Show that the radius of the circle is 6.5225 cm. [3]

(ii) Calculate the reflex angle POQ. [2]

(iii) Find the area of the minor segment POQ. [2]



(b) The diagram in **(a)** is a 2-dimensional view of the body of a teapot with the minor segment being the lid of the teapot.

(i) The volume of the teapot can be calculated using the formula,

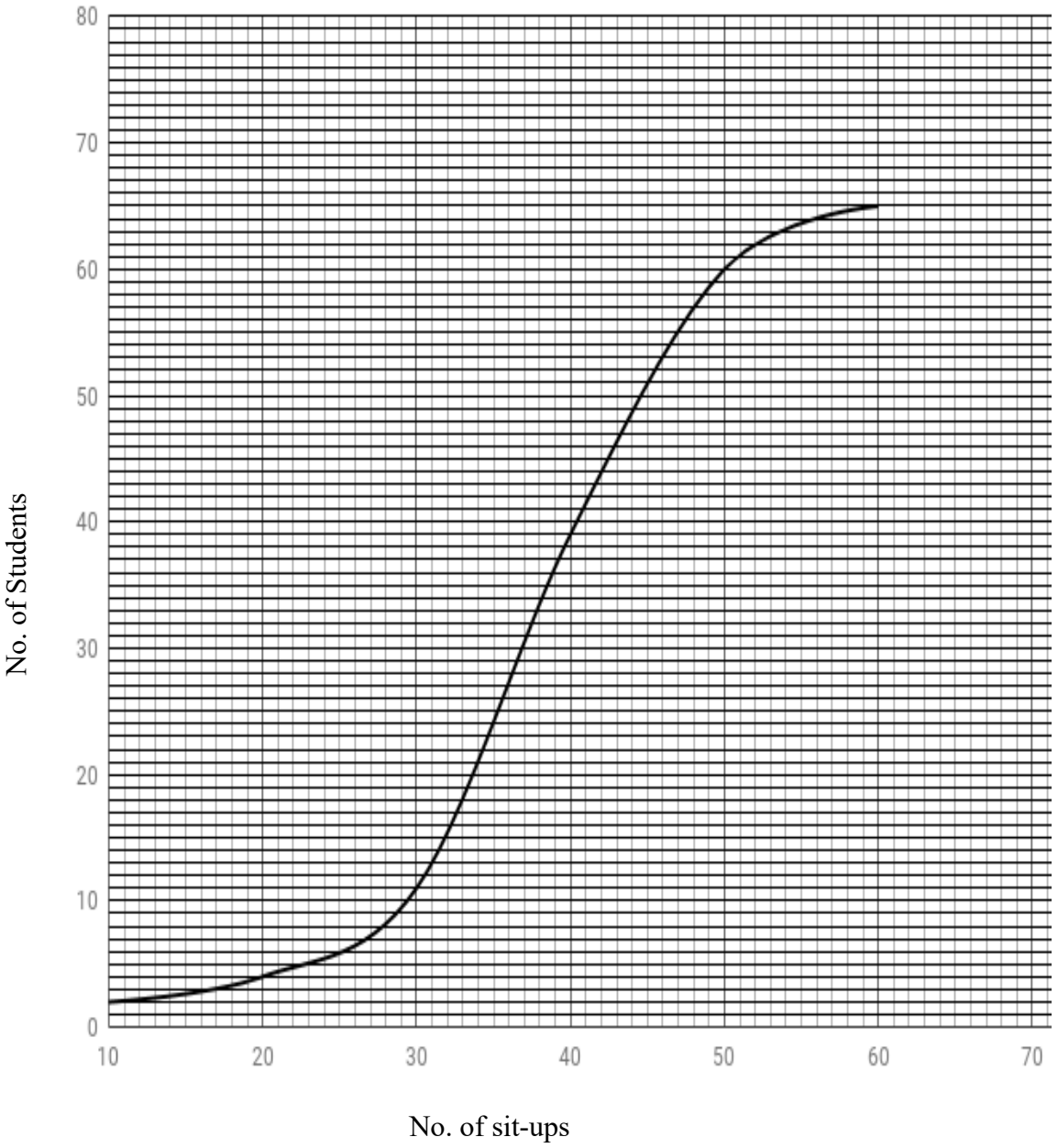
$$V = \frac{\pi}{6} h\left(3c^2 + h^2\right) .$$

h is the vertical height of the teapot measured from the bottom to the opening and passing through the centre, O.
c is radius of the top opening of the teapot.

Calculate the volume of the teapot. [3]

(ii) How many 250ml teacups are needed to contain all the tea in the teapot in part **(b)(i)** given that each cup should be only 60% full? [2]

10 One of the NAPFA test stations is to record the number of sit-ups that each student can do in one minute. The cumulative frequency curve below shows the number of sit-ups by a group of 65 students.



(a) By using the cumulative frequency curve, find the value of a and of b. [2]

Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	2	a	7	28	21	b

10 **(b)** Find the percentage of students who did between 35 and 42 sit-ups inclusive. [1]

(c) Calculate an estimate of

(i) the mean number of sit-ups, [1]

(ii) the standard deviation. [1]

(d) The number of sit-ups by a second group of 65 students is recorded in the table shown below.

Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	12	10	7	14	12	5

(i) Given that the standard deviation for the second group of students is about 16.3, explain briefly which group is more consistent in their performance. [1]

(ii) If the two groups were compared, explain whether the mean or median would be a better measure of central tendency. [1]

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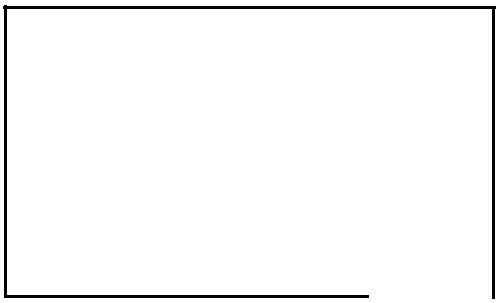
11 For a Parent Teacher Conference, a school has to convert the parade square into a parking lot. The parade square is a rectangular plot of land 40 m by 30 m.

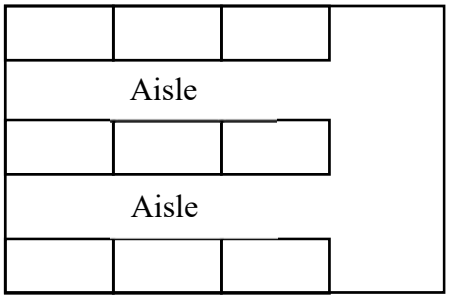


ENTRANCE / EXIT

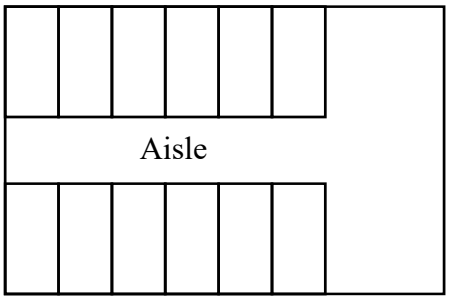
There is an 6 m wide entrance / exit at one corner of the parade square as shown in the sketch above.
You are required to do the parking arrangements.

The Land Transport Authority recommends the following guidelines.
A **Parking Stall** refers to the space for parking of one motorcar, that is, a car parking lot. The space of the stall should be rectangular. The longer side is known as the length and the shorter side is the width.
A **Parking Aisle** refers to an access lane or driveway with adjacent parking stalls.

In parallel parking, the longer side is parallel to the parking aisle or driveway. The aisle for cars to move must be at least 3.6 m.
For two-way traffic flow, the width of the aisle must be at least 6 m.
Each parking stall is 5.4 m by 2.4 m.



In 90° parking, the longer side is perpendicular to the parking aisle or driveway.
The aisle or lane for cars to move must be at least 6 m for one-way traffic flow and at least 6.6 m for two-way traffic flow.
Each parking stall is 4.8 m by 2.4 m.



Propose a possible parking arrangement that would maximise the use of space, showing your calculations clearly. Your proposal must include a sketch, not drawn to scale, indicating the location of the parking stalls, the aisles and the type(s) of parking. You should allow for cars to enter and leave the parade square at any time. You can assume that the cars will not leave in large numbers at any one time, and the parking will be supervised by security guards. [10]

End of Paper

Answer key

1

(a) $\dfrac{9xy}{(2+3y)(2-3y)}$ (b) $\dfrac{6p^3q^2}{r^5}$

(c)(i) $\dfrac{2x-3}{x-5}$ (ii) $x=12$

2

(a) $n=15$
(b) $x=89$
(c)(ii)
They are corresponding angles to lines BE and CD, hence BE and CD are parallel
(iii) 105cm^2

3(a)

Colours	W_1	W_2	W_3	B	P_1	P_2
W_1		W_2, W_1	W_3, W_1	B, W_1	P_1, W_1	P_2, W_1
W_2	W_1, W_2		W_3, W_2	B, W_2	P_1, W_2	P_2, W_2
W_3	W_1, W_3	W_2, W_3		B, W_3	P_1, W_3	P_2, W_3
B	W_1, B	W_2, B	W_3, B		P_1, B	P_2, B
P_1	W_1, P_1	W_2, P_1	W_3, P_1	B, P_1		P_2, P_1
P_2	W_1, P_2	W_2, P_2	W_3, P_2	B, P_2	P_1, P_2	

3(b)(i) $\dfrac{1}{5}$
(ii) $\dfrac{11}{15}$
(iii) $\dfrac{3}{5}$

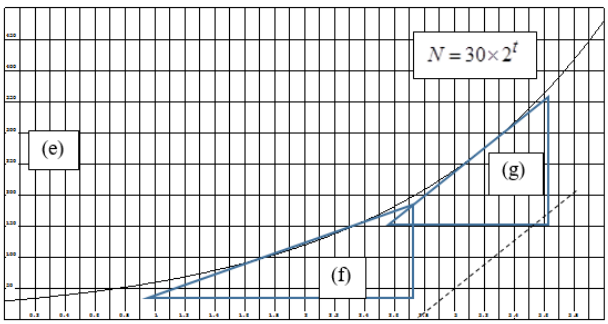
4

(a) 170 (b) 30 (d) $2.9 \le t \le 3.15$

(e) $78 \le$ gradient ≤ 85. At $t=2$, the flies are increasing at 83 flies per day.

(f) $2.8 \le t \le 3.8$

4(c)



5(b)

(i) 32° (ii) 100° (iii) 40° (iv) 80° (v) 18° (vi) 50°

5(c)

By angles in opposite segment property, D is a point on the circle.

6

(a) 356 m (b)12.0 m
(c)(i) 57300m^2 **(ii)** 273m

(d)(i) $\angle RPS = 80°$ **(ii)** 377m

7

(a) $\dfrac{6400}{m}$ cents (b) $(m-5)\left(\dfrac{6400}{m}+40\right)$ cents

(d) $m=80$ *or* $m=-10$

(e) 120 cents or \$1.20

8

(a)(i) $-3\mathbf{q}+\dfrac{3}{2}\mathbf{p}$ **(ii)** $-\mathbf{p}+4\mathbf{q}$

(iii) $2\mathbf{q}-\dfrac{1}{2}\mathbf{p}$

(b)
Since $\overrightarrow{PR}=\dfrac{1}{2}\overrightarrow{PB}$,
$\therefore P, R$ and B are collinear.

(c) (i) 1 (ii) $\dfrac{1}{2}$

9

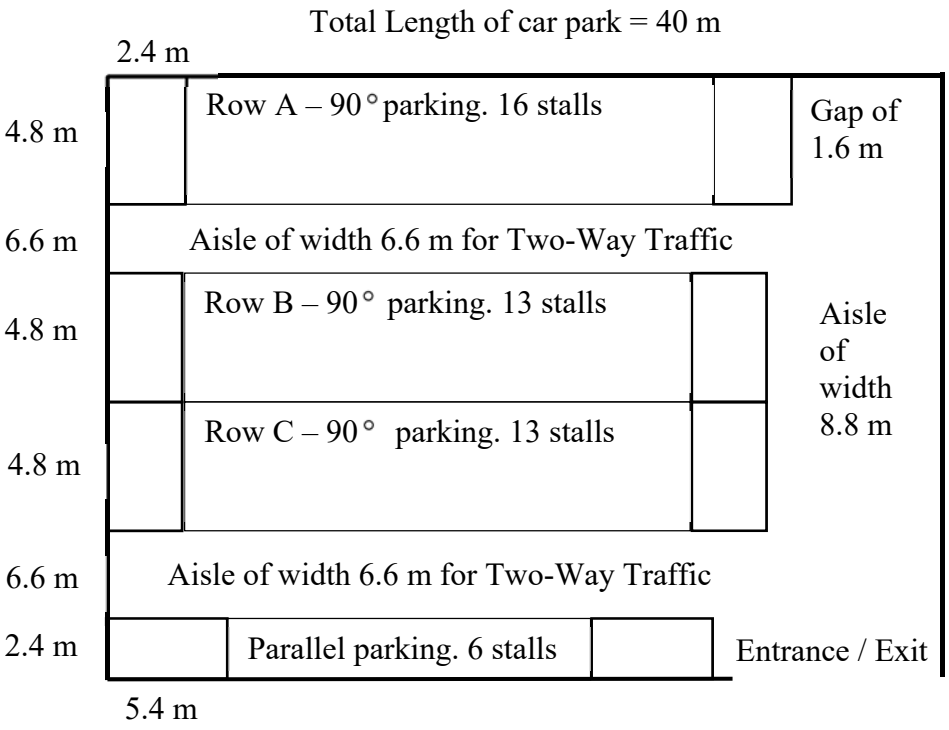
(a) (ii) 4.66 rad or 267.0° (iii) 13.3 cm^2
(b)(i) $\approx 1090\,\text{cm}^3$ (ii) 8

10

(a) $a=2, b=5$
(b) 30.8%
(c) (i) 37.2 (ii) 10.5
(d) (i) Since first group has a smaller standard deviation, so the students in first group are more consistent in their performance.
(ii) The median is a better measure due to the outliers.

Q11

Possible Solution



B1 – sketch is clearly labelled
B1 – layout of parking stalls using parallel parking
B1 – layout of parking stalls using 90° parking
B1 – layout of aisles clearly showing that cars can move in or out at any time.
B1 – aisles are at appropriate width, either 6 m for one-way or 6.6 m for two-way.

	Number of lots in Row A $= \frac{40}{2.4} = 16\frac{2}{3} \approx 16$ Number of lots in Row B or C $= \frac{40-6.6}{2.4} = 13\frac{11}{12} \approx 13$ Length of Row A $= 16 \times 2.4 = 38.4 \text{ m} < 40 \text{ m}$ Length of Row B $= 13 \times 2.4 = 31.2 \text{ m} < 40 \text{ m}$ Length of Row C $= 13 \times 2.4 = 31.2 \text{ m} < 40 \text{ m}$ Number of lots for parallel parking $= \frac{40-6}{5.4} = 6\frac{8}{27} \approx 6$ Length of parallel parking $= 6 \times 5.4 = 32.4 \text{ m} < 36 \text{ m}$	Considering the length M1 A1
	Width of the rows of parking stalls and aisles $= 4.8 \times 3 + 2.4 + 2 \times 6.6$ $= 30 \text{ m}$	Considering the width M1 A1
	Total number of parking stalls $= 16 + 2 \times 13 + 6 = 48$	A1 – Accept minimum as 42
	Remarks – Students who consider only parallel parking or 90° parking but not both get a maximum of 8 marks.	

1 **(a)** Express as a single fraction in its simplest form $\frac{3x}{2-3y}+\frac{6x}{9y^2-4}$. [2]

(b) Simplify $\frac{21p^2q^3r^0}{2r^5}\div\frac{7pq}{4p^2}$, leaving your answer in positive indices. [2]

(c) **(i)** Simplify $\frac{6x^2-x-12}{3x^2-11x-20}$. [2]

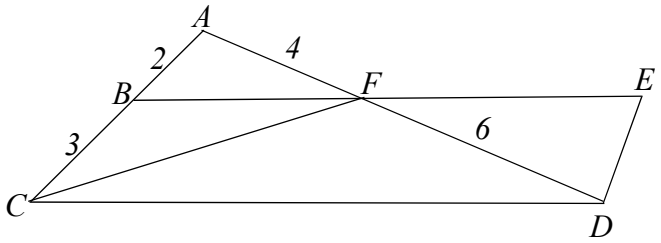
(ii) Hence, or otherwise, solve $\frac{6x^2-x-12}{3x^2-11x-20}=3$. [2]

1(a)	$\frac{3x}{2-3y}+\frac{6x}{9y^2-4}=\frac{3x}{2-3y}-\frac{6x}{4-9y^2}$ $=\frac{3x}{2-3y}-\frac{6x}{(2+3y)(2-3y)}$ $=\frac{3x(2+3y)-6x}{(2+3y)(2-3y)}$ $=\frac{9xy}{(2+3y)(2-3y)}$	M1 A1
1(b)	$\frac{21p^2q^3r^0}{2r^5}\div\frac{7pq}{4p^2}=\frac{21p^2q^3}{2r^5}\times\frac{4p^2}{7pq}$ $=\frac{3p^2q^3}{r^5}\times\frac{2p}{q}$ $=\frac{6p^3q^2}{r^5}$	M1 A1
1(c)(i)	$\frac{6x^2-x-12}{3x^2-11x-20}=\frac{(3x+4)(2x-3)}{(3x+4)(x-5)}$ $=\frac{2x-3}{x-5}$	M1 A1
1(c)(ii)	$\frac{6x^2-x-12}{3x^2-11x-20}=3$ $\frac{2x-3}{x-5}=\frac{3}{1}$ $2x-3=3x-15$ $x=12$	 M1 A1

2 **(a)** Each exterior angle of a regular polygon is $24°$. Find the number of sides of the polygon. [1]

(b) Interior angles of a hexagon are $(x+20)°, 120°, 53°, (2x-24)°, 3x°$ and $17°$. Find the value of x. [2]

(c) In the diagram, $AB = 2$ cm, $BC = 3$ cm, $AF = 4$ cm and $FD = 6$ cm.



(i) Show that ΔACD is similar to ΔABF. [2]

(ii) Explain why BE is parallel to CD. [2]

(iii) Given that the area of $\Delta\ ABF$ is 20 cm^2, calculate the area of $BCDF$. [2]

2(a)	Let n be the number of sides $24n = 360$ $n = 15$	 B1
2(b)	Total interior of hexagon $= (6-2)\times 180° = 720°$ $x + 20° + 120° + 53° + 2x - 24° + 3x + 17° = 720°$ $6x + 186 = 720$ $6x = 534$ $x = 89$	M1 A1
2(c)(i)	$\angle CAD = \angle BAF$ (*common angle*) $\frac{AC}{AB} = \frac{AD}{AF} = \frac{5}{2}$ By SAS Similarity Test, ΔACD is similar to ΔABF.	 M1 A1
2(c)(ii)	Since $\Delta\ ACD$ is similar to $\Delta\ ABF$, $\angle ABF = \angle ACD$ They are corresponding angles to lines BE and CD, hence BE and CD are parallel	 M1 A1
2(c)(iii)	Area of triangle $ACD = \left(\frac{5}{2}\right)^2 \times 20$ $= 125\ cm^2$ Therefore area of quadrilateral $BCDF = 125 - 20$ $= 105\ cm^2$	 M1 A1

3 A wardrobe has 3 white, 1 black and 2 pink shirts. Two shirts are drawn at random, one after another, without replacement.

(a) Draw the possibility diagram to show the outcome of the draw. [2]

(b) Find, as a fraction in its simplest form, the probability that

(i) both shirts are white, [1]

(ii) both shirts are of different colours, [1]

(iii) at least one of the shirts is pink. [1]

3(a)		B2 Award 1 mark if 5 or more cells are correct .

Colours	**W_1**	**W_2**	**W_3**	**B**	**P_1**	**P_2**
W_1		W_2, W_1	W_3, W_1	B, W_1	P_1, W_1	P_2, W_1
W_2	W_1, W_2		W_3, W_2	B, W_2	P_1, W_2	P_2, W_2
W_3	W_1, W_3	W_2, W_3		B, W_3	P_1, W_3	P_2, W_3
B	W_1, B	W_2, B	W_3, B		P_1, B	P_2, B
P_1	W_1, P_1	W_2, P_1	W_3, P_1	B, P_1		P_2, P_1
P_2	W_1, P_2	W_2, P_2	W_3, P_2	B, P_2	P_1, P_2	

o.e.

3(bi)	P (both shirts are white) $= \frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$ Or read directly from the table: P (both shirts are white) $= \frac{6}{30} = \frac{1}{5}$	B1
3(bii)	P (both shirts are of different colours) = 1 – P (both shirts are white or pink) $= 1 - \left(\frac{1}{5} + \frac{2}{6} \times \frac{1}{5} \right) = \frac{11}{15}$ Or read directly from the table: P (both shirts are of different colours) $= \frac{22}{30} = \frac{11}{15}$	B1
3(biii)	P (at least one of the shirts is pink) = 1 – P (no shirts are pink) $= 1 - \left(\frac{4}{6} \times \frac{3}{5} \right) = \frac{3}{5}$ Or read directly from the table: P (at least one of the shirts is pink) = $= \frac{18}{30} = \frac{3}{5}$	B1

4 **Answer the whole of this question on a sheet of graph paper.**

A population of flies increases according to the formula

$$N = 30 \times 2^t$$

where N is the population of flies after t days.

The table shows some corresponding values of variables N and t .

t	1	1.5	2	2.5	3	3.5	4
N	60	84.9	120	k	240	339	480

(a) Find the value of k. [1]

(b) Determine the initial number of flies. [1]

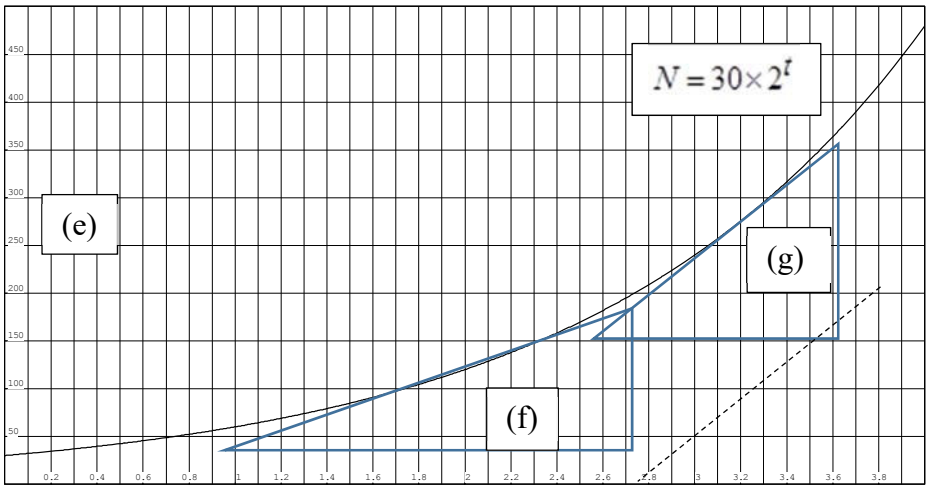
(c) Using a scale of 2 cm to represent 1 unit, draw a horizontal scale for $0 \le t \le 4$.
Using a scale of 2 cm to represent 100 units, draw a vertical scale for $0 \le N \le 500$.
On your axes, plot the points given in the table and join them with a smooth curve. [3]

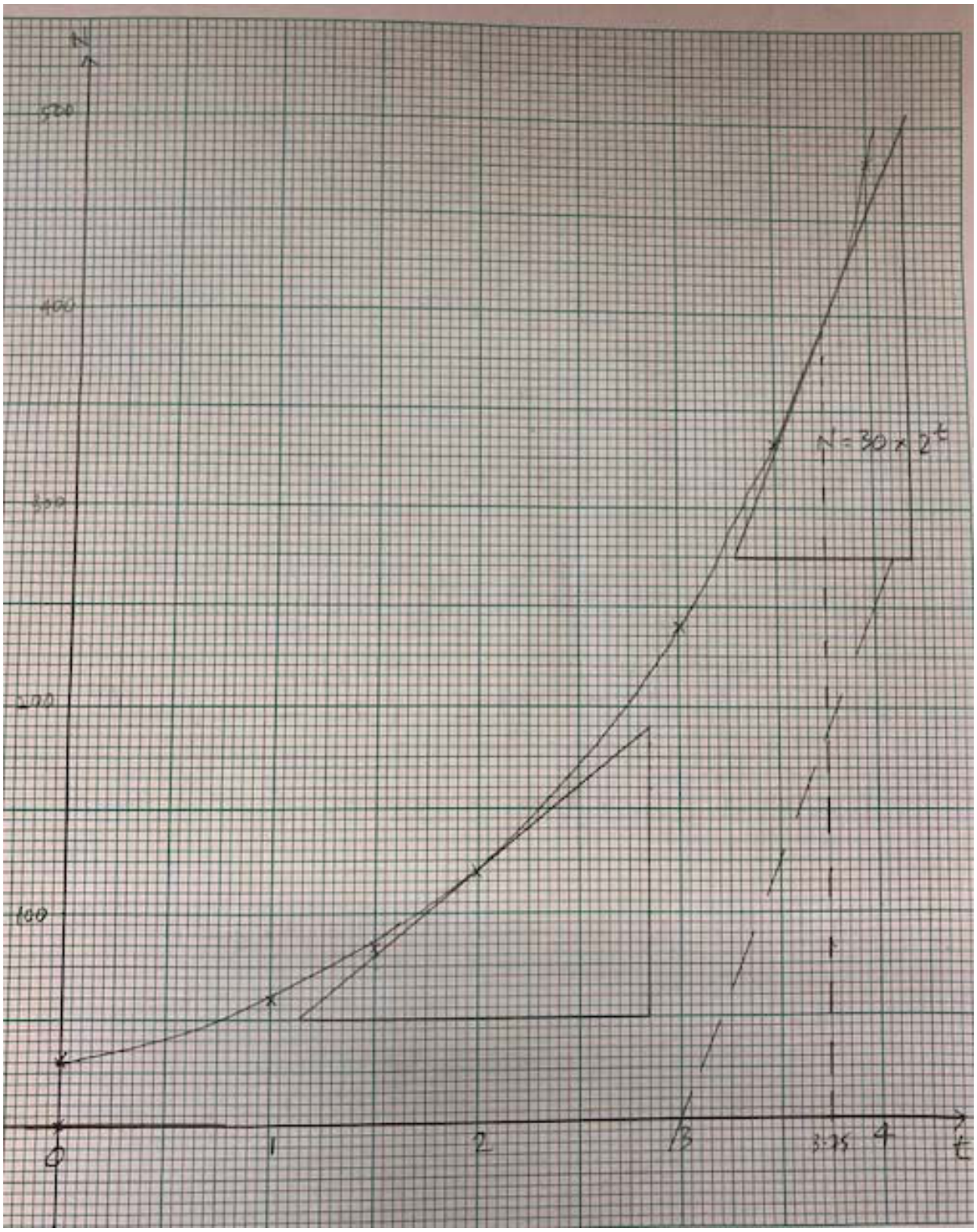
(d) Use your graph to determine the time when the population reaches 250. [1]

(e) By drawing a tangent, find the gradient of the curve at $t = 2$. Explain what this gradient represents. [2]

(f) Use your graph to determine the time when the population is increasing at 200 flies per day. [2]

4(a)	$k = 30 \times 2^{2.5} = 169.706 = 170$	B1
4(b)	When $t = 0$, $N = 30 \times 2^0 = 30$ Initial number of flies is 30.	B1
4(c)	B1 – correct plotting B1 – smooth curve B1 – labelling and correct scale	
4(d)	$2.9 \le t \le 3.15$	B1
4(e)	$78 \le$ gradient ≤ 85. At $t = 2$, the flies are increasing at 83 flies per day.	B1 – tangent B1 - statement
4(f)	$2.8 \le t \le 3.8$	B1- tangent with gradient 200 B1 - value





5 In the diagram, O is the centre of the circle. SAT and BT are tangents to the circle. AP is the diameter. $\angle SAC = 58°$ and $\angle ACB = 50°$.

(a) Show that triangle AOT is congruent to triangle BOT. [2]

(b) Find

(i) $\angle CAO$, [1]

(ii) $\angle AOB$, [1]

(iii) $\angle BAO$, [1]

(iv) $\angle ATB$, [1]

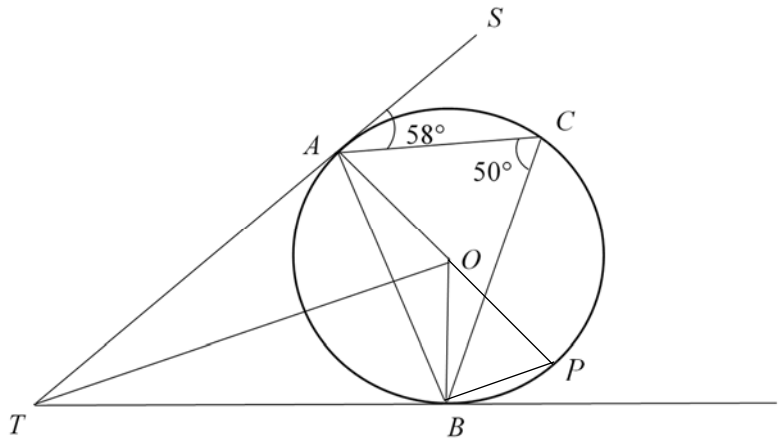
(v) $\angle OBC$, [2]

(vi) $\angle OPB$. [1]

Show your working and give reasons.

(c) A point D is such that $ACBD$ is a quadrilateral where $\angle ADB = 130°$.

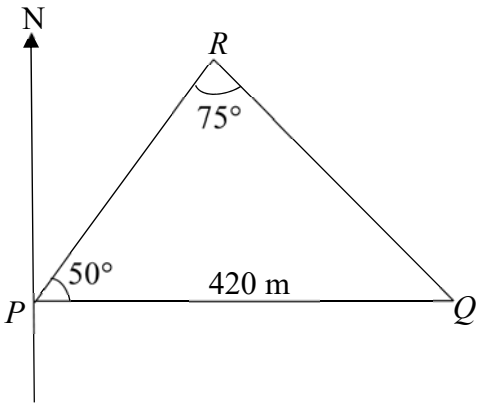
Determine whether D lies on the circumference of the circle. [1]



5(a)	$AO = OB$(radii of circle) $\angle TAO = \angle TBO = 90°$(radius perpendicular to tangent) o.e. $OT = OT$(common side, hypotenuse) By RHS, $\Delta\ AOT$ is congruent to $\Delta\ BOT$.	M1 A1
5(b)(i)	$\angle CAO = 90° - 58°$(radius perpendicular to tangent) $= 32°$	B1
5(b)(ii)	obtuse $\angle AOB = 50° \times 2$($\angle$ at centre $= 2 \times \angle$ at circumference) $= 100°$	B1
5(b)(iii)	$\angle BAO = \frac{180° - 100°}{2}$(isosceles ΔAOB) $= 40°$	B1

5(b)(iv)	$\angle ATB = 360° - \angle TAO - \angle TBO - \text{obtuse}\angle AOB$ $= 360° - 90° - 90° - 100°$(property ofquadrilateral ATBO) $= 80°$ *OR* $\angle AOT = 100° \div 2$ $= 50°$ $\angle ATO = 180° - 90° - 50°$(sum of angles of triangle) $\angle ATB = 40° \times 2$ $= 80°$	B1
5(b)(v)	$\angle ACO = \angle CAO = 32°$(isosceles ΔAOC) $\angle OCB = \angle ACB - \angle ACO$ $= 50° - 32°$ $= 18°$ $\angle OBC = \angle OCB$(isosceles ΔCOB) $= 18°$	M1 A1
5(b)(vi)	$\angle OPB = 50°$($\angle$s in the same segment)	B1
5(c)	By angles in opposite segment property, *D* is a point on the circle. Deduct 1 mark from the whole of question for not stating reason or incorrect reason.	B1

6 In the diagram below, not drawn to scale, *P*, *Q* and *R* are on level ground and *Q* is due east of *P*. $PQ = 420$ m, $\angle RPQ = 50°$ and $\angle PRQ = 75°$.



(a) Find the distance *PR*. [2]

(b) A flag pole is erected at point *P* such that the angle of depression from the top of the flag pole to point *R* is $1.93°$. Calculate the height of the flag pole. [2]

(c) **(i)** Find the area of ΔPQR. [1]

(ii) Hence, find the shortest distance from R to PQ. [2]

The bearing of point S from point P is $120°$. Given $PS = 200$ m, find

(d) **(i)** angle RPS [1]

(ii) the distance RS. [2]

6(a)	$\angle RQP = 180° - 75° - 50° = 55°$ $\frac{\sin 55°}{PR} = \frac{\sin 75°}{420}$ $PR = 356.18$ ≈ 356 m	M1 A1
6(b)	$\tan 1.93° = \frac{\text{height of flagpole}}{356.18}$ height of flagpole $= 12.002$ ≈ 12.0m	M1 A1
6(c)(i)	Area of $\Delta PQR = \frac{1}{2}(356.18)(420)\sin 50°$ $= 57298.4$ $\approx 57300\text{m}^2$	 B1
6(c)(ii)	$\frac{1}{2}(420)h = 57298.4$ $h = 272.85$ $h \approx 273$m	M1 A1
6(d)(i)	$\angle RPS = 80°$	B1
6(d)(ii)	$RS^2 = 356.18^2 + 200^2 - 2(356.18)(200)\cos 80°$ $RS^2 = 142124.19$ $RS = 376.99$ $RS \approx 377$m	M1 A1

7 Peter bought m kiwi fruits for \$64.

(a) Find an expression, in terms of m, for the cost of one kiwi fruit in cents. [1]

(b) Given that 5 of the kiwi fruits were rotten and could not be sold, Peter sold each remaining kiwi fruit at 40 cents more than he paid for it.
Without simplifying, write down an expression in terms of m, for the total amount he received from the sale of the kiwi fruits. [1]

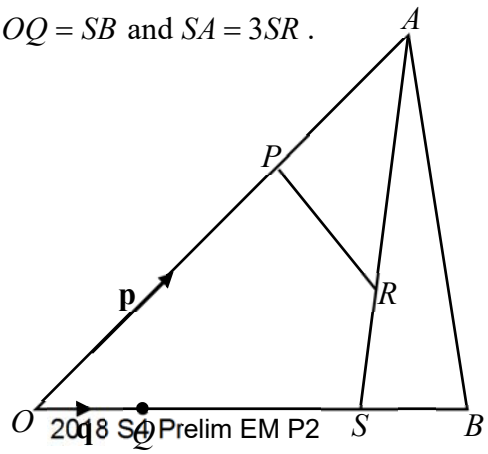
(c) He made a profit of \$26 from the sale of the kiwi fruits.
Write down an equation in m to represent this information, and show that it reduces to $m^2 - 70m - 800 = 0$. [3]

(d) Solve the equation $m^2 - 70m - 800 = 0$. [3]

(e) Find the selling price of a kiwi fruit. [2]

7(a)	Cost of each kiwi fruit = $\frac{6400}{m}$ cents	B1
7(b)	Total sum received = $(m-5)\left(\frac{6400}{m}+40\right)$ cents	B1
7(c)	$(m-5)\left(\frac{6400}{m}+40\right)-6400=2600$ $6400+40m-\frac{32000}{m}-200-6400-2600=0$ $40m-2800-\frac{32000}{m}=0$ $40m^2-2800m-32000=0$ $m^2-70m-800=0$	M1form equations M1expansion A1
7(d)	$m^2-70m-800=0$ $(m-80)(m+10)=0$ $m=80$ *or* $m=-10$	M1 factorisation A2
7(e)	Selling price of each kiwi = $\frac{6400}{80}+40$ $=120$ cents or \$1.20	M1 A1

8 In the diagram below, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. It is given that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OA}$, $\overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OS}$, $OQ = SB$ and $SA = 3SR$.



(a) Express, as simply as possible, in terms of **p** and **q**,

(i) $\overrightarrow{SA}$, [1]

(ii) $\overrightarrow{PB}$, [1]

(iii) $\overrightarrow{PR}$. [2]

(b) Prove that P, R and B are collinear. [2]

(c) Find the numerical value of

(i) $\dfrac{\text{Area of } \Delta APR}{\text{Area of } \Delta ARB}$, [1]

(ii) $\dfrac{\text{Area of } \Delta RSB}{\text{Area of } \Delta APR}$. [2]

8(a)(i)	$\overrightarrow{SA} = \overrightarrow{SO} + \overrightarrow{OA}$ $= -3\mathbf{q} + \frac{3}{2}\mathbf{p}$	 B1
8(a)(ii)	$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB}$ $= -\mathbf{p} + 4\mathbf{q}$	B1
8(a)(iii)	$\overrightarrow{PA} = \frac{1}{2}\mathbf{p}$ $\overrightarrow{AR} = \frac{2}{3}\overrightarrow{AS}$ $= -\frac{2}{3}\left(-3\mathbf{q} + \frac{3}{2}\mathbf{p}\right)$ $= 2\mathbf{q} - \mathbf{p}$ $\overrightarrow{PR} = \frac{1}{2}\mathbf{p} + 2\mathbf{q} - \mathbf{p} = 2\mathbf{q} - \frac{1}{2}\mathbf{p}$	 M1 A1
8(b)	$\overrightarrow{PB} = -\mathbf{p} + 4\mathbf{q}$ $\overrightarrow{PR} = 2\mathbf{q} - \frac{1}{2}\mathbf{p}$ $= \frac{1}{2}(4\mathbf{q} - \mathbf{p})$ $= \frac{1}{2}\overrightarrow{PB}$ Since $\overrightarrow{PR} = \frac{1}{2}\overrightarrow{PB}$, $\therefore P, R$ and B are collinear.	 M1 A1
8(c)(i)	$\dfrac{\text{Area of } \Delta APR}{\text{Area of } \Delta ARB} = 1$	B1

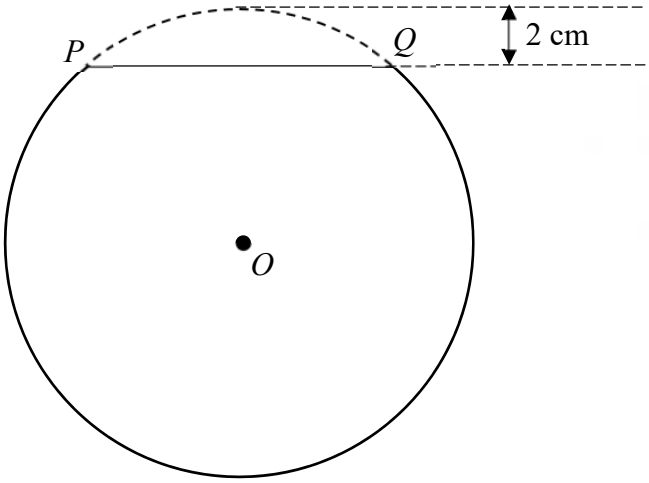
8(c)(ii)	$\frac{\text{Area of } \Delta RSB}{\text{Area of } \Delta ARB} = \frac{1}{2}$ $\frac{\text{Area of } \Delta RSB}{\text{Area of } \Delta APR} = \frac{\text{Area of } \Delta RSB}{\text{Area of } \Delta ARB} \times \frac{\text{Area of } \Delta ARB}{\text{Area of } \Delta APR}$ $= \frac{1}{2} \times \frac{1}{1}$ $= \frac{1}{2}$	M1 A1

9 **(a)** The diagram below shows a circle with centre O. The major arc PQ is 30.4 cm. Given that the straight line PQ is 9.4 cm and the minor segment has a vertical height of 2 cm from the centre of line PQ.

(i) Show that the radius of the circle is 6.5225 cm. [3]

(ii) Calculate the reflex angle POQ. [2]

(iii) Find the area of the minor segment POQ. [2]



(b) The diagram in **(a)** is a 2-dimensional view of the body of a teapot with the minor segment being the lid of the teapot.

(i) The volume of the teapot can be calculated using the formula,

$$V = \frac{\pi}{6} h\left(3c^2 + h^2\right) .$$

h is the vertical height of the teapot measured from the bottom to the opening and passing through the centre, O.

c is radius of the top opening of the teapot.

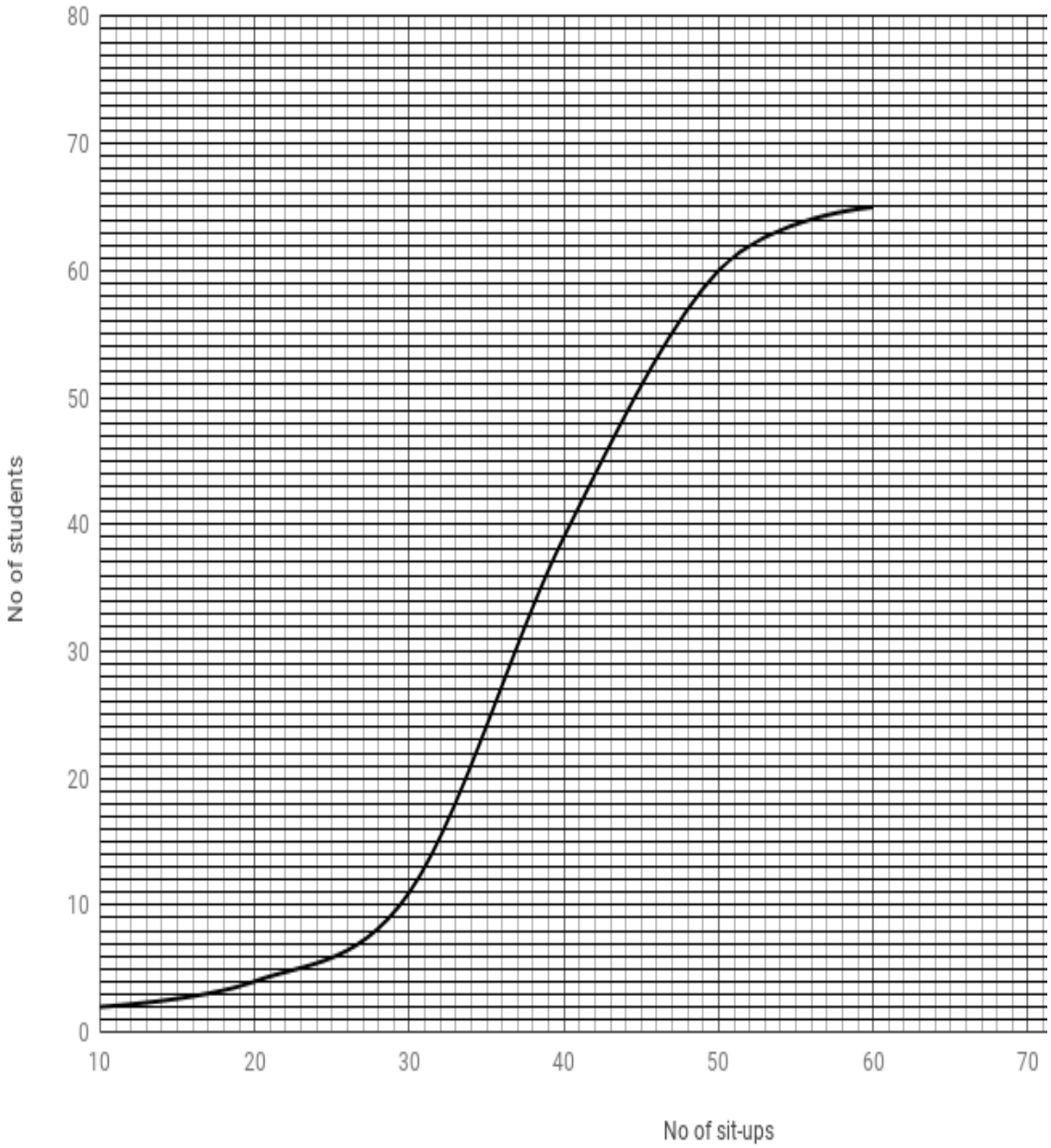
Calculate the volume of the teapot. [3]

(ii) How many 250ml teacups are needed to contain all the tea in the teapot in part **(b)(i)** given that each cup should be only 60% full? [2]

9(ai)	Let the radius of the circle be r cm. $r^2-(r-2)^2=\left(\frac{9.4}{2}\right)^2$ $(r+r-2)(r-r+2)=22.09$ $2(2r-2)=22.09$ $4(r-1)=22.09$ $r=\frac{22.09}{4}+1$ $r=6.5225$ The radius is 6.5225 cm.	M1 forming equation M1 simplify equation A1
9(aii)	$6.5225\times\angle POQ=30.4$ $\angle POQ=\frac{30.4}{6.5225}$ $\angle POQ\approx 4.66078$ rad $\angle POQ\approx 4.66$ rad Or $\frac{\angle POQ}{360°}\times 2\times 6.5225\times\pi=30.4$ $\angle POQ=\frac{30.4\times 360°}{2\times 6.5225\times\pi}$ $\angle POQ\approx 267.043°$ $\angle POQ\approx 267.0°$	M1 A1
9(aiii)	Area of the minor segment POQ $=\frac{1}{2}(6.5225)^2\left[(2\pi-4.66078)-\sin(2\pi-4.66078)\right]$ ≈ 13.268 cm^2 ≈ 13.3 cm^2 Or Area of the minor segment POQ $=\frac{360°-267.043°}{360°}\times\pi\times(6.5225)^2-\frac{1}{2}(6.5225)^2\times\sin(360°-267.043°)$ ≈ 13.268 cm^2 ≈ 13.3 cm^2	M1 A1

9(bi)	$h = 2 \times .65225 - 2$ $h = 11.045\,\text{cm}$ Volume of the teapot $= \frac{\pi}{6}(11.045)\left[3\left(\frac{9.4}{2}\right)^2 + 11.045^2\right]$ ≈ 1088.747 $\approx 1090\,\text{cm}^3$	M1 M1 A1
9(bii)	Number of teacups needed $= \frac{1088.747}{0.6 \times 250}$ ≈ 7.2583 ≈ 8	M1 A1

10 One of the NAPFA test station is to record the number of sit-ups that each student can do in one minute. Below shows the cumulative frequency curve of the number of sit-ups by a group of 65 students.



(a) By using the cumulative frequency curve, find the value of a and of b. [2]

Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	2	a	7	28	21	b

(b) Find the percentage of students who did between 35 and 42 sit-ups inclusive. [1]

(c) Calculate an estimate of

(i) the mean number of sit-ups, [1]

(ii) the standard deviation. [1]

(d) The number of sit-ups by a second group of 65 students is recorded in the table shown below.

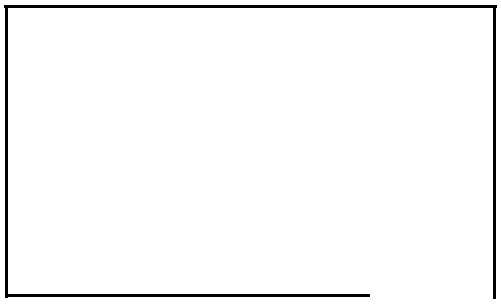
Number of sit-ups	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of students	12	10	7	14	12	5

(i) Given that the standard deviation for the second group of students is about 16.3, explain briefly which group is more consistent in their performance. [1]

(ii) If the two groups were compared, explain whether the mean or median would be a better measure of central tendency. [1]

10(a)	$a = 2, b = 5$	B2
10(b)	Percentage of students $= \frac{44-24}{65} \times 100\% = 30.8\%$	B1
10(c)(i)	Mean number of sit-ups $= 37\frac{2}{13}$ or $\approx 37.154 \approx 37.2$	B1
10(c)(ii)	Standard deviation $= = \sqrt{\frac{96825}{65} - \left(37\frac{2}{13}\right)^2}$ ≈ 10.4502 ≈ 10.5	 B1
10(d)(i)	Since first group has a smaller standard deviation, so the students in first group are more consistent in their performance.	B1
10(d)(ii)	The median is a better measure due to the outliers. Note: Award mark as long as the description is similar to outlier. No mark at all if students compare mean.	B1

11 For a Parent Teacher Conference, a school has to convert the parade square into a parking lot. The parade square is a rectangular plot of land 40 m by 30 m.



ENTRANCE / EXIT

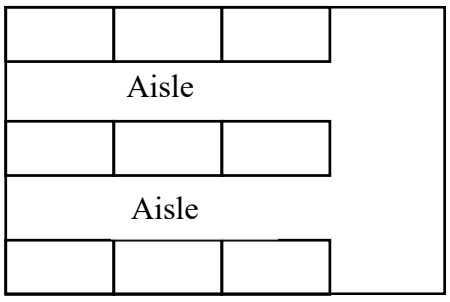
There is an 6 m wide entrance / exit at one corner of the parade square as shown in the sketch above.
You are required to do the parking arrangements.

The Land Transport Authority recommends the following guidelines.

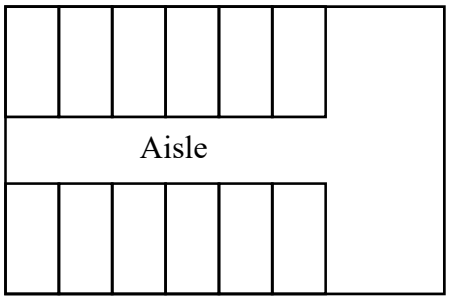
A Parking Stall refers to the space for parking of one motorcar, that is, a car parking lot. The space of the stall should be rectangular. The longer side is known as the length and the shorter side is the width.

A Parking Aisle refers to an access lane or driveway with adjacent parking stalls.

In parallel parking, the longer side is parallel to the parking aisle or driveway.
The aisle for cars to move must be at least 3.6 m. For two-way traffic flow, the width of the aisle must be at least 6 m. Each parking stall is 5.4 m by 2.4 m.

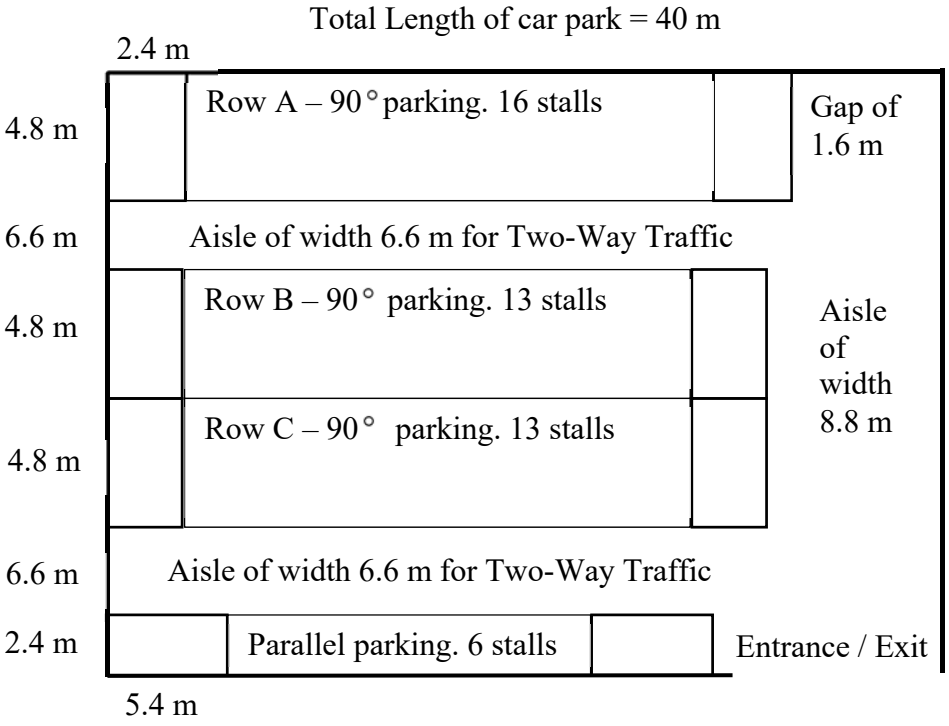


In 90° parking, the longer side is perpendicular to the parking aisle or driveway.
The aisle or lane for cars to move must be at least 6 m for one-way traffic flow and at least 6.6 m for two-way traffic flow.
Each parking stall is 4.8 m by 2.4 m.



Propose a possible parking arrangement that would maximise the use of space, showing your calculations clearly. Your proposal must include a sketch, not drawn to scale, indicating the location of the parking stalls, the aisles and the type(s) of parking. You should allow for cars to enter and leave the parade square at any time. You can assume that the cars will not leave in large numbers at any one time, and the parking will be supervised by security guards. [10]

Possible Solution



B1 – sketch is clearly labelled
B1 – layout of parking stalls using parallel parking
B1 – layout of parking stalls using 90° parking
B1 – layout of aisles clearly showing that cars can move in or out at any time.
B1 – aisles are at appropriate width, either 6 m for one-way or 6.6 m for two-way.

	Number of lots in Row A $= \frac{40}{2.4} = 16\frac{2}{3} \approx 16$ Number of lots in Row B or C $= \frac{40-6.6}{2.4} = 13\frac{11}{12} \approx 13$ Length of Row A $= 16 \times 2.4 = 38.4 \text{ m} < 40 \text{ m}$ Length of Row B $= 13 \times 2.4 = 31.2 \text{ m} < 40 \text{ m}$ Length of Row C $= 13 \times 2.4 = 31.2 \text{ m} < 40 \text{ m}$ Number of lots for parallel parking $= \frac{40-6}{5.4} = 6\frac{8}{27} \approx 6$ Length of parallel parking $= 6 \times 5.4 = 32.4 \text{ m} < 36 \text{ m}$	Considering the length M1 A1
	Width of the rows of parking stalls and aisles $= 4.8 \times 3 + 2.4 + 2 \times 6.6$ $= 30 \text{ m}$	Considering the width M1 A1
	Total number of parking stalls $= 16 + 2 \times 13 + 6 = 48$	A1 – Accept minimum as 42
	Remarks – Students who consider only parallel parking or 90° parking but not both get a maximum of 8 marks.	